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Probabilistic structural damage detection approaches based on structural dynamic response moments

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Abstract. Because of the inevitable uncertainties such as structural parameters, external excitations and measurement noises, the effects of uncertainties should be taken into consideration in structural damage detection. In this paper, two probabilistic structural damage detection approaches are proposed to account for the underlying uncertainties in structural parameters and external excitation. The first approach adopts the statistical moment-based structural damage detection (SMBDD) algorithm together with the sensitivity analysis of the damage vector to the uncertain parameters. The approach takes the advantage of the strength SMBDD, so it is robust to measurement noise. However, it requests the number of measured responses is not less than that of unknown structural parameters. To reduce the number of measurements requested by the SMBDD algorithm, another probabilistic structural damage detection approach is proposed. It is based on the integration of structural damage detection using temporal moments in each time segment of measured response time history with the sensitivity analysis of the damage vector to the uncertain parameters. In both approaches, probability distribution of damage vector is estimated from those of uncertain parameters based on stochastic finite element model updating and probabilistic propagation. By comparing the two probability distribution characteristics for the undamaged and damaged models, probability of damage existence and damage extent at structural element level can be detected. Some numerical examples are used to demonstrate the performances of the two proposed approaches, respectively.

Keywords: structural damage detection; uncertainty; probabilistic approach; statistical moment

1. Introduction

Structural damage detection is an important issue in structural health monitoring. However, one of the main challenges of the practical application in a damage detection method to civil structures is that a significant amount of uncertainties such as the structural parameters of the finite element model, the excitation force acting on the structure, and the measurement errors are inevitably involved in the damage detection procedure for civil structures. Conventional deterministic approaches usually lead to the error and mistake in quantitative damage identification results. Housner (1997) indicated that structural identification within a statistical framework appears to be a promising general approach to structural health monitoring of civil structures in view of inescapable data and modeling uncertainties. Therefore, many studies have been performed in the area of statistical and probabilistic structural damage detection to take the effects of uncertainties into consideration (Xia and Hao 2003, Lu and Law 2007, Li and Law 2008, Zhang *et al.* 2013, Wang *et al.* 2014, Li *et al.* 2016, Ye *et al.*2016)

Previous statistical and probabilistic structural damage detection approaches can be divided into two major categories: Bayesian model updating and stochastic finite element model updating (Xu *et al.* 2010). Beck and Katafygiotis (1998) came up with the idea that the dynamic response data of Bayesian stochastic frame structure could be applied in model updating and dealing with uncertainty. Then, Bayesian probabilistic approaches have been widely used to detect structural damage with consideration of uncertainties (Katafygiotis and Beck 1998, Beck and Au 2002, Yuen and Katafygiotis 2005, Mustafa *et al.* 2007, Yuen 2010, Hao and Betti 2015). Compared with Bayesian based approaches, the probability and statistics characteristics of structural parameters can be obtained by stochastic analyses of test data and the model parameter (Moaveni *et al.* 2005, Xu *et al.* 2010). Especially, it is straightforward and efficient to conduct analytical formulation of probabilistic structural damage detection based on the analysis of response sensitivity to the uncertain parameters (Lu and Law 2007, Li *et al.* 2008, Law and Li 2010, Wang *et al.* 2014). The propagation of each of these uncertainties in updating damage detection is studied based on the response-sensitivity approach. However, the response-sensitivity is sensitive to measurement noise pollution; it is usually requested to adopt Tikhonov regularization (Li *et al.* 2008, Law and Li 2010, Wang *et al.* 2014) to stabilize the inverse solution by the response-sensitivity damage detection approach.

It is well known that measured structural responses are always contaminated by measurement noise but model

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updating exactly reproduces the contaminated structural responses (Yi *et al*, 2012). The results of damage detection from the model updating may become unreliable. To reduce the influence of measurement noise on damage identification, Zhang and Xu (Zhang *et al*. 2008, Xu *et al*. 2009, Zhang *et al*. 2013) proposed a novel statistical moment-based damage detection (SMBDD) method to locate and identify structural damage. A significant superiority of SMBDD lies in that it is robust to measurement noise but sensitive to structural damage as statistical moment of response is not sensitive to measurement noise. Furthermore, Xu *et al.* (2011) investigate stochastic damage detection method for building structures with parametric uncertainties by integrating SMBDD with the probability density evolution method developed by Li and Chen (2004). However, the computational effort is quite involved. Moreover, it is requested by the SMBDD method that the number of measured responses is not less than that of unknown structural parameters to avoid ill-conditioned nonlinear optimization problem for structural damage identification.

In this paper, two probabilistic structural damage detection approaches are proposed to consider uncertainties in structural parameters and external excitation. The first proposed approach adopts the statistical moment-based structural damage detection (SMBDD) with the sensitivity analysis of the damage vector to the uncertain parameters. The approach is robust to measurement noise due to the advantage of SMBDD method. To reduce the number of measurements requested by the SMBDD algorithm, another probabilistic structural damage detection approach is proposed based on the integration of structural damage detection using time segment temporal moment-based structural damage detection (TSTMBDD) with the sensitivity analysis of the damage vector to the uncertain parameters. In both approaches, probability distribution of damage vector is estimated from those of uncertain parameters based on stochastic finite element model updating and probabilistic propagation. The probability of damage existence and damage extent is identified by comparing the two probability distribution characteristics for the undamaged and damaged models. Some numerical examples are used to validate the two proposed approaches, respectively.

2. Probabilistic structural damage detection approaches by integrating SMBDD and sensitivity-based analysis

The first proposed approach for probabilistic structural damage detection is based on the integration of SMBDD and sensitivity analysis of the damage vector to the uncertain parameters.

2.1 Deterministic structural damage detection by SMBDD and sensitivity analysis

The equation of motion of a linear structure under external excitation can be written as

$$\mathbf{M}\ddot{\mathbf{x}}(\mathrm{t}) + \mathbf{C}\dot{\mathbf{x}}(\mathrm{t}) + \mathbf{K}\mathbf{x}(\mathrm{t}) = \mathbf{B}\mathbf{f}(\mathrm{t}) \tag{1}$$

where $\mathbf{x}$, $\dot{\mathbf{x}}$ and $\ddot{\mathbf{x}}$ are the vectors of displacement, velocity, and acceleration responses , respectively, $\mathbf{f}(\mathrm{t})$ is a known external excitation vector, **B** is the position matrix of the $\mathbf{f}(\mathrm{t})$, **M** is the mass matrix of the structures, and **C** is the damp matrix of the structures. It is assumed that matrix **M** and the external excitation **f**(t) are known. In this paper, structural damage id defined as the degradation of structural element stiffness. The structural global stiffness matrix $\mathbf{K}$ is expressed as

$$\mathbf{K} = \sum_{i=1}^{n} \alpha_i \mathbf{K}_i = \alpha_1 \mathbf{K}_1 + \alpha_2 \mathbf{K}_2 + \cdots + \alpha_n \mathbf{K}_n \tag{2}$$

in which α_i is the *i*-th elemental relative stiffness parameter of reference FE model($0 \le \alpha_i \le 1$),n is the number of elements in the structure; $\mathbf{K}_i$ is the *i-th* elemental stiffness matrix.

By differentiating both sides of Eq. (1) with respect to the stiffness parameter α_i, it leads to

$$\mathbf{M}\frac{\partial \ddot{\mathbf{x}}(\mathrm{t})}{\partial \alpha_i} + \mathbf{C}\frac{\partial \dot{\boldsymbol{x}}(\mathrm{t})}{\partial \alpha_i} + \mathbf{K}\frac{\partial \boldsymbol{x}(\mathrm{t})}{\partial \alpha_i} = -\frac{\partial \mathbf{K}}{\partial \alpha_i}\boldsymbol{x}(\mathrm{t}) - \frac{\partial \mathbf{C}}{\partial \alpha_i}\dot{\boldsymbol{x}}(\mathrm{t}) \tag{3}$$

Then, structural response and its derivatives can be obtained from Eqs. (1) and (3), respectively.

In this paper, structural fourth-order statistical moment is used and it is estimated by

$$M_i = \frac{1}{N_s}\sum_{k=1}^{Ns} \boldsymbol{x}_{ik}^4 - \frac{4}{N_s}\overline{\boldsymbol{x}}_i \sum_{k=1}^{Ns} \boldsymbol{x}_{ik}^3 + \frac{6}{N_s}\overline{\boldsymbol{x}}_i^2 \sum_{k=1}^{Ns} \boldsymbol{x}_{ik}^2 - 3\overline{\boldsymbol{x}}_i^4 \tag{4}$$

where N_s is the number of sampling points and $\overline{\boldsymbol{x}}_i$ denotes the average of the *i*-th response.

The sensitivity analysis of the structural fourth-order statistical moment for damage identification is given as

$$\mathbf{S} \cdot \Delta \boldsymbol{\alpha} = \mathbf{M}^m - \mathbf{M}^c \tag{5}$$

where $\mathbf{M}^m$ and $\mathbf{M}^c$ are the vectors of the measured and calculated of structural fourth-order statistical moments, respectively. **S** is the sensitivity matrix of structural statistical moment with respect to structural stiffness parameter defined as

$$\mathrm{S}_{ij} = \frac{\partial M_i}{\partial \alpha_j} = \frac{4}{N_s}\sum_{k=1}^{Ns} x_{ik}^3 \frac{x_{ik}}{\partial \alpha_j} - \frac{4}{N_s}\frac{\partial \overline{x}_i}{\partial \alpha_j}\sum_{k=1}^{Ns} \boldsymbol{x}_{ik}^3 + \frac{12}{N_s}\overline{x}_i \frac{\partial \overline{\boldsymbol{x}}_i}{\partial \alpha_j}\sum_{k=1}^{Ns} \boldsymbol{x}_{ik}^2 + \frac{12}{N_s}\overline{x}_i^2 \sum_{k=1}^{Ns} x_{ik}\frac{\partial x_{ik}}{\partial \alpha_j} - 12\overline{\boldsymbol{x}}_i^3 \frac{\partial \overline{x}_i}{\partial \alpha_j} \tag{6}$$

Therefore, structural element stiffness vector **α**, which is also structural damage vector according to Eq. (2), can be estimated from Eq. (5) in an iterative least-squares procedure as

$$\Delta \boldsymbol{\alpha} = \mathbf{S}^{+}(\mathbf{M}^m - \mathbf{M}^c) \tag{7}$$

in which $\mathbf{S}^{+}$ is the Moore–Penrose generalized inverse of matrix. As higher-order terms are neglected in the first-order sensitivity Eq. (4), an iterative procedure is employed.

2.2 Probabilistic structural damage detection considering uncertainties

Due to the inevitable uncertainties such as structural parameters, external excitations and measurement noises, it is necessary to take the effects of uncertainties into consideration in structural damage detection. In this paper, the uncertainties in both structural parameters and external excitations are considered. Eqs. (1) and (3) can be re-written as

$$\mathbf{M}(\boldsymbol{\Theta})\ddot{\mathbf{x}}(t)+\boldsymbol{C}(\boldsymbol{\Theta})\dot{\mathbf{x}}(t)+\mathbf{K}(\boldsymbol{\Theta})\mathbf{x}(t)=\mathbf{Df}(\boldsymbol{\Psi},t) \tag{8a}$$

$$\boldsymbol{M}(\boldsymbol{\Theta})\frac{\partial\ddot{\boldsymbol{x}}(t)}{\partial\alpha_i}+\boldsymbol{C}(\boldsymbol{\Theta})\frac{\partial\dot{\boldsymbol{x}}(t)}{\partial\alpha_i}+\boldsymbol{K}(\boldsymbol{\Theta})\frac{\partial\boldsymbol{x}(t)}{\partial\alpha_i}=-\frac{\partial\boldsymbol{K}(\boldsymbol{\Theta})}{\partial\alpha_i}\boldsymbol{x}(t)-\frac{\partial\boldsymbol{C}(\boldsymbol{\Theta})}{\partial\alpha_i}\dot{\boldsymbol{x}}(t) \tag{8b}$$

where $\boldsymbol{\Theta}$ is a n-dimension random parameter vector reflecting the uncertainty in the structural parameters with the known probability density function $p_{\boldsymbol{\Theta}}$ (θ), $\boldsymbol{\Psi}$ is a m-dimension random parameter vector with probability density function $p_{\boldsymbol{\Psi}}$ (ψ), reflecting the uncertainty in the known external excitation. All the uncertain parameters both in structures and in external excitations can be written together to form a general uncertain vector $\mathbf{X}=[\boldsymbol{\Theta}\ \boldsymbol{\Psi}]$ with a mean value vector $\mathbf{X}_0=\{X_{q0}\}$.(q=1,2,…,n+m)

Analogous to the formula by other researchers (Xia and Hao 2003, Li and Law 2008, Law and Li 2010), the structural damage vector $\boldsymbol{\alpha}$ is expended by the first-order Taylor series in terms of the general uncertain variable $\mathbf{X}$, it is obtained as

$$\Delta\boldsymbol{\alpha}(\boldsymbol{X})=\Delta\boldsymbol{\alpha}(\boldsymbol{X}_0)+\sum_{q=1}^{n+m}\left.\frac{\partial\Delta\boldsymbol{\alpha}}{\partial X_q}\right|_{X_q=X_{q0}}(X_q-X_{q_0}) \tag{9}$$

in which $\Delta\boldsymbol{\alpha}(\boldsymbol{X}_0)$ can be estimated from Eq. (7) by the deterministic structural damage detection based on SMBDD and sensitivity analysis when the uncertain variable $\boldsymbol{X}$ in its mean value $\boldsymbol{X}_0$ as

$$\Delta\boldsymbol{\alpha}(\boldsymbol{X}_0)=\boldsymbol{S}^+(\boldsymbol{M}-\boldsymbol{M}^c(\boldsymbol{X}_0)) \tag{10}$$

By differentiating both sides of Eq. (5) with respect to the general random variable X_q, it is derived as

$$\frac{\partial\mathbf{S}}{\partial X_q}\cdot\Delta\boldsymbol{\alpha}+\mathbf{S}\cdot\frac{\partial\Delta\boldsymbol{\alpha}}{\partial X_q}=-\frac{\partial\mathbf{M}^c}{\partial X_q} \tag{11}$$

Then, the derivative $\left.\frac{\partial\Delta\boldsymbol{\alpha}}{\partial X_q}\right|_{X_q=X_{q0}}$ can be estimated by least-squares as

$$\left.\frac{\partial\Delta\boldsymbol{\alpha}}{\partial X_q}\right|_{X_q=X_{q0}}=\mathbf{S}^+(\mathbf{X}_0)\left[-\left.\frac{\partial\mathbf{M}^c}{\partial X_q}\right|_{X_q=X_{q0}}-\left.\frac{\partial\mathbf{S}}{\partial X_q}\right|_{X_q=X_{q0}}\Delta\boldsymbol{\alpha}(\mathbf{X}_0)\right] \tag{12}$$

In Eq. (12), Matrix $\mathbf{S}^+(\mathbf{X}_0)$ and vector $\Delta\boldsymbol{\alpha}(\mathbf{X}_0)$ can be obtained from Eqs. (6) and (7) but the terms $\left.\partial\mathbf{M}^c\big/\partial X_q\right|_{X_q=X_{q0}}$ and $\left.\partial\mathbf{S}\big/\partial X_q\right|_{X_q=X_{q0}}$ need to be estimated according to the type of uncertainty. In this paper, the uncertainties in the material mass density of structural model and external excitation are included in the study and they are assumed independent.

2.2.1 Uncertainties in the structural parameter

As an example, the uncertainty of mass density is studied herein. This uncertainty is expressed as

$$m_i=m_{i0}(1+X_{m_i})\quad(i=1,\cdots,n) \tag{13}$$

where m_i is the uncertain element mass density, m_{i0} denotes its nominal value, and X_{m_i} is the uncertain variable with given probabilistic distribution. The sensitivity of structural response with respect to X_{m_i} can be obtained by taking differentiation with respect to X_{m_i} on both sides of Eq. (8(a)) as

$$\mathbf{M}\frac{\partial\ddot{\mathbf{x}}(\mathrm{t})}{\partial X_{m_i}}+\mathbf{C}\frac{\partial\dot{\mathbf{x}}(\mathrm{t})}{\partial X_{m_i}}+\mathbf{K}\frac{\partial\mathbf{x}(\mathrm{t})}{\partial X_{m_i}}=-\frac{\partial\mathbf{M}}{\partial X_{m_i}}\ddot{\mathbf{x}}(\mathrm{t})-\frac{\partial\mathbf{C}}{\partial X_{m_i}}\dot{\mathbf{x}}(\mathrm{t})-\frac{\partial\mathbf{K}}{\partial X_{m_i}}\mathbf{x}(\mathrm{t}) \tag{14}$$

in which $\frac{\partial\mathbf{K}}{\partial X_{m_i}}$ can be estimated by

$$\frac{\partial\mathbf{K}}{\partial X_{m_i}}=\sum_{j=1}^{N}\frac{\partial\mathbf{K}}{\partial\alpha_j}\cdot\frac{\partial\Delta\alpha_j}{\partial X_{m_i}} \tag{15}$$

Differentiating both sides of Eq. (8(b)) with respect to the random variable X_{m_i} gives

$$\mathbf{M}\frac{\partial^2\ddot{\mathbf{x}}(\mathrm{t})}{\partial X_{m_i}\partial\alpha_i}+\mathbf{C}\frac{\partial^2\dot{\mathbf{x}}(\mathrm{t})}{\partial X_{m_i}\partial\alpha_i}+\mathbf{K}\frac{\partial^2\mathbf{x}(\mathrm{t})}{\partial X_{m_i}\partial\alpha_i}=-\frac{\partial\mathbf{M}}{\partial X_{m_i}}\frac{\partial\ddot{\mathbf{x}}(\mathrm{t})}{\partial\alpha_i}-\frac{\partial C}{\partial X_{m_i}}\frac{\partial\dot{\mathbf{x}}(\mathrm{t})}{\partial\alpha_i}-\frac{\partial\mathbf{C}}{\partial\alpha_i}\frac{\partial\dot{\mathbf{x}}(\mathrm{t})}{\partial X_{mi}}-\frac{\partial\mathbf{K}}{\partial X_{m_i}}\frac{\partial\mathbf{x}(\mathrm{t})}{\partial\alpha_i}-\frac{\partial\mathbf{K}}{\partial\alpha_i}\frac{\partial\mathbf{x}(\mathrm{t})}{\partial X_{m_i}} \tag{16}$$

From Eq. (3), $\frac{\partial\ddot{\boldsymbol{x}}}{\partial\alpha_i}$, $\frac{\partial\dot{\boldsymbol{x}}}{\partial\alpha_i}$ and $\frac{\partial\boldsymbol{x}}{\partial\alpha_i}$ can be estimated. However, $\frac{\partial\boldsymbol{x}}{\partial X_{mi}}$, $\frac{\partial\dot{\boldsymbol{x}}}{\partial X_{mi}}$ and $\frac{\partial^2\boldsymbol{x}}{\partial\alpha_i\partial X_{mi}}$ in Eqs. (14) and (16) need to be calculated iteratively from Eqs. (12)-(16) with an initial value $\frac{\partial\boldsymbol{K}}{\partial X_{mi}}=0$. Finally, $\left.\frac{\partial\Delta\boldsymbol{\alpha}}{\partial X_q}\right|_{X_q=X_{q0}}$ can be estimated from Eq. (12).

2.2.2 Uncertainty in the external excitations

Two types of uncertain external excitations are investigated in this paper.

2.2.2.1 Uncertainty in the external exciting magnitudes

Sometimes, the magnitudes of external excitations are uncertain variables. This uncertainty is expressed as

$$f_j = f_{j0} X_{f_j} \qquad (j = 1, \cdots, m) \tag{17}$$

where f_j is the uncertain external excitation, f_{j0} denotes the nominal value, and X_{f_j} is a random variable with given probabilistic distribution. By taking the differentiation with respect to the random variable X_{fi} on both sides of Eqs. (8(a)) and (8(b)), it is derived as

$$\mathbf{M}\frac{\partial \ddot{\mathbf{x}}}{\partial X_{fi}} + \boldsymbol{C}\frac{\partial \dot{\mathbf{x}}}{\partial X_{fi}} + \mathbf{K}\frac{\partial \mathbf{x}}{\partial X_{fi}} = \boldsymbol{B}\mathbf{f}^0 - \frac{\partial \mathbf{K}}{\partial X_{fi}}\boldsymbol{x} - \frac{\partial \mathbf{C}}{\partial X_{fi}}\dot{\boldsymbol{x}} \tag{18}$$

$$\boldsymbol{M}\frac{\partial^2 \ddot{\boldsymbol{x}}}{\partial X_{fi}\partial\alpha_i} + \boldsymbol{C}\frac{\partial^2 \dot{\boldsymbol{x}}}{\partial X_{fi}\partial\alpha_i} + \boldsymbol{K}\frac{\partial^2 \boldsymbol{x}}{\partial X_{fi}\partial\alpha_i} = -\frac{\partial \boldsymbol{K}}{\partial X_{fi}}\frac{\partial \boldsymbol{x}}{\partial \alpha_i} - \frac{\partial \boldsymbol{K}}{\partial \alpha_i}\frac{\partial \boldsymbol{x}}{\partial X_{fi}} - \frac{\partial \boldsymbol{C}}{\partial \alpha_i}\frac{\partial \dot{\boldsymbol{x}}}{\partial X_{fi}} - \frac{\partial \boldsymbol{C}}{\partial X_{fi}}\frac{\partial \dot{\boldsymbol{x}}}{\partial \alpha_i} \tag{19}$$

Analogously, $\dfrac{\partial \boldsymbol{x}}{\partial X_{fi}}$, $\dfrac{\partial \dot{\boldsymbol{x}}}{\partial X_{fi}}$ and $\dfrac{\partial^2 \boldsymbol{x}}{\partial \alpha_i \partial X_{fi}}$ in Eqs. (18) and (19) need to be calculated in an iterative approach.

2.2.2.2 Uncertainty in the Kanai-Tajimi ground excitation

If the structure is subject to ground excitation in the Kanai-Tajimi spectrum, the equation of motion can be expressed by

$$\mathbf{M}(\boldsymbol{\Theta})\ddot{\mathbf{x}}(\mathrm{t}) + \boldsymbol{C}(\boldsymbol{\Theta})\dot{\mathbf{x}}(\mathrm{t}) + \mathbf{K}(\boldsymbol{\Theta})\mathbf{x}(\mathrm{t}) = -\mathbf{M}(\boldsymbol{\Theta})\{\mathbf{I}\}\ddot{\mathrm{x}}_{\mathrm{g}}(\boldsymbol{\Psi}, \mathrm{t}) \tag{20}$$

where $\{\mathbf{I}\}$ is a identity vector and the ground acceleration $\ddot{\mathrm{x}}_{\mathrm{g}}(\boldsymbol{\Psi}, \mathrm{t})$ is in the Kanai-Tajimi spectrum with the spectral density function in the form as

$$S_{\ddot{x}_g}(\omega) = \frac{1 + 4\xi_g^2 (\dfrac{\omega}{\omega_g})^2}{\left[1 - (\dfrac{\omega}{\omega_g})^2\right]^2 + 4\xi_g^2 (\dfrac{\omega}{\omega})^2} S_0 \tag{21}$$

in which ω_g, ξ_g and S_0 are the characteristic parameters of the ground motion. In this study, uncertainty of ω_g is considered by

$$\omega_g = \omega_{g0} X_{\omega_g} \tag{22}$$

where ω_{g0} denotes the nominal value and X_g is the uncertain variable with given probabilistic distribution.

The Kanai-Tajimi ground acceleration $\ddot{\mathrm{x}}_{\mathrm{g}}(\boldsymbol{\Psi}, \mathrm{t})$ can be treated as the output of soil layer excited by the motion of rock stratum, i.e.

$$\ddot{\mathrm{x}}_{\mathrm{g}}(\boldsymbol{\Psi}, \mathrm{t}) = \ddot{\mathrm{u}}(t) + \ddot{\mathrm{U}}(t) \tag{23a}$$

$$\ddot{\mathrm{u}}(\mathrm{t}) + 2\xi_g \omega_g \dot{\mathrm{u}}(\mathrm{t}) + \omega_g^2 \mathrm{u}(\mathrm{t}) = -\ddot{\mathrm{U}}(\mathrm{t}) \tag{23b}$$

and $\ddot{\mathrm{U}}(\mathrm{t})$ is the rock acceleration which is usually regarded as white noise.

Taking derivative with respect to X_g on both sides of Eq. (20) gives

$$\mathbf{M}\frac{\partial \ddot{\mathbf{x}}(\mathrm{t})}{\partial X_{\omega_g}} + \mathbf{C}\frac{\partial \dot{\mathbf{x}}(\mathrm{t})}{\partial X_{\omega_g}} + \mathbf{K}\frac{\partial \mathbf{x}(\mathrm{t})}{\partial X_{\omega_g}} = \mathbf{M}\frac{\partial \ddot{\mathrm{u}}(\mathrm{t})}{\partial X_{\omega_g}} - \frac{\partial \mathbf{C}}{\partial X_{\omega_g}}\dot{\mathbf{x}}(\mathrm{t}) - \frac{\partial \mathbf{K}}{\partial X_{\omega_g}}\mathbf{x}(\mathrm{t}) \tag{24}$$

in which $\dfrac{\partial \mathbf{K}}{\partial X_{\omega_g}}$ can be estimated analogous to Eq. (15), $\left.\dfrac{\partial \ddot{\boldsymbol{u}}(\mathrm{t})}{\partial X_{\omega_g}}\right|_{X_{\omega_g} = X_{g0}}$ can be calculated by differentiating both sides of Eq. (23(b)) with respect to X_g as

$$\frac{\partial \ddot{\mathrm{u}}(\mathrm{t})}{\partial X_{\omega_g}} + 2X_g \xi_g \omega_{g0}\frac{\partial \dot{\mathrm{u}}(\mathrm{t})}{\partial X_g} + X_g{}^2 \omega_{g0}^2 \frac{\partial \mathrm{u}(\mathrm{t})}{\partial X_g} = -2\xi_g \omega_{g0}\dot{\mathrm{u}}(\mathrm{t}) - 2X_g \omega_{g0}^2 \mathrm{u}(\mathrm{t}) \tag{25}$$

By taking the differentiation with respect to the random variable X_g on both sides of Eq. (8(b)), it is also derived as

$$\mathbf{M}\frac{\partial^2 \ddot{\mathbf{x}}(\mathrm{t})}{\partial X_{\omega_g}\partial\alpha_i} + \mathbf{C}\frac{\partial^2 \dot{\mathbf{x}}(\mathrm{t})}{\partial X_{\omega_g}\partial\alpha_i} + \mathbf{K}\frac{\partial^2 \mathbf{x}(\mathrm{t})}{\partial X_{\omega_g}\partial\alpha_i} = -\frac{\partial \mathbf{K}}{\partial \alpha_i}\frac{\partial \mathbf{x}(\mathrm{t})}{\partial X_{\omega_g}} - \frac{\partial \mathbf{K}}{\partial X_{\omega_g}}\frac{\partial \mathbf{x}(\mathrm{t})}{\partial \alpha_i} - \frac{\partial \mathbf{C}}{\partial \alpha_i}\frac{\partial \dot{\mathbf{x}}(\mathrm{t})}{\partial X_{\omega_g}} - \frac{\partial \mathbf{C}}{\partial X_{\omega_g}}\frac{\partial \dot{\mathbf{x}}(\mathrm{t})}{\partial \alpha_i} \tag{26}$$

$\dfrac{\partial^2 \mathbf{x}(\mathrm{t})}{\partial \alpha_i \partial X_{\omega_g}}$ can also be estimated in an iterative procedure.

In summary, $\left.\dfrac{\partial \Delta\boldsymbol{\alpha}}{\partial X_q}\right|_{X_q = X_{q0}}$ can be implemented according to the following steps:

Step1: Estimate $\dfrac{\partial \mathbf{x}(\mathrm{t})}{\partial X_{qi}}$ by Eq. (14), Eq. (18) or Eq. (24), with the initial $\dfrac{\partial \mathbf{K}}{\partial X_{qi}}$ =0.

Step 2: Calculate $\dfrac{\partial^2 \mathbf{x}(\mathrm{t})}{\partial \alpha_i \partial X_{qi}}$ according to Eq. (16), Eq. (19) or Eq. (26);

Step 3: Evaluate $\left.\dfrac{\partial \mathbf{M}^c}{\partial X_q}\right|_{X_q = X_{q0}}$ and $\left.\dfrac{\partial \mathbf{S}}{\partial X_q}\right|_{X_q = X_{q0}}$ from the results of $\dfrac{\partial \mathbf{x}(\mathrm{t})}{\partial X_{qi}}$ and $\dfrac{\partial^2 \mathbf{x}(\mathrm{t})}{\partial X_{qi}\partial \alpha_i}$

Step4: After obtaining $\left.\dfrac{\partial \Delta\boldsymbol{\alpha}}{\partial \mathbf{X}}\right|_{\boldsymbol{X} = \boldsymbol{X}_0}$ from Eq. (12), $\dfrac{\partial \mathbf{K}}{\partial X_{qi}}$ is estimated according to Eq. (15)

Step5: Repeat the Steps 1-4 till the convergence criterion for the iterative estimation $\left.\dfrac{\partial \Delta\boldsymbol{\alpha}}{\partial X_q}\right|_{X_q = X_{q0}}$ is satisfied.

2.2.3 Probability of damage existence and damage extent

According to the linear relation from the first order Taylor expansion in Eq. (9), the probability distribution function of structural element stiffness parameters can be estimated based on the assumed probability distribution of the uncertainties in structural modeling parameters and external excitations.

Then, the probability of damage existence ($\mathrm{PDE_i}$) for the i-th element is defined as

$$\mathrm{PDE_i} = \mathrm{prob}(0 < \alpha_{di} \le L_i) = 1 - \mathrm{prob}(L_i \le \alpha_{ui} < \infty) \quad (27)$$

in which the subscript u and d donate the quantities in the undamaged and damaged models, respectively. The value of PDE is between 0 and 1, larger value suggests greater PDE.

Moreover, structural element damage extent is defined by

$$\mathrm{DE}_i = \frac{\alpha_{ui} - \alpha_{di}}{\alpha_{ui}} \times 100\% \quad (28)$$

2.3 Numerical examples for probabilistic structural damage detection by integrating SMBDD and sensitivity-based analysis

Some numerical examples are used to illustrate the proposed probabilistic structural damage detection approach by integrating SMBDD and sensitivity-based analysis.

2.3.1 Damage detection of a shear building underground excitation with uncertain magnitude

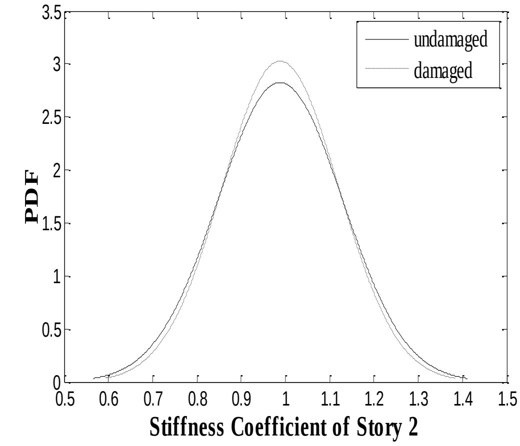
In this section, numerical investigation of a ten-story shear frame building is conducted to verify the proposed approach. Structural parameters of the building assumed as $m_i = 2500\,\mathrm{kg}$, $k_i = 7.0\times10^6\,\mathrm{N/m}$ (i=1,2,..10), Rayleigh damping is applied herein. The first two damping ratios are 3% and Rayleigh damping coefficients are $a = 0.0705$ and $b = 0.0127$.

In this numerical example, The ground acceleration is modeled as the Kanai-Tajimi (K-T) spectrum with the parameters $\omega_g = 15.6$ rad/s and $\xi_g = 0.6$ in Eq. (21), but the magnitude of K-T spectrum is a uncertain variable with nominal value of $S_0 = 4.64\times10^{-1}\,\mathrm{m^2/rads^3}$ and a Gaussian distribution X_f~N(1,0.02) in Eq. (17). All story drifts are measured and measurement noises with 5% and 15% noise-to-signal ratio in root mean square (rms) are considered, respectively.

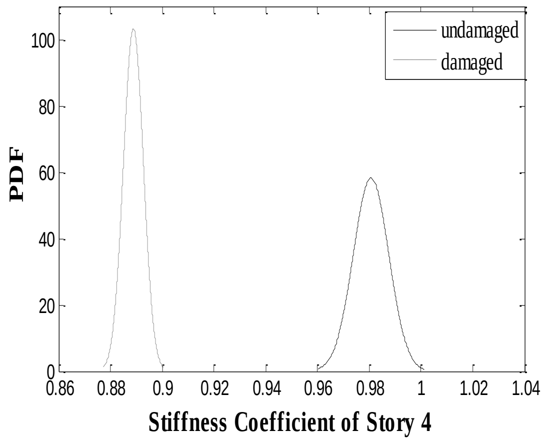
Structural damage is assumed as the 4th story stiffness reduction. In Table 1, the identification results of probability of damage existence (PDE) and damage extent (DE) are presented. It is found the PDE at the 4th story of damaged structure is 100% while the PDEs of intact stories are small.

As seen from Figs. 1(a) and 1(b), the stiffness values corresponding to the peak values of the PDFs are almost at the same position for intact structural elements while stiffness parameters corresponding to the peak values of the PDFs after damage are apparently smaller than those before damage occurrence.

Therefore, the identification results by the proposed approach are quite accurate even with a high measurement noise level of 15% in rms.



(a)



(b)

Fig. 1 (a) The PDF of α_2 with 15% noise and (b) The PDF of α_4 with 15% noise

2.3.2 Damage detection of the ten-story shear frame with uncertainties in both floor mass and ground excitation

The numerical example is similar to the above one except that two independent uncertain variables of the parameter ω_g in Kanai-Tajimi ground excitation and the floor mass are considered simultaneously. The two uncertainties are assumed as independent Gaussian distribution with X_{ω_g} ~ N(1,0.02) and X_m ~ N(0,0.02), respectively. From Eq. (9), the probability distribution of damage vector ***a*** can be estimated based on the linear relationship with respect to the above two independent uncertainties of floor mass parameter (ω_g) in Kanai-Tajimi round excitation with Gaussian distribution functions.

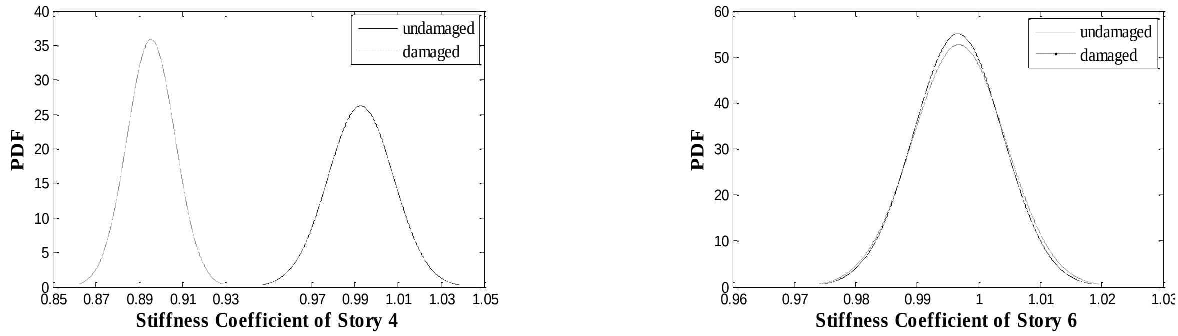
Similar identification results of identification are shown in Table 3 and in Figs. 2(a) and 2(b). From the Table and Figures, it is noted that the identification results by the proposed approach are accurate even with a high noise pollution level of 15% in rms

Table1 Probabilistic damage detection of a shear building under ground excitation with uncertain magnitude

Story No.	DE	5% noise				15% noise			
		α_u	α_d	DE	PDE	α_u	α_d	DE	PDE
1	0	1.00	1.00	0.02%	2.07%	0.99	0.99	0.22%	1.94%
2	0	1.00	1.00	0.00%	4.69%	0.99	0.99	0.03%	3.88%
3	0	1.00	1.00	0.05%	1.36%	0.98	0.99	0.48%	1.28%
4	**-10%**	1.00	**0.90**	**9.94%**	**100.00%**	**0.98**	**0.89**	**9.36%**	**100.00%**
5	0	1.00	1.00	0.03%	2.09%	0.99	0.99	0.41%	0.18%
6	0	1.00	1.00	0.03%	2.12%	0.99	0.99	0.32%	1.23%
7	0	1.00	1.00	0.02%	2.58%	0.99	0.99	0.19%	2.61%
8	0	1.00	1.00	0.01%	3.59%	0.99	0.99	0.12%	1.70%
9	0	1.00	1.00	0.03%	1.61%	0.99	0.99	0.38%	0.22%
10	0	1.00	1.00	0.04%	1.38%	0.99	0.99	0.44%	0.49%

Table 2 Probabilistic damage detection of the shear building with uncertainties in floor mass and ground excitation

Story No.	α_u	α_d	DE	DE	PDE
1	0.99	1.00	0	0.10%	0.00%
2	0.99	0.99	0	0.00%	0.00%
3	0.99	0.99	0	0.22%	0.53%
4	**0.99**	**0.89**	**-10%**	**-9.74%**	**100.00%**
5	0.99	1.00	0	0.14%	0.00%
6	0.99	0.99	0	0.14%	1.19%
7	0.99	1.00	0	0.09%	0.00%
8	1.00	1.00	0	0.04%	0.00%
9	0.99	1.00	0	0.15%	2.39%
10	0.99	0.99	0	0.18%	5.98%



(a) (b)

Fig. 2 (a) The PDF of α_4 with 15% noise and (b) The PDF of α_6 with 15% noise

2.3.3 Damage detection of a continuous beam with uncertain mass density

As an illustration for the identification of other type structure by the proposed approach, a three-span continuous beam shown in Fig. 3 is selected as another numerical example. The continuous beam with 1.5 m long is divided into 12 uniform finite elements with 22 degrees of freedom.

The parameters of each element are selected as: cross section $A = b \times h = 7 \times 10^{-2}\,\mathrm{m}^2$ and bending moment $EI = 2.4 \times 10^6\,\mathrm{N \cdot m^2}$. Then, the equivalent stiffness parameter of the i-th element is $k_i = E_i I_i / l_i = 2.22 \times 10^8\,\mathrm{N/m}$. The damping ratio is 3% and Rayleigh damping coefficients are $a = 0.0705$, $b = 0.0127$.

A white noise excitation is applied at a node in the beam as shown in Fig. 3

In this example, the mass per unit length is considered as uncertain parameter with nominal value of 196 kg/m and a logarithmic Gaussian distribution with mean value m=1 and standard variance υ=0.02 in Eq. (13). All nodal vertical acceleration responses and three angle acceleration responses as shown in Fig. 3 are measured. Also, 5% and 15% noise in rms are considered, respectively.

Structural damage is assumed as 10% reduction of the 2nd element equivalent stiffness parameter. The results of identification are shown in Table 3 and Figs. 4(a) and 4(b). Again, the identification results by the proposed approach are quite accurate.

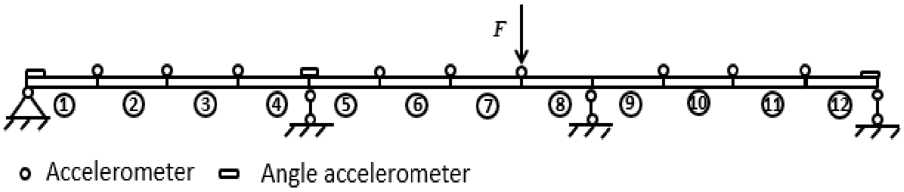
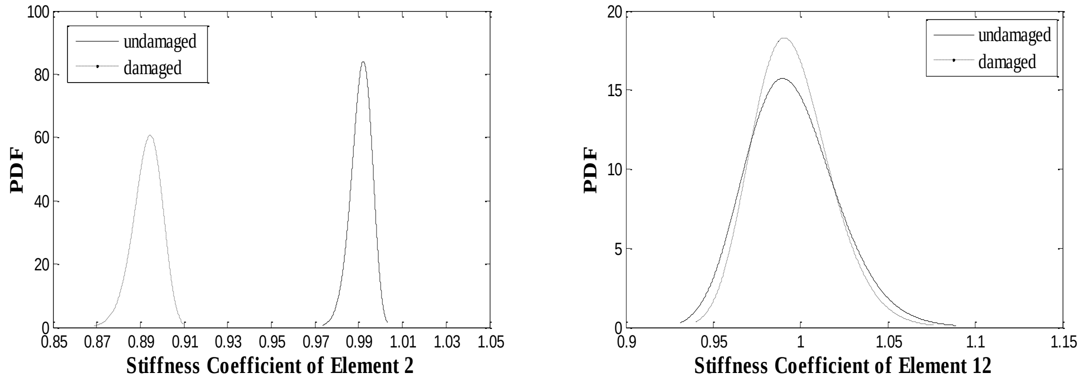


Fig. 3 A three-span continuous beam with uncertain mass density

Table 3 Probabilistic damage detection of a continuous beam with uncertain mass density

Element No.	DE	5% noise				15% noise			
		α_u	α_d	DE	PDE	α_u	α_d	DE	PDE
1	0	1.00	1.00	0.04%	5.33%	1.03	1.04	0.32%	7.37%
2	**-10%**	**1.00**	**0.90**	**9.96%**	**100.00%**	**0.99**	**0.89**	**9.89%**	**100.00%**
3	0	1.00	1.00	0.00%	5.30%	1.00	1.00	-0.01%	1.96%
4	0	1.00	1.00	0.07%	5.56%	1.00	1.00	0.22%	0.06%
5	0	0.99	0.99	-0.03%	3.90%	0.93	0.92	-0.53%	1.84%
6	0	0.99	0.99	-0.01%	8.65%	0.98	0.98	-0.04%	0.00%
7	0	1.00	1.00	0.01%	7.01%	0.98	0.98	0.02%	0.10%
8	0	0.99	0.99	-0.02%	12.66%	0.96	0.96	-0.07%	0.00%
9	0	1.00	1.00	0.04%	2.73%	0.95	0.95	0.20%	1.48%
10	0	1.00	1.00	0.02%	9.56%	1.01	1.01	0.03%	0.41%
11	0	1.00	1.00	-0.02%	2.20%	1.00	1.00	-0.08%	0.00%
12	0	1.00	1.00	0.01%	4.05%	1.00	1.00	0.01%	2.42%



(a) (b)

Fig. 4 (a) The PDF of α_2 with 15% noise and (b) The PDF of α_{12} with 15% noise

3. Probabilistic structural damage detection by integrating TSTMBDD and sensitivity-based analysis

The prominent advantage of the probabilistic damage identification methods based on statistical moment-based damage detection lies in its insensitive to measurement noise but sensitive to structural damage. However, to avoid ill-conditioned nonlinear optimization problem for structural identification, the number of measured responses is requested to be not less than that of unknown structural parameters. Herein, another probabilistic structural damage detection approach is proposed based on the integration of structural damage detection using time segment temporal moment-based structural damage detection (TSTMBDD) with the sensitivity analysis of the damage vector to the uncertain parameters.

3.1 Deterministic structural damage based on time segment temporal moment of measured response time histories

The time history of the *j*-th response of the structure calculated from Eq. (1) can be denoted as $\ddot{x}_j = \left\{\ddot{x}_{j,1}, \ddot{x}_{j,2}, \cdots, \ddot{x}_{j,N}\right\}$ where N is the total number of sampling points. Then, $\ddot{x}_j$ can be split into s time segment as

$$\ddot{x}_j = \left\{\ddot{x}_{j,1}, \ddot{x}_{j,2}, \cdots, \ddot{x}_{j,N_s}; \ddot{x}_{j,N_s+1}, \ddot{x}_{j,N_s+2}, \cdots, \ddot{x}_{j,2Ns}, \cdots, \ddot{x}_{j,(s-1)N_s+1}, \ddot{x}_{j,(s-1)N_s+2}, \cdots, \ddot{x}_{j,sN_s}\right\} \quad (29)$$

in which N_s is the number of sample points in each time segment of response.

Then, the second-order temporal moments of the *j-th* calculated acceleration response in the *p-th* time segment, denoted by $\mathbf{M}^{c}_{j,p}$, can be evaluated by

$$\mathbf{M}^{c}_{j,p}=\frac{1}{N_s}\sum_{i=pN_s+1}^{(p+1)N_s}\ddot{x}^2_{j,i}-(\frac{1}{N_s}\sum_{i=pN_s+1}^{(p+1)N_s}\ddot{x}^2_{j,i})^2\,(j=1,2,3,\cdots,s) \quad (30)$$

Analogously, the second-order temporal moments of the *j-th* measured acceleration response $\ddot{x}^2_{mj,i}$ in the *p-th* time segment denoted by $\mathbf{M}^{m}_{j,p}$, can also be estimated by

$$\mathbf{M}^{m}_{j,p}=\frac{1}{N_s}\sum_{i=pN_s+1}^{(p+1)N_s}\ddot{x}^2_{mj,i}-(\frac{1}{N_s}\sum_{i=pN_s+1}^{(p+1)N_s}\ddot{x}^2_{mj,i})^2\,(j=1,2,3,\cdots,s) \quad (31)$$

Both the calculated and measured acceleration time histories are divided into *s* time segments. When the number of partial measurements is *r*, two sets of time segment temporal moment vector can be obtained as

$$\mathbf{M}^{c}(\alpha)=[\mathbf{M}^{c}_{11}(\alpha),\mathbf{M}^{c}_{12}(\alpha),...\mathbf{M}^{c}_{1s}(\alpha),\mathbf{M}^{c}_{21}(\alpha),\mathbf{M}^{c}_{22}(\alpha),...\mathbf{M}^{c}_{2s}(\alpha),....,\mathbf{M}^{c}_{r1}(\alpha),\mathbf{M}^{c}_{r2}(\alpha),...,\mathbf{M}^{c}_{rs}(\alpha)]^{T} \quad (32a)$$

$$\mathbf{M}^{m}=[\mathbf{M}^{m}_{11},\mathbf{M}^{m}_{12},...\mathbf{M}^{m}_{1s},\mathbf{M}^{m}_{21},\mathbf{M}^{m}_{22},...\mathbf{M}^{m}_{2s},....,\mathbf{M}^{m}_{r1},\mathbf{M}^{m}_{r2},...,\mathbf{M}^{m}_{rs}]^{T} \quad (32b)$$

where α_i is the *i*-th elemental stiffness parameter ($0\le\alpha_i\le1$) in Eq. (2). $\mathbf{M}^{c}(\alpha)$ is the vector of time segment temporal moment of the calculated acceleration responses while $\mathbf{M}^{m}$ is the vector of time segment temporal moment of the measured acceleration responses.

Therefore, the dimension of the time segment temporal moment is $d\bullet s$. Under the condition that $d\bullet s\ge n$, structural element stiffness parameter α_i of reference FE model ($0\le\alpha_i\le1$) in Eq. (2) can be estimated based on the sensitivity analysis of temporal moment with respect to structural stiffness parameter analogues to Sect.2.1.

As an alternative approach, structural element stiffness parameter α_i can also be estimated in a straightforwd numerical evaluation by minimization of the following object function as

$$\mathbf{F}(\boldsymbol{\alpha})=\left\|\mathbf{M}^{m}-\mathbf{M}^{c}(\boldsymbol{\alpha})\right\| \quad (33)$$

with the boundary condition of $0\le\alpha_i\le1$

By tracking the degradation of identified structural parameters, structural element damage can be detected.

3.2 Probabilistic structural damage detection considering uncertainties

When uncertainties such as structural parameters and external excitations are considered, probabilistic approach of structural damage detection should be performed.

Analogous to Sect. 2.2, structural damage vector $\boldsymbol{\alpha}$ is expanded by the first-order Taylor series in terms of the general uncertain variable $\mathbf{X}$, as shown in Eq. (9). In which $\Delta\boldsymbol{\alpha}(\boldsymbol{X}_0)$ can be estimated by the deterministic structural damage detection based on TETMBDD and sensitivity analysis when the uncertain variable $\mathbf{X}$ in its mean value $\mathbf{X}_0$.

Also, $\left.\frac{\partial\Delta\boldsymbol{\alpha}}{\partial X_q}\right|_{X_q=X_{q0}}$ can be estimated analogous to the procedure in Sect. 2.1. Alternatively, it can also be directly evaluated by the central difference as follows

$$\left.\frac{\partial\Delta\boldsymbol{\alpha}}{\partial X_q}\right|_{X_q=X_{q0}}=\frac{\Delta a(X_{q_0}+\delta_q)-\Delta a(X_{q_0}-\delta_q)}{2\delta_q} \quad (34)$$

where δ_q denotes the minor deviation of X_{q_0} , $\Delta\boldsymbol{\alpha}(X_{q_0}+\delta_q)$ and $\Delta a(X_{q_0}-\delta_q)$ are estimated by the deterministic TETMBDD.

Then, the probability distribution function of structural element stiffness parameters can be estimated according to the linear relation with the uncertainties in structural modeling parameters and external excitations.

Finally, the structural probability of damage existence (PDE) and structural element damage extent can be evaluated analogously as those in Sect.2.2.3.

3.3 Numerical examples of probabilistic structural damage detection by integrating TETMBDD and sensitivity-based analysis

In this paper, two numerical examples are use to demonstrate the proposed probabilistic structural damage detection approach by integrating TETMBDD and sensitivity-based analysis.

3.3.1 Damage detection of a continuous beam subject to uncertain excitation

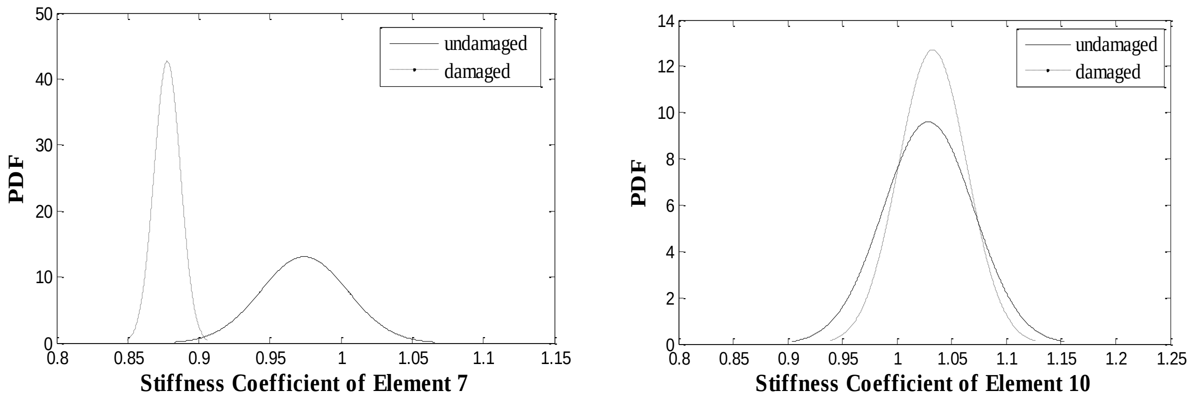
A two span continuous beam is selected as an example to demonstrate the proposed approach. The beam is divided into 10 elements with 16 DOFs. Structural parameters of the beam as: l_i=1.0 m, uniformly distributed mass is 785 kg/m; the equivalent stiffness is $k_i=E_iI_i/l_i=1.11\times10^5$ N/m .

Rayleigh damping is adopted with the first two damping ratios equal to 3% and the two damping coefficients a=0.0705, b=0.0127. A white noise excitation in applied to the 4th node. Only seven acceleration responses in the vertical directions of nodes 2-5, nodes 7-8 and node 10 are used in damage detection by TETMBDD. All the measurements are polluted by white noises with 10% rms. In the TETMBDD, each time history of acceleration response is divided into 100 time segment. Structural damage is assumed as the 10% reduction of k_7.In this example, the magnitude of the white noise excitation is regarded as uncertain variable which is assumed as Gaussian distribution with nominal value S_0 = 15 m^2/rads^3 and $X_f\sim N(1,0.02)$.

In Table 4, the identification results of probability of damage existence (PDE) and damage extent (DE) are presented. It is noted that the PDE at the 8th bar element is 100% while the PDEs of other intact elements remain small values.

Table 4 Probabilistic damage detection of a continuous beam under uncertain excitation

Element No.	α_u	α_d	DE	DE	PDE
1	1.02	1.02	0	0.35%	0.00%
2	1.00	1.00	0	0.01%	19.12%
3	0.97	0.97	0	0.23%	1.11%
4	0.98	0.97	0	-0.30%	10.00%
5	1.02	1.01	0	-0.67%	0.00%
6	1.03	1.04	0	0.64%	0.00%
7	**0.97**	**0.88**	**-10%**	**-9.90%**	**100.00%**
8	0.97	0.96	0%	-0.07%	0.00%
9	1.00	0.99	0	-0.33%	19.19%
10	1.03	1.03	0	0.37%	0.00%



(a) (b)

Fig. 5 (a) The PDF of α_7 with 10% noise and (b)The PDF of α_{10} with 10% noise

In Figs. 5(a) and 5(b), the PDFs of stiffness parameters α_7 and α_{10} in damaged and undamaged structure are illustrated.

Therefore, the identification results by the proposed probabilistic structural damage detection approaches with integrating TSTMBDD and sensitivity-based analysis are quite accurate with a quite high measurement noise level of 10% in rms.

3.3.2 Damage detection of a truss with uncertainties of mass density and excitation magnitude

As shown in Fig. 6, a plane truss consisting of 11 bars with the same cross section area $A = 7.854\times10^{-5}\mathrm{m}^2$, Young's modulus $E = 2\times10^{8}\,\mathrm{N/m}^2$ is subject to two external white noise excitations in the vertical directions at node 3 and node 4, respectively.

Structural global stiffness matrix **K** can be formulated as the summation of each element stiffness matrices, in which $k_i = EA/l_i$ is defined as the equivalent stiffness parameter of the *i*-th truss element. The mass is concentrated on each node. The damping of the truss is assumed as viscous damping.

In this numerical example, uncertainties of bar mass density and the magnitudes of the two white noises excitation are considered. The nominal value of the mass of each horizontal bar is 1.223 kg and that of the inclined bar is 0.872 kg with $X_m \sim N(0,0.02)$. The two white noise excitations are in the same variations with nominal values of power density $S_0 = 0.2\,\mathrm{m}^2/\mathrm{rads}^3$ and $X_f \sim N(1,0.02)$.

Structural damage is assumed as the deduction of the 8-th element

Only five acceleration responses indicated in Fig. 5 are used as the partial measurements for damage detection. The measured acceleration responses are polluted by white noises with 10% rms. In the TETMBDD, each time history of acceleration response is divided into 20 time segment.

In Table 5, the identification results of probability of damage existence (PDE) and damage extent (DE) are presented.

Figs. 7(a) and 7(b) illustrated the PDFs of stiffness parameters α_4 and α_8 in damaged and undamaged structure are illustrated.

Table 5 The results of the truss damage identification with 10% noise

Element No.	α_u	α_d	DE	DE	PDE
1	1.00	1.00	0	0.05%	0.44%
2	1.00	1.00	0	-0.27%	1.49%
3	0.99	0.99	0	0.15%	0.00%
4	0.97	0.97	0	0.04%	0.00%
5	1.01	1.01	0	-0.17%	0.01%
6	1.02	1.02	0	-0.16%	0.00%
7	1.02	1.02	0	0.13%	0.00%
8	**1.00**	**0.91**	**-10%**	**-9.11%**	**100.00%**
9	1.02	1.01	0	-0.15%	0.00%
10	1.02	1.01	0	-0.47%	0.00%
11	1.01	1.01	0	-0.39%	0.01%

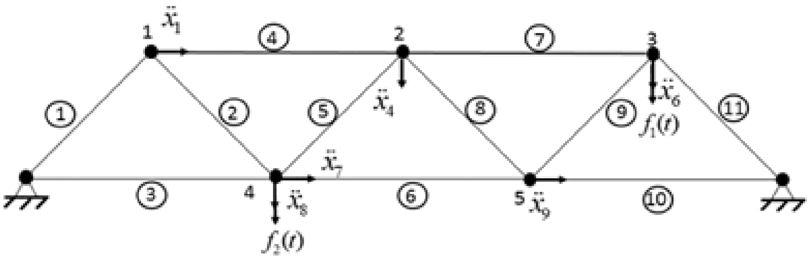
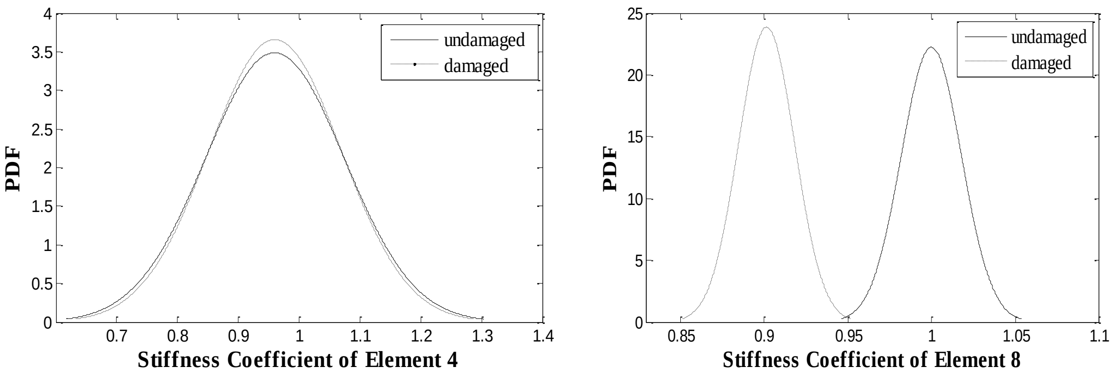


Fig. 6 A plane truss with uncertainmass density and excitation magnitudes



(a) (b)

Fig. 7 (a)The PDF of α_4 with 10% noise and (b) The PDF of α_8 with 10% noise

4. Conclusions

To reduce the influence of measurement noise on previous response-sensitivity based probabilistic structuraldamagedetection approahces, two probabilistic structural damage detection approaches are proposed in this paper. The first proposed approach adopts the statistical moment-based structural damage detection (SMBDD) algorithm together with the sensitivity analysis of the damage vector to the uncertain parameters. Some numerical examples have demonstrate that the identification results of of probability of damage existence (PDE) and damage extentby the proposed approach are quite accurate even with a high measurement noise level of 15% in rms.

The first approach requests the number of measured responses is not less than that of unknown structural parameters. To reduce the number of measurements, another probabilistic structural damage detection approach is proposed. It is based on the integration of structural damage detection using temporal moments in each time segment of measured response time history with the sensitivity analysis of the damage vector to the uncertain parameters. Some numerical results validate that probability of damage existence (PDE) and damage extent (DE) by the proposed approach are satisfactorily even though the structural responses used are incomplete and the measurement noise has a quite high noise-to-signal ratio of 10% in rms. Although the second approach is not so robust to measurement noises as the first approach, it is not sensitive to measurement noise compared with previous response-sensitivity based approaches.

In both approaches, probability distribution of damage vector is estimated from those of uncertain parameters based on stochastic finite element model updating and probabilistic propagation. By comparing the two probability distributioncharacteristics for the undamaged and damaged models, probability of damage existence and damage extentat structural element level can be detected.The proposed approaches can not only locatestructural damage but also identify damage extents without the extensivecomputational efforts, and it can also handle both Gaussian and non-Gaussian uncertain parameters.

In this paper, some numerical examples have demonstrated the effectiveness of the proposed structural dynamic response moment based algorithms. Lab experimental tests are under taken by the authors and results will be published later.

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