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SDRE controller considering Multi Observer applied to nonlinear IPMC model

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Abstract. Ionic Polymer Metal Composite (IPMC) is an electroactive polymer (EAP) and a promising candidate actuator for various potential applications mainly due to its flexible, low voltage/power requirements, small and compact design, and lack of moving parts. Although widely used in industry, this material requires accurate numerical models and knowledge of optimal control methods. This paper presents State-Dependent Riccati Equation (SDRE) approach as one of rapidly emerging methodologies for designing nonlinear controllers. Additionally, the present paper describes a novel method of Multi HGO Observer design. In the proposed design, the calculated position of the IPMC strip accurately tracks the target position, which is illustrated by the experiments. Numerical results and comparison with experimental data are presented and the effectiveness of the proposed control strategy is verified in experiments.

Keywords: IPMC; SDRE; EAP; High Gain Observer; Multi HGO Observer

1. Introduction

Nafion-based IPMC is the best-known and investigated IPMC which consists of a thin polymer membrane with metal electrodes plated on both faces. Researchers consider IPMC as promising candidate for creating artificial muscles, miniature robots and biomedical devices Shahinpoor (1999), Yun (2006). This material belongs to wide group of smart materials. The alternative is for instance piezo smart structures Gupta *et al*. (2011), Alamir (2015). The IPMC exhibits large bending in response to applied voltage, nevertheless electrical control is very difficult due to its strongly nonlinear nature resulting from physical, chemical and environmental variables Bernat and Kolota (2016). When the membrane is hydrated, the state is stimulated with an applied small step potential, both fixed anions and mobile counter-ions are subjected to an electric field, and the counter-ions being able to diffuse towards one of the electrodes. As a result, the composite undergoes an initial fast bending deformation, followed by a slow relaxation.

Several models for the mechanism of IPMC actuation have been described Chen (2009), Newbury and Leo (2002), Bonomo *et al*. (2007), Yun (2006). In this paper, we present a detailed description of the phenomenon of actuation under a 1 to 3V DC signal applied. Following this, we present a summary of the Chen model for IPMC sensing and actuation Chen *et al.* (2009). This approach is based on nonlinear Partial Differential Equations (PDE), which includes ion diffusion, ion migration and electrostatic interactions in the IPMC. A nonlinear circuit model was employed to capture the electrical dynamics. This model covers nonlinear capacitance of IPMC, pseudocapacitance due to adsorption, ion diffusion resistance and nonlinear DC resistance of the polymer. Mechanical properties of IPMC were studied by Farinholt (2005), Newbury (2002). In our study the mechanical part was based on viscoelastic behavior. In addition the relationship between curvature output and bending moment was defined. This approach is suited for small deformations, which are considered in this paper. However, the mechanical modelling was extended to consider large deformation domain measurements.

In this study we formulated a new model connecting features proposed in papers Chen *et al*. (2009), Farinholt (2005), Newbury (2002), and determined the base for a control system design. We also considered optimal control approach, which is a common technique in control system theory Mracek and Cloutier (1998), Lee (2012), Lin (2015). Currently, one of the most advanced methods in this area is State Dependent Riccati Equation SDRE which is a systematic approach to designing nonlinear feedback controllers that approximate the solution of the innite time horizon optimal control problem and can be implemented in real-time for a wide range of applications Banks *et al.* (2007), Mracek and Cloutier (1998).

This paper is the first attempt to solve the SDRE control problem for Ionic Polymer Metal Composite. The primary challenge was to solve Riccati equations taking into account the model nonlinearity. Then, we analyzed the stability of a set point control problem which has not been published till now. The SDRE method requires full state feedback, unfortunately only position and current are available from experimental measurement. Thus, to obtain other state variables, the observer had to be designed.

Currently, one of the most effective observer methods

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for nonlinear systems is High Gain Observer (HGO) Besancon (2007), Wang (2001), Bernat and Stepien (2015), Khalil (2000). In this study, we applied Multi HGO Observer which accurately estimated the position of the IPMC strip and ensured high performance. The SDRE and Multi HGO Observer were coupled to create a closed-loop system, and the effectiveness of the proposed method was confirmed by the measurements.

The IPMC strips were derived by Environmental Robots Incorporated (ERI), which is the world leader in manufacturing ionic polymer metal composites.

2. Model

This study investigated a nonlinear IPMC model developed in Chen *et al*. (2009), Newbury and Leo (2002). The model schema is shown in Fig. 1. Its equivalent state space equations are as follows Chen *et al.* (2009)

$$\frac{dV(t)}{dt} = \frac{\dfrac{U(t)-V(t)}{R_a + R_c} - Y(V(t))}{C_1(V(t)) + C_a(V(t))} \tag{1}$$

where $U(t)$ is input voltage, $V(t)$ is electric potential. Relying on physical phenomena, the model accurately defines nonlinear capacitance $C(V(t))$, virtual capacitance $C_a(V(t))$ which describes the electrochemical adsorption process at the polymer-metal interface, ion diffusion resistance R_c, electrode resistance R_a and nonlinear DC resistance of polymer expressed as current-voltage relationship $Y(V(t))$. For the sake of simplicity, these terms are defined in the appendix. The bending moment produced by IPMC is given by

$$M(V) = W\alpha_0\kappa_e\left(sign(V(t))2h\sqrt{2\Gamma(V(t))} - V(t)\right) \tag{2}$$

where κ_e is dielectric constant, h is thickness and α_0 is coupling constant. The motion is defined by curvature output $y(t)$. It is related to the bending moment by

$$\frac{1}{\alpha_\xi^2 + \omega_\xi^2}\ddot{y}(t) + \frac{2\alpha_\xi}{\alpha_\xi^2 + \omega_\xi^2}\dot{y}(t) + y(t) = \frac{M(V)}{Y_e I} \tag{3}$$

where $I = \frac{2}{3}Wh^3$ is the moment of area inertia and Y_e is the equivalent of Young's modulus of IPMC. Coefficients α_ξ and ω_ξ define the character of relationship between curvature output and bending moment. It is different from work Chen *et al.* (2009) because it includes viscoelastic behavior described for instance in Newbury and Leo (2002).

Based on circuit model (1), output current I_s is defined as

$$I_s(t) = \frac{U(t)-V(t)}{R_a + R_c} \tag{4}$$

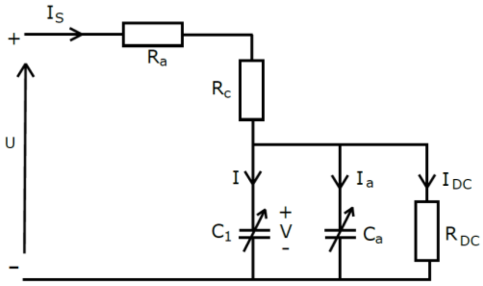


Fig. 1 Circuit model of IPMC, Smart Materials and Structures by IOP Publishing. Reproduced with permission of IOP Publishing in the format Journal via Copyright Clearance Center

3. SDRE control problem

This section presents the proposed nonlinear quadratic optimal control approach for the IPMC actuator based on State-Dependent Riccati Equation. To formulate the optimal control problem, we considered the cost functional with constant weighting matrices given by integral

$$J(u) = \frac{1}{2}\int_0^\infty \left(x^T Qx + Pu^2\right)dt \tag{5}$$

where $x \in R^3$ is a state vector, $u \in R$ is a scalar control variable, $Q \in R^{3x3}$ is symmetric positive definite matrix and $P \in R$ is a positive factor.

The equations presented in the section 2 provide state space representation

$$\dot{x} = A(x)x + B(x)u \tag{6}$$

where

$$A(x) = \begin{bmatrix} 0 & 1 & 0 \\ -\alpha_\xi^2 - \omega_\xi^2 & -2\alpha_\xi & \left(\alpha_\xi^2 + \omega_\xi^2\right)\dfrac{M(V)}{IY_eV(t)} \\ 0 & 0 & \dfrac{\dfrac{-1}{R_a + R_c} - \dfrac{Y(V)}{V(t)}}{C_1(V) + C_a(V)} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ a_{12} & a_{22} & a_{23}(V) \\ 0 & 0 & a_{33}(V) \end{bmatrix} \tag{7}$$

$$B(x) = \begin{bmatrix} 0 \\ 0 \\ \dfrac{1}{(R_a + R_c)(C_1(V) + C_a(V))} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ b(V) \end{bmatrix}$$

where state $x(t)$ is represented by $\begin{bmatrix} y & \dot{y} & V \end{bmatrix}^T$.

In the case of linear systems control the set point control is very simple. However, it requires a mathematical transformation of the state equations for nonlinear systems like it is described by Erdem and Alleyne (2004). We present the set point control methodology of IPMC model below.

Consider the system Eq. (6), where

$$x_r = \begin{bmatrix} y_r \\ \dot{y}_r \\ V_r \end{bmatrix},\ x_r = const \Rightarrow \dot{x}_r = 0 \tag{8}$$

The equilibrium point for reference x_r satisfies

$$0 = A_r(x_r)x_r + B_r(x_r)u_r \tag{9}$$

which is equivalent to

$$\begin{aligned} 0 &= \dot{y}_r \\ 0 &= a_{21}y_r + a_{22}\dot{y}_r + a_{23}(V_r)V_r \\ 0 &= a_{33}(V_r)V_r + b(V_r)u_r \end{aligned} \tag{10}$$

Let us consider error coordinates as

$$e_1 = y - y_r \quad e_2 = \dot{y} \quad e_3 = V - V_r \tag{11}$$

and control input is split into $u = u_r + u_{opt}$ where u_r is reference part and u_{opt} is optimal control which will be determined by SDRE algorithm. Taking into account (6), (10) and (11), we obtain

$$\begin{aligned} \frac{de_1}{dt} &= e_2 \\ \frac{de_2}{dt} &= a_{21}e_1 + a_{22}e_2 + \bar{a}_{23}(e_3)e_3 \\ \frac{de_3}{dt} &= \bar{a}_{33}(e_3)e_3 + \bar{b}(e_3)u_{opt} \end{aligned} \tag{12}$$

where

$$\begin{aligned} \bar{a}_{23}(e_3) &= \frac{a_{21}y_r + a_{22}\dot{y}_r + a_{23}(V_r+e_3)(V_r+e_3)}{e_3} \\ \bar{a}_{33}(e_3) &= \frac{a_{33}(V_r+e_3)(V_r+e_3) + b(V_r+e_3)u_r}{e_3} \\ b(e_3) &= b(V_r+e_3) \end{aligned} \tag{13}$$

The matrix form of the system (12) is following

$$\dot{e} = \bar{A}(e)e + \bar{B}(e)u_{opt} \tag{14}$$

To design a control system and define a control law, firstly we had to determine whether the control of the complete state of the dynamical system was possible Çimen (2010), Lam *et al*. (2012). This information can be obtained by checking controllability of the dynamical system. Essentially, controllability informs about control possibility of a dynamical system from an arbitrary initial state to an arbitrary final state by means of a set of admissible controls. However, the trajectory of the dynamical system, Eq. (6) between initial and final state was not specified. Furthermore, there were no constraints imposed on the control *u* and the state vector *e*.

In order to formulate easily computable algebraic controllability criteria let us introduce the state-dependent controllability matrix W(e) Banks *et al*. (2007), Çimen (2012)

$$W(e) = \left[\bar{B}(e) \quad \bar{A}(e)\bar{B}(e) \quad \bar{A}^2(e)\bar{B}(e)\right] \tag{15}$$

If $W(e)$ (a state-dependent in this case) has full rank then the system is controllable for all e. Parametrized IPMC model in form of Eq. (7) has state-dependent controllability matrix, Eq. (15), given by

$$W(e) = \begin{bmatrix} 0 & 0 & \bar{a}_{23}(e_3) \\ 0 & \bar{a}_{23}(e_3) & a_{22}\bar{a}_{23}(e_3) + \bar{a}_{23}(e_3)\bar{a}_{33}(e_3) \\ 1 & \bar{a}_{33}(e_3) & \bar{a}_{33}^2(e_3) \end{bmatrix} \tag{16}$$

which has full rank for all e. Therefore, the system in eq. (6) is controllable with the parametrization in Eq. (7). In Lin *et al*. (2015) the authors modified the assumption for the solvability of the SDRE algorithm. The newly-summarized lemma has replaced difficult-to-test conditions and accordingly a feasible State-Dependent Coefficient (SDC) matrix can be easily constructed. In this paper we adopt this attempt and compare it with classical SDRE Banks *et al.* (2007). To solve algorithm 1 presented in Lin *et al*. (2015) we take into account IPMC system described as

$$f(x) = \begin{bmatrix} x_2 \\ -(\alpha_\xi^2 + \omega_\xi^2)x_1 - 2\alpha_\xi x_2 + (\alpha_\xi^2 + \omega_\xi^2)\dfrac{M(x_3)}{Y_e I} \\ -\dfrac{x_3}{(R_a + R_c)C(x_3)} - \dfrac{Y(x_3)}{C(x_3)} \end{bmatrix} \tag{17}$$

where $C(x_3) = C_1(x_3) + C_a(x_3)$, $x_1 = y$, $x_2 = \dot{y}$ and $x_3 = V$. Firstly we check whether the pair of vectors $\{x, f\}$ is linearly independent for $x \neq 0$. Due to nonlinearity of functions this condition is verified numerically in the feasible set of x. Next, the auxiliary coefficients λ_k, $x_k = 1,2,3$ are chosen. This enables us to find vector $q_1(x)$ which is orthonormal to vectors x and f. The vectors $q_2(x)$ and $q_3(x)$ are found from relations: $q_i^T(x)(\lambda_i x - f) = 0$ for $i = 2,3$. The auxiliary vector $q_i(x)$ defines matrix $M(x) = [q_1(x) \quad q_2(x) \quad q_3(x)]^T$. Now, it is possible to calculate the matrix $A(x) = M^{-1}(x)DM(x)$ where $D = diag[\lambda_1 \quad \lambda_2 \quad \lambda_3]$. The described procedure has been implemented numerically and solved pointwise for every x. The transformation to error coordinates is performed the same as for classical SDRE. Furthermore, in the experimental section both algorithms are compared.

Relying on works Banks *et al*. (2007), Huang and Lu (1996), the Hamiltonian for the optimal control problem in Eqs. (5) and (6) is given by

$$H = \frac{1}{2}\left(e^T Qe + Pu_{opt}^2\right) + p^T\left(\bar{A}(e)e + \bar{B}(e)u_{opt}\right) \tag{18}$$

The necessary conditions for the optimal control were found to be

$$\dot{p} = -\frac{\partial H}{\partial t} = -Qe - \left[\frac{\partial \overline{A}(e)e}{\partial e}\right]^T p - \left[\frac{\partial \overline{B}(e)u_{opt}}{\partial e}\right]^T p \qquad (19)$$

$$\dot{e} = -\frac{\partial H}{\partial p} = \overline{A}(e)e + \overline{B}(e)u_{opt} \qquad (20)$$

$$\frac{\partial H}{\partial u_{opt}} = Pu_{opt} + \overline{B}^T(e)p = 0 \qquad (21)$$

Assuming that $p = K(e)e$ and using Eq. (21), we obtained a state - dependent feedback control

$$u_{opt} = -P^{-1}\overline{B}^T(e)K(e)e \qquad (22)$$

Substituting the control law into Eq. (20) and differentiating $p = K(e)e$, we have

$$\begin{aligned}&\dot{p} = \dot{K}(e)e + K(e)\dot{e} = \\ &= \dot{K}(e)e + K(e)\overline{A}(e)e - K(e)\overline{B}(e)P^{-1}\overline{B}^T(e)K(e)\end{aligned} \qquad (23)$$

Equating Eqs. (19) and (23), we found a nonlinear differential equation with unknown matrix function $K(e)$

$$\begin{aligned}&\dot{K}(e)e + K(e)\overline{A}(e)e - K(e)\overline{B}(e)P^{-1}\overline{B}^T(e)K(e)e - \\ &Qe - \left[A(e) + \frac{\partial A(e)}{\partial e}e\right]^T K(e)e - \left[\frac{\partial B(e)u}{\partial e}\right]^T K(e)e = 0\end{aligned} \qquad (24)$$

To obtain the state-dependent Riccati equation, Eq. (24) should be divided by e and separated into two parts as

$$\begin{aligned}&\overline{A}^T(e)K(e) + K(e)\overline{A}(e) - \\ &K(e)\overline{B}(e)P^{-1}\overline{B}^T(e)K(e) + Q = 0\end{aligned} \qquad (25)$$

$$\dot{K}(e) - \left[A(e) + \frac{\partial A(e)}{\partial e}e\right]^T K(e) - \left[\frac{\partial B(e)u}{\partial e}\right]^T K(e) = 0 \qquad (26)$$

Eq. (25) is the form of the State Dependent Riccati Equation (SDRE). The nonlinear feedback gain function $K(e)$ can be obtained by solving Eq. (25) and neglecting Eq. (26), which is so - called SDRE necessary condition for optimality Mracek and Cloutier (1998). This is a promising and rapidly emerging methodology to design nonlinear controllers in the context of the nonlinear quadratic regulator problem. To find the solution of Eq. (25), the authors use Frozen Riccati Equation Methods (FRE) of solving SDRE, where the Eq. (25) is treated as typical Algebraic Riccati Equation (ARE) solved for different states Huang and Lu (1996), Banks *et al.* (2007), Çimen (2008). In the recent results of SDRE approach Banks et al. (2007), it is indicated that pointwise stabilizability of $A(x)$ and $B(x)$ does not assure controllability of nonlinear system. The problem of global stability was also studied by Hammett *et al*. (1998). Furthermore, the recent results of work Heydari and Balakrishnan (2015), gives possibility to obtain global asymptotic stability in case of finite time SDRE method. Another issue related with SDRE approach is a recovery of the optimal control. Currently, there are known different techniques which allows to obtain the optimal control, for instance Heydari and Balakrishnan (2013), Lin *et al*. (2015), Huang and Lu (1996).

4. Multi HGO observer

In recent years an interesting technique to estimate plant state is High Gain Observer. Its extension, which causes elimination of peak phenomenon and improves transients, is Multi High Gain Observer. In the case of IPMC it is required to estimate speed $\dot{y}$ based on position y, input voltage U and output current I_s. It is worth noting that potential V is directly available by calculating (4) which requires signals U and I_s and knowledge of resistance $R_a + R_c$.

The Multi HGO Observer approach provides estimation of the state by M independent observers. This results in the estimated state described by M weighted states

$$\hat{x} = \sum_{m=1}^{M} \alpha_m \hat{x}_m \qquad (27)$$

where α_m is weight coefficient, and $\hat{x}_m$ is m observer state. As the result, the estimation is performed in two layers.

The first layer is related to High Gain Observer, which calculates the estimated state $\hat{x}_m$. Its design process is to find gains $L = \begin{bmatrix} p_1/\varepsilon & \dots & p_n/\varepsilon^n \end{bmatrix}$ which creates a closed-loop estimation process described by

$$\frac{de}{dt} = (A_{HGO} - LC_{HGO})e + B_{HGO}\Theta(\hat{x},u) - B_{HGO}\Theta(x,u) \qquad (28)$$

where

$$\begin{aligned}&A_{HGO} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad B_{HGO} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C_{HGO} = \begin{bmatrix} 1 & 0 \end{bmatrix} \\ &\Theta(x,u) = -(\alpha_\xi^2 + \omega_\xi^2)x_1 - 2\alpha_\xi \omega_\xi x_2 + \frac{M(V)}{Y_e I}\end{aligned} \qquad (29)$$

The idea of High Gain Observer is to set up such a small ε that the gain is sufficiently high to eliminate nonlinear terms and thus ensure stability.

The second layer is built by multi observer, which finds the values of α_m weights. This requires additional observation law which is created by introduction auxiliary signal ξ_m. This signal is expressed in terms of measurable signal $\eta_m = y - \hat{y}_m$ as

$$\xi_m(t)=\begin{bmatrix}1 & 0 & 0 & \dots & 0\\ CL & 1 & 0 & \dots & 0\\ CAL & CL & 1 & \dots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ CA^{n-2}L & CA^{n-3}L & CA^{n-4}L & \dots & 1\end{bmatrix}=$$

$$=\begin{bmatrix}\underbrace{\int_{t-t_d}^{t}\dots\int_{t-t_d}^{t}}_{n-1}\eta_k(\tau)d\tau\\ \vdots\\ \eta_m(t)-\eta_m(t-(n-1)t_d)\end{bmatrix} \tag{30}$$

Hence we have a possibility to calculate ξ_m in real time. This signal may be also found as function of $e_m=\hat{x}_m-x$

$$\xi_m(t)=\begin{bmatrix}C\\ CA\\ \vdots\\ CA^{n-1}\end{bmatrix}\int_{t-t_d}^{t}\dots\int_{t-t_d}^{t}e_m(\tau)d\tau \tag{31}$$

which is invertible. Thus, by solving identity

$$0=\sum_{m-1}^{M}\alpha_m\xi_m \tag{32}$$

we obtain $e=\hat{x}-x=0$. Due to noise and other disturbances the above equation cannot be solved directly, that is why Recursive Least Square method is applied.

5. Experiments

The previous section details the theoretical developments of optimal controller relying on the SDRE methodology. Now, the results are examined by experiments. Fig. 2 illustrates the experimental setup. To avoid dehydration of the sample during the experiments, all tests were performed with IPMC immersed in deionized water and clamped at one end. After each experiment, the clamping was polished with alcohol to remove and prevent oxidation. The polymer was subject to voltage excitation generated from the computer through realtime data acquisition board (RTDAC Inteco). A laser displacement sensor (micro-epsilon optoNCDT 1302) was used to measure the bending displacement. We added another small resistor $R_s=1\Omega$, which was serially connected with IPMC. By measuring the voltage V_s, one can calculate current $I_s=\frac{V_s}{R_s}$. The IPMC samples dimensions have an accuracy of ±0,5 [mm] in the length and width, and ±0,5 [mm] in the thickness. Most parameters of the IPMC model can be directly measured, but some of them must be identified by a fitting process. Table 1 shows all the parameters.

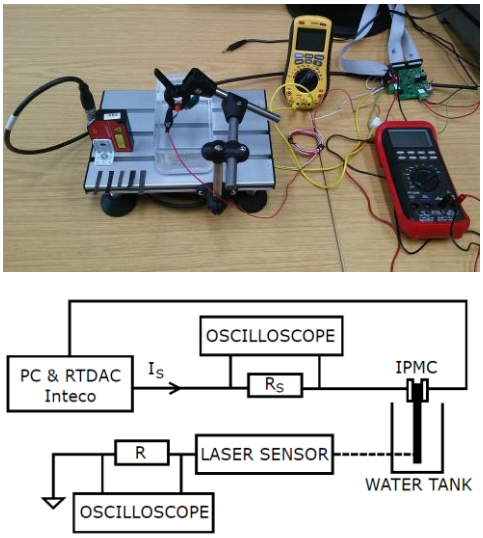


Fig. 2 Experimental setup and its schematic diagram, Smart Materials and Structures by IOP Publishing. Reproduced with permission of IOP Publishing in the format Journal via Copyright Clearance Center

Table 1 Parameters in the model

F	R	T	R_a
96487 $\frac{C}{mol}$	8.3143 $\frac{J}{mol*K}$	300 K	0 Ω
R_c	L	W	H
130 Ω	5 mm	37 mm	100 μm
C^-	κ_e	C^{H+}	K_1
1091 $\frac{mol}{m^3}$	3.35*10^{-6} $\frac{F}{m}$	1*10^{-6} $\frac{mol}{m^3}$	4*10^5
α_0	Y_e	q_1	α_ζ
0.0258 $\frac{J}{C}$	0.62 GPa	0.105 $\frac{\mu C}{cm^2}$	15
Y_1	Y_2	Y_3	ϖ_ζ
0.1*10^{-3} $\frac{A}{V}$	0.6*10^{-3} $\frac{A}{V^2}$	1.8*10^{-3} $\frac{A}{V^3}$	0

5.1 Multi HGO Observer

Firstly, we would like to show the results of state estimation. To obtain IPMC speed, the Multi Observer described in section 4 is applied. The first layer contains $M=3$ observers. Its gains are equal to $L=\left[-1/\varepsilon \quad \dots \quad -1.5/\varepsilon^2\right]$ where $\varepsilon=0.05$. The second layer is applied to estimate weights α_m. Relying on (32), the following linear regressor is constructed

$$y[n_p]=\varphi^T[n_p]\hat{\theta}[n_p]+e_{LS}[n_p] \tag{33}$$

where

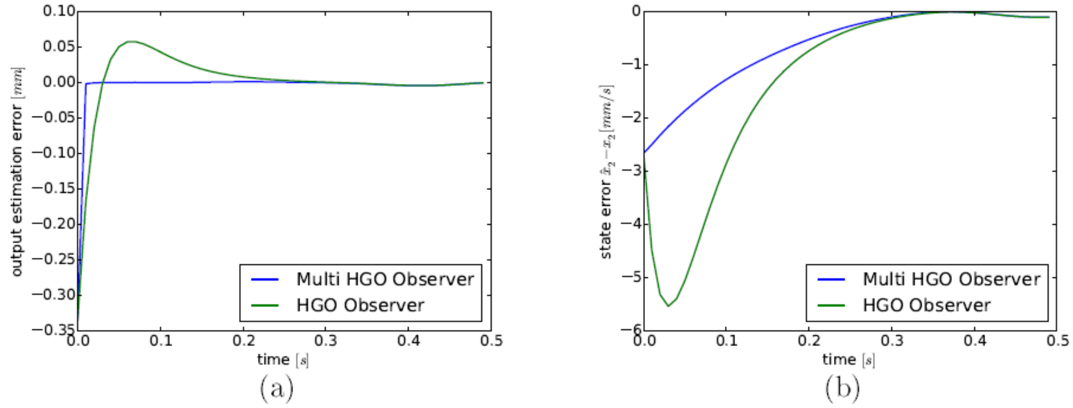


Fig. 3 Output position error (a) and estimated speed error (b) for Multi HGO Observer and High Gain Observer

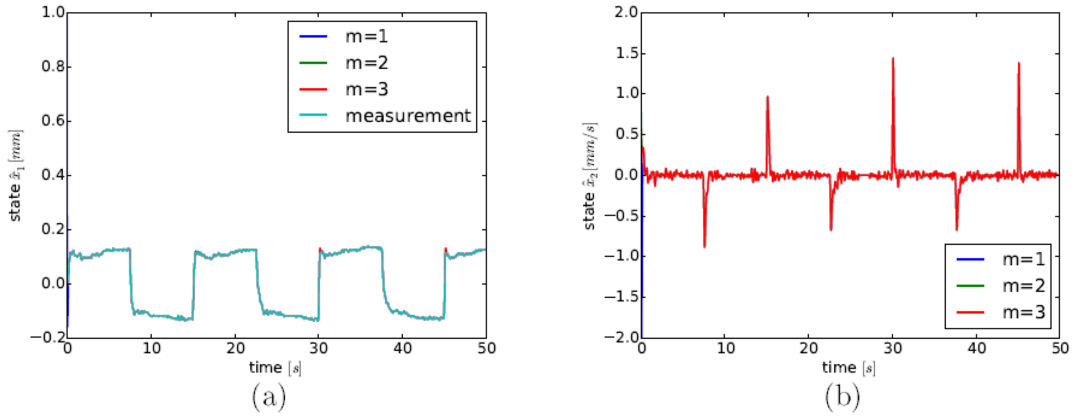


Fig. 4 Estimated position (a) and estimated velocity (b) of Multi HGO Observer under different initial conditions

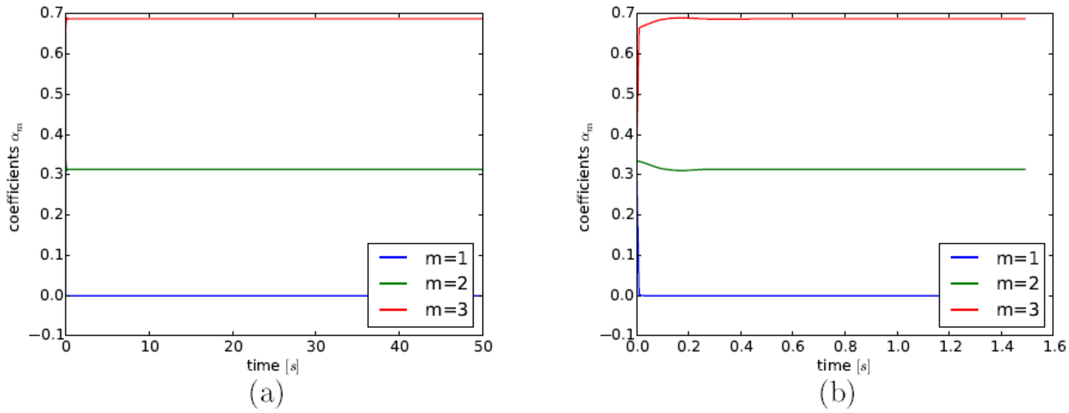


Fig. 5 Multi HGO Observer coefficients m for a long time (a) and for a short time (b)

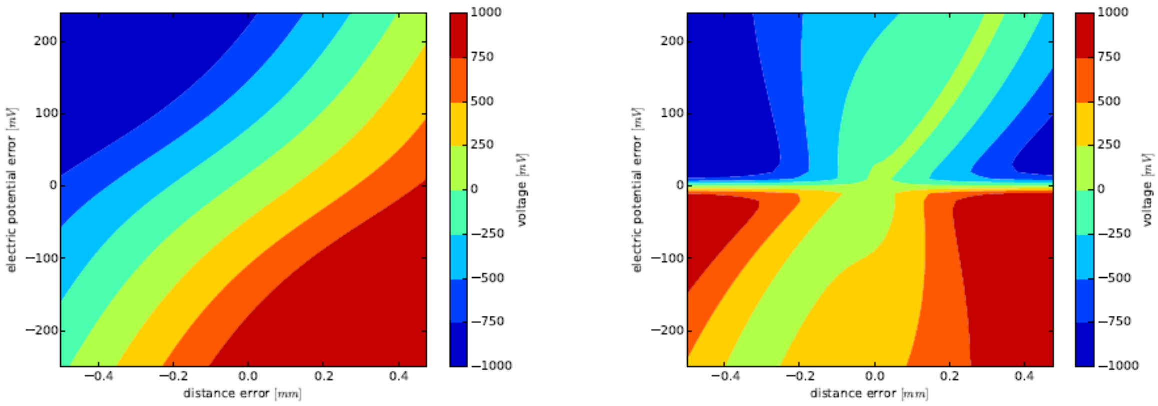
$$\hat{\theta}[n_p] = \left[\hat{\alpha}_1(n_pT_p) \quad \ldots \quad \hat{\alpha}_{n-1}(n_pT_p)\right]^T \tag{34}$$

is the $R^{n\times 1}$ vector with estimated parameters

$$\varphi[n_p] = \begin{bmatrix} \xi_1(n_pT_p) - \xi_{n+1}(n_pT_p) \\ \vdots \\ \xi_n(n_pT_p) - \xi_{n+1}(n_pT_p) \end{bmatrix} \tag{35}$$

is the $R^{n\times n}$ regressor and $y[n_p] = -\xi_{n+1}(n_pT_p)$ is the $R^{1\times 1}$ output. The regressor parameters are estimated by Recursive Least Square algorithm with gain equal to 0.5.

The estimation algorithm was calculated in real time with probe time T_p = 0.01[s]. In Fig. 3(a), the comparison of estimated position error between Multi HGO Observer and High Gain Observer is visible. Multi HGO Observer provides faster convergence rates than HGO observer.



(a) (b)

Fig. 6 Voltage as a function of displacement error and electric potential error for classical SDRE (a) and SDRE by Lin *et al.* (b)

Fig. 3(b) shows the transients of estimated speed error of IPMC by Multi HGO Observer and High Gain Observer. It should be noted that the first one provided overshoot four times lower than the second one.

In order to demonstrate the behavior of Multi HGO Observer, Figs. 4(a) and 4(b) present the estimated position and velocity. In both figures position and speed are represented by three independent observers. The estimated state is calculated by applying weights shown in Fig. 5.

It is worth noting that Multi HGO Observer reduced the initial overshoot in the position error and was capable to recover speed at the start of the process. The main advantage of Multi HGO Observer technique is the transient improvement due to double layer state estimation. One is in the first layer with HGO observers and the second layer consists time varying coefficient, see details in Bernat and Stepien (2015).

5.2 SDRE controller

In the second part we aimed to verify the behavior of SDRE controller for classical approach Banks *et al*. (2007) and new approach Lin *et al*. (2015). To calculate the optimal control law, the following weight factors were chosen

$$P = [1] \quad Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{36}$$

Next, gain K is found by taking into account the following operating range

$$\begin{gathered} -1.5[V[\leq V \leq 1.5[V] \\ -0.5[mm/s] \leq \dot{y} \leq 0.5[mm/s] \\ -1[mm] \leq y \leq 1[mm] \end{gathered} \tag{37}$$

The calculated gain is applied to find feedback voltage from Eq. (22). The calculated voltage is demonstrated in Fig. 6(a). In the case of SDRE parametrization presented by Lin *et al*. (2015), we solved SDRE problem relying on function (17). The coefficients $\lambda_i, i = 1,2,3$ are as follows -0.5, -1.0 and -1.5.

The example voltage feedback is presented in Fig. 6(b). The square waveform function was taken as the reference distance. Its period is equal to 15[s] and amplitude 0.15[mm]. Relying on Eq. (10), the reference electric potential is found taking into account the reference distance and current state.

The controller is run online with probe time T_p = 0.01[s]. The feedback voltage signals are visible in Figs. 7(a) and 7(b). The IPMC distance was measured by laser sensor. It is clear in Figs. 8(a) and 8(b) that IPMC converges to the reference position in both cases (classical SDRE and SDRE by Lin *et al.* (2015)). These figures also demonstrate that IPMC control system is noisy and highly disturbed, which results from the IPMC nature, as it has been reported in several papers previously Chen *et al*. (2009), Farinholt (2005).

The overall control process quality is good taking into account small steady state error. In Figs. 8(c) and 8(d) the control errors are presented for classic SDRE and new approach proposed by Lin *et al.* (2015). Figs. 9(c) and 9(d) show the control errors calculated for electric potential signals. In all presented cases the errors are not higher than 5[%] in the steady state. This figures also show that electric potential follows its reference value. Additionally, the electric potential is much less noisy in comparison with distance signal.

To ensure experiment clarity in Figs. 7(a) and 7(b) the IPMC current transient signals are compared for both SDRE algorithms. Their waveform contain high level peaks which are caused by voltage peaks generated by controller.

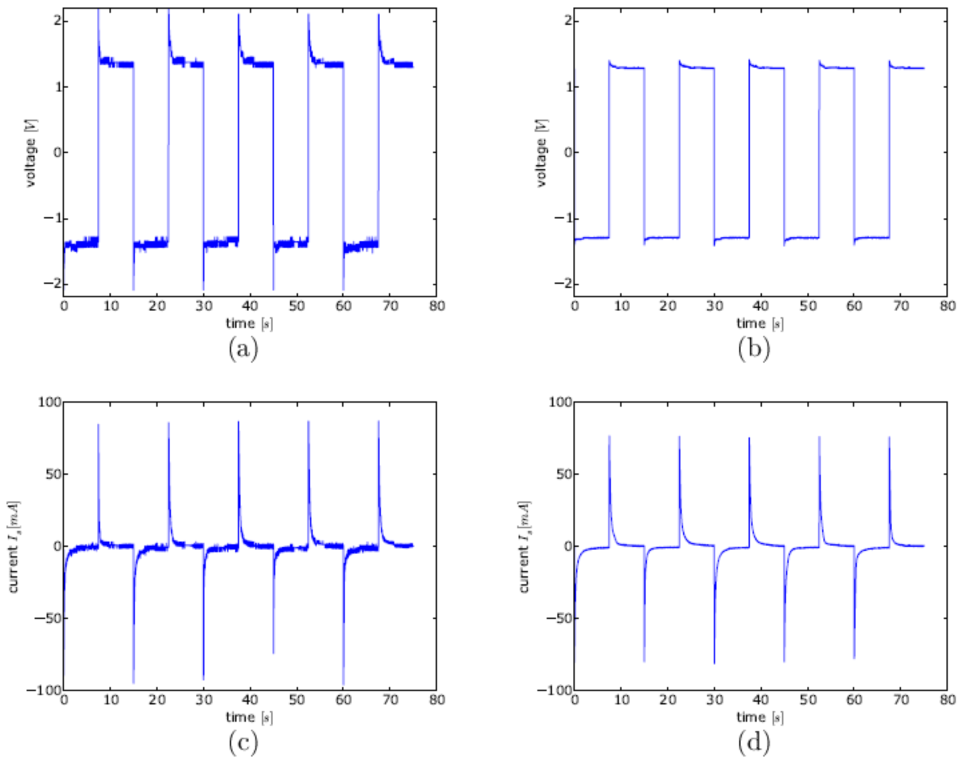


Fig. 7 Voltage transient calculated by classical SDRE controller (a) and SDRE by Lin *et al*. controller (b). The transient of IPMC current under classical SDRE control (c) and SDRE by Lin *et al*. control (d)

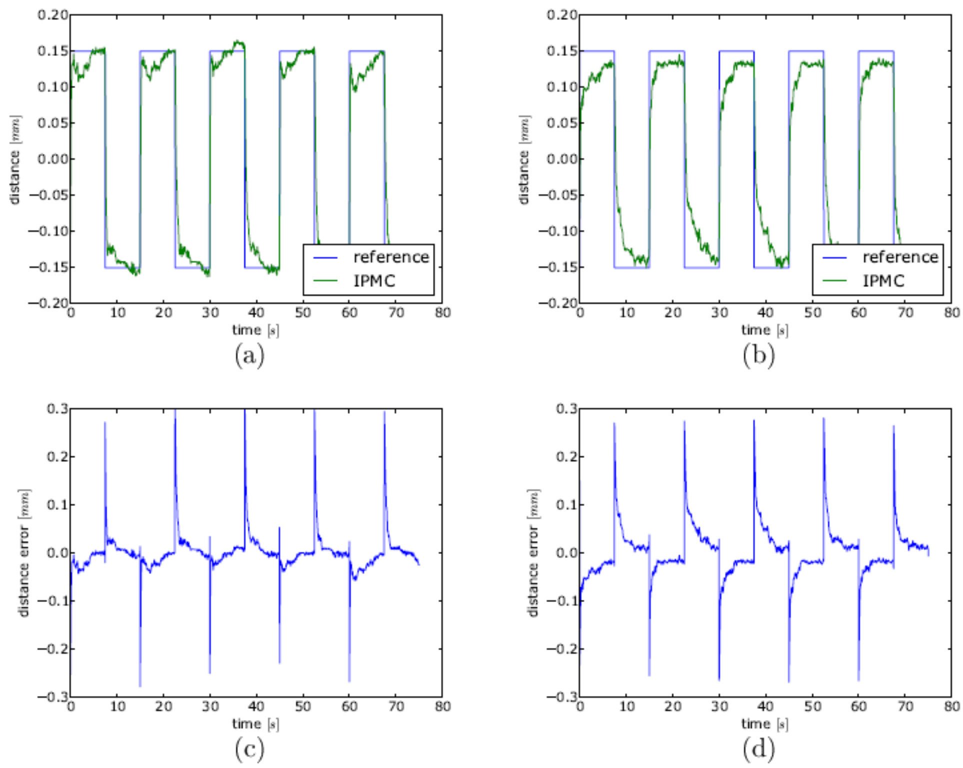


Fig. 8 Comparison of measured distance and its reference value for classical SDRE (a) and SDRE by Lin *et al*. (b). The transient of distance error acquired in the experiment for classical SDRE (c) and SDRE by Lin *et al*. (d)

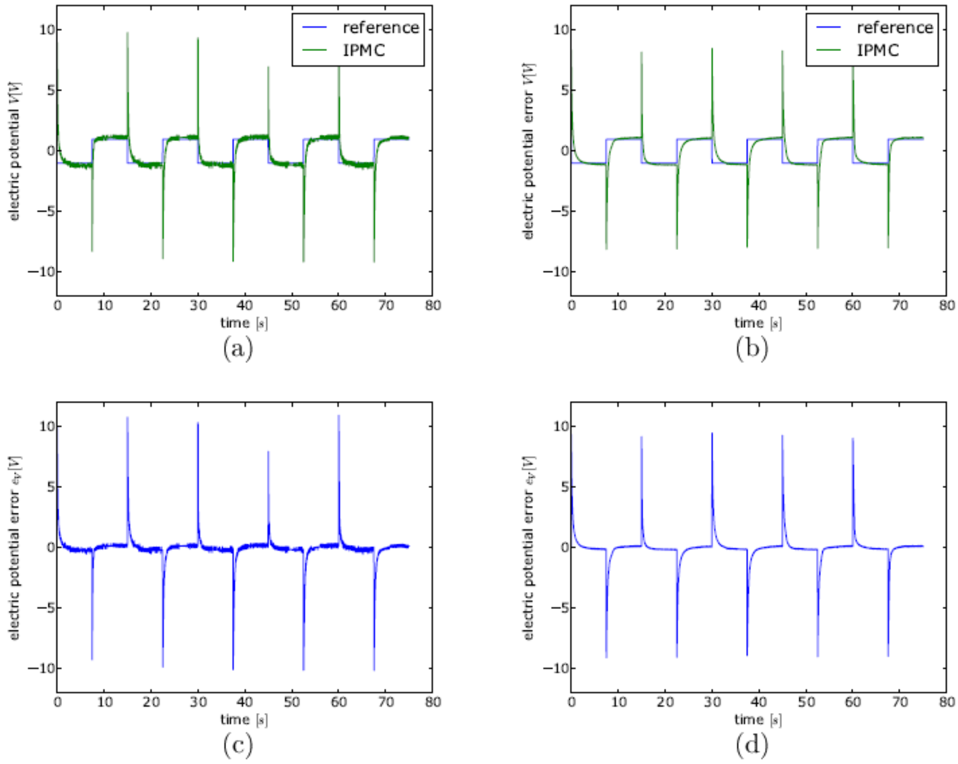


Fig. 9 Comparison of measured electric potential and its reference value for classical SDRE (a) and SDRE by Lin *et al.* (b). Electric potential error for classical SDRE (c) and SDRE by Lin *et al.* (d)

6. Conclusions

Accurate information about the properties and control methods of EAP materials is critical to designers, who are considering the construction of mechanisms or devices using these materials. While some of the EAPs are well known, the IPMCs still require new control methods because have complex, nonlinear behavior associated with the mobility of the cations on the microscopic level.

In this study, the nonlinear model proposed by Chen *et al.* (2009) has been extended to the equations of mechanics and described in the state space, which was a starting point for nonlinear quadratic optimal control design based on State Dependent Riccati Equation. We compare two methods of construction of feasible SDC matrix. Additionally, Multi HGO Observer were used to estimate velocity of IPMC strip based on position, input voltage andoutput current. Both cases were compared and the estimated velocity, position and error signals were demonstrated graphically.

The proposed nonlinear model was validated experimentally and this approach can be used in various applications that require precise control. Future work will be focused on the application to IPMC-actuated biomedical devices and biomimetic robots, which require large deformation of the IPMC.

References

Alamir, A.E. (2015), "Optimal control and design of composite laminated piezoelectric plates", *Smart Struct. Syst.*, **15**(5), 1177-1202.

Banks, H.T., Lewis, B.M. and Tran, H.T. (2007), "Nonlinear feedback controllers and compensators: A state-dependent Riccati equation approach", *Comput. Optim. Appl.*, **37**, 177-218.

Bernat, J. and Kolota, J. (2016), "Sensorless position estimator applied to nonlinear IPMC model", *Smart Mater. Struct.*, **25**(11), 115037.

Bernat, J. and Stepien, S. (2015), "Multi-modelling as new estimation schema for high-gain observers", *Int. J. Control*, **88**(6), 1209-1222.

Besancon, G. (2007), Nonlinear Observers and Applications, volume 363, Springer-Verlag, Inc., Berlin.

Bonomo, C., Fortuna, L., Giannone, P., Graziani, S. and Strazzeri, S. (2007), "A nonlinear model for ionic polymer metal composites as actuators", *Smart Mater. Struct.*, **16**(1), 1-12.

Chen, Z., Hedgepeth, D.R. and Tan, X. (2009), "A nonlinear, control-oriented model for ionic polymer-metal composite actuators", *Smart Mater. Struct.*, **18**(5), 055008.

Çimen, T. (2008), "State-dependent Riccati equation (SDRE) control: A survey", *Proceedings of the 17th World Congress*, IFAC.

Çimen, T. (2010), "Systematic and effective design of nonlinear feedback controllers via the state dependent Riccati equation (SDRE) method", *Annu. Rev. Control*, **34**, 32-51.

Çimen, T. (2012), "Survey of state-dependent Riccati equation in nonlinear optimal feedback control synthesis", *J. Guid. Control Dynam.*, **35**(4), 1025-1047.

Erdem, E.B. and Alleyne, A.G. (2004), "Design of a class of nonlinear controllers via state dependent Riccati equations", *IEEE T. Control Syst. Technol.*, **12**, 133-137.

Farinholt, K.M. (2005), Modeling and Characterization of Ionic Polymer Transducers for Sensing and Actuation - Ph.D. Thesis, Virginia Polytechnic Institute and State University.

Gupta, V., Sharma, M. and Thakur, N. (2011), "Mathematical modeling of actively controlled piezo smart structures: a review", *Smart Struct. Syst*., **8**(3), 275-302.

Hammett, K., Hall, C. and Ridgely, D. (1998), "Controllability issues in nonlinear state-dependent riccati equation control", *J. Guid. Control Dynam.*, **21**(5), 767-773.

Heydari, A. and Balakrishnan, S. (2013), "Path planning using a novel finite horizon suboptimal controller", *J. Guid. Control, Dynam*., **36**(4), 1210-1214.

Heydari, A. and Balakrishnan, S. (2015), "Closed-form solution to finite-horizon suboptimal control of nonlinear systems", *Int. J. Robust Nonlin*., **25**, 2687-2704.

Huang, Y. and Lu, W.M. (1996), "Nonlinear optimal control: Alternatives to Hamilton-Jacobi equation", *Proceedings of the IEEE Conference on Decision and Control*.

Khalil, H.K. (2000), Nonlinear Systems, Prentice Hall, Inc., New Jersey.

Lam, Q.M., Xin, M. and Cloutier, J.R. (2012), "SDRE control stability criteria and convergence issues: where are we today addressing practitioners' concerns?", AIAA Infotech at Aerospace Conference and Exhibit.

Lee, M.H. (2012), "Beam-rotating machinery system active vibration control using a fuzzy input estimation method and LQG control technique combination", *Smart Struct. Syst*., **10**(1), 15-31.

Lin, L.G., Vandewalle, J. and Liang, Y.W. (2015), "Analytical representation of the state-dependent coefficients in the SDRE/SDDRE scheme for multivariable systems", *Automatica*, **59**(6433), 106-111.

Mracek, C. and Cloutier, J.R. (1998), "Control designs for the nonlinear benchmark problem via the state-dependent Riccati equation method", *Int. J. Robust Nonlin*., **8**, 401-433.

Newbury, K. (2002), Characterization, Modeling, and Control of Ionic Polymer Transducers - Ph.D. Thesis, Virginia Polytechnic Institute and State University.

Newbury, K. and Leo, D. (2002), "Electromechanical modeling and characterization of ionic polymer benders", *J. Intel. Mat. Syst. Str*., **13**, 51-60.

Shahinpoor, M. (1999), "Electromechanics of ionoelastic beams as electrically controllable artificial muscles", Smart Structures and Materials : Electroactive Polymer Actuators and Devices, **3669**, 109-121.

Wang, Z., Huang, B. and Unbehauen, H. (2001), "Robust H1 observer design of linear time-delay systems with parametric uncertainty", *Syst. Control Lett*., **42**, 303-312.

Yun, K. (2006), A Novel Three-finger IPMC Gripper for Microscale Applications - Ph.D. Thesis, Texas A&M University.

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A. IPMC model details

Nonlinear capacitance is defined by the following formula

$$C_1(V)=\frac{dQ}{dV}=S\kappa_e\frac{\Gamma'(V)}{\sqrt{2\Gamma(V)}} \tag{38}$$

where Q is the total charge as a function of V and $\Gamma'(V)$ is defined as follows

$$\Gamma'(V)=\frac{b}{a}\left(1-\frac{e^{aV}-1}{aV}\right)\frac{e^{aV}-1-aVe^{aV}}{(e^{aV}-1)^2} \tag{39}$$

$$a\overset{\Delta}{=}\frac{F\left(1-C^-\Delta V\right)}{RT} \tag{40}$$

$$b\overset{\Delta}{=}\frac{F^2C^-\left(1-C^-\Delta V\right)}{RT\kappa_e} \tag{41}$$

where F is Faraday constant, C^- is anion concentrations, R is gas constant, T is temperature. Pseudocapacitance C_a to define adsorption process is as follows

$$C_a(V_a)\overset{\Delta}{=}\frac{q_1SF}{RT}\frac{K_1c^{H+}e^{-\frac{V_aF}{RT}}}{\left(K_1c^{H+}+e^{-\frac{V_aF}{RT}}\right)^2} \tag{42}$$

where: q_1 is physical constant (for H on polycrystalline Pt), K_1 is the chemical rate constant for electrochemical surface process and c^{H+} is the concentration of H^+.

The DC current can be approximated by a series of polynomial function $Y(V)$, and the coefficients Y_1, Y_2, Y_3 can be identified Chen *et al*. (2009).