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Nonlocal thermo-electro-mechanical vibration analysis of smart curved FG piezoelectric Timoshenko nanobeam

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Abstract. To peruse the free vibration of curved functionally graded piezoelectric (FGP) nanosize beam in thermal environment, nonlocal elasticity theory is applied for modeling the nano scale effect. The governing equations are obtained via the energy method. Analytically Navier solution is employed to solve the governing equations for simply supported boundary conditions. Solving these equations enables us to estimate the natural frequency for curved FGP nanobeam under the effect of a uniform temperature change and external electric voltage. The results determined are verified by comparing the results by available ones in literature. The effects of various parameters such as nonlocality, uniform temperature changes, external electric voltage, gradient index, opening angle and aspect ratio of curved FGP nanobeam on the natural frequency are successfully discussed. The results revealed that the natural frequency of curved FGP nanobeam is significantly influenced by these effects.

Keywords: curved nanobeam; thermo-electro-mechanical vibration; piezoelectric nanobeams; functionally graded material; nonlocal elasticity

1. Introduction

Recently, distributed signal processing has received a Nowadays, nano structures such as nanobeams, nano plates and nano membranes, are very attractive field for many researchers due to their improvement of the quality properties.

The classical continuum theory is aptly practical in the mechanical behavior of the macroscopic structures, but it is improperly for the size effect on the mechanical treatments on micro- or nano-scale structure. Nevertheless, the classical continuum theory need to be extended to factor in the nanoscale effects. This problem can be unraveled through the nonlocal elasticity model that presented by Eringen (2002). According to this model, the stress state at a certain point is considered as a function of strain states of all points in its area.

Among of assortment of nano structures, nanobeams have more important applications (Daulton *et al*. 2010, Hu *et al.* 2010). Lots of studies have been performed to investigate the size-dependent response of structural systems based on Eringen's nonlocal elasticity theory (Ebrahimi and Salari 2015a, b, 2016, Ebrahimi *et al.* 2015a, 2016c, Ebrahimi and Nasirzadeh 2015, Ebrahimi and Barati , 2016 a,b, c, d, e, f, Ebrahimi and Hosseini 2016 a, b, c) .

Vibration of carbon nanotubes using nonlocal continuum mechanics theory has been surveyed by Wang and Varadan (2006). However, Zhang *et al.* (2009) have investigated bending, buckling and vibration of micro and nanobeams by nonlocal beam theory. In addition, on bending, buckling and vibration of graphene nanosheets based on the nonlocal theory has been presented by Liu *et al.* (2016). Nevertheless, vibration of nanobeams using nonlocal theory has been developed with an axial load by Li *et al.* (2010). In addition, Eltaher *et al*. (2013), have presented vibration of Euler-Bernoulli nanobeams by employing finite element method. Also, forced vibration analysis of viscoelastic nanobeams embedded in an elastic medium has been presented by seref and Akbas (2016). In addition, Ebrahimi and Shaghaghi (2016) have investigated thermal effects on nonlocal vibrational characteristics of nanobeams with non-ideal boundary conditions. Nonetheless, Murmu and Adhikari (2010) have analyzed the nonlocal transverse vibration of double nanobeam system. In addition Murmu and Adhikari (2010) have presented longitudinal vibration of double nanorod system by using nonlocal theory. However, many authors are attracted to analyse the micro and nano structures in recent years (Akbas 2016)

Recently, piezoelectric nanostructures, have been appealing a great deal of interest from research. These distinguished properties make them apt for potential applications in micro and nano electro-mechanical systems (MEMS & NEMS) such as nano sensors (Wang *et al.* 2004), nano actuators (Juang *et al.* 2007) and nano generators (Wang and Song 2006). Analyzing of the mechanical behavior of piezoelectric nanostructures such as vibration analyze, is an important topic in the design process of the nano devices.

On the static and dynamic stability of beams with an axial piezoelectric actuation has been investigated by Zehetner and Irschik (2008). In addition, Benjeddou (2009) has presented new insights in piezoelectric free-vibrations

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using simplified modeling and analyses. Also, Simulation and experimental analysis of active vibration control of smart beams under harmonic excitation has been developed by Malgaca and Karagulle (2009). However, thermoelectric-mechanical vibration of piezoelectric nanobeams has been investigated by Ke and Wang (2012). In this paper nonlocal Eringen theory and Timoshenko beam model are used. However, dynamics of silicon nanobeams with axial motion subjected to transverse and longitudinal loads considering nonlocal and surface effects has been presented by Shen *et al*. (2017). In addition, Also, Ke *et al.* (2012) have presented nonlinear vibration of the piezoelectric nanobeams based on the nonlocal theory. However, vibration behavior of piezoelectric nanobeams has been developed by Rahmani and Noroozi (2014). Nevertheless, Ke *et al*. (2014) have evaluated Thermo-electro-mechanical vibration of piezoelectric nanoshells. Also, in this paper various boundary conditions are employed. Meanwhile, free vibration of piezoelectric nanobeams in framework of a nonlocal Eringen model have been presented by Jandaghian and Rahmani (2016). In addition, Beni (2016) has investigated size-dependent analysis of piezoelectric nanobeams with considering electro-mechanical coupling. Moreover, Doroushi *et al*. (2011) have surveyed vibration analysis and transient response of an FGPM beam under thermo-electro-mechanical loads using higher-order shear deformation model.

Recently a new class of composite materials known as functionally graded materials (FGMs) (Shen 2016). A typical FGM is an inhomogeneous composite that composed of different parts such as ceramic and metal. Several studies in analysis of FG materials, have been significantly presented by many researchers (Li and Hu 2017, Ebrahimi *et al.* 2017, Ebrahimi and Rastgoo 2008 a, b, c, Ebrahimi 2013, Ebrahimi *et al*. 2008, 2009a,b, 2016a , Ebrahimi and Zia 2015, Ebrahimi and Mokhtari 2015).

Thermo-mechanical vibration analysis of nonlocal temperature-dependent FG nanobeams has been surveyed by Ebrahimi and Salari (2015). In this research, various boundary conditions are considered. Also Ebrahimi and Barati (2016g-v, 2017a,b) and Ebrahimi *et al*. (2017) explored thermal and hygro-thermal effects on nonlocal behavior of FG nanobeams and nanoplates.

Nevertheless, Nazemnezhad and Hosseini-Hashemi (2014), have developed nonlinear free vibration of functionally graded nanobeams by employing nonlocal Eringen theory in their research. In addition, free vibration analysis of functionally graded size-dependent nanobeams has been surveyed by Eltaher *et al*. (2012). Meanwhile, free flexural vibrational behavior of FG nanobeams employing differential transform method has been studied by Ebrahimi and Salari (2015). In addition, Ebrahimi and Shafiei (2016) have presented application of Eringen`s nonlocal elasticity theory for vibration analysis of rotating functionally graded nanobeams. Also, Application of the differential transformation method for nonlocal vibration analysis of functionally graded nanobeams has been developed by Ebrahimi *et al.* (2015). In addition, Ebrahimi and Salari (2015) have presented nonlocal thermo-mechanical vibration analysis of functionally graded nanobeams in thermal environment. Furthermore, a nonlocal higher-order shear deformation beam model for vibration analysis of FG nanobeams has been investigated by Ebrahimi and Barati (2016). In addition, komijani *et al*. (2016), have been presented nonlinear analysis of microstructure-dependent functionally graded piezoelectric material actuators.

Nowadays, considering all the good properties of functionally graded (FG) combined piezoelectric materials, functionally graded piezoelectric material (FGPM) is a material with exceptional properties. Furthermore, with the progress of nanotechnology and the procedures of combining materials in the nanoscale, a set of FGPM nanostructures has been made, which generally includes a variety of beam-type structures. Studies conducted to investigate electric and mechanical properties of FGPMs are not profuse, and they generally use classical continuum models which overlook the scale effect.

Electromechanical bending, buckling, and free vibration analysis of functionally graded piezoelectric nanobeams has been presented by Beni (2016). However, Ebrahimi and Salari (2016), have developed thermo-electrical buckling analysis of FG piezoelectric nanobeams. Moreover, buckling analysis of nonlocal third-order shear deformable FG piezoelectric nanobeams with considering elastic foundation. In addition, Hosseini-Hashemi *et al*. (2014) have surveyed Surface effects on free vibration of piezoelectric functionally graded nanobeams using nonlocal elasticity

In recent years vibration of curved nanobeams and nanorings, have been worked in many empirical experiments and dynamic molecular simulations (Wang and Duan 2008). Hence some researchers are interested in studding of vibration curved nanobeams.

Yan and Jiang (2011) have investigated the electromechanical response of a curved piezoelectric nanobeam with the consideration of surface effects. In addition, a new numerical technique, the differential quadrature method has been developed for dynamic analysis of the nanobeams in the polar coordinate system by Kananipour *et al*. (2014). However, Ebrahimi and Daman (2016) have presented the radial vibration of embedded double-curved-nanobeam-systems. However, Ebrahimi and Daman (2016) have developed investigating surface effects on thermomechanical behavior of embedded circular curved nanosize beams. As well as, dynamic modeling of embedded curved nanobeams incorporating surface effects has been presented by Ebrahimi and Daman (2016). In addition, Ebrahimi and Daman (2017) have studied analytical investigation of the surface effects on nonlocal vibration behavior of nanosize curved beams. However, Wang and Duan (2008) have surveyed the free vibration problem of nanorings/arches. In this research the problem was formulated on the framework of Eringen’s nonlocal theory of elasticity according to allow for the small length scale effect. Furthermore, Out-of-plane frequency analyze of FG circular curved beams in thermal environment has been investigated by Malekzadeh *et al.* (2010). In addition, Hosseini and Rahmani (2016) have surveyed free vibration of shallow and deep curved functionally graded nanobeam based on nonlocal Timoshenko curved beam model.

This present research makes the first achievement to develop the thermoelectric-mechanical vibration of curved functionally graded piezoelectric nanobeams based on nonlocal Timoshenko beam theory. Curvature rather exists in all of the real beams and nanobeams. Moreover, in previous researches in order to streamline of mathematical equations, straight beam models have been used, whilst curved beam models are more practicable than straight ones. To the best of the author's knowledge, there is no record regarding the thermoelectric-mechanical vibration of curved FG piezoelectric nanobeams based on nonlocal Timoshenko curved beam theory. Therefore, there is a strong scientific need to understand the vibration behavior of curved FGP nanobeams in thermal environment. In this study, the size-dependent formulation is developed for the FGPM Timoshenko nanobeam. The thermo-electro-mechanical material properties of the beam is assumed to be graded in the thickness direction based on to the power law distribution. The governing equations and boundary conditions are derived by employing the Hamilton's principle. Then Navier method is used to solution differential equations. Dimensionless natural frequencies are obtained respect to the effect of various parameters such as angle of curvature, external electric voltage, temperatures change, mode numbers, power-law index and nonlocal parameter on vibration of curved FGP nanobeams. Comparison between results of the present research and available data in literature reveals the accuracy of this model.

2. Problem formulation

2.1 The material properties of curved FGP nanobeams

A curved FGP nanobeam made of piezoelectric materials involved PZT-4 and PZT-5H with length L in θ direction and uniform thickness h in z direction, and under an electric potential $\varphi(\theta, z, t)$ as shown in figure 1 is assumed.

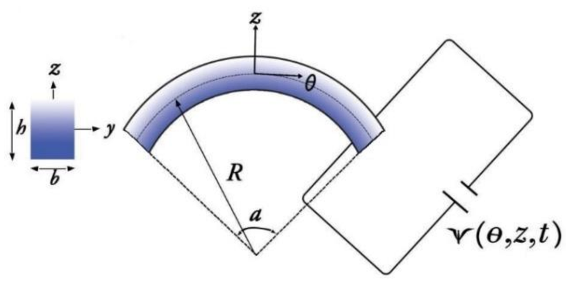


Fig. 1 Geometric of curved FGP nanobeam

The relation between length of circular curved beam (θ) and the angle of curvature of beam (α) can be written as (Setoodeh *et al*. 2015).

$$\theta = R\alpha \tag{1}$$

The effective material properties of the curved FGPM beam are assumed to vary continuously in the thickness direction based on a power-law model. According to this model, the effective material properties, P, can be defined as follow (Komijani *et al*. 2014).

$$P = P_u V_u + P_l V_l \tag{2}$$

where (P_l, P_u) are the properties of materials at the lower surface and upper surface, respectively, in addition, (V_l, V_u) are the corresponding volume fractions related by

$$V_u = \left(\frac{z}{h} + \frac{1}{2}\right)^p \tag{3}$$

$$V_u + V_l = 1 \tag{4}$$

Hence, from equations (2) and (3), the impressive material properties of the curved FGP beam can be defined as

$$P(z) = (P_u - P_l)\left(\frac{z}{h} + \frac{1}{2}\right)^p + P_l \tag{5}$$

where p is the nonnegative variable parameter (power-law exponent). Power-law exponent determines the distribution profile of material through the thickness of the beam in z direction. Based on this distribution, the bottom surface $(z = -h/2)$ pure is PZT-5H, whiles the top surface $(z = h/2)$ of curved FGP nanobeam stands for pure PZT-4.

2.2 Governing equation

Based on Timoshenko beam theory, displacement field in a point of the curved beam model can be remarked as

$$u_\theta(\theta, z, t) = \left(1 + \frac{z}{R}\right)u(\theta, t) + z\varphi(\theta, t) \tag{6a}$$

$$u_z(\theta, z, t) = w(\theta, t) \tag{6b}$$

where w and u interpret the radial and tangential displacement of curved FGP beam. In addition, φ is total bending rotation of cross sections of curved FGP beam. The strains of Timoshenko curved beam theory may be expressed as

$$\varepsilon_{\theta\theta}^0 = \frac{\partial w}{\partial \theta} - \frac{u}{R} \tag{7a}$$

$$\kappa = \frac{\partial \varphi}{\partial \theta} \tag{7b}$$

$$\gamma_{\theta z} = \frac{\partial u}{\partial \theta} - \varphi + \frac{w}{R} \tag{7c}$$

Here γ denotes shear strain in curved beam model.

$$E_\theta = -\frac{\partial \psi}{\partial \theta} = Cos(\beta z)\frac{\partial \psi}{\partial \theta} \tag{8a}$$

$$E_z = -\frac{\partial \psi}{\partial z} = -\beta Sin(\beta z)\psi - \frac{2V_0}{h}e^{i\Omega t} \tag{8b}$$

$$\varepsilon_{\theta\theta} = \left(\varepsilon_{\theta\theta}^0 + z\kappa\right) \tag{9}$$

The energy method (Hamilton's principle) can be employed to derive the governing equations as follow

$$\int_0^t \delta\left(U_s - T + W_{ext}\right) = 0 \tag{10}$$

where U_s, T and W_{ext} are strain energy, kinetic energy and work done by external exerted loads, respectively. The first variation of strain energy U_s can be determined as

$$\delta U_s = \int_V \left(\sigma_{\theta\theta}\delta\varepsilon_{\theta\theta} + \sigma_{\theta z}\delta\gamma_{\theta z} - D_\theta \delta E_\theta - D_z \delta E_z\right)dV \tag{11}$$

By inserting Eqs. (6) and (7) in Eq. (10), first variation of strain energy can be obtained as

$$\begin{aligned}\delta U_s &= \int_0^L \left(N\left(-\frac{\delta u}{R} + \frac{\partial \delta w}{\partial \theta}\right) + M\left(\frac{\partial \delta\varphi}{\partial \theta}\right) + Q\left(\frac{\partial \delta u}{\partial \theta} - \delta\varphi + \frac{\delta w}{R}\right)\right)d\theta \\ &+ \int_0^L \int_{\frac{-h}{2}}^{\frac{h}{2}} \left(D_z \beta Sin(\beta z)\delta\psi - D_\theta Cos(\beta z)\frac{\partial \delta\psi}{\partial \theta}\right)dz d\theta\end{aligned} \tag{12}$$

Here *M*, *N*, and *Q* define bending moment of cross section, axial force, and shear force, respectively. These stress resultants existing in Eq. (12) may be expressed as

$$\begin{aligned} N &= \int_A \sigma_{\theta\theta} dA \\ M &= \int_A \sigma_{\theta\theta} z dA \\ Q &= \int_A K_{shear}\sigma_{\theta z} dA \end{aligned} \tag{13}$$

where K_{Shear} expresses the shear correction factor. In addition, Kinetic energy of Timoshenko curved beam can be calculated as

$$T = \frac{1}{2}\int_0^L \int_A \rho(z)\left(\left(\frac{\partial u_\theta}{\partial t}\right)^2 + \left(\frac{\partial u_z}{\partial \theta}\right)^2\right)dA d\theta \tag{14}$$

Hence, first variation of kinetic energy can be calculated as

$$\delta T = \left[\begin{aligned} & I_0\left(\frac{\partial u}{\partial t}\frac{\partial \delta u}{\partial t}\right) + \left(I_0 + \frac{2I_1}{R} + \frac{I_2}{R^2}\right)\frac{\partial w}{\partial t}\frac{\partial \delta w}{\partial t} \\ & + \left(I_1 + \frac{I_2}{R}\right)\left(\frac{\partial \varphi}{\partial t}\frac{\partial \delta w}{\partial t} + \frac{\partial w}{\partial t}\frac{\partial \delta\varphi}{\partial t}\right) + I_2\frac{\partial \varphi}{\partial t}\frac{\partial \delta\varphi}{\partial t}\end{aligned}\right]d\theta \tag{15}$$

where mass moments of inertias $\left(I_0, I_1, I_2\right)$ are calculated as follows

$$\left(I_0, I_1, I_2\right) = \int_A \rho(z)\left(1, z, z^2\right)dA \tag{16}$$

Whereas the work done by the external loads is defined by W_{ext}

$$\delta W_{ext} = \frac{1}{2}\int_0^L \left(N_E + N_T\right)\frac{\partial w}{\partial \theta}\delta\frac{\partial w}{\partial \theta}d\theta \tag{17}$$

where N_E and N_T are the external electric voltage V_0 and temperature changes ΔT, which can be given as

$$\begin{aligned} N_T &= \int_{\frac{-h}{2}}^{\frac{h}{2}} c_{11}\lambda_1 \Delta T dz \\ N_E &= -\int_{\frac{-h}{2}}^{\frac{h}{2}} 2e_{31}\frac{V_0}{h}dz \end{aligned} \tag{18}$$

By inserting δu, δw, $\delta\varphi$ and $\delta\psi$ coefficients equal to zero, following equation of motions can be determined for curved FGP nanobeam

$$\frac{N}{R} + \frac{\partial Q}{\partial \theta} = I_0\frac{\partial^2 u}{\partial t^2} \tag{19a}$$

$$\begin{aligned}&\frac{\partial N}{\partial \theta} - \frac{Q}{R} - \left(N_T + N_E\right)\frac{\partial^2 w}{\partial \theta^2} \\ &= \left(I_0 + \frac{2I_1}{R} + \frac{I_2}{R^2}\right)\frac{\partial^2 w}{\partial t^2} + \left(I_1 + \frac{I_2}{R}\right)\frac{\partial^2 \varphi}{\partial t^2}\end{aligned} \tag{19b}$$

$$\frac{\partial M}{\partial \theta} + Q = \left(I_1 + \frac{I_2}{R}\right)\frac{\partial^2 w}{\partial t^2} + I_2\frac{\partial^2 \varphi}{\partial t^2} \tag{19c}$$

$$\int_{\frac{-h}{2}}^{\frac{h}{2}} \frac{\partial D_\theta}{\partial \theta}Cos(\beta z)dz + \int_{\frac{-h}{2}}^{\frac{h}{2}} D_z \beta Sin(\beta z)dz = 0 \tag{19d}$$

Boundary conditions that are related to equation of motions are considered as

$$N = 0 \text{ or } w = 0 \text{ at } \theta = 0 \text{ and } \theta = L \tag{20a}$$

$$Q = 0 \text{ or } u = 0 \text{ at } \theta = 0 \text{ and } \theta = L \tag{20b}$$

$$M = 0 \text{ or } \varphi = 0 \text{ at } \theta = 0 \text{ and } \theta = L \tag{20c}$$

$$\int_{\frac{-h}{2}}^{\frac{h}{2}} D_\theta Cos(\beta z)\,dz = 0 \quad \text{or} \qquad (20d)$$
$$\psi = 0 \quad \text{at} \quad \theta = 0 \quad \text{and} \quad \theta = L$$

2.3 The nonlocal elasticity model for curved FG nanobeam

Despite the fundamental equations in classic elasticity theory, Eringen's nonlocal model [1] explains that the stress at a certain point *x* in a body is assumed as a function of strains of all points x' in the near realm. This supposition is very good agreement with experiments of atomic model and lattice dynamics in phonon scattering in which for a nonlocal piezoelectric materials. The basic equations with zero body force can be given as (Ke *et al*. 2012).

$$\sigma_{ij} = \int_V \alpha\left(\left|x'-x\right|,\tau\right)\begin{bmatrix} C_{ijkl}\varepsilon_{kl}(x') - e_{kij}E_k(x') \\ -C_{ijkl}\alpha_{kl}\Delta T \end{bmatrix} dV(x') \qquad (21a)$$

$$D_i = \int_V \alpha\left(\left|x'-x\right|,\tau\right)\begin{bmatrix} e_{ikl}\varepsilon_{kl}(x') \\ +k_{ik}E_k(x') + p_i\Delta T \end{bmatrix} dV(x') \qquad (21b)$$

where σ_{ij} , ε_{ij} , D_i and E_i are the stress, strain, electric displacement and electric field components, respectively; ΔT and α_{kl} are the temperature changes and thermal expansion coefficient, respectively; C_{ijkl} , e_{kij} , k_{ik} and p_i are elastic, piezoelectric, dielectric and pyroelectric constants, respectively; $\alpha\left(\left|x'-x\right|,\tau\right)$ is the nonlocal kernel function and $\left|x'-x\right|$ is the nonnegative distance. $\tau = e_0 a/l$ is given as size coefficient.

However, the relations in Eq. (21) causes the elasticity problems difficult to solve, in addition to possible lack of determinism. Eringen [1] presented in detail properties of non-local kernel $\alpha\left(\left|x'-x\right|\right)$ and evaluated that when a kernel takes a Green's function of linear differential operator

$$L\alpha\left(\left|x'-x\right|\right) = \delta\left(\left|x'-x\right|\right) \qquad (22)$$

By matching the scattering curves with lattice models, Eringen [1] supposed a nonlocal theory with the linear differential operator L expressed as follow

$$L = 1 - (e_0 a)^2 \nabla^2 \qquad (23)$$

where ∇^2 is the Laplacian operator. Therefore, the fundamental relations given by Eq. (21) for nonlocal elasticity may be rewritten by differentiable form as

$$\sigma_{ij} - (e_0 a)^2 \nabla^2 \sigma_{ij} = C_{ijkl}\varepsilon_{kl} - e_{kij}E_k - C_{ijkl}\alpha_{kl}\Delta T \qquad (24a)$$

$$D_i - (e_0 a)^2 \nabla^2 D_i = e_{ikl}\varepsilon_{kl} + k_{ik}E_k + p_i\Delta T \qquad (24b)$$

The parameter $e_0 a$ is the scale coefficient disclosing the nano scale effect on the responses of structures of nanoscale. The nonlocal parameter, $\mu = (e_0 a)$ is experimentally determined for different materials.

For a curved FGPM nanobeam under thermo-electro-mechanical loading in the one dimensional case, the nonlocal fundamental relations (24(a)) and (24(b)) can be streamlined as

$$\sigma_{\theta\theta} - \mu^2 \frac{\partial^2 \sigma_{\theta\theta}}{\partial\theta^2} = c_{11}\varepsilon_{\theta\theta} - e_{31}E_z - c_{11}\lambda_1\Delta T \qquad (25a)$$

$$\sigma_{\theta z} - \mu^2 \frac{\partial^2 \sigma_{\theta z}}{\partial\theta^2} = c_{55}\gamma_{\theta z} - e_{15}E_\theta \qquad (25b)$$

$$D_\theta - \mu^2 \frac{\partial^2 D_\theta}{\partial\theta^2} = e_{15}\gamma_{\theta z} + k_{11}E_\theta \qquad (25c)$$

$$D_z - \mu^2 \frac{\partial^2 D_z}{\partial\theta^2} = e_{31}\varepsilon_{\theta\theta} + k_{33}E_z + p_3\Delta T \qquad (25d)$$

Calculating Eqs. (25) by integrating over cross-section area of the curved beam, force–strain and moment–strain of nonlocal curved FGP Timoshenko beam model will be determined as

$$N - \mu^2 \frac{\partial^2 N}{\partial\theta^2} = A\left(\frac{\partial w}{\partial\theta} - \frac{u}{R}\right) + B\frac{\partial\varphi}{\partial\theta} + B_{31}\psi \qquad (26a)$$

$$M - \mu^2 \frac{\partial^2 M}{\partial\theta^2} = B\left(\frac{\partial w}{\partial\theta} - \frac{u}{R}\right) + D\frac{\partial\varphi}{\partial\theta} + F_{31}\psi \qquad (26b)$$

$$Q - \mu^2 \frac{\partial^2 Q}{\partial\theta^2} = K_{Shear}C\left(\frac{w}{R} + \frac{\partial u}{\partial\theta} - \varphi\right) - K_{Shear}E_{15}\frac{\partial\psi}{\partial\theta} \qquad (26c)$$

$$\int_{\frac{-h}{2}}^{\frac{h}{2}}\left[D_\theta - \mu^2 \frac{\partial^2 D_\theta}{\partial\theta^2}\right]Cos(\beta z)\,dz = K_{Shear}E_{15}\left(\frac{w}{R} + \frac{\partial u}{\partial\theta} - \varphi\right) + X_{11}\frac{\partial\psi}{\partial\theta} \qquad (26d)$$

$$\int_{\frac{-h}{2}}^{\frac{h}{2}}\left[D_z-\mu^2\frac{\partial^2 D_z}{\partial\theta^2}\right]\beta Sin(\beta z)dz = B_{31}\left(\frac{\partial w}{\partial\theta}-\frac{u}{R}\right)+F_{31}\frac{\partial\varphi}{\partial\theta}-X_{33}\psi \tag{26e}$$

where K_{Shear} defined as correction factor and assumed that equal 5/6 . Consequently, coefficients are obtained as

$$\{A,B,D\}=\int_A c_{11}\{1,z,z^2\}dA \tag{27a}$$

$$C=\int_A c_{55}dA \tag{27b}$$

$$\{B_{31},F_{31},E_{15}\}=\int_A\left\{\begin{matrix}e_{31}\beta Sin(\beta z),ze_{31}\beta Sin(\beta z)\\ ,e_{15}Cos(\beta z)\end{matrix}\right\}dA \tag{27c}$$

$$\{X_{11},X_{33}\}=\int_A\left\{\begin{matrix}\varepsilon_{11}Cos^2(\beta z)\\ ,\varepsilon_{33}\beta^2 Sin^2(\beta z)\end{matrix}\right\}dA \tag{27d}$$

By inserting Eqs. (27) into Eqs. (19), nonlocal governing equations of curved FGP Timoshenko nanobeam in terms of displacement can be calculated as

$$\begin{aligned}&A\left(\frac{\partial w}{\partial\theta}-\frac{u}{R}\right)+B\frac{\partial\varphi}{\partial\theta}+B_{31}\psi\\&+K_{Shear}CR\left(\frac{\partial w}{R\partial\theta}+\frac{\partial^2 u}{\partial\theta^2}-\frac{\partial\varphi}{\partial\theta}\right)\\&-K_{Shear}E_{15}R\frac{\partial^2\psi}{\partial\theta^2}=RI_0\left(\frac{\partial^2 u}{\partial t^2}-\mu^2\frac{\partial^4 u}{\partial\theta^2\partial t^2}\right)\end{aligned} \tag{28a}$$

$$\begin{aligned}&AR\left(\frac{\partial^2 w}{\partial\theta^2}-\frac{\partial u}{R\partial\theta}\right)+BR\frac{\partial^2\varphi}{\partial\theta^2}+B_{31}R\frac{\partial\psi}{\partial\theta}\\&-(N_T+N_E)R\frac{\partial^2 w}{\partial\theta^2}+\mu^2(N_T+N_E)R\frac{\partial^4 w}{\partial\theta^4}\\&-K_{Shear}C\left(\frac{w}{R}+\frac{\partial u}{\partial\theta}-\varphi\right)+K_{Shear}E_{15}\frac{\partial\psi}{\partial\theta}\\&=\left(I_0+\frac{2I_1}{R}+\frac{I_2}{R^2}\right)R\frac{\partial^2 w}{\partial t^2}+\left(I_1+\frac{I_2}{R}\right)R\frac{\partial^2\varphi}{\partial t^2}\\&-\mu^2\left(I_0+\frac{2I_1}{R}+\frac{I_2}{R^2}\right)R\frac{\partial^4 w}{\partial\theta^2\partial t^2}\\&-\mu^2\left(I_1+\frac{I_2}{R}\right)R\frac{\partial^4\varphi}{\partial\theta^2\partial t^2}\end{aligned} \tag{28b}$$

$$\begin{aligned}&B\left(\frac{\partial w}{\partial\theta^2}-\frac{\partial u}{R\partial\theta}\right)+D\frac{\partial^2\varphi}{\partial\theta^2}+F_{31}\frac{\partial\psi}{\partial\theta}\\&+K_{Shear}C\left(\frac{w}{R}+\frac{\partial u}{\partial\theta}-\varphi\right)-K_{Shear}E_{15}\frac{\partial\psi}{\partial\theta}\\&=\left(I_1+\frac{I_2}{R}\right)\frac{\partial^2 w}{\partial t^2}+I_2\frac{\partial^2\varphi}{\partial t^2}\\&-\mu^2\left(I_1+\frac{I_2}{R}\right)\frac{\partial^4 w}{\partial\theta^2\partial t^2}-\mu^2 I_2\frac{\partial^4\varphi}{\partial\theta^2\partial t^2}\end{aligned} \tag{28c}$$

$$\begin{aligned}&E_{15}\left(\frac{\partial w}{R\partial\theta}+\frac{\partial^2 u}{\partial\theta^2}-\frac{\partial\varphi}{\partial\theta}\right)\\&+X_{11}\frac{\partial^2\psi}{\partial\theta^2}+B_{31}\left(\frac{\partial w}{\partial\theta}-\frac{u}{R}\right)\\&+F_{31}\frac{\partial\varphi}{\partial\theta}-X_{33}\psi=0\end{aligned} \tag{28d}$$

3. Solution method

In this section, analytical Navier method has been developed to solve the governing equations of curved FGP in regard to find out free vibrational of a simply supported curved FGP nanobeam. However, to define the displacement functions, product of unknown factors and known trigonometric functions has been employed to satisfy the governing equations and boundary conditions at $\theta=0,\ L$ ends. The displacement fields are assumed to be as follows

$$w(\theta,t)=\sum_{n=1}^{\infty}W_n Cos\left(\frac{n\pi}{L}\theta\right)e^{i\omega_n t} \tag{29a}$$

$$u(\theta,t)=\sum_{n=1}^{\infty}U_n Sin\left(\frac{n\pi}{L}\theta\right)e^{i\omega_n t} \tag{29b}$$

$$\begin{aligned}\varphi(\theta,t)&=\sum_{n=1}^{\infty}\phi_n Cos\left(\frac{n\pi}{L}\theta\right)e^{i\omega_n t}\\ \psi(\theta,t)&=\sum_{n=1}^{\infty}\psi_n Sin\left(\frac{n\pi}{L}\theta\right)e^{i\omega_n t}\end{aligned} \tag{29c}$$

where W_n, U_n, ϕ_n and ψ_n are the unknown Fourier factors to be obtained for each n value. The boundary conditions for simply supported curved FGP nanobeam can be given as

$$\begin{aligned}&w(0)=0,\ \frac{\partial w}{\partial\theta}(L)=0,\ u(0)=u(L)=0\\&\frac{\partial\varphi}{\partial\theta}(0)=\frac{\partial\varphi}{\partial\theta}(L)=0,\ \psi(0)=\psi(L)=0\end{aligned} \tag{30}$$

Inserting Eq. (29) into Eqs. (28) respectively, leads to Eqs. (30)

$$\begin{aligned}
&A\left(-\frac{U_n}{R}-\left(\frac{n\pi}{L}\right)W_n\right)-B\left(\frac{n\pi}{L}\right)\phi_n \\
&+B_{31}\psi_n+K_{Shear}CR\begin{bmatrix}-\frac{1}{R}\left(\frac{n\pi}{L}\right)W_n \\ -\left(\frac{n\pi}{L}\right)^2U_n+\left(\frac{n\pi}{L}\right)\phi_n\end{bmatrix} \\
&+K_{Shear}E_{15}R\left(\frac{n\pi}{L}\right)^2\psi_n \\
&=RI_0\left(-\omega_n^2U_n-\mu^2\omega_n^2\left(\frac{n\pi}{L}\right)^2U_n\right)
\end{aligned} \tag{31a}$$

$$\begin{aligned}
&AR\left[-\frac{1}{R}\left(\frac{n\pi}{L}\right)U_n-\left(\frac{n\pi}{L}\right)^2W_n\right] \\
&-BR\left(\frac{n\pi}{L}\right)^2\phi_n+B_{31}R\left(\frac{n\pi}{L}\right)\psi_n \\
&-\left(N_T+N_E\right)R\left(\frac{n\pi}{L}\right)^2W_n-\mu^2\left(N_T+N_E\right)R\left(\frac{n\pi}{L}\right)^4W_n \\
&-K_{Shear}C\left[\frac{1}{R}W_n+\left(\frac{n\pi}{L}\right)U_n-\phi_n\right]+K_{Shear}E_{15}\left(\frac{n\pi}{L}\right)\psi_n \\
&=-\left(I_0+\frac{2I_1}{R}+\frac{I_2}{R^2}\right)R\omega_n^2W_n-\left(I_1+\frac{I_2}{R}\right)R\omega_n^2\phi_n \\
&-\mu^2R\left(\frac{n\pi}{L}\right)^2\left(I_0+\frac{2I_1}{R}+\frac{I_2}{R^2}\right)\omega_n^2W_n \\
&-\mu^2\left(I_1+\frac{I_2}{R}\right)R\left(\frac{n\pi}{L}\right)^2\omega_n^2\phi_n
\end{aligned} \tag{31b}$$

$$\begin{aligned}
&BR\left[-\frac{1}{R}\left(\frac{n\pi}{L}\right)U_n+\left(\frac{n\pi}{L}\right)^2W_n\right] \\
&-RD\left(\frac{n\pi}{L}\right)^2\phi_n+F_{31}R\left(\frac{n\pi}{L}\right)\psi_n \\
&+K_{Shear}CR\left[\frac{1}{R}W_n+\left(\frac{n\pi}{L}\right)U_n-\phi_n\right] \\
&-K_{Shear}E_{15}R\left(\frac{n\pi}{L}\right)\psi_n= \\
&-R\left(I_1+\frac{I_2}{R}\right)\omega_n^2W_n-RI_2\omega_n^2\phi_n \\
&-R\mu^2\left(I_1+\frac{I_2}{R}\right)\left(\frac{n\pi}{L}\right)^2\omega_n^2W_n \\
&-R\mu^2I_2\left(\frac{n\pi}{L}\right)^2\omega_n^2\phi_n
\end{aligned} \tag{31c}$$

$$\begin{aligned}
&-K_{Shear}E_{15}R\begin{bmatrix}-\frac{1}{R}\left(\frac{n\pi}{L}\right)W_n \\ -\left(\frac{n\pi}{L}\right)^2U_n+\left(\frac{n\pi}{L}\right)\phi_n\end{bmatrix} \\
&+X_{11}R\left(\frac{n\pi}{L}\right)^2\psi_n \\
&-RB_{31}\left[-\frac{1}{R}U_n-\left(\frac{n\pi}{L}\right)W_n\right] \\
&+RF_{31}\left(\frac{n\pi}{L}\right)\phi_n+RX_{33}\psi_n
\end{aligned} \tag{31d}$$

By inserting the determinant of the coefficient matrix of the Eqs. (30), the nontrivial analytical method may be determined from the Eq. (32)

$$\left\{\left(\left[K\right]+\Delta T\left[K_T\right]\right)-\omega_n^2\left[M\right]\right\}\begin{Bmatrix}U_n\\W_n\\\phi_n\\\psi_n\end{Bmatrix}=0 \tag{32}$$

Here $\left[K\right]$ and $\left[K_T\right]$ are stiffness matrix and coefficient matrix of temperature change, respectively. And $\left[M\right]$ is the mass matrix. By equaling the obtained determinant from coefficient matrix of above equations, which is a polynomial for ω_n^2, to zero, ω_n is obtained.

4. Results and discussion

In regard to investigate the nano size effect on the thermos-electro vibration of curved nanobeams, the amounts of nonlocality for the curved FGP nanobeams is considered as constant in the numerical results. To this purpose, the properties materials of curved FGP nanobeam made of PZT-4 and PZT-5H, are listed in Table 1. The beam's material composition varies from pure PZT-5H at the bottom surface to pure PZT-4 at the top surface. To validate the results, thermal effect is eliminated and Simply-Simply supported boundary conditions are considered. In addition, material properties are assumed as metal and ceramic for FG curved nanobeams.

The nondimensional fundamental frequencies of the nonlocal FG curved nanobeam without consideration of the piezoelectric properties are compared to the results presented by Hosseini and Rahmani (2012) are listed in Tables 2 and 3 for different power-law index and opening angles.

It is observed that the present results agree very well with the given by Ref [45] and that increasing the nonlocality parameter tends to decrease the natural frequency. The reason is that the presence of the nonlocal effect tends to decrease the stiffness of the nanostructures and hence decrease the values of natural frequencies.

In this section, the effect of voltage on dimensionless natural frequency with investigating different values of nonlocality and aspect ratio can be seen in Fig. 2. Thus, figure 2 clearly demonstrates that with increasing voltage between -0.1 to 0.1, dimensionless natural frequency decreases. However, with the increase the nonlocal parameter μ^2 between 0 and 4 $(nm)^2$ the natural frequency decreases significantly. In addition, Fig. 2 also reveals that the discrepancy between the different values of nonlocality curves decreases when the aspect ratio becomes greater, disclosing that the effect of the aspect ratio is more remarkable in the case of curved FGP nanobeams. Hence the results show that the nonlocal effect is tending to decrease the stiffness of nanobeams and thus decreases the dimensionless natural frequencies.

Table 1 constants of material properties (Doroushi *et al.* 2011)

Properties	PZT-4	PZT-5H
$c_{11}\,(Pa)$	81.3×10^{9}	60.6×10^{9}
$c_{55}\,(Pa)$	25.6×10^{9}	23×10^{9}
$e_{31}\left(C/m^{2}\right)$	-10	-16.604
$e_{15}\left(C/m^{2}\right)$	40.3248	44.9046
$\varepsilon_{11}\left(F/m\right)$	0.6712×10^{-8}	1.5027×10^{-8}
$\varepsilon_{33}\left(F/m\right)$	1.0275×10^{-8}	2.5540×10^{-8}
$\rho\left(Kg/m^{3}\right)$	7500	7500
$\lambda_{1}\left(1/K\right)$	0.2×10^{-5}	1×10^{-5}

Table 2 Comparison of dimensionless natural frequencies of S-S curved FG nanobeams for different amounts of slenderness, mode number and nonlocality where $p=0$ and $\alpha=\pi/3$

L/h	ω_n	$\mu^2=0$		$\mu^2=1$		$\mu^2=2$		$\mu^2=3$		$\mu^2=4$	
		Hosseini and Rahmani 2016	Present	Hosseini and Rahmani 2016	Present	Hosseini and Rahmani 2016	Present	Hosseini and Rahmani 2016	Present	Hosseini and Rahmani 2016	Present
10	$n=1$	8.1991	8.1991	7.8222	7.8222	7.4929	7.4929	7.2020	7.2020	6.9425	6.9425
	$n=2$	35.7451	35.7451	30.2666	30.2666	26.7204	26.7204	24.1855	24.1855	22.2576	22.2576
	$n=3$	77.3993	77.3993	56.3256	56.3256	46.4500	46.4500	40.4308	40.4308	36.2732	36.2732
20	$n=1$	8.2912	8.2912	7.9101	7.9101	7.5771	7.5771	7.2829	7.2829	7.0205	7.0205
	$n=2$	37.2875	37.2875	31.5725	31.5725	27.8733	27.8733	25.2291	25.2291	23.2180	23.2180
	$n=3$	84.3180	84.3180	61.3605	61.3605	50.6022	50.6022	44.0449	44.0449	39.5156	39.5156
50	$n=1$	8.3177	8.3177	7.9353	7.9353	7.6012	7.6012	7.3061	7.3061	7.0429	7.0429
	$n=2$	37.7658	37.7658	31.9776	31.9776	28.2309	28.2309	25.5527	25.5527	23.5159	23.5159
	$n=3$	86.7084	86.7084	63.1000	63.1000	52.0367	52.0367	45.2935	45.2935	40.6359	40.6359

In order to clarify the effect of the opening angle parameter and applied electric voltage on the vibration analysis, Fig. 3 intuitively exhibit the variations of the dimensionless natural frequency of nonlocal curved FGP beam with respect to uniform thermal loadings for different values of opening angles and external electric voltages at constant slenderness ratio $L/h=5$.

Due to the Fig. 3, it obviously can be seen that, the dimensionless natural frequency reduce with high pace while the power exponent in realm between 0 and 2 than that while power exponent in realm from 2 to 10. Whereas, the mentioned results determined also demonstrate that the natural frequency of the nonlocal curved FGP nanobeams are always lower than those of the nonlocal functionally graded piezoelectric straight beam model. With the increase the opening angle from $\pi/6$ to $\pi/3$ the natural frequency decreases significantly. In addition, it clearly can be observed that the positive electric voltage decreases the dimensionless natural frequencies.

Whereas, the negative electric voltage increases the dimensionless natural frequencies. The reason is that compressive and tensile inplane loads are generated in the graded nanobeams by exerting positive and negative voltages, respectively.

In this section, the effect of the small scale parameter and external electric voltage on the frequency analysis of nonlocal FGP curved beam, is demonstrated in Fig. 4 at constant slenderness ratio $L/h=5$. It can be undoubtedly deduced that, the dimensionless natural frequency reduces with high pace while the power exponent in realm between 0 and 2 than that while power exponent in realm from 2 to 10. In addition, the mentioned results obtained also show that the natural frequency of the curved FGP nanobeam model are evermore lower than those of the classical graded piezoelectric curved beam model. With the increase the nonlocality μ^2 between 0 and 4 $(nm)^2$ the natural frequency decreases substantially. The results show that the nonlocal effect is tending to decrease the stiffness of nanobeams and hence decreases the dimensionless natural frequencies.

Table 3 Comparison of dimensionless natural frequency of S-S curved FG nanobeams for different amounts of slenderness, mode number and nonlocality where $p=1$ and $\alpha=\pi/2$

L/h	ω_n	$\mu^2=0$		$\mu^2=1$		$\mu^2=2$		$\mu^2=3$		$\mu^2=4$	
		Hosseini and Rahmani 2016	Present	Hosseini and Rahmani 2016	Present	Hosseini and Rahmani 2016	Present	Hosseini and Rahmani 2016	Present	Hosseini and Rahmani 2016	Present
10	$n=1$	4.5601	4.5601	4.3504	4.3504	4.1673	4.1673	4.0055	4.0055	3.8612	3.8612
	$n=2$	23.7375	23.7375	20.0993	20.0993	17.7444	17.7444	16.0611	16.0611	14.7808	14.7808
	$n=3$	53.2817	53.2817	38.7745	38.7745	31.9762	31.9762	27.8325	27.8325	24.9704	24.9704
20	$n=1$	4.6675	4.6675	4.4530	4.4530	4.2655	4.2655	4.0999	4.0999	3.9522	3.9522
	$n=2$	25.0039	25.0039	21.1716	21.1716	18.6911	18.6911	16.9179	16.9179	15.5694	15.5694
	$n=3$	58.3285	58.3285	42.4472	42.4472	35.0050	35.0050	30.4689	30.4689	27.3356	27.3356
50	$n=1$	4.7208	4.7208	4.5038	4.5038	4.3142	4.3142	4.1466	4.1466	3.9972	3.9972
	$n=2$	25.5362	25.5362	21.6223	21.6223	19.0889	19.0889	17.2780	17.2780	15.9008	15.9008
	$n=3$	60.4005	60.4005	43.9551	43.9551	36.9551	36.9551	31.5512	31.5512	28.3067	28.3067

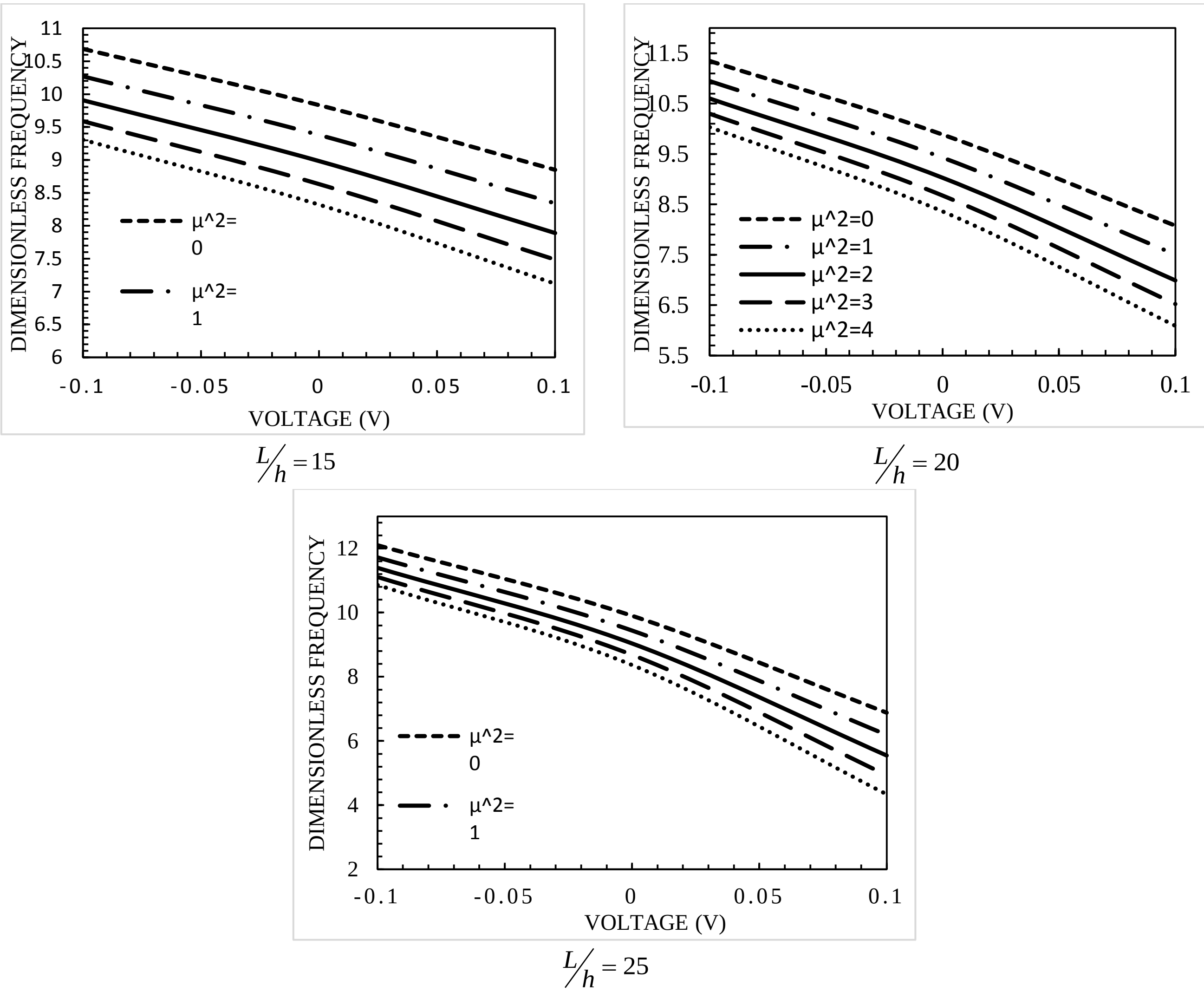


Fig. 2 Variations of the fundamental dimensionless natural frequencies of the curved FGP nanobeam respect to the voltage with different values of nonlocality and aspect ratio $(\alpha=\pi/6, \Delta T=250, p=0)$

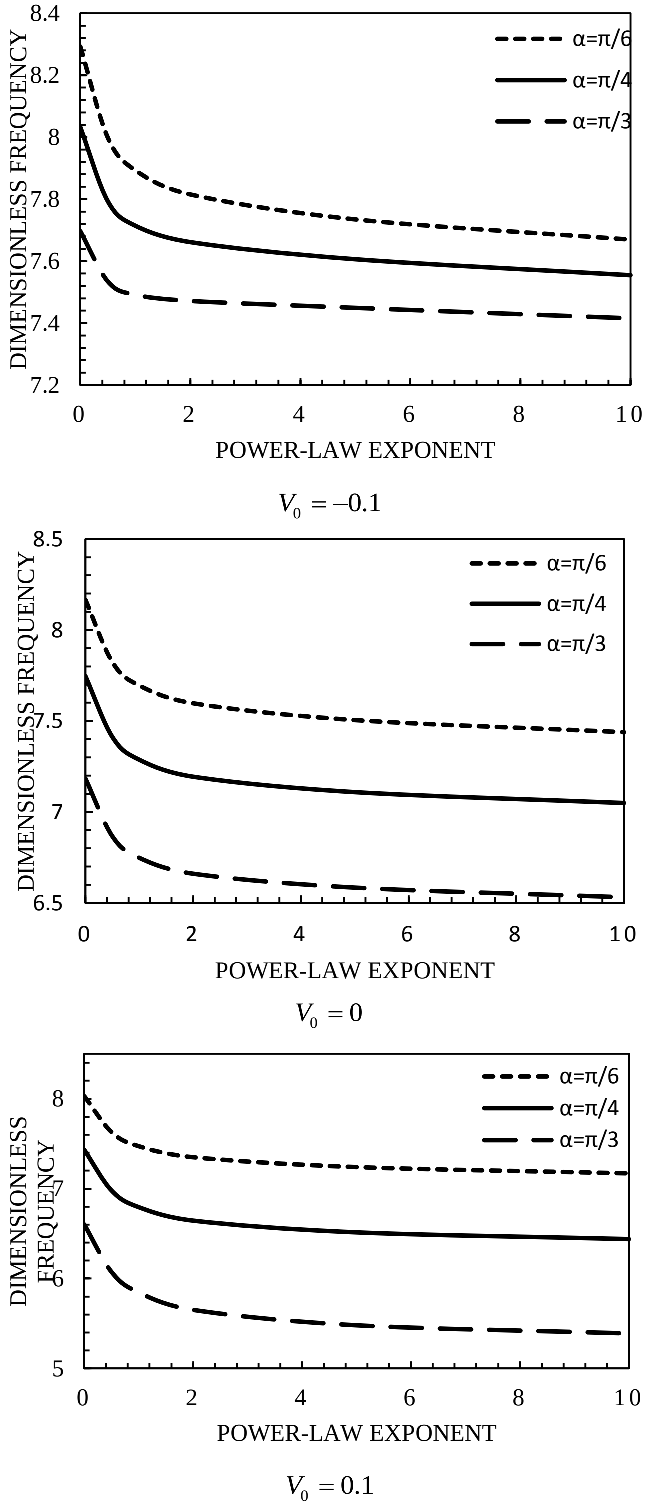
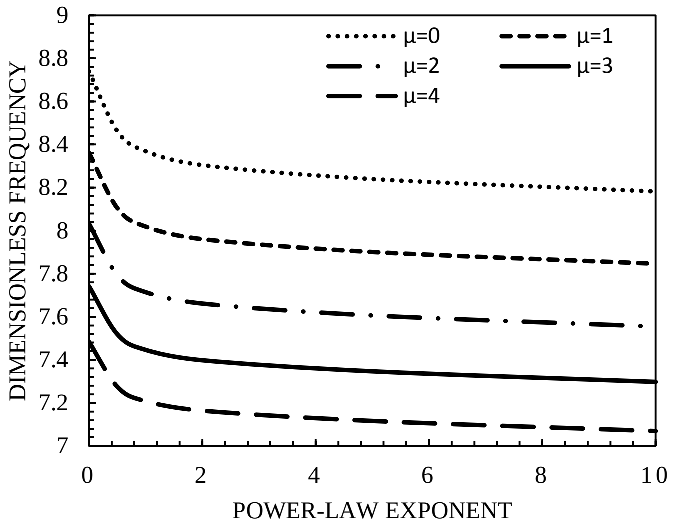
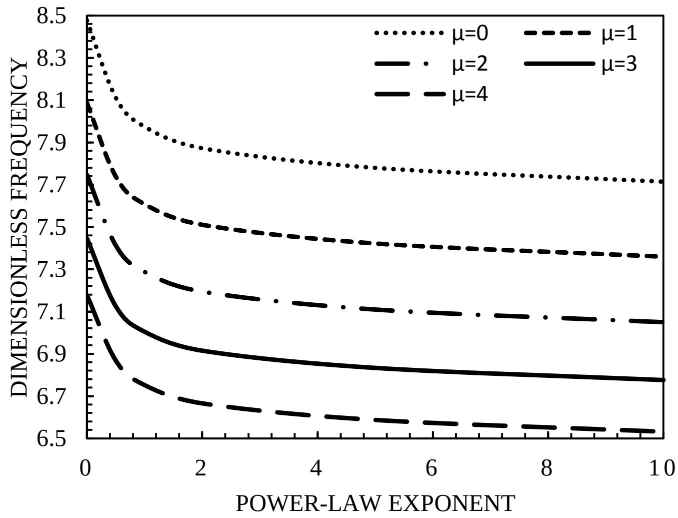


Fig. 3 Variations of the fundamental natural frequency of the curved FGP nanobeam respect to power-law exponent for different amounts of curvatures and external electric voltage $\left(L/h = 5, \mu^2 = 2, \Delta T = 0\right)$.

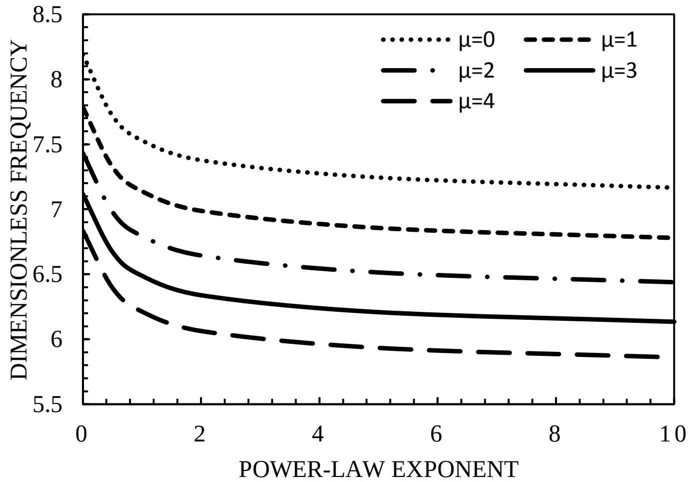
Effects of different values of aspect ratios (L/h) on the free vibration treatment of curved FGP nanobeam for various amounts of nonlocality are presented in Fig. 5. In all figures, results are developed for $(\alpha = \pi/4)$ regardless of the external electric voltage and temperature changes, it may be noted that the amounts of dimensionless natural frequencies increase with the increasing amounts of the slenderness at a constant material distribution. However, with the increase the nonlocality μ^2 m from 0 to 4 $(nm)^2$ the natural frequency decreases obviously.



$V_0 = -0.1$



$V_0 = 0$



$V_0 = 0.1$

Fig. 4 Variations of the fundamental natural frequency of the curved FGP nanobeam respect to power-law exponent for different amounts of nonlocality and external electric voltage $(L/h = 5, \alpha = \pi/4, \Delta T = 0)$

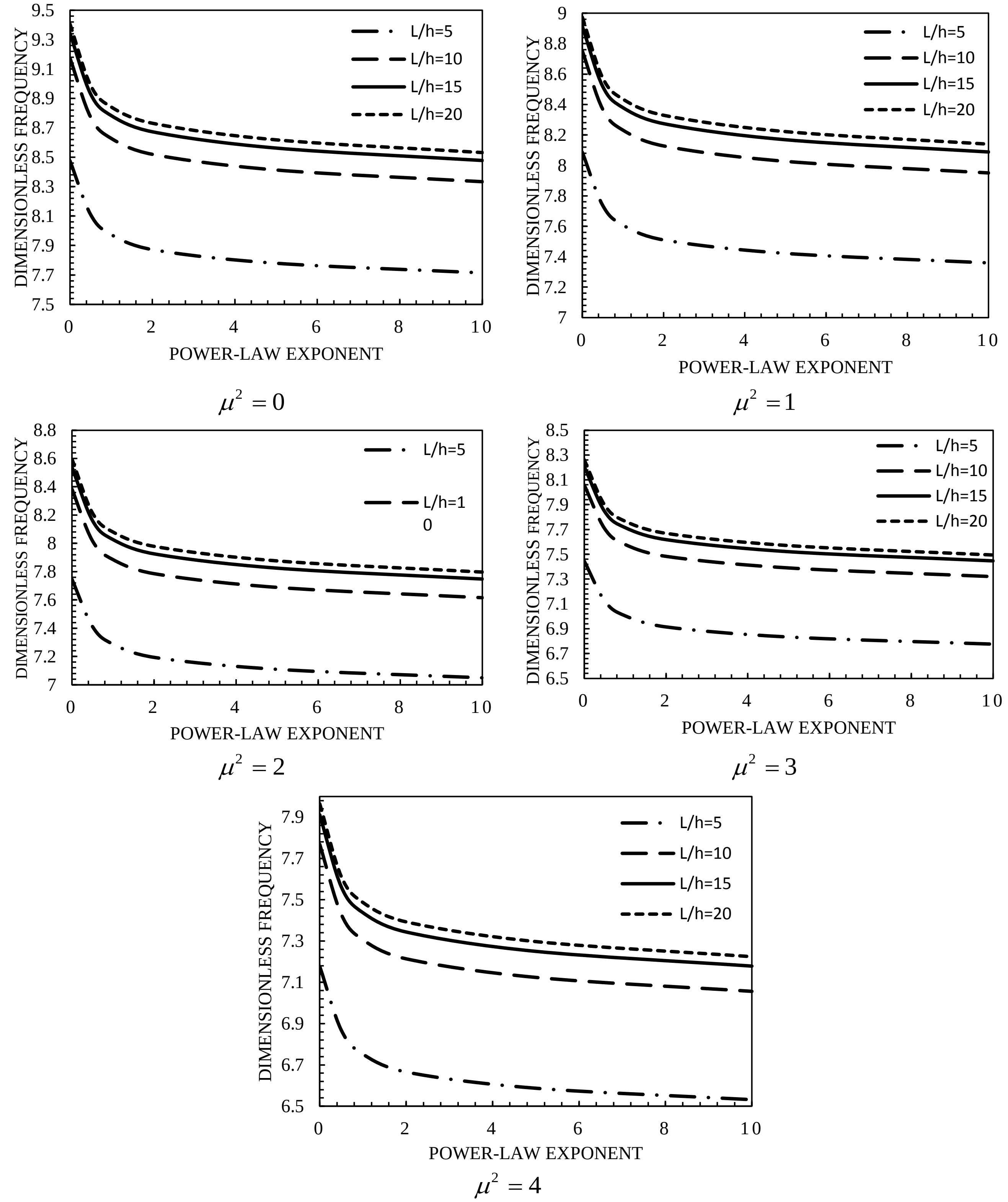
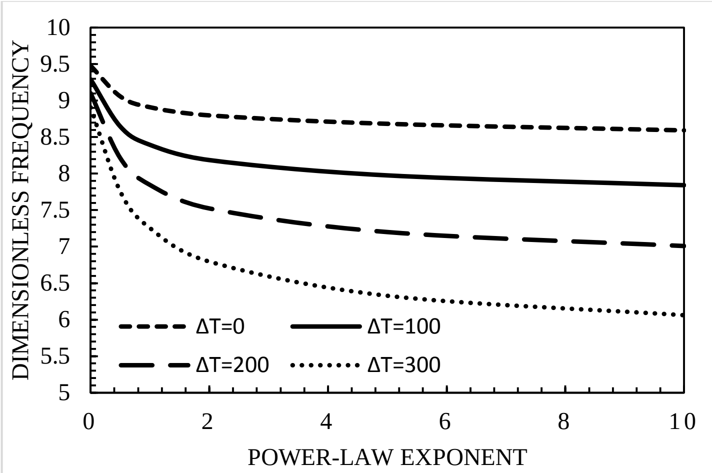
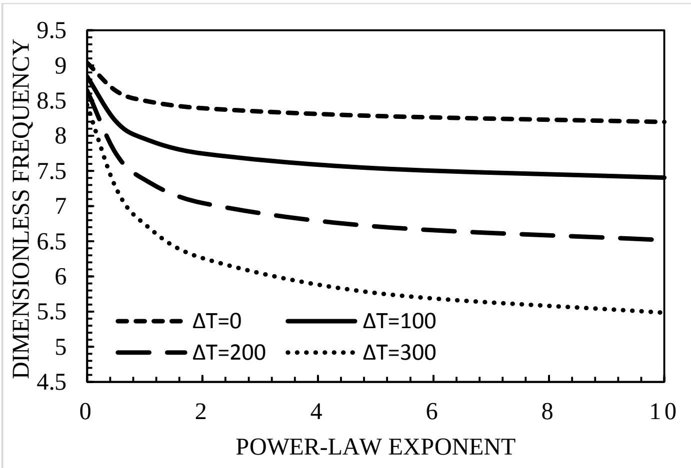


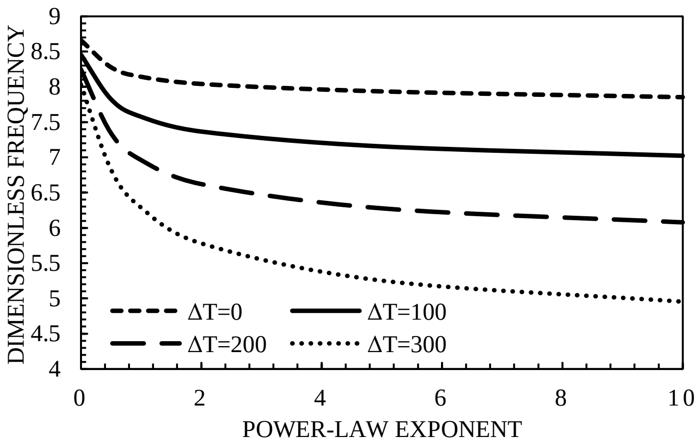
Fig. 5 Variations of the fundamental natural frequency of the curved FGP nanobeam respect to power-law exponent for different amounts of aspect ratio and nonlocality $\left(\alpha = \pi/4,\ V_0 = 0,\ \Delta T = 0\right)$.



$\mu^2 = 0$

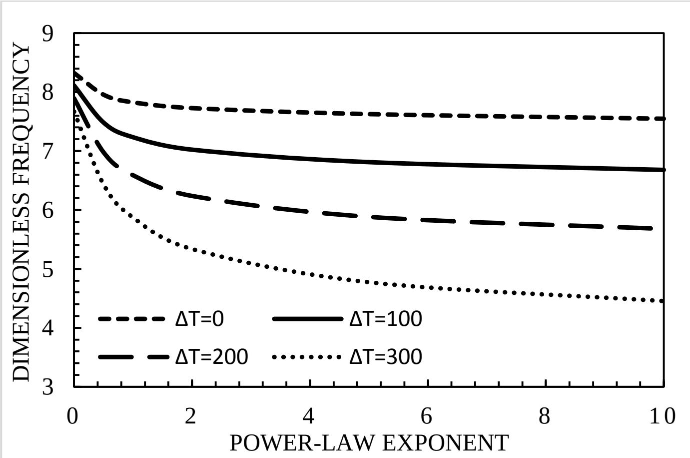


$\mu^2 = 1$

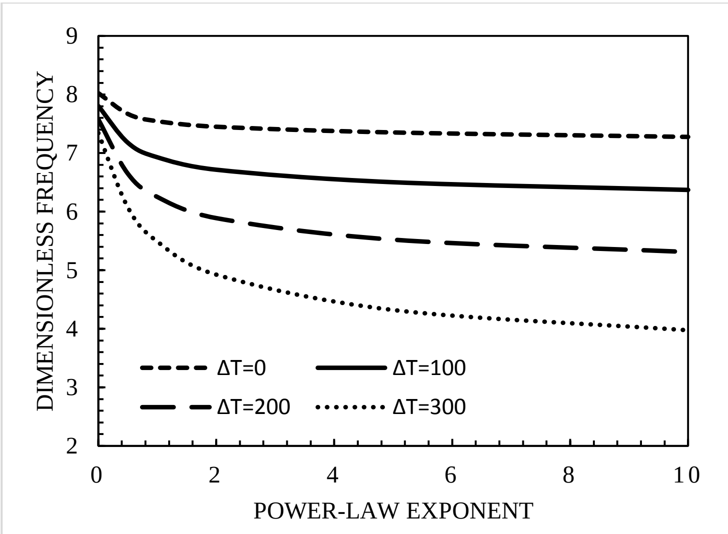


$\mu^2 = 2$

Continued-



$\mu^2 = 3$



$\mu^2 = 4$

Fig. 6 Variations of the fundamental natural frequency of the curved FGP nanobeam respect gradient index for different amounts of temperature changes and nonlocality $\left(L/h = 50,\ \alpha = \pi/4,\ V_0 = 0,\right)$

Table 4 Variation in the fundamental frequency of the S-S curved FGP nanobeams for various amounts of temperature changes, nonlocality, external electric voltage and power-law exponent $\alpha = \pi/6$, $L/h = 20$

		$\Delta T = 0$			$\Delta T = 250$			$\Delta T = 500$		
μ^2	$V_0(v)$	Gradient index			Gradient index			Gradient index		
		0	0.5	1	0	0.5	1	0	0.5	1
	-0.1	11.3734	11.3295	11.3611	11.3463	11.2723	11.2915	11.3191	11.2147	11.2214
0	0	9.9183	9.4816	9.3266	9.8857	9.4089	9.2356	9.8531	9.3354	9.1434
	+0.1	8.1185	6.9990	6.4660	8.0768	6.8936	6.3241	8.0348	6.7864	6.1785
	-0.1	10.9749	10.9622	11.0054	10.9469	10.9034	10.9341	10.9189	10.8441	10.8621
1	0	9.4623	9.0458	8.8978	9.4282	8.9695	8.8024	9.3939	8.8923	8.7056
	+0.1	7.5496	6.3860	5.8165	7.5045	6.2696	5.6572	7.4591	6.1507	5.4926
	-0.1	10.6301	10.6448	10.6982	10.6013	10.5846	10.6253	10.5725	10.5239	10.5517
2	0	9.0640	8.6649	8.5232	9.0283	8.5852	8.4235	8.9925	8.50461	8.3223
	+0.1	7.0385	5.8229	5.2097	6.9898	5.6942	5.0298	6.9408	5.5621	4.8426
	-0.1	10.3283	10.3673	10.4298	10.2988	10.3058	10.3555	10.2693	10.2439	10.2804
3	0	8.7120	8.3285	8.1923	8.6750	8.2455	8.0885	8.6377	8.1615	7.9830
	+0.1	6.5732	5.2970	4.6307	6.5208	5.1542	4.4258	6.4680	5.0069	4.2101
	-0.1	10.0615	10.1223	10.1928	10.0314	10.0597	10.1172	10.0012	9.9965	10.0408
4	0	8.3981	8.0284	7.8971	8.3597	7.9423	7.7894	8.3210	7.8550	7.6797
	+0.1	6.1448	4.7977	4.0654	6.0885	4.6385	3.8285	6.0316	4.4732	3.5750

Table 5 Variation in the fundamental frequency of the S-S curved FGP nanobeams for various amounts of temperature changes, first three mode numbers, external electric voltage and power-law exponent $\alpha = \pi/6$, $L/h = 20$, $\mu^2 = 2$

		$\Delta T = 0$			$\Delta T = 250$			$\Delta T = 500$		
ω_n	$V_0(v)$	Gradient index			Gradient index			Gradient index		
		0	0.5	1	0	0.5	1	0	0.5	1
$n=1$	-0.1	10.6301	10.6448	10.6982	10.6013	10.5846	10.6253	10.5725	10.5239	10.5517
	0	9.0640	8.6649	8.5232	9.0283	8.5852	8.4235	8.9925	8.50461	8.3223
	+0.1	7.0385	5.8229	5.2097	6.9898	5.6942	5.0298	6.9408	5.5621	4.8426
$n=2$	-0.1	30.3711	29.3063	28.9419	30.3609	29.2827	28.9124	30.3507	29.2591	28.8827
	0	29.8460	28.5796	28.1186	29.8347	28.5525	28.0840	29.8234	28.5254	28.0492
	+0.1	29.2620	27.7405	27.1477	29.2495	27.7091	27.1066	29.2369	27.6775	27.0652
$n=3$	-0.1	52.5828	50.6165	49.8978	52.5770	50.6016	49.8786	52.5712	50.5866	49.8592
	0	52.2785	50.1460	49.3481	52.2718	50.1280	49.3244	52.2652	50.1100	49.3005
	+0.1	51.9268	49.5745	48.6613	51.9191	49.5525	48.6312	51.9113	49.5303	48.6009

The fundamental dimensionless frequency as a function of gradient index and temperature changes is presented in Fig. 6 for the curved FGP nanobeam. Similarly, it is disclosed that for a simply-simply curved FGP nanobeam increasing gradient index, temperature change, and nonlocality, leads to reduce the nondimensional natural frequency.

Finally, the vibration of nano-size curved FGP beam under uniform loading temperature changes for different slenderness parameter, nonlocal coefficients, power-law exponent, three cases of electrical loading and first three mode numbers are tabloid in Tables 4 and 5. The similar conclusions are extracted from this table for the effect of the electric voltage parameter on the dimensionless natural frequencies. It can be noted, from Tables 4 and 5 that the dimensionless natural frequency decreases while the gradient index increases. From another perspective, these tables disclose that the dimensionless natural frequency amplifies with the decrease of the power-law exponent parameter. It can also be observed that the natural frequency decreases while temperature change increasing.

In addition, it can be emphasized that the natural frequency decreases by increasing value of external voltage loading. As it can see in Table 5, these situations are evaluated for first three dimensionless natural frequencies.

5. Conclusions

In this study, the Thermo-electro-mechanical frequency of curved FG piezoelectric (FGP) nanobeams based on nonlocal Timoshenko curved beam model is studied with various opening angles in the thermal environment, using the nonlocal elasticity model for the first time. Hamilton's principle is implemented to derive the governing equations and related boundary conditions. Next, the analytically exact solution is used to solve the governing equations for simply supported curved FGP nanobeam. Thermo-electro-mechanical properties of the curved FGP nanobeams are considered to be function of thickness direction via power-law model. Effects of eminent parameters such as external voltage loading, uniform temperature changes, nonlocality, slender, mode numbers, angle of curvature and gradient index are investigated specifically. It is clearly observed that by increasing temperature changes, power-law index, opening angle, voltage value and nonlocal parameter, the dimensionless natural frequencies tend to decrease. In other hand, results are shown that natural frequencies increase with increasing aspect ratio and mode numbers. In addition, results revealed that the curvature angle plays an important role in thermos-electro vibration behavior of FGP nanobeams.

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