

Designing fuzzy systems for optimal parameters of TMDs to reduce seismic response of tall buildings

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Abstract. One of the most reliable and simplest tools for structural vibration control in civil engineering is Tuned Mass Damper, TMD. Provided that the frequency and damping parameters of these dampers are tuned appropriately, they can reduce the vibrations of the structure through their generated inertia forces, as they vibrate continuously. To achieve the optimal parameters of TMD, many different methods have been provided so far. In old approaches, some formulas have been offered based on simplifying models and their applied loadings while novel procedures need to model structures completely in order to obtain TMD parameters. In this paper, with regard to the nonlinear decision-making of fuzzy systems and their enough ability to cope with different unreliability, a method is proposed. Furthermore, by taking advantage of both old and new methods a fuzzy system is designed to be operational and reduce uncertainties related to models and applied loads. To design fuzzy system, it is required to gain data on structures and optimum parameters of TMDs corresponding to these structures. This information is obtained through modeling MDOF systems with various numbers of stories subjected to far and near field earthquakes. The design of the fuzzy systems is performed by three methods: look-up table, the data space grid-partitioning, and clustering. After that, rule weights of Mamdani fuzzy system using the look-up table are optimized through genetic algorithm and rule weights of Sugeno fuzzy system designed based on grid-partitioning methods and clustering data are optimized through ANFIS (Adaptive Neuro-Fuzzy Inference System). By comparing these methods, it is observed that the fuzzy system technique based on data clustering has an efficient function to predict the optimal parameters of TMDs. In this method, average of errors in estimating frequency and damping ratio is close to zero. Also, standard deviation of frequency errors and damping ratio errors decrease by 78% and 4.1% respectively in comparison with the look-up table method. While, this reductions compared to the grid partitioning method are 2.2% and 1.8% respectively. In this research, TMD parameters are estimated for a 15-degree of freedom structure based on designed fuzzy system and are compared to parameters obtained from the genetic algorithm and empirical relations. The progress up to 1.9% and 2% under far-field earthquakes and 0.4% and 2.2% under near-field earthquakes is obtained in decreasing respectively roof maximum displacement and its RMS ratio through fuzzy system method compared to those obtained by empirical relations.

Keywords: Tuned Mass Damper; optimal parameters; frequency and damping; fuzzy systems

1. Introduction

In recent years, many researchers have paid attention to reducing vibrations of civil structures subjected to excitations of natural phenomena such as wind and earthquake. For this purpose, various control systems such as passive, semi- active and active control tools have been developed. Among these techniques, Tuned Mass Damper (TMD) is the most reliable and simplest one. The components of this damper include a mass, a spring, and a viscous damper installed to the structure. Although it is

common for vibrations control of high-rise buildings to focus on the smart material and viscous dampers (Aly *et al.* 2012, Aly *et al.* 2011, Taylor 2010), however, in some buildings, using these tools is faced with a challenge. Nevertheless, for many structures without enough space to install exterior bracing, TMDs can be used efficiently without any significant changes.

In 1909, Frahm (1911) invented a tool to reduce the vibrations due to the volatility of ships. The basis of TMD is composed of this invention operation concept. This tool is just effective while the natural frequency of the absorbers is very close to the vibration frequency of structures, because this tool has no inherent damping. Ormondroyd and Den Hartog (1928) tried to find useful results under the various excitations with different frequencies by connecting the viscous damping to a specified TMD. Finding TMD optimal parameters to best reduce vibrations is one of the most severe problems designers encounter. For this purpose, in 1940 Den Hartog (1947) proposed a close form of optimal parameters including frequency and damping ratio of

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TMDs. However, these relations were stated only based on Single-Degree of Freedom (SDOF) systems without damping. Afterward, original system's damping was considered by researchers (Bishop and Welbourn 1952, Snowdon 1959, Falcon *et al.* 1967, Ioi and Ikeda 1978). Warburton and Ayorinde (1980) represented that for complicated systems the achievement of TMD optimal parameters is possibly faced with some problems as they assume equivalent to SDOF systems. Warburton (1982) offered simple equations for these optimal parameters based on undamped SDOF system subjected to harmonic excitations and random white-noise.

Rana and Soong (1998) designed an optimal TMD to control a specific mode of structures through a numerical approach. To use a damper, there are some inherent limits such as narrow frequency bandwidth and sensitivity of tuning TMD frequency to frequency of the structure and also the difference between the TMD damping and its optimal value. Either incorrect tuning for damping ratio or using a non-optimal one can extremely decrease TMD efficiency (Bakre and Jangid 2004, Hoang and Warnitchai 2005, Lee *et al.* 2006, Li and Qu 2006). Chang (1999) achieved a closed form solution for optimal parameters of TMD corresponding to both wind and earthquake loads. Using numerical search method, Bakre and Jangid (2007) proposed the explicit mathematical relations through the curve-fitting method for TMD optimal parameters. After presenting the meta-heuristic approaches such as genetic algorithm, particle swarm, ant colony algorithm, and harmonic search, these algorithms have been used to solve the optimization issues. The Genetic algorithm has been widely utilized to tune damper parameters (Hadi and Arfiadi 1998, Singh *et al.* 2002, Desu *et al.* 2006, Pourzeynali *et al.* 2007). Leung and Zhang (2009) optimized the mass, damping, and frequency ratio of TMDs installed to the damped SDOF structures by studying the particle swarm algorithm. Bekdas and Nigdeli (2011) employed the harmonic research algorithm to find TMD optimal parameters under the seismic vibrations.

Tuned Liquid Damper (TLD) is a mechanical passive damper designed to reduce random vibration of structures caused by fluid turbulence in a rigid tank. Vibrational energy is dissipated as friction through the liquid boundary layers, the liquid free-surface participation and wave reflections. In these studies, the effect of seismic loading parameters on TLD operation has been investigated in different conditions (Kavand and Zahrai 2006, Zahrai and Kavand 2008).

Mousaad Aly (2014) studied the probable challenges in designing TMD, for instance, ambient temperature and the relative changes of moisture among the other factors. He proposed a method in his study to design TMDs by considering the fundamental unreliability, the optimization goals and the division of input excitations (wind or earthquake).

Numerous experimental and mathematical relations are proposed by different researchers to determine the optimal parameters of TMD. These relations have been mainly extracted through studying the SDOF systems under harmonic excitation. High-rise structure simplification

using SDOF systems and complex loads such as wind or earthquake approximation cause many uncertainties in these relations. On the other hand, in some research, various methods have been presented to recognize these uncertainties to find TMD optimal parameters. However, to apply these methods, the model of the basic structure is required. Furthermore, assigning these obtained optimal parameters to the other structures is impossible. In this study, to consider mentioned uncertainties caused by modeling structures as a SDOF system, a set of MDOF systems with different numbers of stories has been used to design the fuzzy system. It should be mentioned that a set of far and near field earthquakes has been used as base excitations. Therefore, by prediction of these uncertainties and approximation of real structural behavior, the defects observed in relations presented by previous researchers are compensated. In addition, the design of fuzzy systems in order to estimate the TMD optimal parameters allows scholars to employ this proposed system in short time without any information about data used due to the design process. Moreover, this system can be applied to all kind of buildings with various frequencies. Thus, through eliminating inaccuracies of simple relations assumed by previous researchers, the efficient operation of this fuzzy system for all users will be demonstrated.

2. The equations of motion for a MDOF system with TMD

In Fig. 1, the MDOF system equipped with energy dissipation of TMD has been shown. The equation of motion for a linear MDOF system subjected to the external load $\mathbf{P}(t)$ is

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{P}(t) \quad (1)$$

Where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ are the mass, damping and stiffness matrices of structure, respectively and $\mathbf{x}(t)$ is the horizontal displacement vector relative to the ground. Assuming ground acceleration as the applied load leads to

$$\mathbf{P}(t) = -\mathbf{M}\{\mathbf{1}\}\ddot{x}_g(t) \quad (2)$$

Assuming $\mathbf{P}(t)$, from Eq. (2) into Eq. (1), one obtains

$$\ddot{\mathbf{x}}(t) = -\mathbf{M}^{-1} \{ \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) + \mathbf{M}\{\mathbf{1}\}\ddot{x}_g(t) \} \quad (3)$$

Therefore, the acceleration is measured in TMD level (usually, the highest elevation of structure). Then, TMD is analyzed as a SDOF system and the absolute acceleration equals to the summation of the earthquake acceleration and acceleration at TMD level. Thus, TMD force can be obtained by multiplying the TMD mass by absolute acceleration and can be applied to structure at TMD level.

To verify the conducted dynamic analysis by Simulink, a three degree of freedom system is modeled in SAP2000 software and the obtained results are compared to each other. In Fig. 3 this MDOF model in SAP2000 software is shown. It should be noted that, the spring stiffness, the mass related to each degree of freedom and damping ratio are

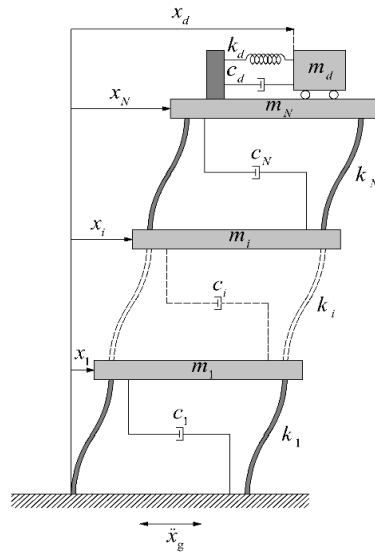


Fig. 1 Schematic model of multi-story building equipped with a TMD

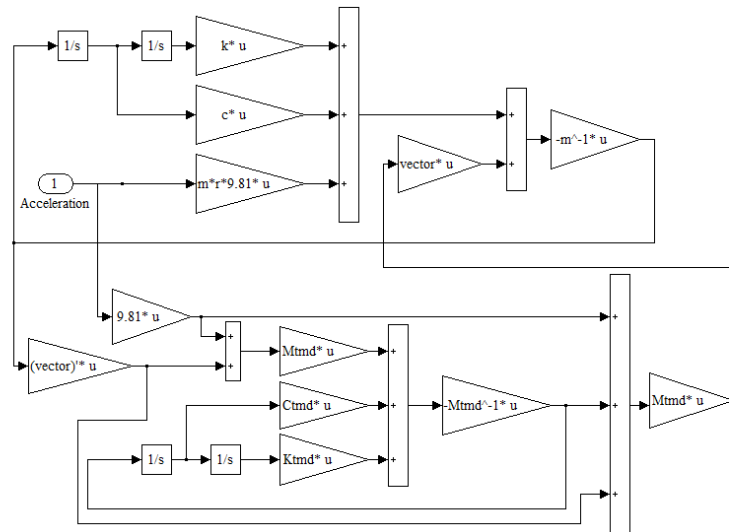


Fig. 2 The model designed in Simulink for a structure with n degrees of freedom with TMD



Fig. 3 MDOF system modeled in SAP2000 software

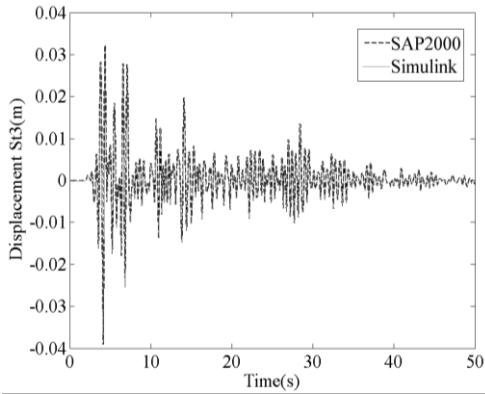


Fig. 4 Time history response for roof displacements obtained by SAP2000 and Simulink software

selected equally as $1 \times 10^8 \text{ N/m}$, $1 \times 10^5 \text{ kg}$ and 0.05 respectively. The frequency, mass and damping ratios of TMD are assumed 0.95, 0.05 and 0.07 respectively. In Fig. 4, roof displacement of structure modeled in SAP2000 and Simulink software under the El Centro earthquake is observed.

3. Fuzzy algorithm

Recently, using fuzzy logic has risen in industrial sector and scientific areas. It can be worked in two different meanings. From the detailed point of view, the fuzzy logic is a logical system generated by the development of a multi-valued logic. However, in a generalized view, the fuzzy logic can be considered equivalent to "the theory of fuzzy sets". The theory of fuzzy sets analyzes an object series in which there is no actual bound. So the object membership issue in a set is not accurate and is determined using the membership functions. In other words, an object can be a member of a set in different orders in this theory. The order of the object membership in a set varies from 0 to 1. In a considered set, zero order shows the non-membership of objects and one order indicates their perfect membership.

One of the great advantages of fuzzy logic usage is the possibility of mapping input data sets to output one. In other words, a fuzzy inference system (FIS) can be created by introducing special rules called fuzzy rules in which the system can determine output by applying input sets.

In this study, taking advantages of three important features of fuzzy logic adjusting to a popular estimate functions, the modelling of the fuzzy system has been established in order to predict TMD optimal parameters. These features include:

- The description of the uncertainties.
- New tool to solve those problems that the probability theory doesn't have any solution for it.
- The fuzzy logic can model any complex non-linear function.
- In fuzzy logic, there is the possibility of using the experts' practices.

In this study, instead of experts' experience, the data sets obtained by modelling different MDOF structures and

gaining TMD optimal parameters, have been used.

3.1 Internal and external data of the fuzzy system

The structures simulated through MATLAB software are laterally subjected to the earthquakes. For this purpose, two far-field and two near-field earthquake records suggested by the International Association for Structural Control can be used. These earthquakes include the 1940 El Centro, 1968 Hachinohe, 1994 Northridge and 1995 Kobe. To increase the number of data for reducing the uncertainties related to loading, four other earthquakes have been used in addition to these records.

To model the fuzzy system, 644 data are used. Required information to design fuzzy system includes optimal parameters of TMD for buildings from 8 to 80 stories subjected to different earthquakes.

3.2 Fuzzy system design using Look-up table optimized by genetic algorithm

The experts in control vibration are among the most reliable sources in the major of fuzzy system design. In this study, to achieve the desired accuracy, a data set is used to estimate the optimal parameters of TMD.

As shown in Fig. 5, a Simulink model like a black box is accessible, as the optimal parameters of TMD are identified for a MDOF system using the trial and error method. However, its contents, i.e. how this parameter for a MDOF structure varies under any special base excitation, is not clear.

In MATLAB software, the specified function is not considered to design the fuzzy system by look-up table method. For this purpose, the design of the fuzzy system is implemented based on the look-up table according to following steps:

1. Classification and identification of fuzzy steps for inputs and outputs.
2. Formation of the possible rules
3. Calculating the score of each rule
4. Removing the opposed and weaker rules

Considering the characteristics of the problem, data are $(\mathbf{X}_p, \mathbf{Y}_p)$ where, $\mathbf{X}_p$ is the input vector and $\mathbf{Y}_p$ is the output vector, $p = 1, 2, \dots, P$ is the number of data or existent pattern to design the fuzzy system.

Parameters used as input and output are

$$\mathbf{X}_p = [\omega_s \quad \bar{m}] \quad (4)$$

$$\mathbf{Y}_p = [\omega_d \quad \xi_d] \quad (5)$$

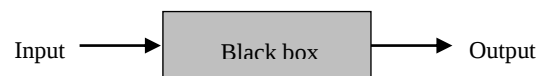


Fig. 5 The system before designing the fuzzy system

where, ω_s is the first mode frequency of MDOF system; $\bar{m}$ indicates the ratio of mass damper to total mass of the MDOF system; ω_d is the TMD frequency, and ξ_d is the TMD damping ratio.

Since the basis of the fuzzy function estimation system is a multi-input-single-output system, a single-output fuzzy system is designed without changing the whole problem conditions for each output. In the fuzzy system of each existent data, a rule is identified. To classify the output and input space as a fuzzy system

$$\begin{cases} x_i^{\min} = \min_p x_{ip} \\ x_i^{\max} = \max_p x_{ip} \end{cases} \rightarrow x_{ip} \in [x_i^{\min}, x_i^{\max}] \quad (6)$$

$$\begin{cases} y^{\min} = \min_p y_p \\ y^{\max} = \max_p y_p \end{cases} \rightarrow y_p \in [y^{\min}, y^{\max}] \quad (7)$$

Where, $i = 1, 2, \dots, n$ is the number of input variables. Then, the changing range of each input is specified. The input and output spaces of the problem are given as follows

$$X = [x_1^{\min}, x_1^{\max}] \times [x_2^{\min}, x_2^{\max}] \times \dots \times [x_n^{\min}, x_n^{\max}] \quad (8)$$

$$Y = [y^{\min}, y^{\max}] \quad (9)$$

Indeed, the objective of the problem is to design a fuzzy system implementing the following mapping

$$f : X \rightarrow Y \quad (10)$$

To allocate the input space to fuzzy sets, it should be noted that a positive membership function is defined for each input

$$\forall x_i \in [x_i^{\min}, x_i^{\max}] \Rightarrow \exists j : \mu_{ij} > 0 \quad (11)$$

Where, μ is the membership order, i is the input variable number, and j is the membership function number.

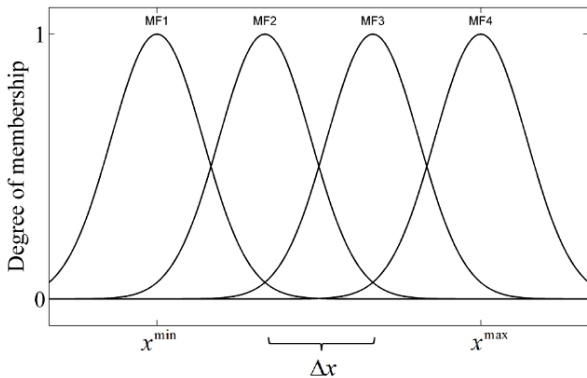


Fig. 6 The uniform distribution of membership functions

In fuzzy system design process using look-up method, because of its non-smart and inadequate information about the distribution of data, the membership functions are mostly uniform distribution functions as shown in Fig. 6. In this case, by assuming Gaussian membership functions, the distance between two consecutive centers of membership functions is obtained

$$\Delta x_i = \frac{x_i^{\max} - x_i^{\min}}{n-1} \quad (12)$$

Where, Δx is the distance between centers of two consecutive membership functions and n is the number of membership functions.

By dividing the input and output data, the possibility of performing their rules is provided. The total number of rules which can be created for a fuzzy system is equal to all the possible combinations of inputs and outputs

$$NR = \underbrace{m_1 \times m_2 \times \dots \times m_n}_{\text{inputs}} \times \underbrace{m_y}_{\text{output}} \quad (13)$$

Where, m is the number of membership functions defined for each input and output variable.

Now, according to the data or patterns prepared for input and output, each of these rules is strengthened.

Eventually, every rule that is further strengthened will remain in the fuzzy system. According to this definition, the accuracy of any rule related to each data should be investigated. In this study, the Mamdani system is used to achieve this goal. For each rule, such as

$$Ru : IF x_1 \text{ is } A_1 \text{ and } x_2 \text{ is } A_2 \text{ THEN } y \text{ is } B \quad (14)$$

Where, A_1 and A_2 are the fuzzy sets related to the input variable and B is the fuzzy set related to the output one. The proposition before "THEN" is called the Antecedent section and the proposition after "THEN" is called the consequent section.

According to Mamdani implication, this rule for input and output is equal to

$$S_r(\mathbf{x}_p, \mathbf{y}_p) = \mu_{A_1}(x_{1p}) \times \mu_{A_2}(x_{2p}) \times \mu_B(y_p) \quad (15)$$

Where, S_r is the degree of rule accuracy (r). That $r = 1, 2, \dots, NR$ and μ is the data membership order in the fuzzy set.

By assigning all data to available rules, the final score of each one can be reproduced by two different methods

$$\begin{cases} S_r^{\text{Total}} = \sum_{p=1}^P S_r(\mathbf{x}_p, \mathbf{y}_p) \\ S_r^{\text{Total}} = \max_p S_r(\mathbf{x}_p, \mathbf{y}_p) \end{cases} \quad (16)$$

Since all input and output combinations are considered as the possible rules, there are some rules that have the same antecedent section, but the consequent section is different. Indeed, these rules are opposed to each other; therefore, that rules having lower score, are removed.

Therefore, the initial fuzzy system is created by the look-up table method. Since the designed fuzzy system is

kind of Mamdani, in MATLAB environments there is no function to optimize the Mamdani fuzzy system. Then, through an innovative method using genetic algorithm, the weights of rules are optimized. In Fig. 7, the reduction process of MSE related to frequency data by the genetic algorithm is shown.

To achieve better evaluation among design methods of the fuzzy system, here a statistical comparison of TMD optimal parameters identified by Simulink analysis and optimal results proposed by the fuzzy system is drawn.

Table 1 presents the information related to statistical investigation of the results of the fuzzy system designed by look-up table method.

Where, ω_d and ξ_d are the frequency and the optimal damping ratio of TMD respectively and $\bar{\mu}$ is the average difference between actual optimal parameters and fuzzy system results. The difference between both quantities is due to the estimation of considered functions; σ is the standard deviation of errors, MSE is the Mean Square of Errors, $RMSE$ is the Root Mean Square of Errors, and R is the correlation coefficient between actual optimal parameters and the fuzzy system results. It is clear that as much as this amount is closer to 1, it indicates more accurate results of the fuzzy system design.

3.3 Fuzzy system design by grid partitioning the data space optimized by ANFIS

In this method, a fuzzy system is designed by simulating the models according to the actual behavior of real structures. As shown in Fig. 8, the output obtained from fuzzy system or any other model, is always different from the real system.

To minimize the errors of these both system outputs, the fuzzy system parameters are adjusted in which the cost function is minimized. So, to design and optimize the fuzzy system using existent data, the ANFIS (Adaptive Neuro-Fuzzy inference system) toolbox in MATLAB software is designed.

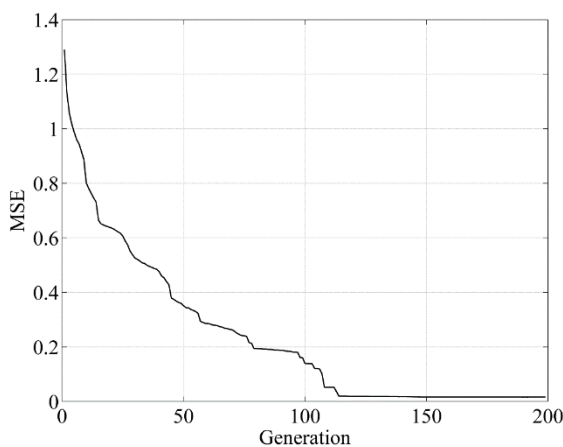


Fig. 7 The process of minimizing the MSE by genetic algorithm

Table 1 Statistical parameters based on look-up table method

	$\omega_d(\text{rad / s})$	$\xi_d(\%)$
$\bar{\mu}$	-0.0859	0.0010
σ	0.5321	0.0405
MSE	0.2901	0.0016
RMSE	0.5386	0.0405
R	0.9254	0.6290

Table 2 Statistical parameters based on the method of grid partitioning of data space

	$\omega_d(\text{rad / s})$	$\xi_d(\%)$
$\bar{\mu}$	0.0000	0.0000
σ	0.3059	0.0394
MSE	0.0934	0.0016
RMSE	0.3057	0.0394
R	0.9751	0.6535

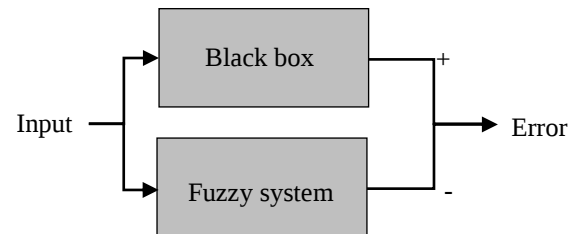


Fig. 8 Designing the fuzzy system with minimizing the errors

In this process, a crude form of a fuzzy algorithm is designed as Takagi Sugeno (TSK). Then, through existent data, with the aim of minimizing the cost function, their parameters are optimized while the weight of produced rules is the same. The contrast between TSK system and Mamdani system presented in look-up table method, is that in Mamdani system the antecedent and consequence sections of rules are considered as the fuzzy propositions, however the consequence section in TSK system is expressed as the function of the inputs. The information related to statistical investigation of the fuzzy system results designed by the grid partitioning method is presented in Table 2.

3.4 Designing fuzzy system based on clustering optimized using the ANFIS

In designing the fuzzy system based on clustering, the available data are classified. Then, due to similarity of each group members, it is tried to offer an approximately identical rule for that group.

One of the advantages of using clustering methods is the increase and reduction of complexity, in case of necessity. For instance, in areas of high population density of data, the density of membership function and rules has risen. In contrast, if there is some disperse data in data sets, only one membership function can be satisfied in that area.

Table 3 Statistical parameters based on clustering method

	$\omega_d(\text{rad} / \text{s})$	$\xi_d(\%)$
$\bar{\mu}$	0.0000	0.0000
σ	0.2992	0.0387
MSE	0.0894	0.0015
RMSE	0.2989	0.0387
R	0.9762	0.6683

Table 4 Required parameters for designing the fuzzy system to estimate the TMD damping

Input 1 $\bar{m}$			Input 2 ω_s			Output 1 ξ_d			
MFs	$\bar{\sigma}$	$\bar{c}$	MFs	$\bar{\sigma}$	$\bar{c}$	MFs	Linear Parameters		
m1	0.0067	0.0610	w1	0.4034	3.0515	xsi1	-9.7083	-0.0220	0.8183
m2	0.0373	0.0511	w2	0.3879	1.2499	xsi2	2.2959	-0.4965	0.5502
m3	0.0133	0.0758	w3	0.3636	1.4948	xsi3	-0.9081	0.0597	0.1501
m4	0.0393	0.0314	w4	0.3611	2.6071	xsi4	1.2101	-0.0332	0.1908
m5	0.0343	0.0421	w5	0.3631	1.8063	xsi5	2.2214	-0.4142	0.9263
m6	0.0073	0.0413	w6	0.6665	4.7958	xsi6	4.4765	-0.3398	1.8666
m7	-0.0002	0.0455	w7	0.3398	2.1880	xsi7	0.0000	0.0000	0.0000
m8	0.0314	0.0548	w8	1.1560	7.2925	xsi8	1.1154	-0.0090	0.1517
m9	0.0161	0.0543	w9	0.9294	6.2014	xsi9	1.2496	-0.0458	0.4879
m10	0.0076	0.0365	w10	0.4669	3.5648	xsi10	-	-0.1445	0.9221
m11	0.0175	0.0487	w11	0.7774	5.4176	xsi11	0.7773	-0.0657	0.3786
m12	0.0059	0.0451	w12	0.5394	4.0500	xsi12	-1.9420	-1.1008	4.5851

Table 5 Required parameters for designing the fuzzy system to estimate the TMD frequency

Input 1 $\bar{m}$			Input 2 ω_s			Output 1 ω_d			
MFs	$\bar{\sigma}$	$\bar{c}$	MFs	$\bar{\sigma}$	$\bar{c}$	MFs	Linear Parameters		
m1	0.0346	0.0551	w1	0.4332	1.2542	fr1	-8.2518	23.5355	-20.0633
m2	0.0147	0.0826	w2	0.5516	3.8524	fr2	28.7752	1.2457	-4.2178
m3	0.0244	0.0396	w3	0.4759	3.2818	fr3	11.9798	1.5566	-2.9127
m4	0.0247	0.0386	w4	0.4081	1.4320	fr4	31.8011	-10.9922	11.8030
m5	0.0204	0.0277	w5	0.3907	2.4622	fr5	20.2428	-8.1205	26.5513
m6	0.0075	0.0437	w6	0.6448	4.4289	fr6	37.8436	1.9109	-5.5324
m7	0.0131	0.0411	w7	0.9678	6.1597	fr7	-18.6847	2.1963	-6.8075
m8	0.0354	0.0459	w8	0.7984	5.2994	fr8	-10.7933	0.9855	0.1441
m9	0.0241	0.0343	w9	0.3537	2.0494	fr9	76.4566	-19.1229	36.1779
m10	0.0164	0.0447	w10	1.2038	7.2640	fr10	3.4747	0.3726	2.6418
m11	0.0466	0.0500	w11	0.4317	2.8933	fr11	-25.3585	0.8095	2.3080
m12	0.0329	0.0539	w12	0.3840	1.7356	fr12	-49.1084	15.2672	-31.6863

In every clustering problem, for $x_1, x_2, \dots, x_i, \dots, x_n \in \mathbb{R}^d$ data, that are the members of $A_1, A_2, \dots, A_j, \dots, A_k$ clustering sets, the x_i is a member of A_j if and only if c_j is the closest cluster center to x_i .

A data arrangement is suitable for clustering as the distance between the cluster members and cluster center is minimized. Indeed, a cluster center is proper when placed in the members' mass center.

There are two methods of subtractive clustering and clustering by fuzzy c-means in MATLAB software making it possible to use fuzzy logic. In other words, each data with any membership degree can belong to several clusters. In this study, the c-means method is used to cluster the data.

In Table 3, the results obtained by the statistical study of designed fuzzy system using the data clustering method is presented.

To compare design methods of fuzzy system and choose the best one, the Gaussian membership functions are used for inputs and outputs of Mamdani fuzzy system and linear membership functions are used for TSk fuzzy system. In all three fuzzy system design methods, the equal rules (12 rules) are established.

Tables 4 and 5 present parameters and Table 6 presents required rules to design the fuzzy system in order to estimate the TMD damping and frequency respectively. This fuzzy system is designed based on data clustering and optimizing parameters by ANFIS.

In Tables 4 and 5, the parameters $\bar{\sigma}, \bar{c}$ are related to Gaussian membership function. This membership function is defined as follows

$$f(x; \bar{\sigma}, \bar{c}) = e^{\frac{-(x-\bar{c})^2}{2\bar{\sigma}^2}} \quad (17)$$

Table 6 Required rules for designing the fuzzy system to estimate the TMD damping and frequency

If ($\bar{m}$ is m1) and (ω_s is w1) then (ξ_d is xsi1)	If ($\bar{m}$ is m1) and (ω_s is w1) then (ω_d is fr1)
If ($\bar{m}$ is m2) and (ω_s is w2) then (ξ_d is xsi2)	If ($\bar{m}$ is m2) and (ω_s is w2) then (ω_d is fr2)
If ($\bar{m}$ is m3) and (ω_s is w3) then (ξ_d is xsi3)	If ($\bar{m}$ is m3) and (ω_s is w3) then (ω_d is fr3)
If ($\bar{m}$ is m4) and (ω_s is w4) then (ξ_d is xsi4)	If ($\bar{m}$ is m4) and (ω_s is w4) then (ω_d is fr4)
If ($\bar{m}$ is m5) and (ω_s is w5) then (ξ_d is xsi5)	If ($\bar{m}$ is m5) and (ω_s is w5) then (ω_d is fr5)
If ($\bar{m}$ is m6) and (ω_s is w6) then (ξ_d is xsi6)	If ($\bar{m}$ is m6) and (ω_s is w6) then (ω_d is fr6)
If ($\bar{m}$ is m7) and (ω_s is w7) then (ξ_d is xsi7)	If ($\bar{m}$ is m7) and (ω_s is w7) then (ω_d is fr7)
If ($\bar{m}$ is m8) and (ω_s is w8) then (ξ_d is xsi8)	If ($\bar{m}$ is m8) and (ω_s is w8) then (ω_d is fr8)
If ($\bar{m}$ is m9) and (ω_s is w9) then (ξ_d is xsi9)	If ($\bar{m}$ is m9) and (ω_s is w9) then (ω_d is fr9)
If ($\bar{m}$ is m10) and (ω_s is w10) then (ξ_d is xsi10)	If ($\bar{m}$ is m10) and (ω_s is w10) then (ω_d is fr10)
If ($\bar{m}$ is m11) and (ω_s is w11) then (ξ_d is xsi11)	If ($\bar{m}$ is m11) and (ω_s is w11) then (ω_d is fr11)
If ($\bar{m}$ is m12) and (ω_s is w12) then (ξ_d is xsi12)	If ($\bar{m}$ is m12) and (ω_s is w12) then (ω_d is fr12)

Table 7 Specifications earthquake records for time history analysis

Earthquake	PGA	Station	t_d (s)	Fault Type	M_w	Site Data		R_{JB} (km)
						V_s (m/s)	NC	
Duzce, Turkey	0.81 g	Bolu	55.85	Strike-slip	7.1	326	D	12
Hector Mine	0.33 g	Hector	45.30	Strike-slip	7.1	685	C	10.4
Imperial valley	0.35 g	Delta	100.1	Strike-slip	6.5	275	D	22
Kobe, Japan	0.48 g	Nishi-Akashi	40.95	Strike-slip	6.9	609	C	7.1
Irpinia, Italy	0.32 g	Sturmo	39.34	Normal	6.9	1000	B	6.8
Cape Mendocino	0.66 g	Petrelia	35.90	Thrust	7.0	713	C	0.0
Landers	0.79 g	Lucerne	48.10	Strike-slip	7.3	685	C	2.2
Loma Prieta	0.51 g	Saratoga - Aloha	39.97	Strike-slip	6.9	371	C	7.6

M_w = Moment magnitude, NC = NEHRP Class, R_{JB} = Closest horizontal distance to rupture plane, t_d = Duration

In designing the fuzzy system using three methods as look-up table, grid partitioning the data space, and clustering, it has been found that the method based on clustering has higher accuracy in estimating the parameters. Therefore, in the following, this method is used to estimate the TMD parameters

4. Fuzzy system design for TMD optimal parameters to reduce seismic

In this research, a TMD is used to control the vibrations of a MDOF structure in which its optimal damping and frequency parameters are proposed by the fuzzy system. The intended structure has 15 stories with horizontal degrees of freedom. Whereas the destructive influences of the earthquake are due to horizontal vibrations, in this study, it is assumed that all degrees of freedom are in the horizontal direction (Guclu and Yazici 2008).

In this building, a TMD with two percent of mass ratio is used to control the dynamic vibrations. To conduct time history analysis, eight different earthquake records along East-West direction offered by 'guidelines FEMA P695' are applied to this building. Among these selected earthquakes, the Duzce Turkey, Hector Mine, Imperial valley and Kobe are far-field earthquakes and the Irpinia Italy, Cape Mendocino, Landers and Loma Prieta are near-field ones.

These records were not used in fuzzy system design process. It should be mentioned that their peak accelerations are scaled to 0.5 g. In Table 7, the related details of the earthquakes are given.

Dynamic properties of the investigated structure presented by Guclu and Yazici (2008) and used as a benchmark structure in this research, are

$$m_1 = 450000, m_2 = m_3 = \dots = m_{15} = 345600 \quad (\text{kg})$$

$$k_1 = 18050, k_2 = k_3 = \dots = k_{15} = 340400 \quad (\text{kN/m})$$

$$c_1 = 26170, c_2 = c_3 = \dots = c_{15} = 293700 \quad (\text{Ns/m})$$

To study the TMD operation, the criterion of the minimum root mean square of displacements is used:

$$C = \frac{\|X_c\|}{\|X_{uc}\|} \quad (18)$$

$$\| \cdot \| = \sqrt{\frac{1}{t_f} \int_0^{t_f} (\cdot)^2 dt} \quad (19)$$

Where, X is the horizontal displacement, c and uc indexes show the controlled state, and the uncontrolled one respectively. Then, the evaluation is performed by dividing the controlled responses to uncontrolled one by C parameter. In Table 8, the undamped frequencies related to the first three modes of the 15-degree of freedom system are given.

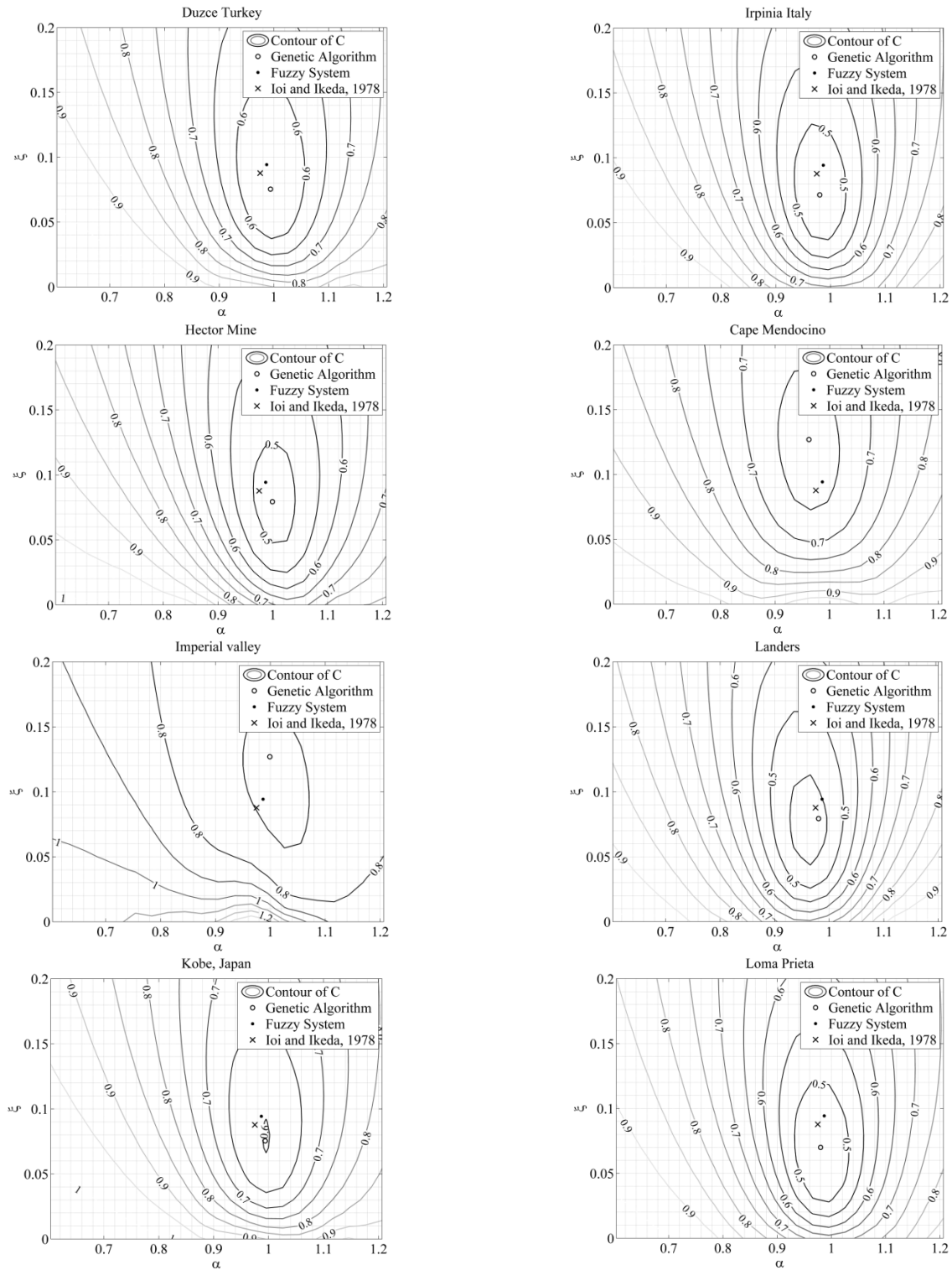


Fig. 9 Parametric study of TMD characteristics to find the optimal value

Table 8 Frequencies of the system with 15 degrees of freedom

Mode No.	1st	2nd	3rd
f (Hz)	0.26	1.10	2.07

The contours related to parametric study of TMD characteristics are illustrated in Fig. 9 where they have less width in frequency ratio (α) direction compared to those in damping (ξ) direction. The reason is insensitivity of TMDs to their damping ratio while least alteration in TMD frequency can strongly affect its performance. This is the problem discussed in optimal design of the TMDs. In Fig.

9, the optimal parameters proposed by the genetic algorithm, empirical relation (Brock 1946, Ioi and Ikeda 1978) and fuzzy system are shown.

To compare the optimal parameters, parametric study as a numerical method, genetic algorithm as a heuristic method and existent formulas are utilized. In parametric approach, frequency and damping ratio vary by 0.01 and 0.05 steps respectively, within the range of possible optimal parameters. Therefore, after 400 times analyzing the structure, these parameters are identified.

In genetic algorithm, the TMD optimal parameters are recognized due to assuming maximum generation of 50 and producing 8 chromosomes in each generation by 400 times analyzing the structure. In FIS method, TMD optimal parameters are obtained with admissible precision without any analysis. In this method, using TMD mass ratio and first mode frequency of structure, the optimal parameters are obtained.

Also in Ioi and Ikeda's relation, the optimum parameters of TMD are obtained without analysis of structure, in the same way. The optimal properties of a TMD system implemented in an undamped (SDOF) system are determined as (Brock 1946)

$$\alpha_{opt} = \frac{1}{1 + \bar{m}} \quad (20)$$

$$\xi_{opt} = \sqrt{\frac{3\bar{m}}{8(1 + \bar{m})}} \quad (21)$$

Where, α_{opt} represents the ratio of the optimal TMD frequency to the natural frequency of the SDF system; and ξ_{opt} denotes the optimal TMD damping ratio.

The empirical formulations, for a damped SDOF system, should be modified as following (Ioi and Ikeda 1978)

$$\bar{\alpha}_{opt} = \alpha_{opt} - (0.241 + 1.7\bar{m} - 2.6\bar{m}^2)\xi_s - (1 - 1.9\bar{m} + \bar{m}^2)\xi_s^2 \quad (22)$$

$$\bar{\xi}_{opt} = \xi_{opt} + (0.13 + 0.12\bar{m} + 0.4\bar{m}^2)\xi_s - (0.01 + 0.9\bar{m} + 3\bar{m}^2)\xi_s^2 \quad (23)$$

Where, ξ_s is the damping ratio of SDOF system.

In Table 9, optimal parameters of TMD for the structure subjected to different earthquakes identified by genetic algorithm are given. Optimal parameters of TMD proposed by the fuzzy system are equal to: $\xi_{TMD} = 0.094$, $\omega_{TMD} = 1.635$ (rad/s).

The values of frequency and damping ratio are optimized, if they vary in each time to minimize the response of structure. For this purpose, active and semi-active control systems in structures have been invented. Since in this research it is assumed that the control system is passive and unable to have variable parameters, the optimal values are determined such that the control system has perfect operation within the whole vibrations. The damping ratio of TMD while helping to reduce structural response would reduce the displacement of TMD as well, in order not to excite the structure in the last seconds of

earthquake when the ground motions decrease. Now, considering these two objectives (reduction in structure responses and TMD displacement in last seconds), optimal values can vary between the amounts related to these two cases.

According to the parameters of TMD identified by genetic algorithm and fuzzy systems, there is very little difference between average optimum parameters of TMD. This similarity is not accidental; because to produce the required data for designing fuzzy system, a series of near and far-field earthquakes were used. Actually, the designed fuzzy system provides optimal parameters with regard to reconciliation between the near and far earthquakes. Therefore, the optimum parameters of TMD represented by fuzzy systems are more reliable than those of other methods without any analysis of structure to provide TMD parameters.

In Tables 10 and 11 the results obtained by four far-field and four near-field earthquakes are presented respectively. According to these Tables, all three methods can reduce the RMS of displacements appropriately. Among these used methods, genetic algorithms and deterministic method always obtain the accurate results due to employing structural analysis to obtain optimum parameters of TMD. In the same number of analysis, genetic algorithm because of the intelligent search among the possible values of TMD parameters provides a more accurate answer in comparison with the others.

In Fig. 10, responses of the roof displacement obtained by uncontrolled analysis, controlled analysis using TMD through Ioi and Ikeda formulas and controlled analysis using TMD through fuzzy logic system, are shown.

In the early seconds of starting the vibrations, both displacements are almost the same. The reason of this result is the shortage of time to activate the TMD. However, after passing enough time, the mass damper obtains the required acceleration to control the structure vibrations. Since the frequency and damping parameters are tuned correctly, the TMD displacement is set in the opposite direction of roof motion and leads to reduce the vibrations of the structure.

Table 9 Optimal parameters of TMD provided by genetic algorithm

Earthquake	ξ_d	ω_d
Duzce, Turkey	0.071	1.625
Hector Mine	0.127	1.595
Imperial valley	0.079	1.625
Kobe, Japan	0.076	1.649
Irpinia, Italy	0.075	1.647
Cape Mendocino	0.079	1.656
Landers	0.127	1.656
Loma Prieta	0.070	1.625
Average	0.088	1.635
Standard deviation	0.024	0.021

Table 10 Comparison between controlled acceleration and displacement by various methods due to far-field earthquakes

Earthquake	Top floor response	Without TMD	Contour	GA	FIS	Ioi and Ikeda, 1978
Duzce, Turkey	Max displacement (m)	0.175	0.161	0.162	0.159	0.161
	RMS displacement	0.087	0.048	0.048	0.048	0.049
	Max acceleration (m/s^2)	5.263	5.237	5.234	5.237	5.237
	RMS acceleration	0.586	0.525	0.526	0.523	0.525
Hector Mine	Max displacement (m)	0.452	0.316	0.316	0.316	0.316
	RMS displacement	0.230	0.110	0.110	0.111	0.113
	Max acceleration (m/s^2)	5.740	5.709	5.708	5.708	5.710
	RMS acceleration	1.208	1.054	1.054	1.053	1.057
Imperial valley	Max displacement (m)	0.715	0.488	0.491	0.508	0.518
	RMS displacement	0.254	0.174	0.175	0.175	0.178
	Max acceleration (m/s^2)	5.939	5.931	5.926	5.932	5.933
	RMS acceleration	1.208	1.055	1.045	1.059	1.066
Kobe, Japan	Max displacement (m)	0.191	0.178	0.177	0.177	0.178
	RMS displacement	0.102	0.058	0.057	0.058	0.059
	Max acceleration (m/s^2)	4.621	4.633	4.633	4.632	4.633
	RMS acceleration	1.158	1.108	1.110	1.103	1.105

Table 11 Comparison between controlled acceleration and displacement by various methods due to near-field earthquakes

Earthquake	Top floor Response	Without TMD	Contour	GA	FIS	Ioi and Ikeda, 1978
Irpinia, Italy	Max displacement (m)	1.549	1.289	1.289	1.290	1.290
	RMS displacement	0.938	0.435	0.431	0.441	0.437
	Max acceleration (m/s^2)	6.656	6.502	6.506	6.503	6.507
	RMS acceleration	2.764	1.532	1.537	1.547	1.549
Cape Mendocino	Max displacement (m)	0.414	0.377	0.377	0.377	0.377
	RMS displacement	0.226	0.142	0.142	0.143	0.144
	Max acceleration (m/s^2)	5.443	5.439	5.439	5.438	5.439
	RMS acceleration	1.138	0.997	0.979	0.989	0.994
Landers	Max displacement (m)	0.337	0.284	0.286	0.287	0.287
	RMS displacement	0.204	0.092	0.089	0.091	0.089
	Max acceleration (m/s^2)	5.324	5.215	5.226	5.219	5.229
	RMS acceleration	0.828	0.654	0.652	0.654	0.654
Loma Prieta	Max displacement (m)	0.284	0.277	0.276	0.276	0.277
	RMS displacement	0.141	0.068	0.067	0.067	0.068
	Max acceleration (m/s^2)	4.572	4.588	4.587	4.587	4.587
	RMS acceleration	0.717	0.632	0.641	0.633	0.639

It is noteworthy that to use the genetic algorithm, a model of real structure is required to obtain the accurate result by repeated analyses. However, in empirical formulas and fuzzy system, TMD optimal parameters are proposed with acceptable accuracy without any analysis and just based on prior analysis conducted for different structures under various earthquakes. To give analytical and empirical formulas, the loading and building structures are simplified. In fact, empirical and analytical relations for structures are faced with two types of uncertainty. The first one is structural uncertainty (usually SDOF systems) and the other one is loading uncertainty (usually assumed harmonic).

In contrast, to design fuzzy system, a series of different MDOF models and far-field and near-field earthquakes were used. This justifies the results of the fuzzy system to have less uncertainty. So the results can be used with more reliability for structures

Whereas the compared control system in all above states is exactly the same, a considerable reduction in responses is not expected. However, the efficient operation of TMD is provided by optimal parameters gained using fuzzy system, even though it is with a little bit of reduction. This improvement in results is because of unreliability to Ioi and Ikeda formula. Thus to design fuzzy system, the far and near-field earthquakes as inputs and MDOF system as structure have been used. Although the proposed method has mainly been effective in reducing displacements, under most earthquake records it has reduced maximum acceleration compared to the Ioi and Ikeda's empirical relations.

In Fig. 11, the roof acceleration resulted by uncontrolled analysis, controlled analysis using TMD through Ioi and Ikeda formulas and controlled analysis using TMD through fuzzy logic system, are shown. There is no tangible difference in reduced accelerations between these two control methods. However, it should be mentioned that it is declined a little after starting the earthquake controlled acceleration using TMD.

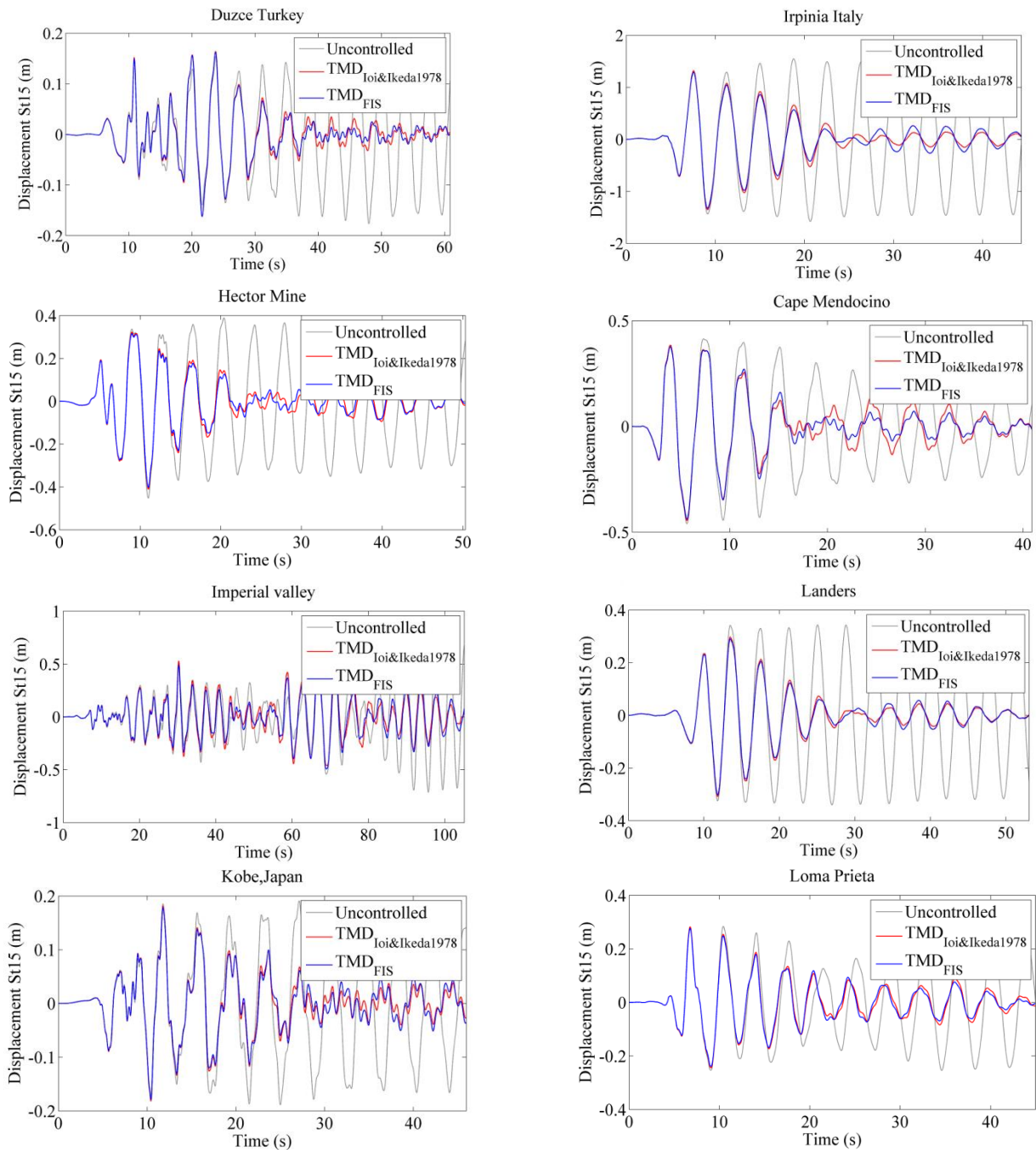


Fig. 10 Comparison between roof displacement of controlled and uncontrolled cases

5. Conclusions

In this study, it was tried to estimate optimal parameters of the TMD without performing the structural analysis and just through general information of structure. Therefore, to design a fuzzy system, the data sets including general information about MDOF systems subjected to different far and near-field earthquakes were used.

Designing the fuzzy system by three methods i.e., the look-up table, grid partitioning the data space, and clustering indicated that the results of all three methods

were almost the same. However, the proportional major operation of the designed fuzzy system based on clustering method optimized by ANFIS illustrated its ability of classifying and identifying the system behavior.

To select the best fuzzy systems, statistical comparison was made between the optimal parameters of TMD identified by the Simulink analysis, and parameters proposed by the fuzzy system. Therefore, the fuzzy system based on data clustering with ERMS of 0.2989 for frequency and 0.0387 for damping ratio was used in the rest of this research as the best method.

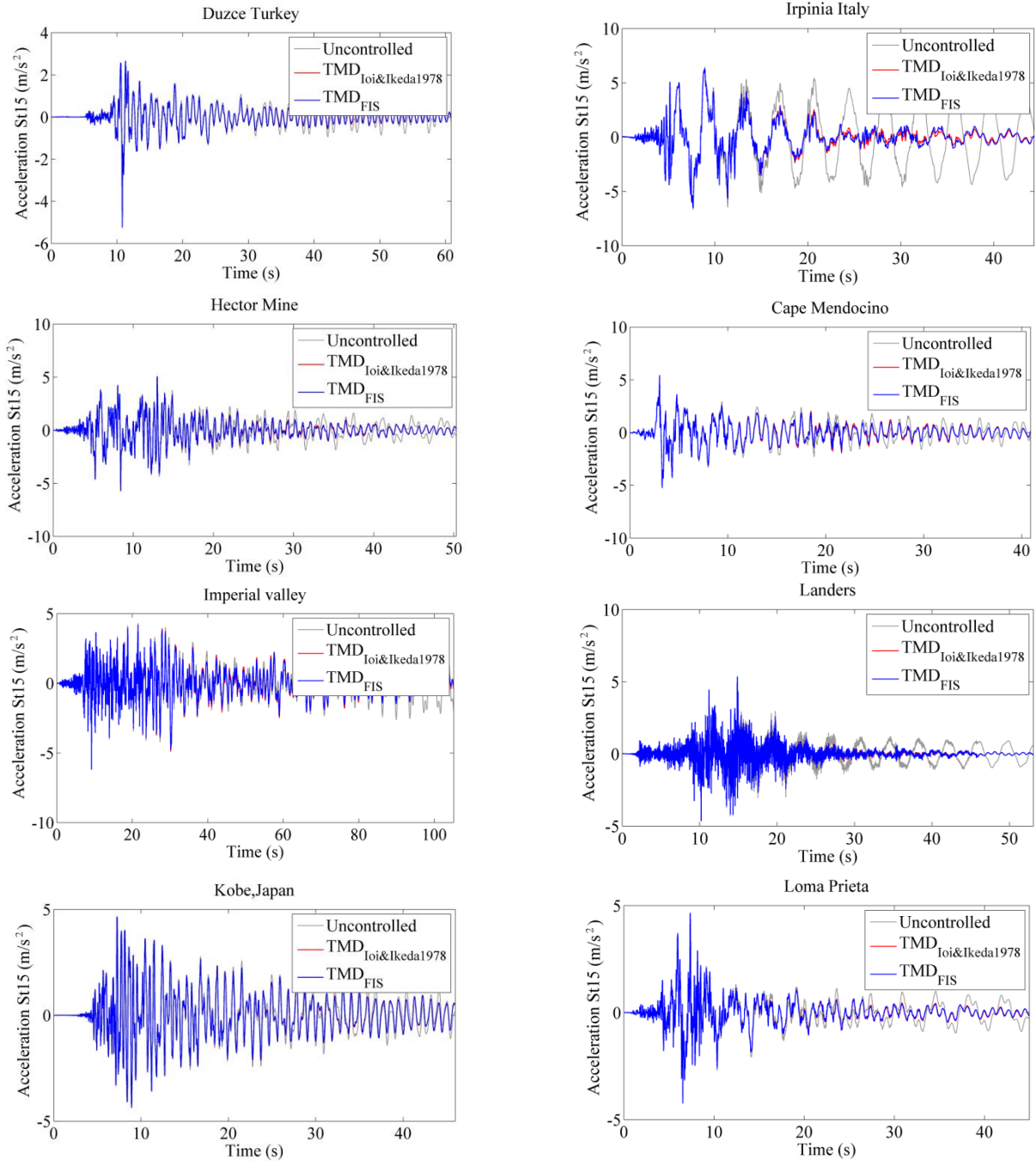


Fig. 11 Comparison between roof acceleration of controlled and uncontrolled cases

The ERMS ratios of frequency were given 0.3057 and 0.5386, and ERMS ratios of damping were given 0.0394 and 0.0405 for grid partitioning and look-up table method respectively.

After that, In order to evaluate the performance of the designed fuzzy system, a 15-story benchmark structure was studied under four near-field and four far-field earthquakes. The optimal parameters of TMD were obtained for this structure using four methods, i.e. parametric study as a numerical method, genetic algorithm as a heuristic method, fuzzy system as a system able to consider unreliability and empirical relation. The results showed that TMD optimal

parameters gained from the fuzzy method are more accurate compared to the empirical relationships considered as approximate methods without structural analysis. Based on obtained results, the progress up to 1.9% and 2% under far-field earthquakes and 0.4% and 2.2% under near-field earthquakes was obtained in decreasing respectively roof maximum displacement and its RMS ratio through fuzzy system method compared to those obtained by empirical relations. In addition, the averages of optimal parameters identified by genetic algorithm were approximately equal to the parameters identified by the fuzzy system. This means that, those parameters identified by the fuzzy system were

the average values due to the near and far-field earthquakes. So, the optimal parameters provided by the fuzzy system can be used to design TMD with more reliability.

The error in frequency has more contribution to divert the correct response in structure. The error in damping estimation that was clear in all three methods is due to the less sensitivity of TMD to the damping ratio compared to the structure frequency. Therefore, the fuzzy system can be employed to estimate the TMD damping ratio without worrying about the modeling errors. Thus, by combining the existent advantages of estimating the TMD optimal parameters based on both old simplified and modern methods requiring a perfect model of structure, the fuzzy system approach proposed a novel technique to consider the uncertainties while being practical like presented equations. Finally, for further achievement and reliability in designed fuzzy system operation, a wider range of structures and earthquakes should be investigated to increase efficiency of nonlinear decision-making process.

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