Smart Structures and Systems, Vol. 2, No. 3 (2006) 237-251
DOI: http://dx.doi.org/10.12989/sss.2006.2.3.237

Applications of piezoelectric sensors in geotechnical engineering

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(Received April 29, 2005, Accepted February 6, 2006)

Abstract. Piezoelectric sensors have many applications in geotechnical engineering, especially in characterizing soils through measurement of wave velocities. Since mechanical properties of a material are closely associated with wave velocities, piezoelectric sensors provide a reliable and non-destructive method for the determination of soil properties. This paper presents results of recent research on measuring stiffness of a wide range of soils such as clay, sand, and gravel, characterizing anisotropic properties of soil induced by external loading, measuring stiffness of base and subgrade materials in the pavement, determining soil properties in a centrifuge model during the flight of a centrifuge, and understanding wave propagation in granular materials under micro-gravity environment using this technique.

Keywords: geotechnical engineering; piezoelectric sensors; stiffness; wave velocity.

1. Introduction

In recent years, the technique of using piezoelectric sensors to measure wave velocities in granular materials has been developed. These sensors are made of piezoceramic materials, in which as an electrical excitation is applied to a transmitter element, it produces mechanical vibrations, gnerating shear or primary waves in the soil. The mechanical vibration starts almost instantaneously when an electrical pulse is applied. Similarly, as a wave receiver, a mechanical vibration of the element leads to an electrical output. The electrical output starts practically at the instant when the mechanical vibration is initiated. The arrival of the wave is shown by a sudden increase in electric voltage on the receiver. Therefore, the velocity of primary and shear waves can be determined by measuring its travel time and the distance between the wave transmitter and receiver. The sensors that generate and receive shear waves are called bender elements while the sensors for primary waves are called extender elements. Each sensor consists of two pizeoceramic plates bonded together onto a center shim as shown in Fig. 1. The element arranged in series as shown in Fig. 1(a) can generate an output voltage twice as much as by an individual layer under the same level of vibration. This arrangement is good for a receiver. On the other hand, for the same motion, a 2-layer element arranged for parallel operation, as shown in Fig. 1(b), needs only half the voltage. Thus, an applied electrical field causes maximum deformation, making this arrangement suitable for a transmitter. Assuming the end of an element is fixed, the free-

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end deflection x and the maximum force output F by a bender element shown in Fig. 1 (b) (when used as a transmitter and with no soils around) are

$$x = 3d(L/t)^2(1 + t_s/t)VK \tag{1}$$

$$F = 0.75\ Ed(t/L)\ (1 + t_s/t)WVK \tag{2}$$

in which d is the piezoelectric strain constant, E is the elastic modulus of the piezoceramic material, L is the cantilever length of the element, t is the thickness of the sensor, t_s is the thickness of the center shim, V is the voltage applied, K is an empirical number, and W is the width of the sensor. Similarly for a bender element shown in Fig. 1(a) (when used as a receiver and without soils around), the output voltage is

$$V = 1.5\ g\ [(FL)/(Wt)]\ (1 - t_s^2/t^2)K \tag{3}$$

in which g is the piezoelectric voltage constant.

When piezoelectric sensors are buried inside soils, as is the case in the applications reported here, the relationships between deflection, force, and voltage output become much more complicated due to the dynamic interaction between an element and surrounding soils. However,

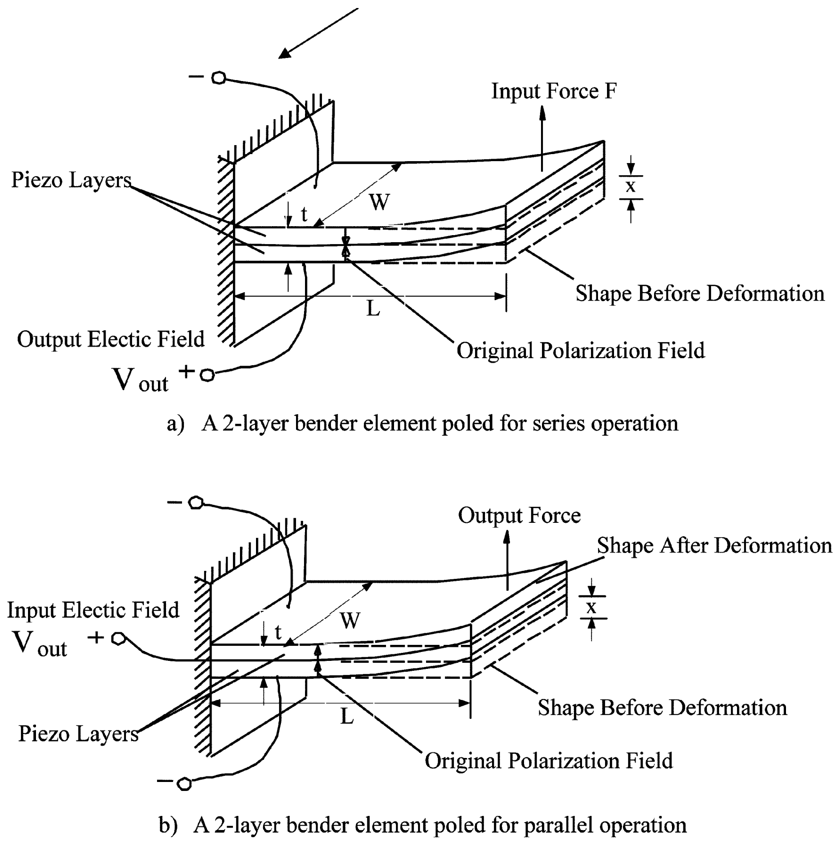


Fig. 1 Bender element transmitter and receiver

for the measurement of wave velocity, we do not need to know the exact relationships. All we need to know is when the mechanical vibration starts on a transmitter (which corresponds to the moment when the voltage is applied) and when the waves reach a receiver (which corresponds to the moment when an output voltage is recorded). This can be done quite easily with today's modern electronic devices.

Since the maximum strain generated by a sensor in the surrounding soil is of the order of 10^{-3} % as reported by Dyvik and Madshus (1985), the stress-strain relationship is well within the elastic range of granular materials. An advantage of this test is that the measurement and computation are more direct and simpler than those in a resonant column test. This technique has been used by a number of researchers such as Dyvik and Madshus (1985), Thomann and Hryciw (1990), Jovicic, *et al.* (1996), Viggiani and Atkinson (1995), Hryciw and Thomann (1993), Jovicic and Coop (1998), Zeng and Ni (1999), and Zeng, *et al.* (2003) to measure the stiffness and stress ratio in granular materials (mainly soils) in the laboratory. The sensors can be coated with waterproof and protective materials so that they can be used in harsh environments (such as underneath water). Therefore, there is a wide range of possible applications for piezoelectric sensors in geotechnical engineering.

Supposing that the distance between the tip of a wave transmitter and the tip of a wave receiver is L and the time for the shear wave and primary wave to travel this distance is t_s and t_p, respectively, the average velocities of shear and primary waves are

$$v_s = L/t_s \tag{4}$$

$$v_p = L/t_p \tag{5}$$

The elastic shear modulus $G_{\max}$ and constrained modulus M of a soil would be

$$G_{\max} = \rho v_s^2 \tag{6}$$

$$M = \rho v_p^2 \tag{7}$$

in which ρ is the mass density of the granular material. The relationship between Young's modulus, constrained modulus, and shear modulus is

$$E = [(3M/G_{\max} - 4)/(M/G_{\max} - 1)]G_{\max} \tag{8}$$

The Poisson's ratio of the material can also be calculated as

$$\mu = [(M/G_{\max} - 2)/(2M/G_{\max} - 2)] \tag{9}$$

Since the elastic moduli of a soil are related to the density, effective confining pressure, and stress history, this technique has a wide range of applications in geotechnical engineering such as measuring elastic modulus of sand (Zeng and Ni 1999), clay, and gravelly soils (Zeng and Haselton 2003), determining stress-induced anisotropy in soils (Zeng and Ni 1999), characterizing soil properties in the foundation of a pavement (Zeng, *et al.* 2003), measuring soil properties during the flight of a centrifuge (Fu, *et al.* 2004), and study the properties of granular materials under micro-gravity. Due to page limits, the results of these studies are summarized here but the details on the tests and applicability of the technique can be found in the above mentioned references.

2. Measurement of elastic modulus of soils

Traditionally, elastic moduli of soils are measured in the laboratory using a resonant device (Drnevich 1985) in which a column of soil is vibrated by a sinusoidal exciting force with varying frequencies until the resonant frequency is found. Then the elastic moduli (including Young's modulus and shear modulus) can be calculated. Using piezoelectric sensors, wave velocity can be measured directly in the soil and elastic moduli can be calculated using the equations listed above. A test setup that incorporating bender elements into a resonant column device is shown in Fig. 2. Then the experimental results from these two methods can be used to check against each other.

As shown in Fig. 2, a wave generator is used to trigger the vibration of a transmitter, which produces waves in the soil. A receiver on the other side of the sample produces an electric output when the wave has arrived. Both the driving signal and receiving signals can be recorded on an oscilloscope, from which the travel time of the wave can be determined. An example of signals recorded by a bender transmitter and receiver is shown in Fig. 3. The technique of reversing poles is used to make sure the received signals are indeed produced by shear waves. As the distance between the transmitter and receiver can be measured, the wave velocity can be calculated. Based on the study by Dyvik and Madshus (1985), a test using piezoelectric sensors produces results similar to that of a resonant column test (within 5% of each other). For instance, the shear modulus of a sand measured by bender elements and resonant column device conducted recently in our laboratory is shown in Fig. 4, which shows very good agreement. Thus this technique is reliable.

One advantage of using piezoelectric sensors over resonant column device is that it can test soils with large particles such as gravelly soil. Typical resonant column device can test a soil sample with a diameter of about 6.35 cm. Thus it is not suitable for tests on soils with gravels that can have particle size as large as a few centimeters. On the other hand, it is quite easy to build large size samples when using piezoelectric sensors. One example of such tests is report by Zeng and Haselton (2003). They

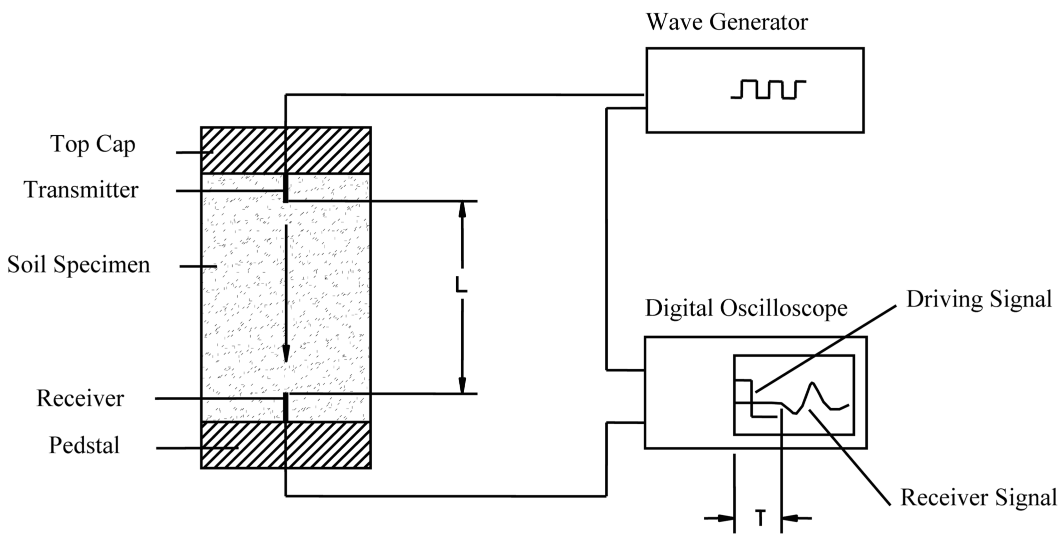


Fig. 2 Test setup for incorporating piezoelectric sensors in a resonant column device

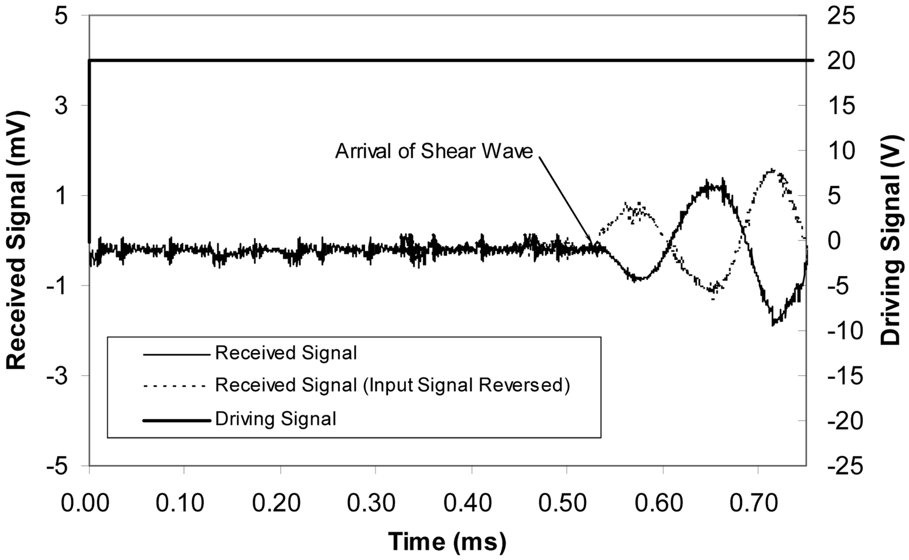


Fig. 3 Example of signals recorded by transmitter and receiver

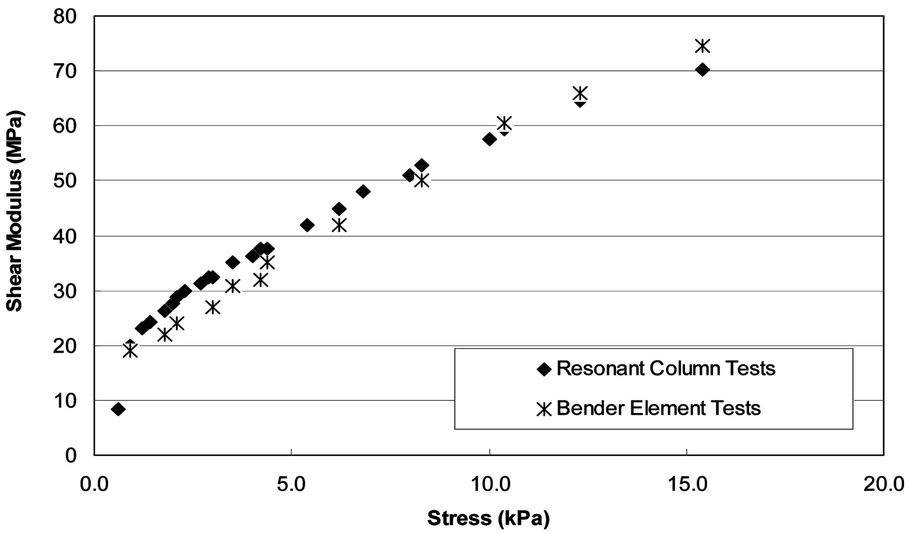


Fig. 4 Comparison of G_{max} measured by the resonant column tests and bender element tests

built a large oedometer with an inside diameter of 30.7 cm, which is large enough to test gravelly soils. Typical results are shown in Fig. 5, which presents the shear modulus measured in different directions in a soil sample with 50% gravel content. It shows a clear dependence of shear modulus on axial stress as the modulus increases nonlinearly with axial stress. It also shows that shear modulus is different in different inclined directions with the shear modulus in the vertical shear plane the highest. In addition, it shows that during unloading, the shear modulus is higher than that during loading at the same stress level.

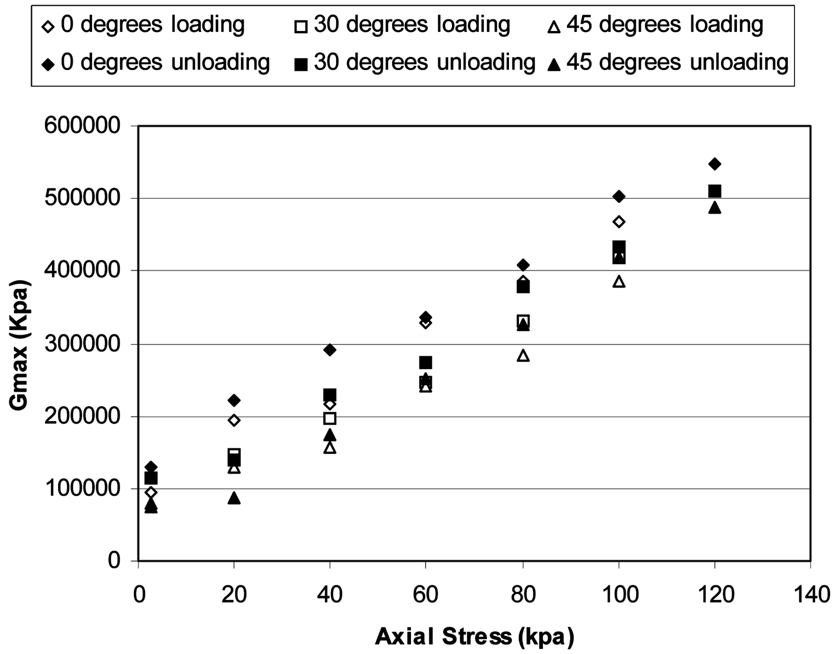


Fig. 5 Experimental results of shear modulus on a soil with 50% gravel (0 degree means wave propagates in the vertical direction, 30 and 45 degrees means wave propagates in directions with 30 and 45 degrees inclination to the vertical, respectively)

3. Stress-induced anisotropy in soils

One of the most difficult problems in elastic properties of soils is the stress-induced anisotropy. It states that even with a fabric isotropic soil, the elastic modulus of soil can show anisotropy when the external loading is not isotropic. In the past, measurement of soil stiffness in different directions is quite difficult to be carried out. However, using piezoelectric sensors, all that need to be done are measuring shear wave velocities in different directions, which is quite straightforward. Using a test setup shown in Fig. 6, it is possible to measure shear wave velocities in four different directions and hence determine the shear modulus in the corresponding shear planes. In the soil sample, two pairs of bender elements are installed in one vertical plane. Using the pair of elements in the vertical direction, it is possible to measure shear modulus in the vertical plane. Similarly, the pair of elements in horizontal direction would determine the shear modulus in the horizontal shear plane. In addition, when waves generated by the transmitter in the horizontal direction are received by the receiver in the vertical direction, shear modulus in an inclined direction is derived. Shear modulus on another inclined shear plane can be derived when waves generated by the transmitter in the vertical direction are received by the receiver in the horizontal direction. Therefore, in one test setup, one can measure shear modulus in four shear planes, which can be used to investigate stress-induced anisotropy in soils (Zeng and Ni 1999). Experimental results of shear modulus in four different shear planes in a remoulded saturated clay sample (which has isotropic fabric) are shown in Fig. 7, which indicate strong anisotropy induced by external loading.

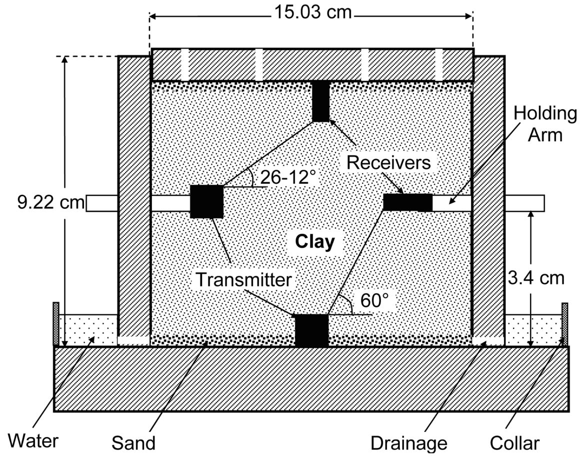


Fig. 6 Experimental setup used to study stress-induced anisotropy in clay

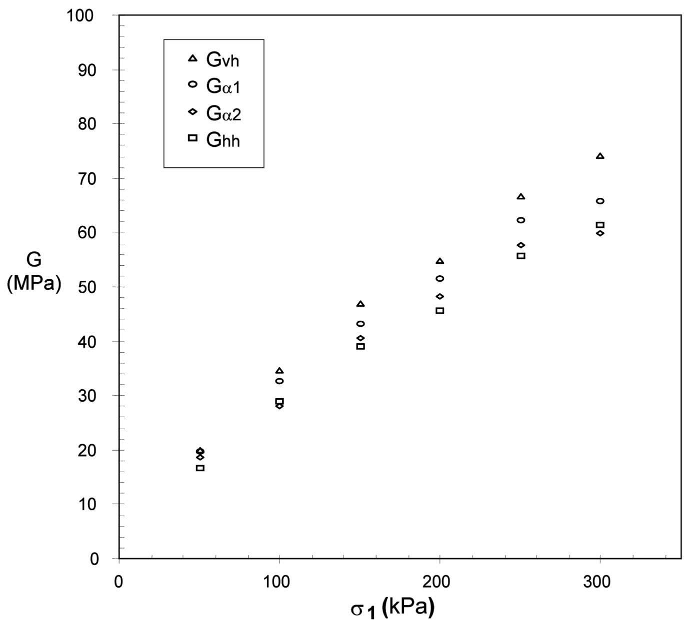


Fig. 7 $G_{\max}$ on four shear planes versus effective vertical stress (G_{vh}–verical transmitter to vertical receiver, $G_{\alpha 1}$–vertical transmitter to horizontal receiver, $G_{\alpha 2}$–horizontal transmitter to vertical receiver, G_{hh}–horizontal transmitter to horizontal receiver)

4. Characterization of pavement foundation

Base and subgrade soil stiffness (shear modulus, elastic moduls, and Poisson's ratio) is an important parameter in the design and construction of highway pavement. During and after pavement construction, it is very important and cost-effective to have a reliable technique that can measure the stiffness of in-situ base and subgrade layers accurately and quickly. For a pavement foundation under

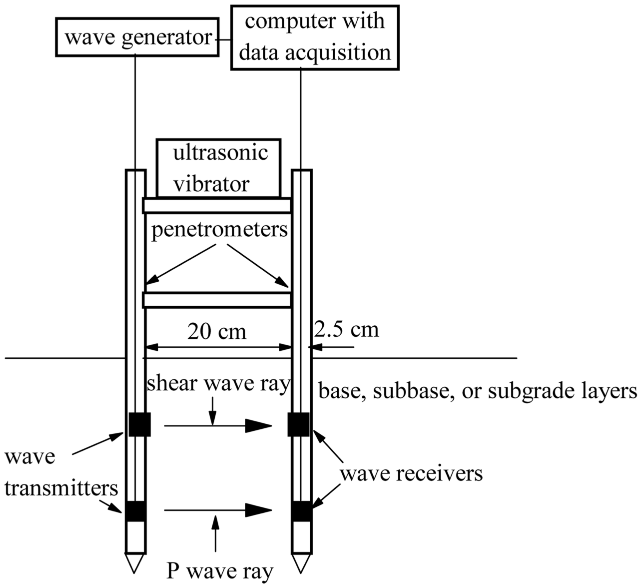


Fig. 8 Experimental setup for a cone penetrometer equipped with piezoelectric sensors

construction, the quick measurement will allow engineers to check whether the construction is up to the specification. For pavement design especially the recently adopted mechanistic design approach, the actual stiffness parameters measured in the field can be used to evaluate the performance. The new highway construction guide proposed by AASHTO (American Association for State Highway and Transportation Officials) requires such measurements be conducted. At present, there is no such device available in the field. Engineers have to conduct laboratory tests which may take days to finish. In addition, it is not possible to create conditions in the laboratory identical to that in the field. In many cases, engineers have to rely on crude empirical formulae or index parameters to estimate these parameters.

A new field-testing technique has been developed to measure small-strain moduli (elastic modulus and shear modulus) and Poisson's ratio of soils utilizing piezoelectric sensors. The device used in this study is shown in Fig. 8, which includes a pair of cone penetrometers, and each is fitted with one bender and one extender element. It can be driven into the base, subbase and subgrade layers using an ultrasonic vibrator to prevent damage to the sensors. The sensors are housed inside the penetrometers and coated with waterproof materials so that they can be used in harsh environment in the field. One set of the elements is used as wave transmitters while the other as wave receivers. The distances between the transmitters and receivers are controlled by the frame supporting the penetrometers and can be measured accurately. A pulse generated by a function generator is used to activate the transmitters. Vibration of the transmitters produces P and S waves that propagate through the soil in the horizontal direction and are captured by the receivers. Signals produced by the receivers are recorded on a computer with data acquisition system, from which the travel time of the waves from the tip of a transmitter to the tip of a receiver can be determined. Then from the measured P and S wave velocities elastic modulus, shear modulus, and Poisson's ratio of soils can be determined. The technique is applicable to a wide range of materials used in base layer construction such as compacted aggregates, cemented aggregates mixtures, lime stabilized soil, as well as natural soils.

For a pavement under construction, the penetrometers can be driven into the compacted sublayer and the natural soils below to measure the stiffness parameters at different depths. The test is simple enough that a technician with some training can carry it out. The test can be completed within a couple of minutes. Therefore, engineers at the site can check whether the construction is up to the design standard or use the measured values to conduct mechanistic design for the pavement. For existing pavement, two pilot holes can be cored through the upper asphalt or Portland pavement layer and then the penetrometers can be pushed in to determine the stiffness parameters of the pavement, which has subjected to many cycles of traffic load and gone through years of weather changes. This is very useful information before re-surfacing of the pavement to determine the thickness of the pavement.

Laboratory tests were conducted to evaluate the applicability of the new device. Tests were conducted on specimens of soil prepared in a steel container 25.4 cm in diameter and 50.8 cm in height. The steel container has enough lateral stiffness to create a K_0 stress condition that is typical in the field. Test specimens were prepared by pouring the soils into the container using a hopper. The height and rate of pouring were kept constant so as to achieve uniformity in the sample. After a sample is ready, the cone penetrometer is pushed into the soil slowly until the piezoelectric sensors reach the specified depth. Tests are then conducted to measure the velocities of *S* and *P* waves. Then, the cone penetrometer is pushed to deeper positions in the soil to obtain other measurements. The sample container used in the laboratory limited the measurement of all three parameters for a depth of about 25 cm, while the constrained modulus could be obtained at deeper locations and the shear modulus at shallower locations. For tests in the field deeper sections of subgrade can be characterized but in the laboratory, the depth was restricted by the height of the model container. Measurement of velocities of *P* and *S* waves was made at the same locations (though not at the same time) so that Poisson's ratio at that depth can be calculated. We have also successfully conducted tests in gravelly soils such as crushed stones.

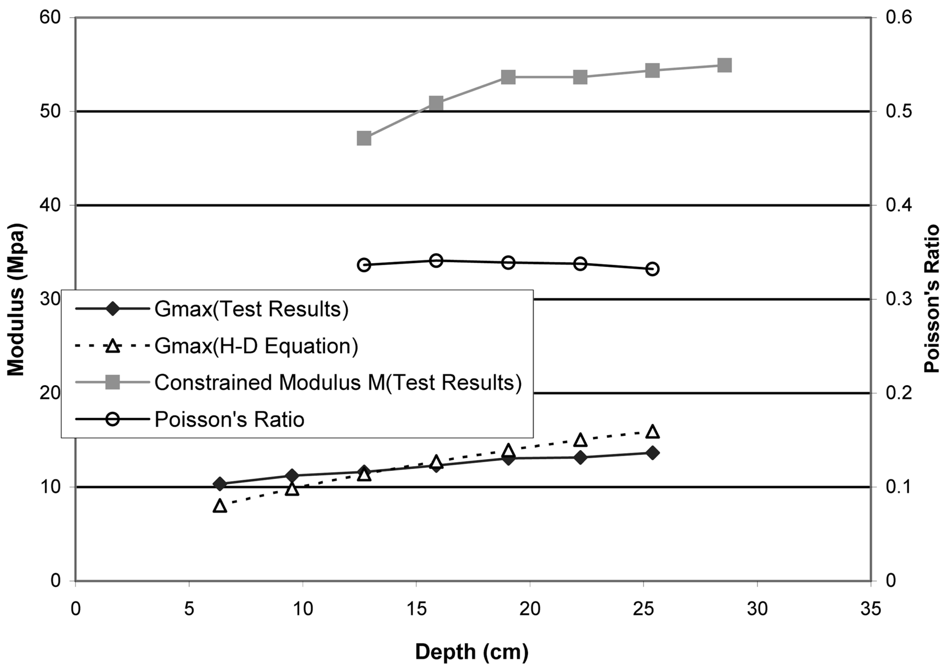


Fig. 9 Test results on Delaware clay using cone penetrometer (dry density = 1590 kg/m^3, void ratio = 0.70)

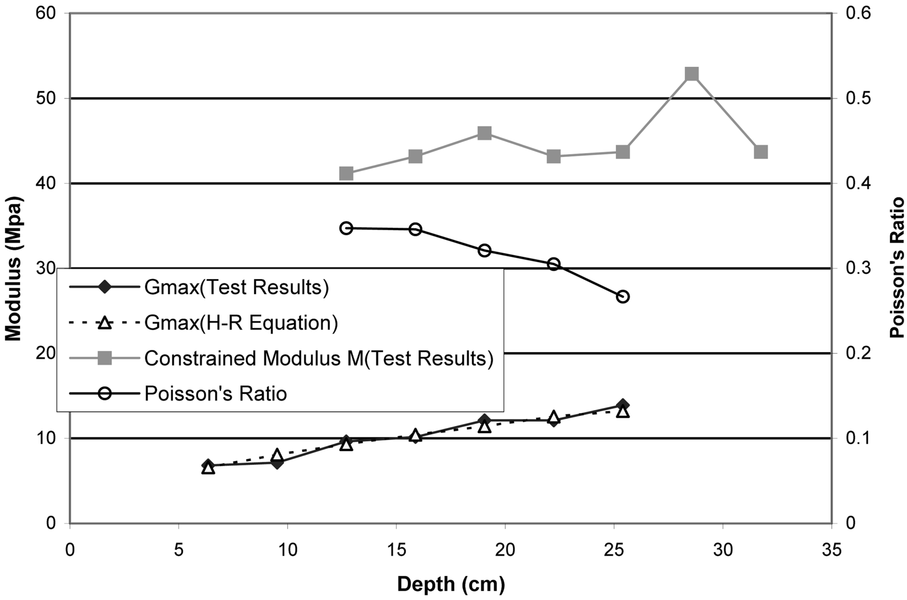


Fig. 10 Test results on Nevada sand using cone penetrometer (dry density = 1600 kg/m^3, void ratio = 0.69)

To ensure the repeatability of experimental data, each test was conducted on a second sample prepared following the same procedures and very good repeatability of results was achieved. Typical results are shown in Figs. 9 and 10. The sample of Delaware clay had a dry density of 1590 kg/m^3 and a void ratio of 0.70. The sample of Nevada sand was prepared with a dry density of 1600 kg/m^3 and a void ratio of 0.69. From the data recorded by the sensors on the cone penetrometer, it was found that the shear modulus and constrained modulus increase gradually with depth as the effective confining pressure increases. The Poisson's ratio of Delaware clay was about 0.33, which is reasonable for a dry clay. For Nevada sand, Poisson's ratio varies between 0.27 to 0.35, which is quite reasonable compared with typical values for dry sands. Also shown in the figures are the results of shear modulus calculated using two commonly known empirical formulae: Hardin and Richart (1963) equation for sand and Hardin and Drnevich (1972) for clay, which show very good agreement with experimental results.

5. Measuring soil properties during the flight of a centrifuge

Centrifuge modeling provides a useful and powerful experimental tool for the study of geotechnical problems. It uses small-scale models subjected to a centrifugal acceleration of many times the gravitational acceleration to simulate prototype problems that are difficult to test at full scale. The principles of centrifuge modeling are well understood now. By reducing the dimensions of a prototype structure by a factor of N while at the same time increasing the body force induced by centrifugal acceleration by the same scale in a model, the stresses in the model will be the same as those at the corresponding points in the prototype. Therefore, the model is expected to have a response similar to that of the prototype, if the scaling laws are strictly followed.

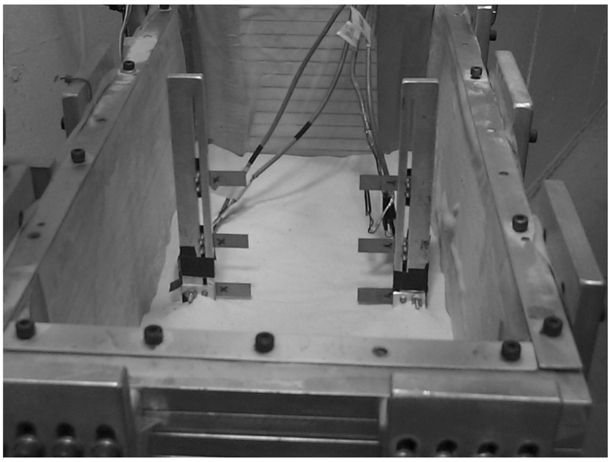


Fig. 11 Bender elements in a centrifuge model

Measuring soil properties during the flight of a centrifuge model is important for the interpretation and analysis of experimental data. Traditionally, measurements of soil properties in a centrifuge model were carried out before or after the flight of a centrifuge model. However, as soil properties depend on the effective stresses in soil, the parameters measured under 1g conditions do not represent the situation during a centrifuge test. Recently, Fu, *et al.* (2004) described the application of the bender element technique to the measurement of shear wave velocities during the flight, spin-up and spin-down of a centrifuge. Three model tests were conducted on a dry sand deposit. The experimental setup (before the sand layer was prepared) is shown in Fig. 11. The shear wave velocities in horizontal and inclined planes were measured, which were used to calculate the shear moduli in the corresponding shear planes. Because the wave transmitter and receiver are placed at the same depth, the wave velocity

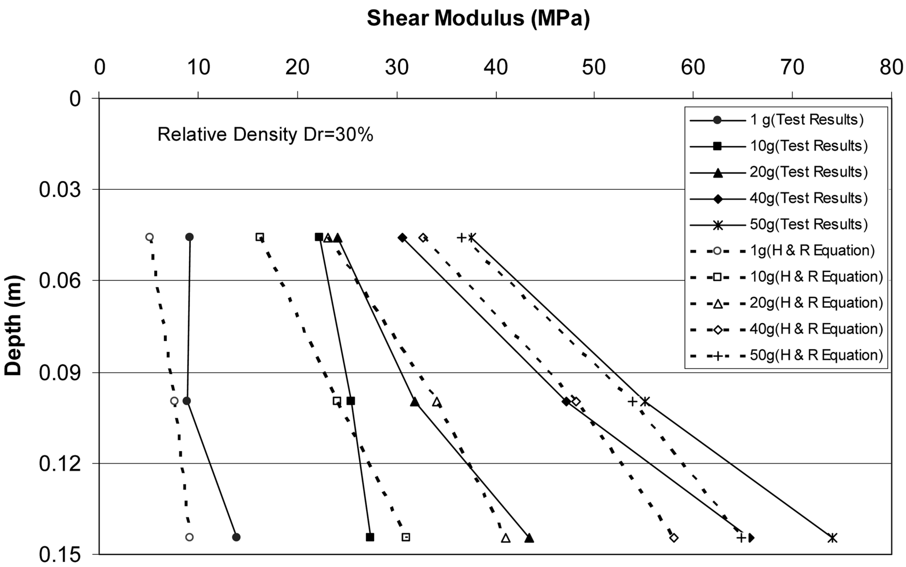


Fig. 12 Comparison between test results and results from empirical equation

measured is a good indication of soil stiffness at the depth. In the past, when a transmitter and a receiver were placed in vertical direction, only average wave velocity can be measured.

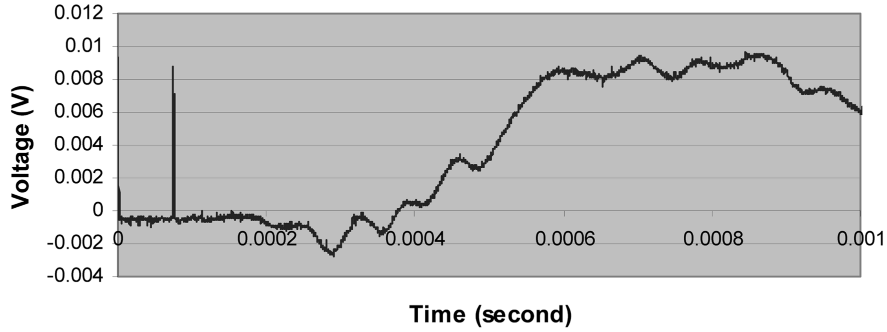
Examples of experimental data are presented in Fig. 12, which shows that shear modulus increase with depth and centrifugal acceleration. Test results have good repeatability and agree well with calculations conducted with a commonly used empirical equation proposed by Hardin and Richard (1963). This study shows that the bender element technique has a great potential as a reliable and inexpensive method to measure an important soil property during the flight of a centrifuge.

6. Determining properties of granular material under microgravity

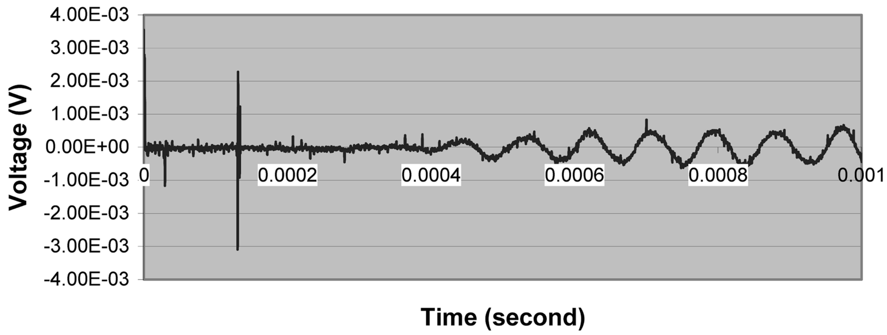
Wave propagation in granular materials depends critically on inter-particle contacting stress. Under normal gravity, it is widely observed that wave velocity increases with confining pressure. In some physical processes such as the liquefaction of soils or flow of two-phase materials (such as a solid powder and liquid mixture), the contacting stresses between solid particles are extremely small. The process to create such state is highly unstable under normal gravity. Therefore, it is difficult to measure properties of granular materials under such conditions. In addition, to understand behavior of granular materials under micro-gravity is very important for NASA's mission of exploration in the outer space.

It is quite straight forward to achieve a near zero contacting stress state for a uniform sample of granular materials in the NASA's Drop Tower or KC-135 facility. With support from NASA Glenn Center and Case Western Reserve University, tests were conducted on the 2.2-second Drop Tower at NASA Glenn Center. Details about the experiments were reported by Zeng, *et al*. (2005). The granular material used was glass beads with a diameter of 6 mm. The cell containing the glass beads is made of aluminum alloy. The dimensions of the cell are 9.83 cm in diameter and 7.3 cm in height. There is one piezoelectric sensor attached to the base of the cell, which was used as the wave transmitter. On the cap of the cell, three piezoelectric sensors were used as wave receivers, making it possible to measure wave velocities in three directions. The distance between the tip of the wave transmitter and receiver was about 4 to 6 cm. The natural frequency of elements was 12,000 Hz. The test setup on the drop tower rig

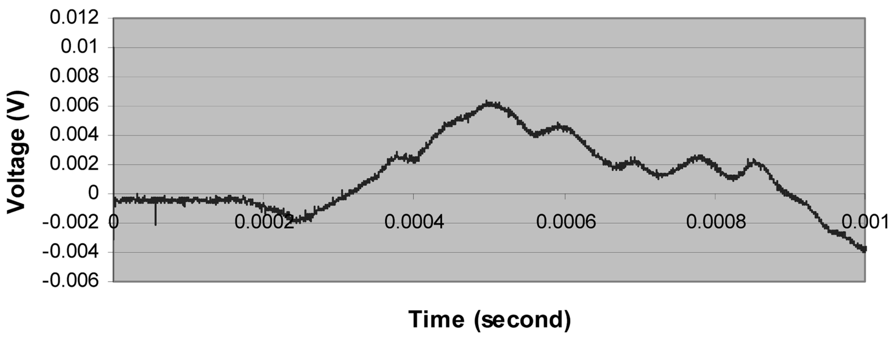
Fig. 13 Assembly of test setup on the drop tower rig at NASA Glenn



a) Output from an extender element under 1g condition before drop test



b) Output from an extender element under micro-gravity



c) Output from an extender element under 1g condition after drop test

Fig. 14 Text results of wave propagation in glass beads under 1g and micro-gravity

at NASA Glenn is shown in Fig. 13. During a test, the rig is dropped from the top of the tower and the free fall lasts for about 2.2 seconds, creating a micro-gravity environment in the sample. At about 1.2 seconds into the free fall, the tests are conducted. Since each test only lasts for a fraction of a second, the facility provides enough time of micro-gravity environment to characterize materials properties.

The test generates some very interesting results. The results of one test on a loosely compacted sample of glass beads are shown in Fig. 14. The experiments show that it is possible to measure *P* wave velocity under micro-gravity environment, which is not surprising. For example, for the results shown in Fig. 14, the calculated *P* wave velocity under 1 g condition was 230 m/s, while the *P* wave velocity under micro-gravity was 116 m/s, a significant reduction. After the drop test, the *P* wave velocity was measured again under 1 g condition, which was increased to 256 m/s as the result of increased density due to compaction. There were obvious differences in the frequency characteristics of the receiving signals as well. At 1 g, the received signals showed two dominant frequencies, one possibly due to the natural frequency of the sensor itself while the other due to surrounding granular materials. On the other hand, under micro-gravity, the receiving signal only showed the frequency of the sensor itself since the granular materials now offer little restriction to the vibration of the sensor. It was also possible to measure shear wave velocity, which is contradicting most existing theories on shear wave propagation. One possible explanation is that shear force chain is transmitted in the granular material through dilation of the material under shear, as has been observed in triaxial test conducted on the Space Shuttle (Sture, *et al.* 1998).

7. Conclusions

Piezoelectric sensors have been successfully used by geotechnical engineers to determine the stiffness of a wide range of soils under different conditions as demonstrated by the examples described in the paper. The method is reliable, non-destructive, relatively simple, economical, and quick to apply. Experimental results on stiffness of soils are repeatable and the accuracy is quite good when comparing with results produced by other methods. Therefore, it is expected that it will become a very popular testing method in geotechnical engineering. The research results reported here show that:

1) This technique can be applied to different types of soils including clay, sand, and gravels. It can be incorporated into devices for triaxial test, oedometer test and resonant column test. Thus it is superior to the traditionally used resonant column test.
2) Since this technique can measure shear wave velocity in different shear planes, it provide a convenient way to investigate anisotropic properties of soils induced by anisotropy in fabric or external loading.
3) When mounting on cone penetrometers, piezoelectric sensors can quickly determine the stiffness parameters of subbase materials of a pavement, and hence improve the design and construction quality control of pavements.
4) This technique has been successfully adopted on a centrifuge. It provides a reliable method to determine the properties of soils in a centrifuge model during the flight of a centrifuge. Therefore, it can benefit data analysis and interpretation.
5) Since the test only takes a short period of time to finish, it is possible to use piezoelectric sensors in tests using the drop tower at NASA Glenn to study mechanical properties of granular materials under micro-gravity.

Like any other experimental technique, the method of using piezoelectric sensors also has its limitations. Firstly, the sensors need to be in contact with a number of soil particles to generate strong enough signals, which puts a limitation on how small the size of a sensor can be reduced to. Secondly, the sensor is made of piezoceramic which is brittle. When working under harsh environment such as in gravel and crushed stone, the sensors could be damaged. Finally, the presence of moisture can cause electric short-circuit and damage the sensor. Therefore, when working under water table, several layers of water-proofing coating is needed to apply to the sensors, which will reduce the sensitivity of response.

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