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Free vibration analysis of embedded nanosize FG plates using a new nonlocal trigonometric shear deformation theory

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Abstract. In this work, free vibration analysis of size-dependent functionally graded (FG) nanoplates resting on two-parameter elastic foundation is investigated based on a novel nonlocal refined trigonometric shear deformation theory for the first time. This theory includes undetermined integral variables and contains only four unknowns, with is even less than the conventional first shear deformation theory (FSDT). Mori–Tanaka model is employed to describe gradually distribution of material properties along the plate thickness. Size-dependency of nanosize FG plate is captured via the nonlocal elasticity theory of Eringen. By implementing Hamilton's principle the equations of motion are obtained for a refined four-variable shear deformation plate theory and then solved analytically. To show the accuracy of the present theory, our research results in specific cases are compared with available results in the literature and a good agreement will be demonstrated. Finally, the influence of various parameters such as nonlocal parameter, power law indexes, elastic foundation parameters, aspect ratio, and the thickness ratio on the non-dimensional frequency of rectangular FG nanoscale plates are presented and discussed in detail.

Keywords: nonlocal elasticity theory; FG nanoplate; free vibration; refined theory; elastic foundation

1. Introduction

Nanostructures are widely employed in micro- and nano-scale devices and systems such as biosensors, atomic force, microscopes, micro-electro-mechanical systems (MEMS) and nanoelectro-mechanical systems (NEMS) because of their superior mechanical, chemical, and electronic characteristics. In such applications, small scale influences are often demonstrated. These influences can be captured via size-dependent continuum mechanics such as strain gradient theory (Nix and Gao 1998, Lam *et al.* 2003, Aifantis 1999), modified couple stress theories (Koiter 1969, Mindlin and Tiersten 1962, Toupin 1962), and nonlocal elasticity theory (Eringen 1972). Among these theories, the nonlocal elasticity theory is introduced by Eringen is the most commonly employed theory. Contrary to the local theories, which consider that the stress at a point is a function of strain at that point, the nonlocal elasticity theory considers that the stress at a point is a function of strains at all points in the continuum (Eltaher *et al*. 2016a, b). Functionally graded materials (FGMs) are the novel generation of new advanced composite materials, whose mechanical characteristics are varied smoothly in the spatial direction microscopically to improve the overall structural performance (Bouderba *et al*. 2013, Tounsi *et al*. 2013, Zidi *et al*. 2014, Ait Amar Meziane *et al*. 2014, Bousahla *et al*. 2014, Sallai *et al*. 2015, Meksi *et al*. 2015, Ait Atmane *et al.* 2015, Bouchafa *et al*. 2015, Tebboune *et al*. 2015, Bellifa *et al*. 2016, Bouderba *et al*. 2016, Beldjelili *et al*. 2016, El-Hassar *et al*. 2016, Bousahla *et al*. 2016). Nanotechnology is also interested with fabrication of functionally graded (FG) materials and engineering structures at a nanoscale, which enables a novel generation of materials with revolutionary characteristics and devices with enhanced functionality. Recently, the use of FG materials has broadly been spread in micro- and nano-structures such as micro- and nano-electromechanical systems (MEMS and NEMS) (Witvrouw and Mehta 2005, Lee *et al*. 2006, Hasanyan *et al*. 2008, Mohammadi-Alasti *et al*. 2011, Zhang and Fu 2012), thin films in the form of shape memory alloys (Fu *et al*. 2003, Lu *et al*. 2011), and atomic force microscopes (AFMs) to achieve high sensitivity and desired performance (Rahaeifard *et al*. 2009). In such applications, size influences have been experimentally seen (Fleck *et al*. 1994, Stolken and Evans 1998, Chong *et al*. 2001, Lam *et al*. 2003). Since the dimension of these structural devices typically falls below micron- or nano-scale in at least one direction, an essential feature triggered in these devices is that their mechanical characteristics such as Young's modulus, flexural rigidity, and so on are size-dependent.

Referring to the mechanical investigation of size-

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dependent plate structures, linear free flexural vibration behavior of size dependent FG nanoplates is analyzed by Natarajan *et al.* (2012) by employing the isogeometric based finite element method. In this research, they used the nonlocal constitutive relation based on Eringen's differential form of nonlocal elasticity theory. The dynamic properties of FG beam with power law material graduation in the axial or the transversal directions were investigated by Alshorbagy *et al*. (2011). Eltaher *et al.* (2012) studied the free vibration analysis of FG size-dependent nanobeams. Eltaher *et al*. (2014a) investigated the vibration of nonlinear graduation of nano-Timoshenko beam considering the neutral axis position. By employing an exact analytical formulation, dynamic analysis of thick circular/ annular FG Mindlin nanoscale plates is studied by Hosseini-Hashemi *et al.* (2013). Eltaher *et al*. (2014b) discussed mechanical behavior of higher order gradient nanobeams. The resonance responses of FG micro/nanoplates via Kirchhoff plate model is investigated by Nami and Janghorban (2014). In this work, they adopted the nonlocal elasticity theory and strain gradient theory with one gradient parameter to include the small scale influences. Eltaher *et al.* (2014c) presented static and buckling analysis of FG Timoshenko nanobeams. Daneshmehr and Rajabpoor (2014) used a nonlocal higher order plate theory for buckling analysis of FG nano-plates under biaxial in-plane loadings using generalized differential quadrature (GDQ). Based on a modified couple stress theory, a model for sigmoid functionally graded material (S-FGM) nanoscale plates resting on elastic medium is proposed by Jung *et al.* (2014). Al-Basyouni *et al.* (2015) studied the size dependent bending and vibration behavior of FG micro beams based on modified couple stress theory and neutral surface position. Rahmani and Pedram (2014) investigated the size effects on the vibration of FG nanobeams based on nonlocal Timoshenko beam theory. Bedroud *et al*. (2015) studied the axisymmetric/asymmetric stability of moderately thick circular and annular FG nanoplates under uniform compressive in-plane loads. Zare *et al*. (2015) examined the natural frequencies of a FG nanoplate for different combinations of boundary conditions. Belkorissat *et al*. (2015) discussed the vibration properties of FG nano-plate using a new nonlocal refined four variable model. Larbi Chaht *et al.* (2015) presented both bending and buckling analyses of FG size-dependent nanoscale beams including the thickness stretching effect. Zemri *et al.* (2015) studied the mechanical response of FG nanoscale beam using a refined nonlocal shear deformation theory beam theory. Ahouel *et al.* (2016) analyzed the size-dependent mechanical behavior of FG trigonometric shear deformable nanobeams including neutral surface position concept. Bounouara *et al.* (2016) presented a nonlocal zeroth-order shear deformation theory for free vibration of FG nanoscale plates resting on elastic foundation. Hamed *et al*. (2016) analyzed the free vibration of symmetric and sigmoid functionally graded nanobeams.

In this work, a new nonlocal trigonometric shear deformation theory is proposed for the free vibration analysis of simply supported size-dependent FG nanoplates on elastic foundation. The use of the integral term in the displacement field led to a reduction in the number of unknowns and equations of motion. Implementing Hamilton's principle, the nonlocal equations of motion are obtained and they are solved via Navier solution method. Comparisons with analytical solutions and the results from the existing literature are provided for two-constituent metal–ceramic nanoplates and the good agreement between the results of this paper and those available in literature validated the presented formulation. Numerical results are presented to serve as benchmarks for the application and the design of nanoelectronic and nano-drive devices, nanooscillators, and nanosensors, in which nanobeams act as basic elements. They can also be useful as valuable sources for validating other approaches and approximate methods.

2. Theoretical formulation

2.1 Mori–Tanaka FGM plate model

According to Mori–Tanaka homogenization technique the local effective material properties of the FG nanoplate (Fig. 1) such as effective local bulk modulus (K) and shear modulus (G) can be computed (Belabed *et al*. 2014, Valizadeh *et al*. 2013, Houari *et al*. 2016):

$$\frac{K-K_m}{K_c-K_m}=\frac{V_c}{1+\left(1-V_c\right)\dfrac{3\left(K_c-K_m\right)}{3K_m+4G_m}} \tag{1a}$$

$$\frac{G-G_m}{G_c-G_m}=\frac{V_c}{1+\left(1-V_c\right)\dfrac{\left(G_c-G_m\right)}{G_m+f_1}} \tag{1b}$$

where

$$f_1=\frac{G_m\left(9K_m+8G_m\right)}{6\left(K_m+2G_m\right)} \tag{2}$$

where, V_i ($i=c,m$) is the volume fraction of the phase material. The subscripts c and m represent the ceramic and metal phases, respectively. The volume fractions of the ceramic and metal phases are related by $V_c+V_m=1$, and V_c is written as

$$V_c(z)=\left(\frac{2z+h}{2h}\right)^n, \quad n\geq 0 \tag{3}$$

with n in Eq. (3) is the gradient index which determines the material distribution through the thickness of the plate and z is the distance from the mid-plane of the FG nanoplate. Fig. 2 plots the distribution of the volume fraction of the ceramic phase within the thickness direction z for the FG plate. The effective Young's modulus E and Poisson's ratio ν can be computed from the following equations

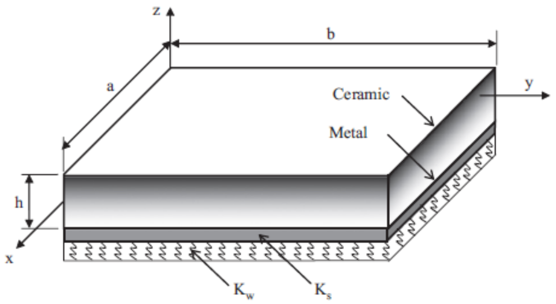

Fig. 1 Schematic representation of a rectangular FG plate resting on elastic foundation

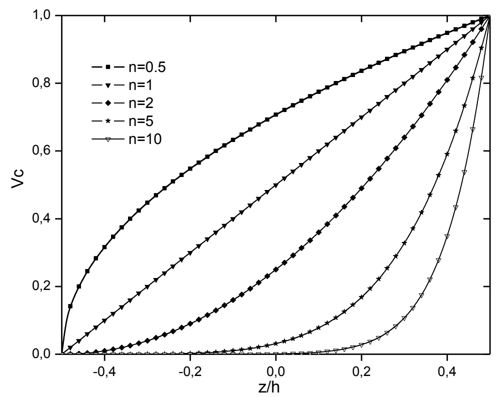

Fig. 2 Variation of ceramic phase through the thickness of the plate

$$E=\frac{9KG}{3K+G} \tag{4a}$$

$$\nu=\frac{3K-2G}{2(3K+G)} \tag{4b}$$

The effective mass density ρ is calculated from the rule of mixtures as (Natarajan *et al*. 2011, Hebali *et al*. 2014, Bourada *et al*. 2015, Attia *et al*. 2015, Bennai *et al*. 2015, Tounsi *et al*. 2016)

$$\rho=\rho_c V_c+\rho_m V_m \tag{5}$$

2.2 Kinematic relations

In this article, further simplifying assumptions are made to the conventional higher order shear deformation theories (HSDTs) so that the number of unknowns is reduced. The kinematic of the conventional HSDTs is given by (Mahi *et al*. 2015)

$$u(x,y,z,t)=u_0(x,y,t)-z\frac{\partial w_0}{\partial x}+f(z)\theta_x(x,y,t) \tag{6a}$$

$$v(x,y,z,t)=v_0(x,y,t)-z\frac{\partial w_0}{\partial y}+f(z)\theta_y(x,y,t) \tag{6b}$$

$$w(x,y,z,t)=w_0(x,y,t) \tag{6c}$$

where u_0 ; v_0 ; w_0 , θ_x , θ_y are five variables displacements of the mid-plane of the plate, $f(z)$ presents shape function representing the variation of the transverse shear strains and stresses across the thickness. In this paper a novel displacement field with four unknowns is proposed (Bourada *et al*. 2016, Hebali *et al*. 2016, Merdaci *et al*. 2016

$$u(x,y,z,t)=u_0(x,y,t)-z\frac{\partial w_0}{\partial x}+k_1 f(z)\int\varphi(x,y,t)\,dx \tag{7a}$$

$$v(x,y,z,t)=v_0(x,y,t)-z\frac{\partial w_0}{\partial y}+k_1 f(z)\int\varphi(x,y,t)\,dy \tag{7b}$$

$$w(x,y,z,t)=w_0(x,y,t) \tag{7c}$$

The constants k_1 and k_2 depends on the geometry and the proposed theory of present study has a cosines function in the form

$$f(z)=\frac{z\left(\pi+2\cos\left(\frac{\pi z}{h}\right)\right)}{(2+\pi)} \tag{8}$$

Nonzero strains of the four variable plate model are expressed as follows

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}+z\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}+f(z)\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix},\quad \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=g(z)\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix} \tag{9}$$

where

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}=\begin{Bmatrix}\frac{\partial u_0}{\partial x}\\ \frac{\partial v_0}{\partial x}\\ \frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}\end{Bmatrix},\quad \begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}=\begin{Bmatrix}-\frac{\partial^2 w_0}{\partial x^2}\\ -\frac{\partial^2 w_0}{\partial y^2}\\ -2\frac{\partial^2 w_0}{\partial x\partial y}\end{Bmatrix}, \tag{10a}$$

$$\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} k_1\theta \\ k_2\theta \\ k_1 \dfrac{\partial}{\partial y}\int \theta\, dx + k_2 \dfrac{\partial}{\partial x}\int \theta\, dy \end{Bmatrix},$$

$$\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} k_1\int \theta\, dy \\ k_2\int \theta\, dx \end{Bmatrix},$$

and

$$g(z) = \frac{df(z)}{dz} \tag{10b}$$

The integrals employed in the above equations shall be resolved by a Navier solution and can be expressed by

$$\frac{\partial}{\partial y}\int \theta\, dx = A'\frac{\partial^2\theta}{\partial x\partial y}, \quad \frac{\partial}{\partial x}\int \theta\, dy = B'\frac{\partial^2\theta}{\partial x\partial y}, \quad \int \theta\, dx = A'\frac{\partial\theta}{\partial x}, \quad \int \theta\, dy = B'\frac{\partial\theta}{\partial y} \tag{11}$$

In which the coefficients A' and B' are determined according to the type of solution considered, in this case via Navier. Thus, A' and B' are expressed by:

$$A' = -\frac{1}{\alpha^2}, \quad B' = -\frac{1}{\beta^2}, k_1 = \alpha^2, k_2 = \beta^2 \tag{12}$$

where α and β are defined in Eq. (29).

2.3 Equations of motion

Through Hamilton's principle, in which the motion of an elastic structure in the time interval $t_1 < t < t_2$ is so that the integral with respect to time of the total potential energy is extremum (Benachour *et al*. 2011, Ait Yahia *et al*. 2015, Tagrara *et al*. 2015, Boukhari *et al*. 2016)

$$0 = \int_0^t (\delta U_p + \delta U_f - \delta K)\, dt \tag{13}$$

where δU_p and δU_f are the variations of strain energy of the plate and foundation, respectively; and δK is the variation of kinetic energy.

The virtual strain energy can be computed as

$$\begin{aligned} \delta U &= \int_V \left[\sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \tau_{xy}\delta\gamma_{xy} + \tau_{yz}\delta\gamma_{yz} + \tau_{xz}\delta\gamma_{xz}\right] dV \\ &= \int_A \left[N_x \delta\varepsilon_x^0 + N_y \delta\varepsilon_y^0 + N_{xy}\delta\gamma_{xy}^0 + M_x^b \delta k_x^b + M_y^b \delta k_y^b + M_{xy}^b \delta k_{xy}^b \right. \\ &\quad \left. + M_x^s \delta k_x^s + M_y^s \delta k_y^s + M_{xy}^s \delta k_{xy}^s + S_{yz}^s \delta\gamma_{yz}^s + S_{xz}^s \delta\gamma_{xz}^0\right] dA = 0 \end{aligned} \tag{14}$$

where A is the top surface and the stress resultants N, M, and S are expressed by

$$\left(N_i, M_i^b, M_i^s\right) = \int_{-h/2}^{h/2} (1, z, f)\sigma_i\, dz, \quad (i = x, y, xy)$$
$$\text{and} \quad \left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} g\left(\tau_{xz}, \tau_{yz}\right) dz \tag{15}$$

The virtual strain energy of the elastic medium can be calculated by

$$\delta U_f = \int_A \left[K_w w\delta w + K_s\left(\frac{\partial w}{\partial x}\frac{\partial\delta w}{\partial x} + \frac{\partial w}{\partial y}\frac{\partial\delta w}{\partial y}\right)\right] dxdy \tag{16}$$

where K_w and K_s are the transverse and shear stiffness coefficients of the elastic medium, respectively.

The virtual kinetic energy of the plate can be calculated by

$$\begin{aligned} \delta K &= \int_V \left[\dot u\,\delta\dot u + \dot v\,\delta\dot v + \dot w\,\delta\dot w\right]\rho(z)\, dV \\ &= \int_A \Big\{ I_0\left[\dot u_0 \delta\dot u_0 + \dot v_0 \delta\dot v_0 + \dot w_0 \delta\dot w_0\right] \\ &\quad - I_1\left(\dot u_0 \frac{\partial\delta\dot w_0}{\partial x} + \frac{\partial\dot w_0}{\partial x}\delta\dot u_0 + \dot v_0\frac{\partial\delta\dot w_0}{\partial y} + \frac{\partial\dot w_0}{\partial y}\delta\dot v_0\right) \\ &\quad + J_1\left((k_1A')\left(\dot u_0\frac{\partial\delta\dot\theta}{\partial x} + \frac{\partial\dot\theta}{\partial x}\delta\dot u_0\right) + (k_2B')\left(\dot v_0\frac{\partial\delta\dot\theta}{\partial y} + \frac{\partial\dot\theta}{\partial y}\delta\dot v_0\right)\right) \\ &\quad + I_2\left(\frac{\partial\dot w_0}{\partial x}\frac{\partial\delta\dot w_0}{\partial x} + \frac{\partial\dot w_0}{\partial y}\frac{\partial\delta\dot w_0}{\partial y}\right) + K_2\left((k_1A')^2\left(\frac{\partial\dot\theta}{\partial x}\frac{\partial\delta\dot\theta}{\partial x}\right) + (k_2B')^2\left(\frac{\partial\dot\theta}{\partial y}\frac{\partial\delta\dot\theta}{\partial y}\right)\right) \\ &\quad - J_2\left((k_1A')\left(\frac{\partial\dot w_0}{\partial x}\frac{\partial\delta\dot\theta}{\partial x} + \frac{\partial\dot\theta}{\partial x}\frac{\partial\delta\dot w_0}{\partial x}\right) + (k_2B')\left(\frac{\partial\dot w_0}{\partial y}\frac{\partial\delta\dot\theta}{\partial y} + \frac{\partial\dot\theta}{\partial y}\frac{\partial\delta\dot w_0}{\partial y}\right)\right)\Big\} dA \end{aligned} \tag{17}$$

where dot-superscript convention indicates the differentiation with respect to the time variable t; and (I_i, J_i, K_i) are mass inertias defined by

$$\left(I_0, I_1, I_2\right) = \int_{-h/2}^{h/2} \left(1, z, z^2\right)\rho(z)\, dz \tag{18a}$$

$$\left(J_1, J_2, K_2\right) = \int_{-h/2}^{h/2} \left(f, z\,f, f^2\right)\rho(z)\, dz \tag{18b}$$

By substituting Eqs. (13), (15) and (16) into Eq. (12) and setting the coefficients of δu_0, δv_0, δw_0 and $\delta\theta$ to zero, the following Euler–Lagrange equation can be obtained

$$\begin{aligned} &\delta u_0: \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0\ddot u_0 - I_1\frac{\partial\ddot w_0}{\partial x} + k_1A'J_1\frac{\partial\ddot\theta}{\partial x} \\ &\delta v_0: \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_0\ddot v_0 - I_1\frac{\partial\ddot w_0}{\partial y} + k_2B'J_1\frac{\partial\ddot\theta}{\partial y} \\ &\delta w_0: \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x\partial y} + \frac{\partial^2 M_y^b}{\partial y^2} - K_w w_0 + K_s\nabla^2 w_0 = I_0\ddot w_0 \\ &\qquad + I_1\left(\frac{\partial\ddot u_0}{\partial x} + \frac{\partial\ddot v_0}{\partial y}\right) - I_2\nabla^2\ddot w_0 + J_2\left(k_1A'\frac{\partial^2\ddot\theta}{\partial x^2} + k_2B'\frac{\partial^2\ddot\theta}{\partial y^2}\right) \\ &\delta\theta: -k_1M_x^s - k_2M_y^s - (k_1A' + k_2B')\frac{\partial^2 M_{xy}^s}{\partial x\partial y} + k_1A'\frac{\partial S_{xz}^s}{\partial x} + k_2B'\frac{\partial S_{yz}^s}{\partial y} = -J_1\left(k_1A'\frac{\partial\ddot u_0}{\partial x} + k_2B'\frac{\partial\ddot v_0}{\partial y}\right) \\ &\qquad - K_2\left((k_1A')^2\frac{\partial^2\ddot\theta}{\partial x^2} + (k_2B')^2\frac{\partial^2\ddot\theta}{\partial y^2}\right) + J_2\left(k_1A'\frac{\partial^2\ddot w_0}{\partial x^2} + k_2B'\frac{\partial^2\ddot w_0}{\partial y^2}\right) \end{aligned} \tag{19}$$

where $\nabla^2 = (\partial^2/\partial x^2) + (\partial^2/\partial y^2)$

is the Laplacian operatorin2D Cartesian coordinate system.

2.4 The nonlocal elasticity model for FG nanoplate

According to Eringen's nonlocal elasticity theory (1972), the stress state at a point inside a body is regarded to be a function of strains of all points in the neighbor regions. For homogeneous elastic solids, the nonlocal stress-tensor components σ_{ij} at each point x in the solid can be expressed as

$$\sigma_{ij}(x) = \int_{\Omega} \alpha\left(\left|x' - x\right|, \tau\right) t_{ij}(x') d\Omega(x') \tag{20}$$

where $t_{ij}(x')$ are the components available in local str ess tensor at point x which are associated to the strain tensor components ε_{kl} as:

$$t_{ij} = C_{ijkl}\varepsilon_{kl} \tag{21}$$

The concept of Eq. (19) is that the nonlocal stress at any point is a weighting average of the local stress of all near points, and the nonlocal kernel $\alpha\left(\left|x' - x\right|, \tau\right)$ considers the influence of the strain at the point x' on the stress at the point x in the elastic body. The parameter α is an internal characteristic length (e.g., lattice parameter, granular distance, the length of C–C bonds). Also $\left|x' - x\right|$ is Euclidean distance and τ is a constant value as follows

$$\tau = \frac{e_0 a}{l} \tag{22}$$

which presents the relation of a characteristic internal length, and a characteristic external length, l (e.g., crack length and wavelength) using a constant, e_0, dependent on each material. The value of e_0 is experimentally evaluated by comparing the scattering curves of plane waves with those of atomistic dynamics. In the nonlocal model of elasticity, the points undergo translational motion as in the classical case, but the stress at a point depends on the strain in a region near that point. As for physical interpretation, the nonlocal model introduces long range interactions between points in a continuum model. Such long range interactions occur between charged atoms or molecules in a solid. Eringen (1972, 1983) numerically determined the functional form of the kernel. By appropriate selection of the kernel function, Eringen shown that the nonlocal constitutive equation given in integral form (see Eq. (19)) can be represented in an equivalent differential form as (Heireche *et al*. 2008, Berrabah *et al*. 2013, Benguediab *et al.* 2014, Adda Bedia *et al*. 2015, Aissani *et al*. 2015, Besseghier *et al*. 2015, Akbas 2016, Ebrahimi and Shaghaghi 2016):

$$\left(1 - (e_0 a)\nabla^2\right)\sigma_{kl} = t_{kl} \tag{23}$$

In which ∇^2 is the Laplacian operator. Hence, the scale length $e_0 a$ considers the effects of small size on the behavior of nanostructures. Thus, the constitutive relations of nonlocal theory for a FG nanoplate can be written as

$$\left(1 - \mu\nabla^2\right)\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{66} & 0 & 0 \\ 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & C_{44} \end{bmatrix}\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{24}$$

In which $\mu = (e_0 a)^2$ and the stiffness coefficients, C_{ij}, can be expressed as

$$C_{11} = C_{22} = \frac{E(z)}{1 - \nu(z)^2}, \quad C_{12} = \frac{\nu\, E(z)}{1 - \nu(z)^2}, \qquad C_{44} = C_{55} = C_{66} = \frac{E(z)}{2\left[1 + \nu(z)\right]}, \tag{25}$$

Integrating Eq. (23) over the plate's cross-section area yields the force–strain and the moment–strain of the nonlocal refined FG plates as follows

$$\left(1 - \mu\nabla^2\right)\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x^b \\ M_y^b \\ M_{xy}^b \\ M_x^s \\ M_y^s \\ M_{xy}^s \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 & B_{11}^s & B_{12}^s & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 & B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} & 0 & 0 & B_{66}^s \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 & D_{11}^s & D_{12}^s & 0 \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 & D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} & 0 & 0 & D_{66}^s \\ B_{11}^s & B_{12}^s & 0 & D_{11}^s & D_{12}^s & 0 & H_{11}^s & H_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 & D_{12}^s & D_{22}^s & 0 & H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & B_{66}^s & 0 & 0 & D_{66}^s & 0 & 0 & H_{66}^s \end{bmatrix}\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ k_x^b \\ k_y^b \\ k_{xy}^b \\ k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} \tag{26a}$$

$$\left(1 - \mu\nabla^2\right)\begin{Bmatrix} S_{xz}^s \\ S_{yz}^s \end{Bmatrix} = \begin{bmatrix} A_{55}^s & 0 \\ 0 & A_{44}^s \end{bmatrix}\begin{Bmatrix} \gamma_{xz}^0 \\ \gamma_{yz}^0 \end{Bmatrix} \tag{26b}$$

Where the cross-sectional rigidities are defined as follows

$$\left(A_{ij}, B_{ij}, D_{ij}, B_{ij}^s, D_{ij}^s, H_{ij}^s\right) = \int_{-h/2}^{h/2} C_{ij}\left(1, z, z^2, f(z), z\, f(z), f^2(z)\right) dz, \quad (i, j = 1,2,6) \tag{27a}$$

$$A_{ij}^s = \int_{-h/2}^{h/2} C_{ij}\left[g(z)\right]^2 dz, \quad \left(i, j = 4,5\right) \tag{27b}$$

The nonlocal equations of motion of FG nanoplates in terms of the displacement can be derived by substituting Eqs. (25), into Eqs. (18) as follows

$$\begin{aligned} & A_{11}\frac{\partial^2 u_0}{\partial x^2} + A_{66}\frac{\partial^2 u_0}{\partial y^2} + \left(A_{12} + A_{66}\right)\frac{\partial^2 v_0}{\partial x\,\partial y} - B_{11}\frac{\partial^3 w_0}{\partial x^3} - \left(B_{12} + 2B_{66}\right)\frac{\partial^3 w_0}{\partial x\,\partial y^2} \\ & + \left(B_{66}^s\left(k_1 A' + k_2\, B'\right)\right)\frac{\partial^3 \theta}{\partial x\,\partial y^2} + \left(B_{11}^s k_1 + B_{12}^s\, k_2\right)\frac{\partial \theta}{\partial x} = \left(1 - \mu\nabla^2\right)\left[I_0 \ddot{u}_0 - I_1\, d_1 \ddot{w}_0 + J_1\, A' k_1 d_1 \ddot{\theta}\right], \end{aligned} \tag{28a}$$

$$A_{22}\frac{\partial^2 v_0}{\partial y^2}+A_{66}\frac{\partial^2 v_0}{\partial x^2}+(A_{12}+A_{66})\frac{\partial^2 u_0}{\partial x\,\partial y}-B_{22}\frac{\partial^3 w_0}{\partial y^3}-(B_{12}+2B_{66})\frac{\partial^3 w_0}{\partial x^2\partial y}$$
$$+\left(B_{66}^s\,(k_1A'+k_2\,B')\right)\frac{\partial^3\theta}{\partial x^2\partial y}+\left(B_{22}^s k_2+B_{12}^s k_1\right)\frac{\partial\theta}{\partial y}=\left(1-\mu\nabla^2\right)\left[I_0\ddot{v}_0-I_1\,d_2\ddot{w}_0+J_1\,B'k_2\,d_2\ddot{\theta}\right], \tag{28b}$$

$$B_{11}\frac{\partial^3 u_0}{\partial x^3}+(B_{12}+2B_{66})\frac{\partial^3 u_0}{\partial x\,\partial y^2}+(B_{12}+2B_{66})\frac{\partial^3 v_0}{\partial x^2\,\partial y}+B_{22}\frac{\partial^3 v_0}{\partial y^3}$$
$$-D_{11}\frac{\partial^4 w_0}{\partial x^4}-2(D_{12}+2D_{66})\frac{\partial^4 w_0}{\partial x^2\partial y^2}-D_{22}\frac{\partial^4 w_0}{\partial y^4}+\left(D_{11}^s\,k_1+D_{12}^s\,k_2\right)\frac{\partial^2\theta}{\partial x^2}$$
$$+2\left(D_{66}^s\,(k_1A'+k_2B')\right)\frac{\partial^4\theta}{\partial x^2\partial y^2}+\left(D_{12}^s k_1+D_{22}^s k_2\right)\frac{\partial^2\theta}{\partial y^2}+\left(1-\mu\nabla^2\right)\left(-K_w w_0+K_s\nabla^2 w_0\right)=$$
$$\left(1-\mu\nabla^2\right)\left[I_0\ddot{w}_0+I_1\,(d_1\ddot{u}_0+d_2\,\ddot{v}_0)-I_2\,(d_{11}\ddot{w}_0+d_{22}\ddot{w}_0)+J_2\left(k_1A'\,d_{11}\ddot{\theta}+k_2\,B'\,d_{22}\ddot{\theta}\right)\right] \tag{28c}$$

$$-\left(B_{11}^s k_1+B_{12}^s k_2\right)\frac{\partial u_0}{\partial x}-\left(B_{66}^s\,(k_1A'+k_2B')\right)\frac{\partial^3 u_0}{\partial x\partial y^2}-\left(B_{66}^s\,(k_1A'+k_2B')\right)\frac{\partial^3 v_0}{\partial x^2\partial y}-\left(B_{12}^s k_1+B_{22}^s k_2\right)\frac{\partial v_0}{\partial y}$$
$$+\left(D_{11}^s k_1+D_{12}^s k_2\right)\frac{\partial^2 w_0}{\partial x^2}+2\left(D_{66}^s\,(k_1A'+k_2B')\right)\frac{\partial^4 w_0}{\partial x^2\partial y^2}+\left(D_{12}^s k_1+D_{22}^s k_2\right)\frac{\partial^2 w_0}{\partial y^2}$$
$$-H_{11}^s k_1^2\,\theta-H_{22}^s\,k_2^2\,\theta-2H_{12}^s\,k_1\,k_2\theta-\left((k_1A'+k_2B')^2 H_{66}^s\right)\frac{\partial^4\theta}{\partial x^2\partial y^2}+A_{44}^s\,(k_2B')^2\frac{\partial^2\theta}{\partial y^2}+A_{55}^s\,(k_1\,A')^2\frac{\partial^2\theta}{\partial x^2}=$$
$$\left(1-\mu\nabla^2\right)\left[-J_1\,(k_1\,A'd_1\ddot{u}_0+k_2\,B'd_2\ddot{v}_0)+J_2\,(k_1\,A'd_{11}\ddot{w}_0+k_2\,B'd_{22}\ddot{w}_0)-K_2\left((k_1A')^2\,d_{11}\ddot{\theta}+(k_2\,B')^2 d_{22}\ddot{\theta}\right)\right] \tag{28d}$$

3. Solution procedures

Here, on the basis of the Navier technique, an analytical solution of the equations of motion for free vibration of a simply supported FG nanoplate is presented. To satisfy governing equations of motion and the simply supported boundary condition, the displacement variables are adopted to be of the form

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta \end{Bmatrix}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{Bmatrix} U_{mn}e^{i\omega t}\,\cos(\alpha\,x)\sin(\beta\,y) \\ V_{mn}e^{i\omega t}\,\sin(\alpha\,x)\cos(\beta\,y) \\ W_{mn}e^{i\omega t}\,\sin(\alpha\,x)\sin(\beta\,y) \\ X_{mn}e^{i\omega t}\,\sin(\alpha\,x)\sin(\beta\,y) \end{Bmatrix} \tag{29}$$

where ω is the frequency of free vibration of the plate, $\sqrt{i}=-1$ the imaginary unit.
with

$$\alpha=m\pi/a \quad \text{and} \quad \beta=n\pi/b \tag{30}$$

Substituting Eqs. (28) into Eqs. (27) respectively, le ads to

$$\left(\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix}-\lambda\,\omega^2\begin{bmatrix} m_{11} & 0 & m_{13} & m_{14} \\ 0 & m_{22} & m_{23} & m_{24} \\ m_{13} & m_{23} & m_{33} & m_{34} \\ m_{14} & m_{24} & m_{34} & m_{44} \end{bmatrix}\right)\begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ X_{mn} \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{31}$$

where

$$S_{11}=-\left(A_{11}\alpha^2+A_{66}\beta^2\right)\ , \qquad S_{12}=-\alpha\beta\,(A_{12}+A_{66})\ ,$$
$$S_{13}=\alpha\left(B_{11}\alpha^2+B_{12}\beta^2+2B_{66}\beta^2\right)\ ,$$
$$S_{14}=\alpha\left(k_1B_{11}^s+k_2B_{12}^s-(k_1A'+k_2B')B_{66}^s\beta^2\right),$$
$$S_{22}=-\left(A_{66}\alpha^2+A_{22}\beta^2\right)\ ,$$
$$S_{23}=\beta\left(B_{22}\beta^2+B_{12}\alpha^2+2B_{66}\alpha^2\right)\ ,$$
$$S_{24}=\beta\left(k_2B_{22}^s+k_1B_{12}^s-(k_1A'+k_2B')B_{66}^s\alpha^2\right)$$
$$S_{33}=-\left(D_{11}\alpha^4+2(D_{12}+2D_{66})\alpha^2\beta^2+D_{22}\beta^4\right)+\lambda\left(K_w+K_s\left(\alpha^2+\beta^2\right)\right)\ ,$$
$$S_{34}=-k_1\left(D_{11}^s\alpha^2+D_{12}^s\beta^2-2A'D_{66}^s\alpha^2\beta^2\right)-k_2\left(D_{22}^s\beta^2+D_{12}^s\alpha^2-2B'D_{66}^s\alpha^2\beta^2\right),$$
$$S_{44}=k_1^2\left(-H_{11}^s-A'^2\,H_{66}^s\alpha^2\beta^2-A'^2\,A_{55}^s\alpha^2\right)+k_2^2\left(-H_{22}^s-B'^2\,H_{66}^s\alpha^2\beta^2-B'^2\,A_{44}^s\beta^2\right)$$
$$-k_1k_2\left(2H_{12}^s+2A'B'H_{66}^s\alpha^2\beta^2\right)$$

$$\begin{gathered} m_{11}=-I_0\,,\ \ m_{13}=\alpha\,I_1\,,\ \ m_{14}=\frac{J_1k_1}{\alpha}\ , \\ m_{22}=-I_0\,,\ \ m_{23}=\beta\,I_1\,,\ \ m_{24}=\frac{J_2k_2}{\beta}\ , \\ m_{33}=-I_0-I_2(\alpha^2+\beta^2)\quad m_{34}=-J_2\,(k_1+k_2)\,, \\ m_{44}=\frac{-K_2\left(k_1^2\,\beta^2+k_2^2\,\alpha^2\right)}{\alpha^2\beta^2},\quad \lambda=1+\mu\left(\alpha^2+\beta^2\right) \end{gathered} \tag{32}$$

4. Numerical results and discussions

In this section, various numerical and illustrative results are presented to study the size-dependent free vibration response of embedded FG nanoplates modeled based on a novel four-variable shear deformation theory. The material properties of the FG nanoplate vary within the thickness direction according to Mori–Tanaka homogenization technique. The top surface of the plate is ceramic rich (Si_3N_4) and the bottom surface is metal rich (SUS304). The mass density ρ and the Young's modulus E are: ρ_c = 2370 kg/m^3, E_c = 348.43e^9 N/m^2 for Si_3N_4 and ρ_m = 8166 kg/m^3, E_m = 201.04e^9 N/m^2 for SUS304. Poisson's ratio ν is considered to be constant and taken as 0.3 for the present work. For convenience, the following dimensionless quantities are used in presenting the numerical results in graphical and tabular forms:

$$\begin{gathered} \overline{\omega}=\omega h\sqrt{\frac{\rho_c}{G_c}}\ ,\ \ \hat{\omega}=\omega h\sqrt{\frac{\rho_m}{E_m}}\ ,\quad k_w=\frac{K_w a^4}{D_m}\ , \\ k_s=\frac{K_s a^2}{D_m}\ ,\ \ D_m=\frac{E_m h^3}{12\left(1-\nu^2\right)} \end{gathered} \tag{33}$$

The correctness of the presented dynamic results of simply supported FG nanoplates is compared with those presented by Belkorissat *et al*. (2015), Aghababaei and Reddy (2009), Natarajan *et al*. (2012) as well as Bounouara et al (2016) and the results are tabulated in Table 1 to 3.

In the first example, simply supported homogeneous nanoplates with different values of nonlocal parameter, the plate thickness and the plate aspect ratio are considered. The results given in Table 1 are compared with those provided by Belkorissat et al. (2015), Bounouara et al (2016) and Aghababaei and Reddy (2009). It can be seen that the present numerical results are in very good agreement with the results available in the literature.

In the second example, FG nanoplates ($n=5$) with different values of nonlocal parameter, aspect ratio (a/b)

and side-to-thickness ratio (a/h) are investigated. The natural frequencies calculated using the present nonlocal refined trigonometric shear deformation theory, are compared with those of Belkorissat et al. (2015) in Table 2. Again, very good agreement is found between the results.

The dimensionless fundamental frequency of square FG nanoplates are provided in Table 3 for different values of nonlocal parameter, the plate thickness, the gradient index (n) and foundation parameters (k_w , k_s). It can be seen that the dimensionless fundamental frequency increases when foundation parameters increase. Compared to the Winkler parameter effect, it can be observed that the vibration responses of FG nanoplates are more affected by Pasternak foundation parameter than the Winkler parameter. It is also remarked that with the introduction of elastic foundations, the plate becomes stiffer, while, the nonlocal parameter makes the plate softer. In addition, it can be seen that the increase of the gradient index (n) leads to a reduction of frequency. This is due to the fact that the gradient index yields a decrease in the stiffness of the FG nanoplate.

To demonstrate the effects of elastic foundation parameters on the vibration behavior of FG nanoplate individually, Figs. 3 and 4 present variations of the frequency ratio with respect to Winkler and Pasternak constants, respectively at $a/h=10$ and $n=5$. From Fig. 3 it is observed that there is a significant influence of the nonlocal parameter on the vibration response of FG nanoplates supported by elastic foundation. The fundamental frequency ratios including nonlocal model are always smaller than the local model ($\mu=0$). This implies that the use of the local theory for investigating the FG nanoplates would lead to an over-prediction of the frequency. Further, with increasing the nonlocal parameter (μ) values, the frequencies computed by non-local theory become smaller compared to local theory. Furthermore, it is observed that the increase of the Winkler modulus coefficient leads to an increase in the frequency ratio. This increasing trend is related to the stiffness of the elastic foundation. With important values of Winkler coefficient the rate of increase of frequency ratio diminishes.

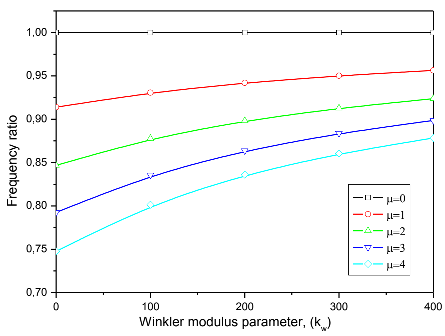

Fig. 3 Effect of Winkler modulus parameter on the frequency ratio of FG square nanoplate for various nonlocal parameters ($k_s=0, a/h=10, n=5$)

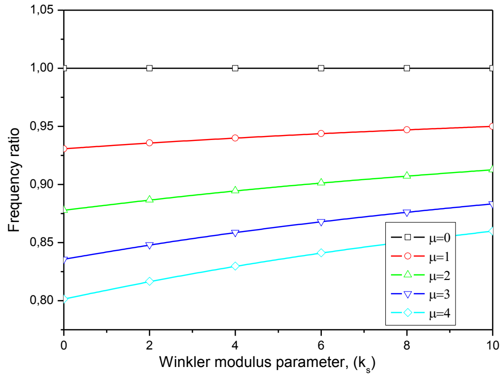

Fig. 4 Effect of Pasternak shear modulus parameter on the frequency ratio of FG square nano-plate for various nonlocal parameters ($k_w=100, a/h=10, n=5$)

This implies that nonlocal influence on vibration response of FG nanoplates looses its importance as the Winkler coefficient values increase. Thus, although the nonlocal influence makes the nanoplates softer, the external elastic foundation “grips” the nanoplates and forces it to be stiffer. Hence, it can be concluded that the nonlocal influence becomes more significant in the case of plates without elastic foundation.

The variation of frequency ratio for first mode with shear modulus parameter is presented in Fig. 4. The frequency ratio increases with increasing the shear modulus coefficient. However, the frequency ratios including the nonlocal model are always smaller than the local model. With higher nonlocal parameter (μ) values the frequencies becomes comparatively less. Contrary to the variation of frequency ratio with Winkler coefficient, which is nonlinear, the variation of frequency ratio with Pasternak shear coefficient is linear in nature.

The effect of nonlocal parameter on the dimensionless frequency of nonlocal square FG plates with and without elastic foundation is plotted in Fig. 5. It is seen that the dimensionless frequency of FG nanoplate decreases due to the fact that presence of nonlocality makes the plate structure more flexible. Therefore, nonlocal plate model provides lower frequency results compared to local plate model.

The variation of the nonlocal frequency with the Winkler modulus coefficient is plotted in Fig. 6 for different values of a/h. It can be seen that with increasing the Winkler modulus parameter the nonlocal frequency increases in linear manner. Moreover, it is observed that the change in nonlocal frequency of nanoscale plate is significantly affected by the side-to-thickness ratio a/h. For a thin plate ($a/h=100$) the influence of nonlocal parameter on frequency is less compared to thick plate ($a/h=10$). Hence side-to-thickness ratio of nanoscale plate plays a considerable role in examining free vibration response of nanoscale plates resting on elastic foundation.

Table 1 Comparison of fundamental frequency ($\bar{\omega} = \omega h\sqrt{\rho / G}$) of nano-plate ($a = 10,\ E = 30\times10^6,\ \rho = 1,\ \nu = 0.3$)

a/b	a/h	μ	Present	Ref [a]	Ref[b]	TSDT[c]	FSDT[c]	CPT[c]
1	10	0	0.0930	0.0930	0.0930	0.0935	0.0930	0.0963
		1	0.0850	0.0850	0.0850	0.0854	0.0850	0.0880
		2	0.0788	0.0787	0.0787	0.0791	0.0788	0.0816
		3	0.0737	0.0737	0.0737	0.0741	0.0737	0.0763
		4	0.0696	0.0695	0.0695	0.0699	0.0696	0.0720
		5	0.0660	0.0659	0.0659	0.0663	0.0660	0.0683
	20	0	0.0239	0.0238	0.0238	0.0239	0.0239	0.0241
		1	0.0218	0.0218	0.0218	0.0218	0.0218	0.0220
		2	0.0202	0.0202	0.0202	0.0202	0.0202	0.0204
		3	0.0189	0.0189	0.0189	0.0189	0.0189	0.0191
		4	0.0178	0.0178	0.0178	0.0179	0.0178	0.0180
		5	0.0169	0.0169	0.0169	0.0170	0.0169	0.0171
2	10	0	0.0588	0.0588	0.0588	0.0591	0.0589	0.0602
		1	0.0556	0.0555	0.0555	0.0557	0.0556	0.0568
		2	0.0527	0.0527	0.0527	0.0529	0.0527	0.0539
		3	0.0503	0.0503	0.0503	0.0505	0.0503	0.0514
		4	0.0482	0.0481	0.0481	0.0483	0.0482	0.0493
		5	0.0463	0.0463	0.0463	0.0464	0.0463	0.0473
	20	0	0.0150	0.0149	0.0149	0.0150	0.0150	0.0150
		1	0.0141	0.0141	0.0141	0.0141	0.0141	0.0142
		2	0.0134	0.0134	0.0134	0.0134	0.0134	0.0135
		3	0.0128	0.0127	0.0127	0.0128	0.0128	0.0129
		4	0.0122	0.0122	0.0122	0.0123	0.0123	0.0123
		5	0.0118	0.0117	0.0117	0.0118	0.0118	0.0118

[a] Bounouara *et al* (2016)
[b] Belkorissat *et al.* (2015)
[c] Aghababaei and Reddy (2009)

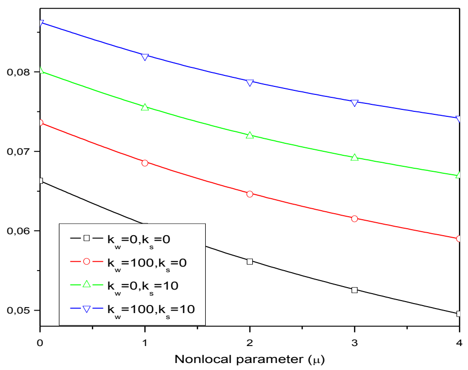

Fig. 5 Effect of nonlocal parameter on the frequency of FG square nano-plate with and without elastic foundation ($a/h = 10$, $n = 5$)

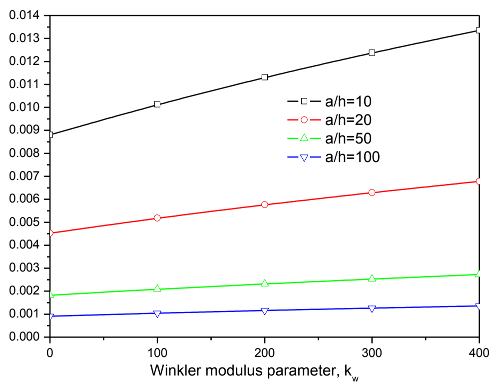

Fig. 6 Effect of Winkler modulus parameter on the nonlocal frequency of FG square nano-plate for various ratios a/h ($k_s = 0$, $\mu = 2$, $n = 5$)

Table 2 Comparison of natural frequency of FG nano-plate $(a=10,\ n=5)$

a/b	a/h	μ	Mode 1		Mode 2		Mode 3	
			Ref[(a)]	Present	Ref[(a)]	Present	Ref[(a)]	Present
1	10	0	0.0432	0.0437	0.1029	0.1041	0.1915	0.1940
		1	0.0395	0.0399	0.0842	0.0852	0.1358	0.1376
		2	0.0366	0.0370	0.0730	0.0738	0.1110	0.1125
		4	0.0323	0.0327	0.0596	0.0604	0.0861	0.0872
	20	0	0.0111	0.0112	0.0274	0.0277	0.0536	0.0542
		1	0.0101	0.0103	0.0224	0.0227	0.0380	0.0385
		2	0.0094	0.0095	0.0194	0.0197	0.0310	0.0314
		4	0.0083	0.0084	0.0158	0.0161	0.0241	0.0244
2	10	0	0.1029	0.1041	0.1574	0.1594	0.2397	0.2431
		1	0.0842	0.0852	0.1177	0.1192	0.1587	0.1609
		2	0.0730	0.0738	0.0980	0.0993	0.1269	0.1287
		4	0.0596	0.0604	0.0772	0.0782	0.0968	0.0982
	20	0	0.0274	0.0277	0.0432	0.0437	0.0688	0.0696
		1	0.0224	0.0227	0.0323	0.0327	0.0455	0.0461
		2	0.0194	0.0197	0.0269	0.0272	0.0364	0.0368
		4	0.0158	0.0161	0.0212	0.0215	0.0277	0.0281

[(d)] Belkorissat *et al.* (2015)

Fig. 7 presents the variation of nonlocal frequency versus Pasternak modulus parameter with different values of a/h. The consideration of the Pasternak foundation provides results higher than those with the consideration of Winkler foundation. The nonlocal frequency increases with increasing the Pasternak modulus parameter. The variation is found to be nonlinear in nature. However, it is observed that the change in nonlocal frequency is more affected by lower side-to-thickness ratios values ($a/h=10$) as shown in the Fig. 7.

Fig. 8 shows the variation of nonlocal frequency versus the aspect ratios a/b with and without the presence of elastic foundations. It can be seen from this figure that increasing the aspect ratio a/b reduces the value of nonlocal frequency. Furthermore, it is found that the presence of elastic foundations increases the nonlocal frequency and hence makes the plate stiffer.

In Fig. 9, the variation of nonlocal frequency versus Winkler modulus parameter is presented for different vibrational modes. It is found that the nonlocal frequency increases linearly with increasing the Winkler parameter and the computed frequencies become more important for vibrational modes.

Fig. 10 shows the variation of nonlocal frequency versus Pasternak modulus parameter for different vibrational modes. It can be seen that the nonlocal frequency increases in nonlinear manner with increasing the Pasternak parameter. Again, the nonlocal frequencies are found to more considerable for vibrational modes.

Fig. 11 illustrated the influence of the gradient index (n) on the non-dimensional nonlocal frequency of the three first modes of FG square nano-plat for various values of the nonlocal parameter. It can be observed that the non-dimensional frequency decreases with increasing the gradient index. This is due to the fact that an increase in the gradient index yields a decrease in the stiffness of the FG nano-plate. There is an abrupt change in the responses when the gradient index changes from 0 to 2.

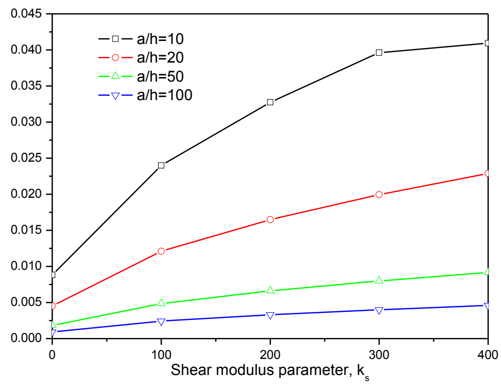

Fig. 7 Effect of Shear modulus parameter on the nonlocal frequency of FG square nano-plate for various ratios a/h ($k_w=0$, $\mu=2$, $n=5$)

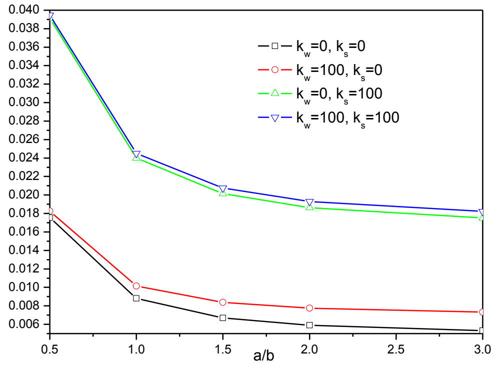

Fig. 8 Effect of aspect ratios a/b on the nonlocal frequency of FG square nano-plate for different values of elastic foundation parameters ($a/h=10$, $\mu=2$, $n=5$)

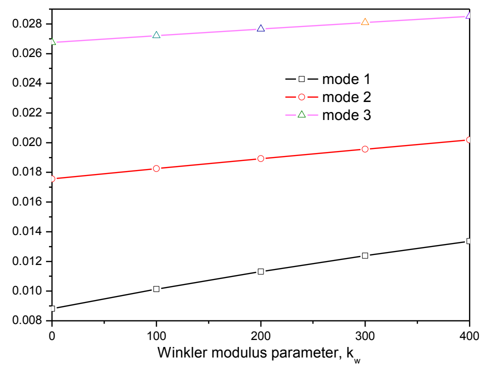

Fig. 9 Effect of Winkler modulus parameter on the nonlocal frequency of FG square nano-plate for the first three modes ($a/h=10$, $k_s=0$, $\mu=2$, $n=5$)

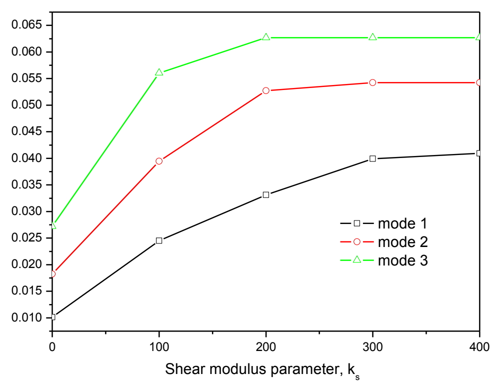

Fig. 10 Effect of Shear modulus parameter on the nonlocal frequency of FG square nano-plate for the first three modes ($a/h=10$, $k_w=100$, $\mu=2$, $n=5$)

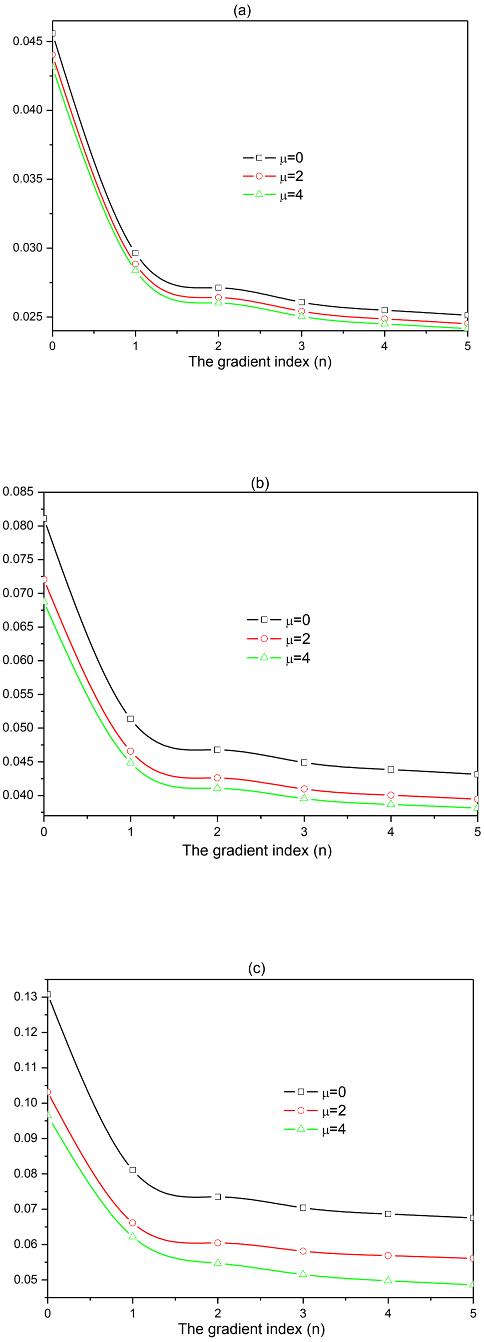

Fig. 11 Effect of the gradient (n) and the nonlocal parameter (μ) on dimensionless frequency for a simply supported square FG nano-plate with ($a/h=10$, $k_w=k_s=100$): (a) first mode,(b) second mode and (c) third mode

Table 3 Dimensionless frequency $(\hat{\omega})$ of FG square nanoplate

k_w	k_s	a/h	μ	Gradient index (n)					
				0		2		5	
				Ref[a]	Present	Ref[a]	Present	Ref[a]	Present
0	0	10	0	0.1409	0.1410	0.0717	0.0733	0.0655	0.0663
			1	0.1288	0.1289	0.0655	0.0670	0.0599	0.0606
			2	0.1193	0.1194	0.0607	0.0621	0.0555	0.0561
			3	0.1117	0.1117	0.0568	0.0581	0.0519	0.0525
			4	0.1053	0.1054	0.0536	0.0548	0.0490	0.0496
		20	0	0.0361	0.0362	0.0184	0.0188	0.0168	0.0170
			1	0.0330	0.0331	0.0168	0.0172	0.0153	0.0156
			2	0.0306	0.0307	0.0156	0.0159	0.0142	0.0144
			3	0.0286	0.0287	0.0146	0.0149	0.0133	0.0135
			4	0.0270	0.0270	0.0137	0.0141	0.0125	0.0127
0	20	10	0	0.1793	0.1793	0.0980	0.1002	0.0908	0.0919
			1	0.1699	0.1699	0.0936	0.0957	0.0868	0.0878
			2	0.1628	0.1629	0.0903	0.0923	0.0839	0.0848
			3	0.1573	0.1574	0.0877	0.0897	0.0815	0.0825
			4	0.1529	0.1529	0.0856	0.0876	0.0797	0.0806
		20	0	0.0456	0.0456	0.0249	0.0255	0.0231	0.0234
			1	0.0432	0.0432	0.0237	0.0243	0.0220	0.0223
			2	0.0413	0.0414	0.0229	0.0234	0.0212	0.0215
			3	0.0399	0.0400	0.0222	0.0228	0.0206	0.0209
			4	0.0388	0.0388	0.0217	0.0222	0.0202	0.0204
100	0	10	0	0.1516	0.1516	0.0792	0.0810	0.0728	0.0736
			1	0.1403	0.1404	0.0736	0.0753	0.0677	0.0685
			2	0.1317	0.1318	0.0694	0.0710	0.0639	0.0646
			3	0.1248	0.1249	0.0660	0.0675	0.0608	0.0615
			4	0.1192	0.1192	0.0633	0.0647	0.0583	0.0590
		20	0	0.0387	0.0388	0.0202	0.0207	0.0186	0.0188
			1	0.0358	0.0359	0.0188	0.0193	0.0173	0.0175
			2	0.0336	0.0336	0.0177	0.0181	0.0163	0.01651
			3	0.0319	0.0319	0.0168	0.0172	0.0155	0.0157
			4	0.0304	0.0305	0.0161	0.0165	0.0148	0.0151
100	20	10	0	0.1877	0.1878	0.1036	0.1059	0.0962	0.0973
			1	0.1788	0.1789	0.0994	0.1017	0.0924	0.0935
			2	0.1721	0.1722	0.0963	0.0985	0.0896	0.0907
			3	0.1669	0.1670	0.0939	0.0960	0.0875	0.0885
			4	0.1627	0.1628	0.0920	0.0941	0.0858	0.0867
		20	0	0.0477	0.0477	0.0263	0.0269	0.0244	0.0247
			1	0.0454	0.0454	0.0252	0.0258	0.0234	0.0237
			2	0.0436	0.0437	0.0244	0.0250	0.0227	0.0230
			3	0.0423	0.0423	0.0238	0.0243	0.0221	0.0224
			4	0.0412	0.0412	0.0233	0.0238	0.0217	0.0220

[a] Bounouara *et al*. (2016)

5. Conclusions

In this work, vibration behavior of the FG nanoplates resting on two-parameter elastic foundation is studied within the framework of a new nonlocal trigonometric shear deformation plate theory. Mechanical properties of the FG nanoplates vary gradually according to Mori–Tanaka model. Via Hamilton's principle and nonlocal constitutive relations of Eringen, the nonlocal differential equations of motion are obtained. Then, these equations are solved by employing Navier analytical method. Finally, it is indicated that vibration responses of FG nanoplates are affected by various parameters such as elastic foundation constants, nonlocal parameter, gradient index, aspect and side-to-thickness ratios. It is found that the presence of nonlocality diminishes the plate rigid ity and reduces the frequency of FG nanoplates. Contra ry to the nonlocal parameter, Winkler and Pasternak el astic foundation parameters enhance the plate structure and increase the frequencies. The formulation lends itse lf particularly well to stretching effects (Fekrar *et al*. 2014, Hamidi *et al*. 2015, Meradjah *et al*. 2015, Draiche *et al*. 2016, Bennoun *et al*. 2016), which will be considered in the near future.

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