Smart Structures and Systems, *Vol. 19, No. 3 (2017) 289-297*
DOI: https://doi.org/10.12989/sss.2017.19.3.289

Thermal buckling analysis of cross-ply laminated plates using a simplified HSDT

Abdelbaki Chikh[1,2], Abdelouahed Tounsi*[1,2,3], Habib Hebali[1,2] and S. R. Mahmoud[4,5]

[1]Université Ibn Khaldoun, BP 78 Zaaroura, 14000 Tiaret, Algérie
[2]Material and Hydrology Laboratory, University of Sidi Bel Abbes, Faculty of Technology, Civil Engineering Department, Algeria
[3]Laboratoire de Modélisation et Simulation Multi-échelle, Département de Physique, Faculté des Sciences Exactes, Département de Physique, Université de Sidi Bel Abbés, Algeria
[4]Department of Mathematics, Faculty of Science, King Abdulaziz University, Saudi Arabia
[5]Mathematics Department, Faculty of Science, University of Sohag, Egypt

(Received October 31, 2016, Revised December 7, 2016, Accepted December 8, 2016)

Abstract. This work presents a simplified higher order shear deformation theory (HSDT) for thermal buckling analysis of cross-ply laminated composite plates. Unlike the existing HSDT, the present one has a new displacement field which introduces undetermined integral terms and contains only four unknowns. Governing equations are derived from the principle of the minimum total potential energy. The validity of the proposed theory is evaluated by comparing the obtained results with their counterparts reported in literature. It can be concluded that the proposed HSDT is accurate and simple in solving the thermal buckling behavior of laminated composite plates.

Keywords: thermal buckling; cross ply-laminated plates; HSDT; analytical modelling

1. Introduction

Nowadays, the employ of composite materials has been experimented a great interest in civil, aerospace, automobile and other engineering industries (Panjehpour *et al*. 2011, 2016, Draiche *et al*. 2014, Nedri *et al*. 2014, Pradhan and Chakraverty 2015, Bellifa *et al*. 2016, Ait Yahia *et al*. 2015, Ebrahimi and Dashti 2015, Bennoun *et al*. 2016, Draiche *et al*. 2016, Houari *et al*. 2016, Tounsi *et al*. 2016). Some composite materials improve the resistance of engineering structures in front of relative higher variations of temperature and moisture contents. Unfortunately, the structural response of composite materials is not easy to understand and hence a mathematical model of composite structures is always an important starting point.

The mechanical response of composite materials has been investigated by different models over the last and present century. For example, the classical plate theory (CPT) was extended to the first order-shear-deformation-theory (FSDT) in Reissner (1945) and Mindlin (1951) to determine the shear deformation influence for thick plates by a constant transverse shear strain component with a shear correction coefficient.

The limitations of CPT and FSDTs favored the development of higher order shear deformation theories to avoid the employ of shear correction coefficients, to introduce effect of cross sectional warping and to get the realistic distribution of the transverse shear strains and stresses within the thickness of plates. An extensive review of laminated plate models can be found in Reissner (1985), Noor and Burton (1989), Mallikarjuna and Kant (1993), Ghugal and Shimpi (2002), Carrera (2003), Reddy and Arciniega (2004), Wanji and Zhen (2008), Demasi (2008, 2009a, b, c, d, e) and Kreja (2011). Reddy (1984a) has proposed well-known higher order shear deformation theory by assuming polynomial functions in-terms of thickness coordinate. Soldatos (1992) developed a hyperbolic shear deformation theory for homogenous monoclinic plates whereas Mahi *et al*. (2015) proposed a new hyperbolic shear deformation theory for bending and vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates. A novel inverse hyperbolic shear deformation theory is presented by Grover *et al*. (2013). Hebali *et al*. (2014) have developed a new quasi 3D-hyperbolic shear deformation theory for the bending and free vibration analysis of functionally graded plates. Belabed *et al*. (2014) presented an efficient and simple higher order shear and normal deformation theory for functionally graded plates. A novel simple shear and normal deformations theory is proposed by Bourada *et al*. (2015) for functionally graded beams. Ait Atmane *et al*. (2015) presented a new computational shear displacement model for dynamic analysis of FG beams with porosities. Al-Basyouni *et al*. (2015) discussed the size dependent bending and vibration analysis of FG micro beams based on a simple HSDT, modified couple stress theory and neutral surface position. Karama *et al*. (2003, 2009) developed an exponential function in terms of thickness coordinate for laminated composite beam and plates. Bousahla *et al*. (2014) presented a new higher order shear and normal deformation theory based on neutral surface position for

*Corresponding author, Ph.D.
E-mail: tou_abdel@yahoo.com

http://www.techno-press.com/journals/sss&subpage=7 ISSN: 1738-1584 (Print), 1738-1991 (Online)

bending analysis of advanced composite plates. Ait Amar Meziane *et al*. (2014) presented an efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions. Nguyen *et al*. (2015) proposed a refined higher-order shear deformation theory for bending, vibration and buckling analysis of FG sandwich plates. Belkorissat *et al*. (2015) studied vibration properties of FG nano-plate using a new nonlocal refined four variable model. Sayyad and Ghugal (2012a, b) also investigated an exponential shear deformation theory for the bending, buckling and dynamic analysis of isotropic plates. Sayyad (2013) used the exponential shear deformation theory for the bending analysis of orthotropic plates. Versino *et al*. (2013) have been proposed a refined zigzag theory for the investigation of homogeneous, multilayer composite and sandwich plates. Sturzenbecher and Hofstetter (2011) discussed bending response of cross-ply laminated composites by employing an accurate and efficient plate theory. Finite element analysis of laminated sandwich plate based on an improved higher order zigzag plate model is investigated by Pandit *et al*. (2010). Shooshtari and Razavi (2010) have used an analytical solution for linear and nonlinear vibrations of composite and fiber metal laminated rectangular plates. Kar *et al*. (2015) analyzed the nonlinear flexural response of laminated composite flat panel under hygro-thermo-mechanical loading. Recently, Bourada *et al*. (2016) studied the buckling behavior of isotropic and orthotropic plates by proposing a new four variable refined plate theory. Saidi *et al*. (2016) presented a simple hyperbolic shear deformation theory for vibration analysis of thick FG rectangular plates resting on elastic foundations. Boukhari *et al*. (2016) presented an efficient shear deformation theory for wave propagation of FG plates. Bounouara *et al*. (2016) developed a nonlocal zeroth-order shear deformation theory for free vibration of FG nanoscale plates resting on elastic foundation. It is noted that many studies can be find on the buckling, post-buckling and thermo-mechanical behaviors of composite structures with and without smart material in the open literature (Panda and Singh 2009, Panda and Singh 2010a, b, Panda and Singh 2011, Panda and Singh 2013a, b, c, d, Tounsi *et al*. 2013, Bouderba *et al*. 2013, Zidi *et al*. 2014, Kar and Panda 2014, Mahapatra and Panda 2015, Mahapatra *et al*. 2015a, b, Panda and Katariya 2015, Singh and Panda 2015, Meradjah *et al*. 2015, Kar and Panda 2015a, b, Attia *et al*. 2015, Tebboune *et al*. 2015, Hamidi *et al*. 2015, Katariya and Panda 2016, Mahapatra and Panda 2016, Mahapatra *et al*. 2016a, b, Mehar and Panda 2016a, b, Beldjelili *et al*. 2016, Bousahla *et al*. 2016, Bouderba *et al*. 2016, El-Hassar *et al*. 2016, Kar *et al*. 2016, Mehar *et al*. 2016, Kar and Panda 2016a, b, c, Ahouel *et al*. 2016, Mehar and Panda 2017).

In this paper, a hyperbolic shear deformation theory is employed to develop the analytical solution for the thermal buckling analysis of cross-ply composite multilayered plates under uniform temperature rise. The present theory differs from other higher order theories because, in present theory the displacement field which include undetermined integral terms and contains only four unknowns which is not considered by the other researchers. The results of the present model are compared with the known data in the literature.

2. Analytical modeling

2.1 Kinematics and constitutive equations

The kinematic of the novel theory is proposed as follows (Merdaci *et al*. 2016, Hebali *et al*. 2016)

$$u(x,y,z)=u_0(x,y)-z\frac{\partial w_0}{\partial x}+k_1\, f(z)\int\theta(x,y)\,dx \tag{1a}$$

$$v(x,y,z)=v_0(x,y)-z\frac{\partial w_0}{\partial y}+k_2\, f(z)\int\theta(x,y)\,dy \tag{1b}$$

$$w(x,y,z)=w_0(x,y) \tag{1c}$$

where $u_0(x,y)$, $v_0(x,y)$, $w_0(x,y)$ and $\theta(x,y)$ are the four unknown displacement functions of middle surface of the plate. The last unknown is a mathematical term that allows obtaining the rotations of the normal to the mid-plate about the x and y axes (as in the ordinary HSDT). Note that the integrals do not have limits. In the present work is considered terms with integrals instead of terms with derivatives. The constants k_1 and k_2 depends on the geometry.

In this paper, the proposed HSDT is obtained by putting

$$f(z)=z-\frac{2z\sinh\left(\dfrac{z^2}{h^2}\right)}{2\sinh\left(\dfrac{1}{4}\right)+\cosh\left(\dfrac{1}{4}\right)} \tag{2}$$

The linear strain relations obtained from the displacement model of Eqs. (1a)-(1c), valid for thin, moderately thick and thick plates under consideration are as follows

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}+z\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}+f(z)\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix},\qquad \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=g(z)\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix}, \tag{3}$$

where

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial u_0}{\partial x}\\ \dfrac{\partial v_0}{\partial x}\\ \dfrac{\partial u_0}{\partial y}+\dfrac{\partial v_0}{\partial x}\end{Bmatrix},\quad \begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}=\begin{Bmatrix}-\dfrac{\partial^2 w_0}{\partial x^2}\\ -\dfrac{\partial^2 w_0}{\partial y^2}\\ -2\dfrac{\partial^2 w_0}{\partial x\partial y}\end{Bmatrix}, \tag{4a}$$

$$\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} k_1\theta \\ k_2\theta \\ k_1 \dfrac{\partial}{\partial y}\int \theta\, dx + k_2 \dfrac{\partial}{\partial x}\int \theta\, dy \end{Bmatrix},$$

$$\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} k_1 \int \theta\, dy \\ k_2 \int \theta\, dx \end{Bmatrix},$$

and

$$g(z) = \frac{df(z)}{dz} \tag{4b}$$

The integrals employed in the above expressions shall be resolved by a Navier solution and can be written by

$$\frac{\partial}{\partial y}\int \theta\, dx = A' \frac{\partial^2 \theta}{\partial x \partial y}, \quad \frac{\partial}{\partial x}\int \theta\, dy = B' \frac{\partial^2 \theta}{\partial x \partial y}, \qquad \int \theta\, dx = A' \frac{\partial \theta}{\partial x}, \quad \int \theta\, dy = B' \frac{\partial \theta}{\partial y} \tag{5}$$

where the parameters A' and B' are defined according to the type of solution used, in this case via Navier. Hence, A' and B' are given by

$$A' = -\frac{1}{\alpha^2}, \quad B' = -\frac{1}{\beta^2}, \quad k_1 = \alpha^2, \quad k_2 = \beta^2 \tag{6}$$

where α and β are given in expression (17).

The constitutive relations for a laminated plate considering the thermal influences can be expressed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} & 0 & 0 \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} & 0 & 0 \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \overline{Q}_{44} & \overline{Q}_{45} \\ 0 & 0 & 0 & \overline{Q}_{45} & \overline{Q}_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x - \alpha_x \Delta T \\ \varepsilon_y - \alpha_y \Delta T \\ \gamma_{xy} - \alpha_{xy} \Delta T \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{7}$$

where $\overline{Q}_{ij}$ are the transformed material constants given in Reddy (1997). α_x, α_y and α_{xy} are the transformed coefficients of thermal expansion and ΔT is the uniform constant temperature difference.

2.2 Governing equations

The equilibrium equations of plates under thermal loadings may be obtained on the basis of the stationary potential energy (Reddy 1984b). The equilibrium equations are deduced as

$$\begin{aligned} &\delta u_0: \ \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \\ &\delta v_0: \ \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \\ &\delta w_0: \ \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} + N_x^0 \frac{\partial^2 w_0}{\partial x^2} + 2N_{xy}^0 \frac{\partial^2 w_0}{\partial x \partial y} + N_y^0 \frac{\partial^2 w_0}{\partial y^2} = 0 \\ &\delta \theta: -k_1 M_x^s - k_2 M_y^s - (k_1 A' + k_2 B') \frac{\partial^2 M_{xy}^s}{\partial x \partial y} + k_1 A' \frac{\partial S_{xz}^s}{\partial x} + k_2 B' \frac{\partial S_{yz}^s}{\partial y} = 0 \end{aligned} \tag{8}$$

By employing the constitutive relations, the stress and moment resultants are given as

$$\left(N_i, M_i^b, M_i^s\right) = \int_{-h/2}^{h/2} (1, z, f)\sigma_i dz, \quad (i = x, y, xy)$$
$$\text{and} \quad \left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} g\left(\tau_{xz}, \tau_{yz}\right) dz \tag{9}$$

Substituting Eq. (7) into Eq. (9) and integrating within the thickness of the plate, the stress resultants are expressed as

$$\begin{Bmatrix} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} \\ \begin{Bmatrix} M_x^b \\ M_y^b \\ M_{xy}^b \end{Bmatrix} \\ \begin{Bmatrix} M_x^s \\ M_y^s \\ M_{xy}^s \end{Bmatrix} \end{Bmatrix} = \begin{bmatrix} \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} & \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} & \begin{bmatrix} B_{11}^s & B_{12}^s & B_{16}^s \\ B_{12}^s & B_{22}^s & B_{26}^s \\ B_{16}^s & B_{26}^s & B_{66}^s \end{bmatrix} \\ \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} & \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} & \begin{bmatrix} D_{11}^s & D_{12}^s & D_{16}^s \\ D_{12}^s & D_{22}^s & D_{26}^s \\ D_{16}^s & D_{26}^s & D_{66}^s \end{bmatrix} \\ \begin{bmatrix} B_{11}^s & B_{12}^s & B_{16}^s \\ B_{12}^s & B_{22}^s & B_{26}^s \\ B_{16}^s & B_{26}^s & B_{66}^s \end{bmatrix} & \begin{bmatrix} D_{11}^s & D_{12}^s & D_{16}^s \\ D_{12}^s & D_{22}^s & D_{26}^s \\ D_{16}^s & D_{26}^s & D_{66}^s \end{bmatrix} & \begin{bmatrix} H_{11}^s & H_{12}^s & H_{16}^s \\ H_{12}^s & H_{22}^s & H_{26}^s \\ H_{16}^s & H_{26}^s & H_{66}^s \end{bmatrix} \end{bmatrix} \begin{Bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} \\ \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} \\ \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} \end{Bmatrix} - \begin{Bmatrix} \begin{Bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{Bmatrix} \\ \begin{Bmatrix} M_x^{bT} \\ M_y^{bT} \\ M_{xy}^{bT} \end{Bmatrix} \\ \begin{Bmatrix} M_x^{sT} \\ M_y^{sT} \\ M_{xy}^{sT} \end{Bmatrix} \end{Bmatrix} \tag{10a}$$

$$\begin{Bmatrix} S_{yz}^s \\ S_{xz}^s \end{Bmatrix} = \begin{bmatrix} A_{44}^s & A_{45}^s \\ A_{45}^s & A_{55}^s \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} \tag{10b}$$

and stiffness components are given as

$$(A_{ij}, B_{ij}, B_{ij}^s, D_{ij}, D_{ij}^s, H_{ij}^s) = \int_{-h/2}^{h/2} \overline{Q}_{ij} (1, z, f(z), z^2, z\, f(z), f(z)^2) dz, \ (i, j = 1,2,6), \tag{10c}$$

$$A_{ij}^s = \int_{-h/2}^{h/2} \overline{Q}_{ij} \left[g(z)\right]^2 dz, \ (i, j = 4,5) \tag{10d}$$

The stress and moment resultants, $\{N^T\}$, $\{M^{bT}\}$ and $\{M^{sT}\}$ to thermal loading are defined by

$$\left\{N^T\right\} = \int_{-h/2}^{h/2} \{\beta\} T dz, \quad \left\{M^{bT}\right\} = \int_{-h/2}^{h/2} \{\beta\} Tz dz,$$
$$\left\{M^{sT}\right\} = \int_{-h/2}^{h/2} \{\beta\} T f(z) dz \tag{11}$$

where

$$\{\beta\} = \begin{Bmatrix} Q_{11}\alpha_x + Q_{12}\alpha_y \\ Q_{12}\alpha_x + Q_{22}\alpha_y \\ 0 \end{Bmatrix} \tag{12}$$

To determine the stability equations and study the thermal stability behavior of the composite plate, the adjacent equilibrium criterion is employed (Brush and Almroth 1975). By using this approach, the governing stability equations are given as

$$\begin{aligned}
&\delta u_0: \ \frac{\partial N_x^1}{\partial x}+\frac{\partial N_{xy}^1}{\partial y}=0\\
&\delta v_0: \ \frac{\partial N_{xy}^1}{\partial x}+\frac{\partial N_y^1}{\partial y}=0\\
&\delta w_0: \ \frac{\partial^2 M_x^{b1}}{\partial x^2}+2\frac{\partial^2 M_{xy}^{b1}}{\partial x\partial y}+\frac{\partial^2 M_y^{b1}}{\partial y^2}+N_x^0\frac{\partial^2 w_0^1}{\partial x^2}+2N_{xy}^0\frac{\partial^2 w_0^1}{\partial x\partial y}+N_y^0\frac{\partial^2 w_0^1}{\partial y^2}=0\\
&\delta\theta: -\mathrm{k}_1 M_x^{s1}-\mathrm{k}_2 M_y^{s1}-(k_1A'+k_2B')\frac{\partial^2 M_{xy}^{s1}}{\partial x\partial y}+k_1A'\frac{\partial S_{xz}^{s1}}{\partial x}+k_2B'\frac{\partial S_{yz}^{s1}}{\partial y}=0
\end{aligned} \tag{13}$$

where N_x^0, N_{xy}^0 and N_y^0 are the pre-buckling forces.

2.3 Thermal stability solution

For anti-symmetric cross-ply plates, the following stiffness coefficients are identically zero

$$\begin{aligned}
&A_{16}=A_{26}=D_{16}=D_{26}=B_{16}^s=B_{26}^s=\\
&D_{16}=D_{26}=D_{16}^s=D_{26}^s=H_{16}^s=H_{26}^s=A_{45}^s=0\\
&B_{12}=B_{66}=B_{12}^s=B_{66}^s=0
\end{aligned} \tag{14}$$

For symmetric cross-ply laminated plates, the following plate stiffness to have zero values

$$\begin{aligned}
&A_{16}=A_{26}=D_{16}=D_{26}=B_{16}^s=B_{26}^s\\
&=D_{16}=D_{26}=D_{16}^s=D_{26}^s=H_{16}^s=H_{26}^s=A_{45}^s=0\\
&B_{ij}=0, \qquad i,j=1,2,6
\end{aligned} \tag{15}$$

Rectangular plates are generally classified in accordance with the type of the used support. We are here concerned with the exact solution of Eqs. (13) for a simply supported laminated plate. Based on the Navier procedure, the following expansions of displacements are used to automatically respect the simply supported boundary conditions of plate

$$\begin{Bmatrix} u_0^1 \\ v_0^1 \\ w_0^1 \\ \theta^1 \end{Bmatrix}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{Bmatrix} U_{mn}^1\cos(\alpha\, x)\sin(\beta\, y) \\ V_{mn}^1\sin(\alpha\, x)\cos(\beta\, y) \\ W_{mn}^1\sin(\alpha\, x)\sin(\beta\, y) \\ X_{mn}^1\sin(\alpha\, x)\sin(\beta\, y) \end{Bmatrix} \tag{16}$$

where U_{mn}^1, V_{mn}^1, W_{mn}^1 and X_{mn}^1 are coefficients, and α and β are expressed as

$$\alpha=m\pi/a \quad \text{and} \quad \beta=n\pi/b \tag{17}$$

Substituting Eq. (16) into Eq. (13), one obtains

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix}\begin{Bmatrix} U_{mn}^1 \\ V_{mn}^1 \\ W_{mn}^1 \\ X_{mn}^1 \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{18}$$

3. Numerical results and discussion

Several examples are presented to show the accuracy and efficiency of the proposed theory. In the considered examples, both symmetric and anti-symmetric cross-ply thick rectangular plates are examined and the following material properties are assumed:

a) *Isotropic plate* (Brush and Almroth 1975)

$$E_1=E_2, \ \nu=0.3, \ \alpha_1=\alpha_2=\alpha_0=10\times10^{-6}$$

b) *Orthotropic lamina*

• Material A (Brush and Almroth 1975)

$$E_1=15E_2, \ G_{12}=G_{13}=0.5E_2,$$
$$G_{23}=0.3356E_2, \ \nu_{12}=0.3, \ \alpha_1=0.015\alpha_2$$

• Material B (Shiau *et al*. 2010)

$$E_1=22.5\,\text{GPa}, \ E_2=1.17\,\text{GPa}, \ G_{12}=0.66\,\text{GPa},$$
$$\nu_{12}=0.22, \ \alpha_1=-0.04\times10^{-6}\,°C, \ \alpha_2=16.7\times10^{-6}\,°C$$

• Material C (Shiau *et al*. 2010)

$$E_1=155\,\text{GPa}, \ E_2=8.07\,\text{GPa}, \ G_{12}=4.55\,\text{GPa}, \ G_{23}=3.25\,\text{GPa},$$
$$\nu_{12}=0.22, \ \alpha_1=-0.07\times10^{-6}\,°C, \ \alpha_2=30.1\times10^{-6}\,°C$$

• Material D (Thangaratnam *et al*. 1989)

$$E_1=20E_2, \ G_{12}=G_{13}=G_{23}=0.5E_2, \ \nu_{12}=0.25, \ \alpha_1=2\alpha_2$$

In the first example, the obtained results of critical temperatures ($\alpha_0\Delta T$) of simply supported square isotropic plates are compared with those reported by Noor and Burton (1992), Matsunaga (2005), Singh *et al*. (2013) and Bouazza *et al*. (2016) in Table 1. The square plates with different thickness-side ratio (a/h=2, 10/3, 4, 5, 20/3, 10, 20,100) are examined with considering the uniform temperature load. Table 1 demonstrates a good agreement between the present results and those reported by Noor and Burton (1992), Matsunaga (2005), Singh *et al*. (2013) and Bouazza *et al*. (2016).

In the second comparison, orthotropic plate with simply supported boundary conditions is considered. Table 2 presents comparisons of the critical temperatures ($\alpha_0\Delta T$) for simply supported square orthotropic plates with the solutions of Noor and Burton (1992), Matsunaga (2005),

Table 1 Comparisons of critical temperatures for simply supported square isotropic plates subjected to uniform temperature rise

a/h	Noor and Burton (1992)	Matsunaga (2005)	Singh *et al*. (2013)	Bouazza *et al*. (2016)	Present
2	–	0.1253	–	0.1325	0.1325
10/3	$0.7193\ 10^{-1}$	$0.7193\ 10^{-1}$	–	$0.7569\ 10^{-1}$	$0.7569\ 10^{-1}$
4	$0.5600\ 10^{-1}$	$0.5600\ 10^{-1}$	$0.5860\ 10^{-1}$	$0.5853\ 10^{-1}$	$0.5853\ 10^{-1}$
5	$0.3990\ 10^{-1}$	$0.3990\ 10^{-1}$	$0.4134\ 10^{-1}$	$0.4132\ 10^{-1}$	$0.4132\ 10^{-1}$
20/3	$0.2468\ 10^{-1}$	$0.2468\ 10^{-1}$	–	$0.2527\ 10^{-1}$	$0.2527\ 10^{-1}$
10	$0.1183\ 10^{-1}$	$0.1183\ 10^{-1}$	$0.1198\ 10^{-1}$	$0.1198\ 10^{-1}$	$0.1198\ 10^{-1}$
20	$0.3109\ 10^{-2}$	$0.3109\ 10^{-2}$	$0.3120\ 10^{-2}$	$0.3119\ 10^{-2}$	$0.3119\ 10^{-2}$
100	$0.1264\ 10^{-3}$	$0.1264\ 10^{-3}$	$0.1256\ 10^{-3}$	$0.1265\ 10^{-3}$	$0.1265\ 10^{-3}$

Table 2 Comparisons of critical temperatures for simply supported square orthotropic plates subjected to uniform temperature rise, Material A

a/h	Noor and Burton (1992)	Matsunaga (2005)	Singh *et al*. (2013)	Bouazza *et al*. (2016)	Present
2	–	0.2761	–	0.3001	0.3001
10/3	0.2057	0.2057	–	0.2276	0.2276
4	0.1777	0.1777	0.1878	0.1973	0.1973
5	0.1436	0.1436	0.1506	0.1591	0.1591
20/3	0.1029	0.1029	–	0.1124	0.1124
10	$0.5782\ 10^{-1}$	$0.5782\ 10^{-1}$	$0.5918\ 10^{-1}$	$0.6125\ 10^{-1}$	$0.6125\ 10^{-1}$
20	$0.1739\ 10^{-1}$	$0.1739\ 10^{-1}$	$0.1752\ 10^{-1}$	$0.1773\ 10^{-1}$	$0.1773\ 10^{-1}$
100	$0.7463\ 10^{-3}$	$0.7463\ 10^{-3}$	$0.7463\ 10^{-3}$	$0.7469\ 10^{-3}$	$0.7469\ 10^{-3}$

Table 3 Comparisons of critical temperatures for simply supported square cross-ply laminated composite plates [0°/90°] subjected to uniform temperature rise, Material A

a/h	Matsunaga (2005)	Bouazza *et al*. (2016)	Present
2	0.3198	0.3646	0.3001
10/3	0.2114	0.2342	0.2276
4	0.1729	0.1891	0.1973
5	0.1302	0.1399	0.1591
20/3	$0.8524\ 10^{-1}$	$0.8966\ 10^{-1}$	0.1124
10	$0.4310\ 10^{-1}$	$0.4428\ 10^{-1}$	$0.6125\ 10^{-1}$
20	$0.1177\ 10^{-1}$	$0.1186\ 10^{-1}$	$0.1773\ 10^{-1}$
100	$0.4856\ 10^{-3}$	$0.4858\ 10^{-3}$	$0.7469\ 10^{-3}$

Table 4 Comparisons of critical temperatures for simply supported square cross-ply laminated composite plates [0°/90°/0°] subjected to uniform temperature rise, Material A

a/h	Noor and Burton (1992)	Matsunaga (2005)	Singh *et al*. (2013)	Bouazza *et al*. (2016)	Present
2	–	0.3298	–	0.3755	0.3755
10/3	–	0.2447	–	0.2852	0.2852
4	0.2140	0.2133	0.2253	0.2486	0.2486
5	0.1763	0.1752	0.1828	0.2022	0.2022
20/3	–	0.1287	–	0.1446	0.1446
10	$0.7467\ 10^{-1}$	$0.7442\ 10^{-1}$	$0.7433\ 10^{-1}$	$0.7990\ 10^{-1}$	$0.7990\ 10^{-1}$
20	$0.2308\ 10^{-1}$	$0.2291\ 10^{-1}$	$0.2308\ 10^{-1}$	$0.2342\ 10^{-1}$	$0.2342\ 10^{-1}$
100	$0.9961\ 10^{-3}$	$0.9910\ 10^{-3}$	$0.9917\ 10^{-3}$	$0.9919\ 10^{-3}$	$0.9919\ 10^{-3}$

Singh *et al*. (2013) and Bouazza *et al*. (2016). Again, a good agreement is demonstrated between the obtained results and those of Noor and Burton (1992), Matsunaga (2005), Singh *et al*. (2013) and Bouazza *et al*. (2016). It is observed that the minimum critical temperatures of orthotropic square plates correspond to m=1, n=2. However, the minimum critical temperatures of isotropic square plates correspond to m=n=1.

In Table 3, authors presents comparisons of the critical temperatures ($\alpha_0\Delta T$) for a square two-ply cross-ply laminated composite plates [0°/90°] with different values of plate width-to-thickness ratio a/h with those of Matsunaga (2005) and Bouazza *et al*. (2016). Examination of Table 3 also reveals that, the present theory performs as well as Matsunaga (2005) and Bouazza *et al*. (2016).

Table 4 shows the critical temperatures ($\alpha_0\Delta T$) for a three-ply cross-ply laminated composite plates [0°/90°/0°] under uniform temperature change where a comparison is carried out with those of Noor and Burton (1992), Matsunaga (2005), Singh *et al*. (2013) and Bouazza *et al*. (2016). A good agreement is also confirmed from this examination.

Figs. 1 and 2 show the variations of non-dimensional critical temperature versus side-to-thickness ratio of $[0°/90°/90°/0°]_S$ symmetric square plate and [0°/90°] anti-symmetric square plate, respectively. The results are compared with those of the refined hyperbolic shear deformation theory used by Bouazza *et al*. (2016) and CPT. The non-dimensional critical temperature is defined by $T^* = T_{cr} a^2 h \alpha_2 / \pi^2 D_{22}$. It can be observed that the results of the present theory are in excellent agreement with those of Bouazza *et al*. (2016) for all values of a/h. However, since the transverse shear deformation influences of plate

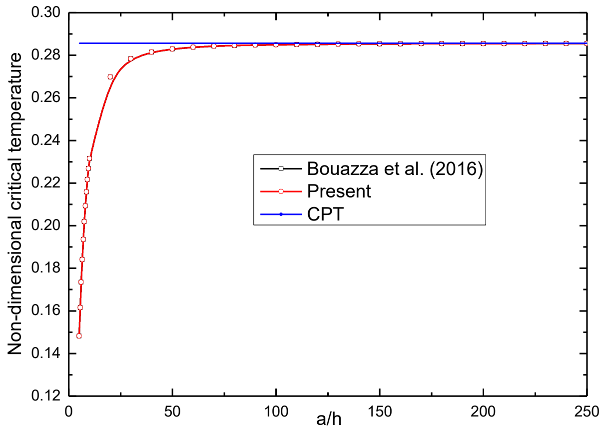

Fig. 1 Non-dimensional critical temperature versus side-to-thickness ratio of a symmetric square laminated plate [0°/90°/90°/0°]S ($T^* = T_{cr} a^2 h \alpha_2 / \pi^2 D_{22}$). Material C

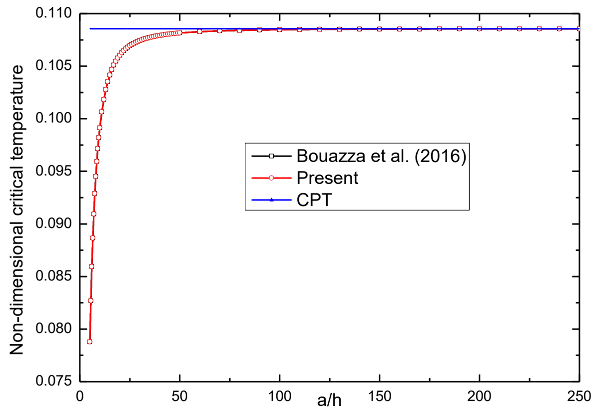

Fig. 2 Non-dimensional critical temperature versus side-to-thickness ratio of a anti-symmetric square laminated plate [0°/90°] ($T^* = T_{cr} a^2 h \alpha_2 / \pi^2 D_{22}$). Material C

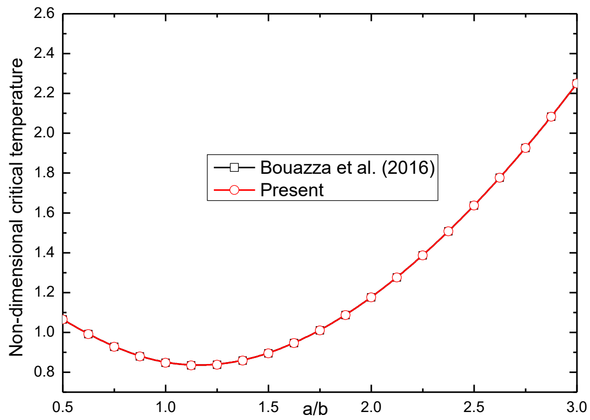

Fig. 3 Non-dimensional critical temperature for cross-ply symmetric laminates plates (0°/90°)s versus aspect ratios of plates ($T_{cr}\alpha_2 10^4$). Material D

are not considered in the CPT, the values of non-dimensional critical temperature predicted by CPT are independent of a/h. It is seen that the non-dimensional critical temperature increases with increasing the thickness ratio a/h, while the CPT overestimates the non-dimensional critical buckling temperature of laminated plate. The difference between HSDTs and CPT is found to be considerable for thick plates (a/h<50), and negligible for thin plates.

Fig. 3 presents the comparisons of the critical temperatures obtained via the proposed theory and those computed using the refined hyperbolic shear deformation theory used by Bouazza *et al.* (2016). The variation of critical temperature of cross-ply symmetric laminates subjected to a constant temperature rise for different aspect ratios is presented also in Fig. 3. It can be concluded that the obtained results are in excellent agreement with those obtained by the theory of Bouazza *et al.* (2016) for all values of aspect ratios a/b. It can be seen from Fig. 3 that the critical temperature decreases when a/b varies from 0.5 to 1.0, while this temperature increases when the aspect ratios a/b becomes larger than 1.

4. Conclusions

In this work, a simplified HSDT is proposed for thermal buckling analysis of simply supported isotropic, orthotropic and cross-ply laminated plates. By considering some additional simplifying suppositions to the existing HSDTs, with the consideration of an undetermined integral term, the number of unknowns and governing equations of the proposed HSDT are reduced by one, and thus, make this formulation simple and efficient to use. The obtained results were compared with the solutions of several theories. It is concluded that the results of the proposed theory has an excellent agreement with the other theories employed for comparison for thermal buckling problems.

References

Ahouel, M., Houari, M.S.A., Adda Bedia, E.A. and Tounsi, A. (2016), "Size-dependent mechanical behavior of functionally graded trigonometric shear deformable nanobeams including neutral surface position concept", *Steel Compos. Struct.*, **20**(5), 963-981.

Ait Amar Meziane, M., Abdelaziz, H.H. and Tounsi, A. (2014), "An efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions", *J. Sandw. Struct. Mater.*, **16**(3), 293-318.

Ait Atmane, H., Tounsi, A., Bernard, F. and Mahmoud, S.R. (2015), "A computational shear displacement model for vibrational analysis of functionally graded beams with porosities", *Steel Compos. Struct.*, **19**(2), 369-384.

Ait Yahia, S., Ait Atmane, H., Houari, M.S.A. and Tounsi, A. (2015), "Wave propagation in functionally graded plates with porosities using various higher-order shear deformation plate theories", *Struct. Eng. Mech.*, **53**(6), 1143-1165.

Al-Basyouni, K.S., Tounsi, A. and Mahmoud, S.R. (2015), "Size dependent bending and vibration analysis of functionally graded micro beams based on modified couple stress theory and neutral surface position", *Compos. Struct.*, **125**, 621-630.

Attia, A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2015), "Free vibration analysis of functionally graded plates with temperature-dependent properties using various four variable refined plate theories", *Steel Compos. Struct.*, **18**(1), 187-212.

Belabed, Z., Houari, M.S.A., Tounsi, A., Mahmoud, S.R. and Anwar Bég, O. (2014), "An efficient and simple higher order shear and normal deformation theory for functionally graded material (FGM) plates", *Compos.: Part B*, **60**, 274-283.

Beldjelili, Y., Tounsi, A. and Mahmoud, S.R. (2016), "Hygro-thermo-mechanical bending of S-FGM plates resting on variable elastic foundations using a four-variable trigonometric plate theory", *Smart Struct. Syst.*, **18**(4), 755-786.

Belkorissat, I., Houari, M.S.A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2015), "On vibration properties of functionally graded nano-plate using a new nonlocal refined four variable model", *Steel Compos. Struct.*, **18**(4), 1063-1081.

Bellifa, H., Benrahou, K.H., Hadji, L., Houari, M.S.A. and Tounsi, A. (2016), "Bending and free vibration analysis of functionally graded plates using a simple shear deformation theory and the concept the neutral surface position", *J. Braz. Soc. Mech. Sci. Eng.*, **38**(1), 265-275.

Bennoun, M., Houari, M.S.A. and Tounsi, A. (2016), "A novel five variable refined plate theory for vibration analysis of functionally graded sandwich plates", *Mech. Adv. Mater. Struct.*, **23**(4), 423-431.

Bouazza, M., Lairedj, A., Benseddiq, N., Khalki, S. (2016), "A refined hyperbolic shear deformation theory for thermal buckling analysis of cross-ply laminated plates", *Mech. Res. Commun.*, **73**, 117-126.

Bouderba, B., Houari, M.S.A. and Tounsi, A. (2013), "Thermomechanical bending response of FGM thick plates resting on Winkler-Pasternak elastic foundations", *Steel Compos. Struct.*, **14**(1), 85-104.

Bouderba, B., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2016), "Thermal stability of functionally graded sandwich plates using a simple shear deformation theory", *Struct. Eng. Mech.*, **58**(3), 397-422.

Boukhari, A., Ait Atmane, H., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2016), "An efficient shear deformation theory for wave propagation of functionally graded material plates", *Struct. Eng. Mech.*, **57**(5), 837-859.

Bounouara, F., Benrahou, K.H., Belkorissat, I. and Tounsi, A. (2016), "A nonlocal zeroth-order shear deformation theory for free vibration of functionally graded nanoscale plates resting on elastic foundation", *Steel Compos. Struct.*, **20**(2), 227-249.

Bourada, M., Kaci, A., Houari, M.S.A. and Tounsi, A. (2015), "A new simple shear and normal deformations theory for functionally graded beams", *Steel Compos. Struct.*, **18**(2), 409-423.

Bourada, F., Amara, K. and Tounsi, A. (2016), "Buckling analysis of isotropic and orthotropic plates using a novel four variable refined plate theory", *Steel Compos. Struct.*, **21**(6), 1287-1306.

Bousahla, A.A., Houari, M.S.A., Tounsi, A. and Adda Bedia, E.A. (2014), "A novel higher order shear and normal deformation theory based on neutral surface position for bending analysis of advanced composite plates", *Int. J. Comput. Meth.*, **11**(6), 1350082.

Bousahla, A.A., Benyoucef, S., Tounsi, A. and Mahmoud, S.R. (2016), "On thermal stability of plates with functionally graded coefficient of thermal expansion", *Struct. Eng. Mech.*, **60**(2), 313-335.

Brush, D.O. and Almroth, B.O. (1975), *Buckling of bars, plates, and shells*, New York: McGraw-Hill.

Carrera, E. (2003), "Historical review of zig-zag theories for multilayered plates and shells", *Appl. Mech. Rev.*, **56**(3), 287-308.

Demasi, L. (2008), "13 hierarchy plate theories for thick and thin composite plates: the generalized unified formulation", *Compos. Struct.*, **84**(3), 256-270.

Demasi, L. (2009a), "16 mixed plate theories based on the generalized unified formulation. Part I: governing equations", *Compos. Struct.*, **87**(1), 1-11.

Demasi, L. (2009b), "16 mixed plate theories based on the generalized unified formulation. Part II: layerwise theories", *Compos. Struct.*, **87**(1), 12-22.

Demasi, L. (2009c), "16 mixed plate theories based on the generalized unified formulation. Part III: advanced mixed high order shear deformation theories", *Compos. Struct.*, **87**(3), 183-194.

Demasi, L. (2009d), "16 mixed plate theories based on the generalized unified formulation. Part IV: zig-zag theories", *Compos. Struct.*, **87**(3), 195–205.

Demasi, L. (2009e), "16 mixed plate theories based on the generalized unified formulation. Part V: results", *Compos. Struct.*, **88**, 1-16.

Draiche, K., Tounsi, A. and Khalfi, Y. (2014), "A trigonometric four variable plate theory for free vibration of rectangular composite plates with patch mass", *Steel Compos. Struct.*, **17**(1), 69-81.

Draiche, K., Tounsi, A. and Mahmoud, S.R. (2016), "A refined theory with stretching effect for the flexure analysis of laminated composite plates", *Geomech. Eng.*, **11**(5), 671-690.

Ebrahimi, F. and Dashti, S. (2015), "Free vibration analysis of a rotating non-uniform functionally graded beam", *Steel Compos. Struct.*, **19**(5), 1279-1298.

El-Hassar, S.M., Benyoucef, S., Heireche, H. and Tounsi, A. (2016), "Thermal stability analysis of solar functionally graded plates on elastic foundation using an efficient hyperbolic shear deformation theory", *Geomech. Eng.*, **10**(3), 357-386.

Ghugal, Y.M. and Shimpi, R.P. (2002), "A review of refined shear deformation theories for isotropic and anisotropic laminated plates", *J. Reinf. Plast. Compos.*, **21**(9), 775-813.

Grover, N., Maiti, D.K. and Singh, B.N. (2013), "A new inverse hyperbolic shear deformation theory for static and buckling analysis of laminated composite and sandwich plates", *Compos. Struct.*, **95**, 667-675.

Hamidi, A., Houari, M.S.A., Mahmoud, S.R. and Tounsi, A. (2015), "A sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates", *Steel Compos. Struct.*, **18**(1), 235-253.

Hebali, H., Tounsi, A., Houari, M.S.A., Bessaim, A. and Adda Bedia, E.A. (2014), "A new quasi-3D hyperbolic shear deformation theory for the static and free vibration analysis of functionally graded plates", *J. Eng. Mech.*, ASCE, **140**(2), 374-383.

Hebali, H., Bakora, A., Tounsi, A. and Kaci, A. (2016), "A novel four variable refined plate theory for bending, buckling, and vibration of functionally graded plates", *Steel Compos. Struct.*, **22**(3), 473-495.

Houari, M.S.A., Tounsi, A., Bessaim, A. and Mahmoud, S.R. (2016), "A new simple three -unknown sinusoidal shear deformation theory for functionally graded plates", *Steel Compos. Struct.*, **22**(2), 257-276.

Kar, V.R. and Panda, S.K. (2014), "Large deformation bending analysis of functionally graded spherical shell using FEM", *Struct. Eng. Mech.*, **53**(4), 661-679.

Kar, V.R. and Panda, S.K. (2015a), "Free vibration responses of temperature dependent functionally graded curved panels under thermal environment", *Lat. Am. J. Solids Struct.*, **12**(11), 2006-2024.

Kar, V.R. and Panda, S.K. (2015b), "Thermoelastic analysis of functionally graded doubly curved shell panels using nonlinear finite element method", *Compos. Struct.*, **129**, 202-212.

Kar, V.R. and Panda, S.K. (2016a), "Nonlinear thermomechanical behavior of functionally graded material cylindrical/hyperbolic/elliptical shell panel with temperature-dependent and temperature-independent properties", *J. Press. Vessel Technol.*, **138**(6), 061202.

Kar, V.R. and Panda, S.K. (2016b), "Post-buckling analysis of shear deformable FG shallow spherical shell panel under uniform and non-uniform thermal environment", *J. Therm. Stress.*, 1-15.

Kar, V.R. and Panda, S.K. (2016c), "Post-buckling behaviour of shear deformable functionally graded curved shell panel under edge compression", *Int. J. Mech. Sci.*, **115**, 318-324.

Kar, V.R., Mahapatra, T.R. and Panda, S.K. (2016), "Effect of different temperature load on thermal postbuckling behaviour of functionally graded shallow curved shell panels", *Compos. Struct.*, 1236-1247.

Kar, V.R., Mahapatra, T.R. and Panda, S.K. (2015), "Nonlinear flexural analysis of laminated composite flat panel under hygro-thermo-mechanical loading", *Steel Compos. Struct.*, **19**(4), 1011-1033.

Karama, M., Afaq, K.S. and Mistou, S. (2003), "Mechanical behavior of laminated composite beam by the new multilayered laminated composite structures model with transverse shear stress continuity", *Int. J. Solids Struct.*, **40**(6), 1525-1546.

Karama, M., Afaq, K.S. and Mistou, S. (2009), "A new theory for laminated composite plates", *Proc. Inst. Mech. Eng., Part L: J. Mater.: Des. Appl.*, **223**(2), 53-62.

Katariya, P.V. and Panda, S.K. (2016), "Thermal buckling and vibration analysis of laminated composite curved shell panel", *Aircraft Eng. Aerosp. Technol.*, **88**(1), 97-107.

Kreja, I. (2011), "A literature review on computational models for laminated composite and sandwich panels", *Cent. Eur. J. Eng.*, **1**(1), 59-80.

Mahapatra, T.R. and Panda, S.K. (2015), "Thermoelastic vibration analysis of laminated doubly curved shallow panels using non-linear FEM", *J. Therm. Stress.*, **38**(1), 39-68.

Mahapatra, T.R., Kar, V.R. and Panda, S.K. (2015a), "Nonlinear free vibration analysis of laminated composite doubly curved shell panel in hygrothermal environment", *J. Sandw. Struct. Mater.*, **17**(5), 511-545.

Mahapatra, T.R., Panda, S.K. and Kar, V.R. (2015b), "Geometrically nonlinear flexural analysis of hygro-thermo-elastic laminated composite doubly curved shell panel", *Int. J. Mech. Mater. Des.*, **12**(2), 153-171.

Mahapatra, T.R., Panda, S.K. and Kar, V.R. (2016a), "Large amplitude vibration analysis of laminated composite spherical panels under hygrothermal environment", *Int. J. Struct. Stab. Dyn.*, **16**(2), 1450105.

Mahapatra, T.R., Panda, S.K. and Kar, V.R. (2016b), "Nonlinear flexural analysis of laminated composite panel under hygro-thermo-mechanical loading - A micromechanical approach", *Int. J. Comput. Meth.*, **13**(3), 1650015.

Mahapatra, T.R. and Panda, S.K. (2016), "Nonlinear free vibration analysis of laminated composite spherical shell panel under elevated hygrothermal environment: A micromechanical approach", *Aerosp. Sci. Technol.*, **49**, 276-288.

Mahi, A., Adda Bedia, E.A. and Tounsi, A. (2015), "A new hyperbolic shear deformation theory for bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates", *Appl. Math. Model.*, **39**(9), 2489-2508.

Mallikarjuna, Kant, T. (1993), "A critical review and some results of recently developed refined theories of fiber-reinforced laminated composites and sandwiches", *Compos. Struct.*, **23**(4), 293-312.

Matsunaga, H. (2005), "Thermal buckling of cross ply laminated composite and sandwich plates according to a global higher order deformation theory", *Compos. Struct.*, **68**(4), 439-454.

Mehar, K. and Panda, S.K. (2016a), "Geometrical nonlinear free vibration analysis of FG-CNT reinforced composite flat panel under uniform thermal field", *Compos. Struct.*, **143**, 336-346.

Mehar, K. and Panda, S.K. (2016b), "Thermoelastic analysis of FG-CNT reinforced shear deformable composite plate under various loadings", *Int. J. Comput. Meth.*, 1750019.

Mehar, K., Panda, S.K., Dehengia, A. and Kar, V.R. (2016), "Vibration analysis of functionally graded carbon nanotube reinforced composite plate in thermal environment", *J. Sandw. Struct. Mater.*, **18**(2), 151-173.

Mehar, K. and Panda, S.K. (2017), "Numerical investigation of nonlinear thermomechanical deflection of functionally graded CNT reinforced doubly curved composite shell panel under different mechanical loads", *Compos. Struct.*, **161**, 287-298.

Meradjah, M., Kaci, A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2015), "A new higher order shear and normal deformation theory for functionally graded beams", *Steel Compos. Struct.*, **18**(3), 793-809.

Merdaci, S., Tounsi, A. and Bakora, A. (2016), "A novel four variable refined plate theory for laminated composite plates", *Steel Compos. Struct.*, **22**(4), 713-732.

Mindlin, R.D. (1951), "Influence of rotary inertia and shear on flexural motions of isotropic elastic plates", *J. Appl. Mech., Trans ASME*, **18**(1), 31-38.

Nedri, K., El Meiche, N. and Tounsi, A. (2014), "Free vibration analysis of laminated composite plates resting on elastic foundations by using a refined hyperbolic shear deformation theory", *Mech. Compos. Mater.*, **49**(6), 629-640.

Nguyen, K.T., Thai, T.H. and Vo, T.P. (2015), "A refined higher-order shear deformation theory for bending, vibration and buckling analysis of functionally graded sandwich plates", *Steel Compos. Struct.*, **18**(1), 91-120.

Noor, A.K. and Burton, W.S. (1989), "Refinement of higher-order laminated plate theories", *Appl. Mech. Rev.*, **42**(1), 1-13.

Noor, A.K. and Burton, W.S. (1992), "Three-dimensional solutions for thermal buckling multilayered anisotropic plates", *J. Eng. Mech.*, **118**(4), 683-701.

Panda, S.K. and Singh, B.N. (2009), "Thermal post-buckling behaviour of laminated composite cylindrical/hyperboloid shallow shell panel using nonlinear finite element method", *Compos. Struct.*, **91**(3), 366-374.

Panda, S.K. and Singh, B.N. (2010a), "Thermal post-buckling analysis of a laminated composite spherical shell panel embedded with shape memory alloy fibres using non-linear finite element method", *Proc. IMechE Part C: J. Mech. Eng. Sci.*, **224**(4), 757-769.

Panda, S.K. and Singh, B.N. (2010b), "Nonlinear free vibration analysis of thermally post-buckled composite spherical shell panel", *Int. J. Mech. Mater. Des.*, **6**(2), 175-188.

Panda, S.K. and Singh, B.N. (2011), "Large amplitude free vibration analysis of thermally post-buckled composite doubly curved panel using nonlinear FEM", *Finite Element. Anal. Des.*, **47**(4), 378-386.

Panda, S.K. and Singh, B.N. (2013a), "Post-buckling analysis of laminated composite doubly curved panel embedded with SMA fibers subjected to thermal environment", *Mech. Adv. Mater. Struct.*, **20**(10), 842-853.

Panda, S.K. and Singh, B.N. (2013b), "Thermal postbuckling behavior of laminated composite spherical shell panel using NFEM", *Mech. Based Des. Struct. Machines*, **41**(4), 468-488.

Panda, S.K. and Singh, B.N. (2013c), "Nonlinear finite element analysis of thermal post-buckling vibration of laminated composite shell panel embedded with SMA fibre", *Aerosp. Sci. Technol.*, **29**(1), 47-57.

Panda, S.K. and Singh, B.N. (2013d), "Large amplitude free vibration analysis of thermally post-buckled composite doubly curved panel embedded with SMA fibers", *Nonlin. Dyn.*, **74**(1), 395-418.

Panda, S.K. and Katariya, P.V. (2015), "Stability and free vibration behaviour of laminated composite panels under thermo-mechanical loading", *Int. J. Appl. Comput. Math.*, **1**(3), 475-490.

Pandit, M.K., Sheikh, A.H. and Singh, B.N. (2010), "Analysis of laminated sandwich plates based on an improved higher order zigzag theory", *J. Sandw. Struct. Mat.*, **12**, 307-326.

Panjehpour, M., Farzadnia, N., Anwar, M.P. and Ali, A.A.A. (2011), "FRP sheets contribution in common repair techniques of concrete structures with emphasis on concrete columns", *Int. J. Sustain. Constr. Eng. Technol.*, **2**(2), 54-61.

Panjehpour, M., Farzadnia, N., Demirboga, R. and Ali, A.A.A. (2016), "Behavior of high-strength concrete cylinders repaired with CFRP sheets", *J. Civ. Eng. Manage.*, **22**(1), 56-64.

Pradhan, K.K. and Chakraverty, S. (2015), "Free vibration of functionally graded thin elliptic plates with various edge supports", *Struct. Eng. Mech.*, **53**(2), 337-354.

Reddy, J.N. and Arciniega, R.A. (2004), "Shear deformation plate and shell theories: from Stavsky to present", *Mech. Adv. Mater. Struct.*, **11**(6), 535-582.

Reddy, J.N. (1984a), "A simple higher order theory for laminated composite plates", *ASME J. Appl. Mech.*, **51**(4), 745-752.

Reddy, J.N. (1997), *Mechanics of laminated composite plate: theory and analysis*, New York: CRC Press.

Reddy, J.N. (1984b), *Energy principles and variational methods in applied mechanics*, New York: John Wiley.

Reissner, E. (1945), "The effect of transverse shear deformation on the bending of elastic plates", *J. Appl. Mech., Trans ASME*, **12**(2), 69-77.

Reissner, E. (1985), "Reflection on the theory of elastic plates", *Appl. Mech. Rev.*, **38**(11), 1453-1464.

Saidi, H., Tounsi, A. and Bousahla, A.A. (2016), "A simple hyperbolic shear deformation theory for vibration analysis of thick functionally graded rectangular plates resting on elastic foundations", *Geomech. Eng.*, **11**(2), 289-307.

Sayyad, A.S. and Ghugal, Y.M. (2012a), "Bending and free vibration analysis of thick isotropic plates by using exponential shear deformation theory", *Appl. Comput. Mech.*, **6**(1), 65-82.

Sayyad, A.S. and Ghugal, Y.M. (2012b), "Buckling analysis of thick isotropic plates by using exponential shear deformation theory", *Appl. Comput. Mech.*, **6**(2), 185-196.

Sayyad, A.S. (2013), "Flexure of thick orthotropic plates by exponential shear deformation theory", *Lat. Am. J. Solids Struct.*, **10**(3), 473-490.

Shiau, L.C., Kuo, S.Y. and Chen, C.Y. (2010), "Thermal buckling behavior of composite laminated plates", *Compos. Struct.*, **92**(2), 508-514.

Singh, S., Singh, J. and Shukla, K.K. (2013), "Buckling of laminated composite plates subjected to mechanical and thermal loads using meshless collocations", *J. Mech. Sci. Technol.*, **27**(2), 327-336.

Singh, V.K. and Panda, S.K. (2015), "Large amplitude free vibration analysis of laminated composite spherical shells embedded with piezoelectric layers", *Smart Struct. Syst.*, **16**(5), 853-872.

Shooshtari, A. and Razavi, S. (2010), "A closed form solution for linear and nonlinear free vibrations of composite and fiber metal laminated rectangular plates", *Compos. Struct.*, **92**(11), 2663-2675.

Soldatos, K.P. (1992), "A transverse shear deformation theory for homogeneous monoclinic plates", *Acta Mech.*, **94**(3), 195-220.

Sturzenbecher, R. and Hofstetter, K. (2011), "Bending of cross-ply laminated composites: an accurate and efficient plate theory based upon models of Lekhnitskii and Ren", *Compos. Struct.*, **93**(3), 1078-1088.

Tebboune, W., Benrahou, K.H., Houari, M.S.A. and Tounsi, A. (2015), "Thermal buckling analysis of FG plates resting on elastic foundation based on an efficient and simple trigonometric shear deformation theory", *Steel Compos. Struct.*, **18**(2), 443-465.

Thangaratnam, K.R. and Palaninathan, Ramachandran, J. (1989), "Thermal buckling of composite laminated plates", *Comput. Struct.*, **32**(5), 1117-1124.

Tounsi, A., Houari, M.S.A. and Bessaim, A. (2016), "A new 3-unknowns non-polynomial plate theory for buckling and vibration of functionally graded sandwich plate", *Struct. Eng. Mech.*, **60**(4), 547-565.

Tounsi, A., Houari, M.S.A., Benyoucef, S. and Adda Bedia, E.A. (2013), "A refined trigonometric shear deformation theory for thermoelastic bending of functionally graded sandwich plates", *Aerosp. Sci. Technol.*, **24**(1), 209-220.

Versino, D., Gherlone, M., Mattone, M., Sciuva, M.D. and Tessler, A. (2013), "C0 triangular elements based on the Refined Zigzag Theory for multilayer composite and sandwich plates", *Compos. B*, **44**(1), 218-230.

Wanji, C. and Zhen, W. (2008), "A selective review on recent development of displacement-based laminated plate theories", *Recent Pat. Mech. Eng.*, **1**(1), 29-44.

Zidi, M., Tounsi, A., Houari, M.S.A., Adda Bedia, E.A. and Anwar Bég, O. (2014), "Bending analysis of FGM plates under hygro-thermo-mechanical loading using a four variable refined plate theory", *Aerosp. Sci. Technol.*, **34**, 24-34.

CC