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Nonlinear and linear thermo-elastic analyses of a functionally graded spherical shell using the Lagrange strain tensor

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Abstract. This research tries to present a nonlinear thermo-elastic solution for a functionally graded spherical shell subjected to mechanical and thermal loads. Geometric nonlinearity is considered using the Lagrange or finite strain tensor. Non-homogeneous material properties are considered based on a power function. Adomian's decomposition method is used for calculation of nonlinear results. Nonlinear results such as displacement can be evaluated for sphere in terms of different indexes of non-homogeneity. A comprehensive comparison between linear and nonlinear results and evaluation of the percentage of difference between them can be performed in this paper. The obtained results indicate that the improvement of the results due to usage of nonlinear analysis is depending on the non-homogeneous index.

Keywords: thermo-elastic; nonlinear analysis; non-homogenous index; mechanical loads; strain

1. Introduction

One of basic relations in continuum mechanics is strain-displacement relations. For problems with very small displacement, one can use infinitesimal strain tensor relation that is a linear equation. For problems required more accurate results, using the finite or Lagrange strain tensor is proposed. This relation due to existence of nonlinear terms tends to a nonlinear differential equation. Although consideration of nonlinear differential equations and solution of that yields more confidential results rather than linear results, due to inexistence of a comprehensive analytical method for solution of nonlinear problems, the researchers and designer prefer to use a linear solution. One of the important aims of this study is nonlinear solution of a functionally graded sphere. A literature review can express necessity of study of this problem.

Yamanouchi, Koizumi *et al.* (1990) presented the concept of functionally graded materials for the first time in Japan. Woo and Meguid (2001) investigated the nonlinear analysis of functionally graded plates and shallow shells. They proposed an analytical solution for the coupled large deflection of FG plates and shallow shells. Tutuntu and Ozturk (2001) studied the elastic analysis of functionally graded hollow structures. Benjeddou, Deu *et al.* (2002) proposed an exact two-dimensional analytical solution for the free-vibration analysis of simply supported piezoelectric adaptive plates. Chen, Lu *et al.* (2002) studied a FG piezoceramic hollow sphere by using 3D electro elastic formulation. Jabbari, Sohrabpour *et al.* (2002) presented thermoelastic analysis of a functionally graded cylinder under thermal and mechanical loads. They studied the effect of different values of non-homogenous index on the radial displacement and stress. Transient electro elastic analysis of a non-homogenous sphere based on a power function was analytically investigated by Ding, Wang *et al.* (2003).

The piezo thermoelastic solution of a functionally graded hollow cylinder was presented by Ying and Zhi-Fei (2005). Thermo-electro-elastic transient analysis of functionally graded piezoelectric hollow structures was analytically investigated under thermal, mechanical and electrical loads by Dai and Wang (2005). Higher order shear deformation theory was employed for analysis of the problem. Layer wise theory was used for analysis of the functionally graded composite plates in the cylindrical bending that subjected to thermomechanical loadings by Tahani and Mirzababaee (2008). The non-linear strain–displacement relations were used to study the effect of geometric nonlinearity. The equilibrium equations were solved exactly by using the perturbation technique. Hojjati and Safari (2008) studied elastic solution of a rotating disk with non-uniform distribution of thickness and density. They used homotopy perturbation and Adomian's decomposition method for solving the governing equation. These two methods are the adequate tools for solving the non-homogenous linear and nonlinear differential equations. Allahverdizadeh, Naei *et al.* (2008a, b) investigated the nonlinear behavior of thin circular FG plates. The analysis was assumed to be axisymmetric and solution was derived based on a semi-analytical approach. Banerjee, Deu *et al.* (2008) presented the analytical and numerical solutions for the large deflection analysis of a cantilever beam with geometric nonlinearity. Adomian

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decomposition and shooting methods were employed as analytical and numerical methods for analysis of the nonlinear system, respectively. The obtained results using shooting method were compared with Adomian decomposition results.

Malekzadeh and Vosoughi (2009) investigated the large amplitude vibration of composite beams on the nonlinear elastic foundation. The foundation was supposed that has cubic nonlinearity with shearing layer. Khabbaz, Manshadi *et al.* (2009) investigated the nonlinear analysis of FG plates under pressure based on the higher-order shear deformation theory. They used the first and higher order shear deformation theory to investigate the large deflection of FG plate. The effect of the thickness and non-homogenous index was investigated on the distribution of the displacements and stresses. Khoshgoftar, Arani *et al.* (2009) presented the comprehensive thermoelastic analysis of a functionally graded piezoelectric cylindrical shell under electrical and mechanical loads. They supposed all mechanical and electrical properties to be variable along the thickness direction. They considered three different materials for those analyses.

GhannadPour and Alinia (2006) investigated the large deflection analysis of a rectangular FG plate. Von Karman theory was employed for the large deflection analysis. The solution was obtained using minimization of the total potential energy. The effect of material non-homogeneity was investigated on the stresses and deformations. Hui-Shen Shen (2007) considered the nonlinear response of a FG plate due to heat conduction. It was assumed that the plate to be shear deformable. Higher order shear deformation theory was employed for analysis of the problem. Linear and nonlinear analyses of FG and FGP structures have been performed recently by Arefi and Rahimi (2011a, b, 2012a, b, 2014), Rahimi, Arefi *et al*. (2012) and Arefi, Rahimi *et al*. (2012). Arefi and Khoshgoftar (2014) studied thermo-piezo-elastic analysis of a functionally graded piezoelectric sphere and presented the effect of different non-homogenous indexes on the electro mechanical responses of the system. The influence of piezoelectric materials on the nonlinear behaviour of a functionally graded piezoelectric cylinder and sphere has been investigated by Arefi (2013) and Arefi and Nahas (2014).

This paper uses the geometric nonlinearity equations to present the nonlinear differential equations of a functionally graded spherical shell subjected to mechanical and thermal loads. After presentation of nonlinear differential equation, a nonlinear solution is proposed for differential equation. A comparison between linear and nonlinear radial displacement can be presented.

2. Nonlinear formulation for thermoelastic analysis

The nonlinearity is considered in strain displacement relations using the finite or Lagrange strain tensor. The strain-displacement relation using finite strain tensor can be presented as follows (Lai, Rubin *et al.* 1999, Arefi and Rahimi 2011a, Arefi and Rahimi 2012a, Arefi 2013, Arefi and Nahas 2014)

$$\{\varepsilon\} = \frac{1}{2}\{\nabla\vec{u} + \nabla\vec{u}^T + (\nabla\vec{u})^T(\nabla\vec{u})\} \tag{1}$$

in which ε, $\vec{u}$ are strain tensor and displacement vector.

For symmetric condition, we have radial and circumferential strain components as follows (Arefi and Rahimi 2011a, Arefi and Rahimi 2012a, Arefi and Khoshgoftar 2014)

$$\varepsilon_r = \frac{\mathrm{d}u}{\mathrm{d}r} + \frac{1}{2}\left(\frac{\mathrm{d}u}{\mathrm{d}r}\right)^2, \quad \varepsilon_\theta = \varepsilon_\phi = \frac{u}{r}, \tag{2}$$

where u is radial displacement. The constitutive equations in spherical coordinate system by considering the thermal effects have the following form (Tutuncu and Ozturk 2001, Arefi and Khoshgoftar 2014)

$$\begin{aligned} \sigma_r &= \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_r + 2\nu\varepsilon_\theta] - \frac{\alpha T E}{1-2\nu} \\ \sigma_\theta = \sigma_\phi &= \frac{E}{(1+\nu)(1-2\nu)}[\nu\varepsilon_{rr} + \varepsilon_{\theta\theta}] - \frac{\alpha T E}{1-2\nu} \end{aligned} \tag{3}$$

For a functionally graded material with power function distribution, modulus of elasticity, E, has following form: $E = E_0 r^n$, where n is non-homogenous index. Substitution of this gradation and strain components into constitutive equations result in (Tutuncu and Ozturk 2001)

$$\begin{aligned} \sigma_r &= \frac{E_0 r^n}{(1+\nu)(1-2\nu)}\left[(1-\nu)\left(\frac{\mathrm{d}u}{\mathrm{d}r} + \frac{1}{2}\left(\frac{\mathrm{d}u}{\mathrm{d}r}\right)^2\right) + 2\nu\frac{u}{r}\right] - \frac{\alpha T E_0 r^n}{1-2\nu} \\ \sigma_\theta = \sigma_\phi &= \frac{E_0 r^n}{(1+\nu)(1-2\nu)}\left[\nu\left(\frac{\mathrm{d}u}{\mathrm{d}r} + \frac{1}{2}\left(\frac{\mathrm{d}u}{\mathrm{d}r}\right)^2\right) + \frac{u}{r}\right] - \frac{\alpha T E_0 r^n}{1-2\nu} \end{aligned} \tag{4}$$

Substitution of stress components in equilibrium equation present final nonlinear differential equations of the system as follows (Tutuncu and Ozturk 2001)

$$\frac{\mathrm{d}\sigma_{rr}}{\mathrm{d}r} + 2\frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0 \tag{5}$$

or

$$\begin{aligned} &\frac{\mathrm{d}^2u}{\mathrm{d}r^2} + (n+2)\frac{1}{r}\frac{\mathrm{d}u}{\mathrm{d}r} + 2\left(\frac{n\nu}{1-\nu} - 1\right)\frac{u}{r^2} + \left[n + \frac{2(1-2\nu)}{1-\nu}\right]\frac{1}{2r}\left(\frac{\mathrm{d}u}{\mathrm{d}r}\right)^2 \\ &\qquad + \frac{\mathrm{d}u}{\mathrm{d}r}\frac{\mathrm{d}^2u}{\mathrm{d}r^2} = \frac{\alpha(1+\nu)}{1-\nu}\left(\frac{nT}{r} + \frac{\mathrm{d}T}{\mathrm{d}r}\right) \end{aligned} \tag{6}$$

The governing nonlinear differential equations of the system can be solved using Adomian's Decomposition Method. In this method, the nonlinear differential equation must be decomposed into some linear and nonlinear operators. In the general state, a nonlinear differential equation may be decomposed as follows (Banerjee, Bhattacharya *et al.* 2008)

$$L[u] + N[u] + R[u] = g \tag{7}$$

where L is the largest linear differential operator, N is nonlinear operators, R is the other operators and g is the remained functions in nonlinear and non-homogenous differential equation.

By considering the nonlinear differential equation of the current problem, we have

$$\begin{aligned} L &:= \frac{\mathrm{d}^2(\ldots)}{\mathrm{d}r^2} \\ R &:= (n+2)\frac{1}{r}\frac{\mathrm{d}(\ldots)}{\mathrm{d}r} + 2\left(\frac{n\nu}{1-\nu} - 1\right)\frac{(\ldots)}{r^2} \\ N &:= \left[n + \frac{2(1-2\nu)}{1-\nu}\right]\frac{1}{2r}\left(\frac{\mathrm{d}(\ldots)}{\mathrm{d}r}\right)^2 + \frac{\mathrm{d}(\ldots)}{\mathrm{d}r}\frac{\mathrm{d}^2(\ldots)}{\mathrm{d}r^2} \\ g &:= \frac{\alpha(1+\nu)}{1-\nu}\left(\frac{nT}{r} + \frac{\mathrm{d}T}{\mathrm{d}r}\right) \end{aligned} \tag{8}$$

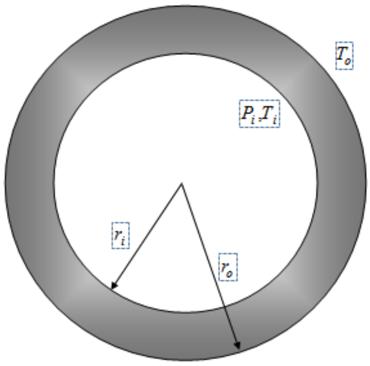

Fig. 1 The schematic of a functionally graded spherical shell

The solution procedure can be started by evaluation of linearized solution as zero'th order solution of the problem. By setting u_0 as zero'th order solution, the new solution of the problem can be considered by multiplication of differential equation with L^{-1} and evaluating the u as follows

$$u = -L^{-1}\left[N[u] + R[u]\right] + g \tag{9}$$

In a successive approximation manner, we will have higher-order solutions as follows

$$u_q = -L^{-1}\left[N\left[u_{q-1}\right] + R\left[u_{q-1}\right]\right] + g + C_{q1}r + C_{q2} \tag{10}$$

Due to two times integration, in every step, a linear term must be considered at the end of solution ($C_{q1}r + C_{q2}$). In every step, homogenized boundary conditions must be considered for evaluation of the constants of linear term. The reason for applying the homogenous boundary condition is that main boundary conditions have been applied on the zero'th order solution and no longer needs them. Final solution of the system is summation of obtained solutions in Eq. (10).

3. Linear solution

After eliminating the nonlinear terms in nonlinear differential equation, Eq. (6), we have

$$\frac{\mathrm{d}^2u}{\mathrm{d}r^2} + (n+2)\frac{1}{r}\frac{\mathrm{d}u}{\mathrm{d}r} + 2\left(\frac{n\nu}{1-\nu} - 1\right)\frac{u}{r^2} = \frac{\alpha(1+\nu)}{1-\nu}\left(\frac{nT}{r} + \frac{\mathrm{dT}}{\mathrm{d}r}\right) \tag{11}$$

Changing the variable of equation from r to s simplifies finding the solution. If we consider $r = \mathrm{e}^s$, we will get homogenized linear differential equations of the system as follows

$$\frac{\mathrm{d}^2u_h(s)}{\mathrm{d}s^2} + (n+1)\frac{du_h(s)}{ds} + 2\left(\frac{n\nu}{1-\nu} - 1\right)u_h(s) = 0 \tag{12}$$

Linear solution is containing homogenous and particular solutions as follows

$$\begin{gathered} u_h = C_1\mathrm{e}^{\left(-\lambda+\sqrt{\lambda^2-\mu}\right)s} + C_2\mathrm{e}^{\left(-\lambda-\sqrt{\lambda^2-\mu}\right)s}, \\ u_p = \frac{\mathrm{e}^{\left(-\lambda+\sqrt{\lambda^2-\mu}\right)s}\int\psi(s)\mathrm{e}^{\left(\lambda-\sqrt{\lambda^2-\mu}\right)s}\mathrm{d}s - \mathrm{e}^{\left(-\lambda-\sqrt{\lambda^2-\mu}\right)s}\int\psi(s)\mathrm{e}^{\left(\lambda+\sqrt{\lambda^2-\mu}\right)s}\mathrm{d}s}{2\sqrt{\lambda^2-\mu}} \end{gathered} \tag{13}$$

where

$$\lambda = \frac{n+1}{2}, \quad \mu = 2\left(\frac{n\nu}{1-\nu} - 1\right), \quad \psi(s) = \frac{\alpha(1+\nu)}{1-\nu}\mathrm{e}^s\left(nT + \frac{\mathrm{dT}}{\mathrm{d}s}\right) \tag{14}$$

In order to find particular solution of the problem, solution of heat transfer equation is required. Symmetric and steady state heat transfer equation in spherical coordinate system has the following form (Jabbari, Bahtui *et al.* 2002).

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(kr^2\frac{\mathrm{d}T(r)}{\mathrm{d}r}\right) = 0 \tag{15}$$

This differential equation can be solved analytically by employing the boundary conditions. With constant temperatures at inner and outer radii and power function of thermal conductivity coefficient as $k = k_0r^m$ (Arefi 2015), we will have the following form of solution

$$T(r) = C_1^Tr^{-(m+1)} + C_2^T \tag{16}$$

where C_1^T and C_2^T are constants that determined from the boundary conditions at the inner ($T(r_i) = T_i$) and outer ($T(r_o) = T_o$) surfaces of the sphere. They are

$$C_1^T = \frac{T_o - T_i}{r_o^{-(m+1)} - r_i^{-(m+1)}}, \quad C_2^T = \frac{T_ir_o^{-(m+1)} - T_or_i^{-(m+1)}}{r_o^{-(m+1)} - r_i^{-(m+1)}} \tag{17}$$

Using thermal conductivity equation, we will have particular solution as follows

$$u_p = \frac{\alpha(1+\nu)}{1-\nu}\left(\frac{n-m-1}{\mu+m^2-2m\lambda}C_1^T\mathrm{e}^{-ms} + \frac{n}{1+\mu+2\lambda}C_2^T\mathrm{e}^s\right) \tag{18}$$

We can suppose particular solution as summation of two terms as follows

$$u_p = D_1\mathrm{e}^{-ms} + D_2\mathrm{e}^s \tag{19}$$

The substitution of above solution in differential equation of the system yields constants of particular solution (D_1, D_2) as follows

$$D_1 = \frac{\alpha(1+\nu)(n-m-1)}{(1-\nu)(\mu+m^2-2m\lambda)}C_1^T, \quad D_2 = \frac{\alpha n(1+\nu)}{(1-\nu)(1+\mu+2\lambda)}C_2^T \tag{20}$$

After performing a thermal analysis, a complete thermoelastic analysis can be performed. We recall homogenous and particular solutions from Eqs. (13) and (19) as follows

$$u = C_1r^{\beta_1} + C_2r^{\beta_2} + D_1r^{-m} + D_2r \tag{21}$$

where $\beta_{1,2} = -\lambda \pm \sqrt{\lambda^2 - \mu}$. By employing the required boundary conditions for a pressure vessel subjected to internal pressure and with no outer pressure, we will have

$$\sigma_r|_{r=r_i} = -P, \quad \sigma_r|_{r=r_o} = 0 \tag{22}$$

Calling the linearized linear stress from Eq. $(4)_1$ and substitution of radial displacement in radial stress yields

$$\begin{aligned} \sigma_r = \frac{E_0r^n}{(1+\nu)(1-2\nu)}\Big\{&[\beta_1(1-\nu) + 2\nu]C_1r^{\beta_1-1} \\ &+ [\beta_2(1-\nu) + 2\nu]C_2r^{\beta_2-1} \\ &- [m(1-\nu) + 2\nu]D_1r^{-(m+1)} + (1+\nu)D_2 \\ &- \alpha(1+\nu)\left(C_1^Tr^{-(m+1)} + C_2^T\right)\Big\} \end{aligned} \tag{23}$$

By employing the required boundary conditions from Eq. (22), the constants C_1 and C_2 can be evaluated. After determination of linear results, the process of nonlinear analysis can be started using Eqs. (8)-(10).

4. Numerical results and discusions

In this stage, the numerical results of linear analysis including thermal and mechanical solutions can be presented. The temperature distribution along the radial direction is plotted in terms of different values of non-homogenous index in Fig. 2. The inner and outer radii are 0.6 and 1 and inner and outer temperatures are 250 and 20 centigrade. One can conclude that with increase of non-homogeneous index, the temperature is increased. This increase is due to decrease of heat conductivity coefficient with increase of non-homogeneous index that decrease flux of temperature.

After determination of temperature distribution, the elastic results can be presented. Numerical constants are considered as $E = 200\,GPa$, $\nu = 0.3$. The non-homogenous index can vary between± 2.

Shown in Fig. 3 is the radial distribution of radial displacement for different values of non-homogenous index using a linear analysis. As observed in this figure, the radial displacement increases with increasing the non-homogenous index. This increasing is due to decrease of modulus of elasticity based on power function distribution and consequently decrease of stiffness of spherical shell.

After performing a linear analysis and using the results of them, at this stage we can evaluate the nonlinear results. Nonlinear radial displacement along the radial direction for different values of non-homogenous index is plotted in Fig. 4.

Shown in Figs. 5 and 6 are the radial distributions of radial and circumferential stresses along the thickness direction of sphere for various values of non-homogeneous index of functionally graded materials. It is observed with increase of non-homogeneous index, the radial stress is increased.

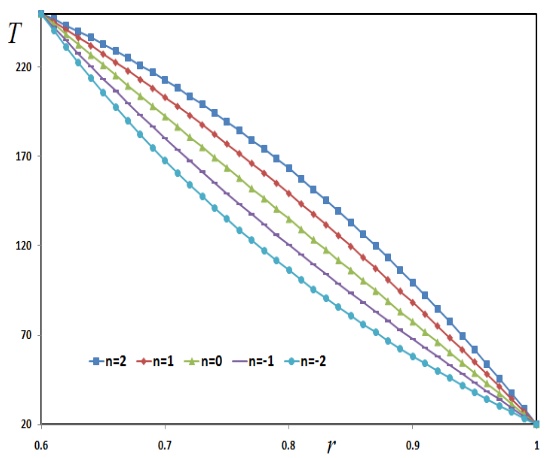

Fig. 2 The radial distribution of temperature for different values of non-homogeneous index

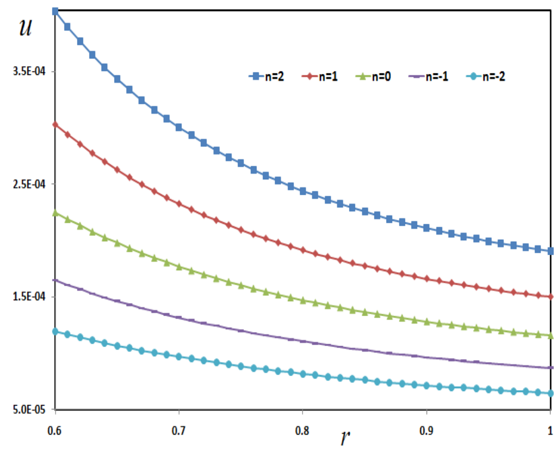

Fig. 3 The radial distribution of radial displacement for different values of non-homogeneous based on a linear analysis

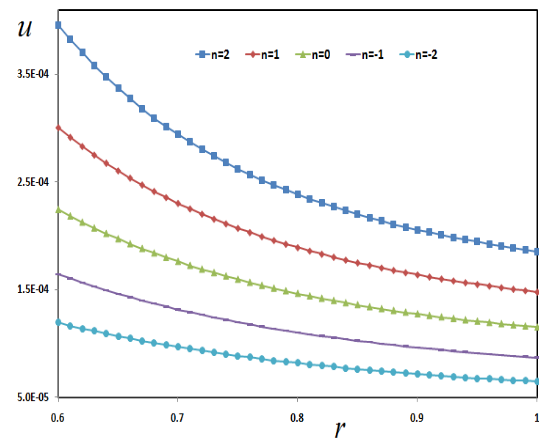

Fig. 4 The radial distribution of radial displacement for different values of non-homogeneous index based on a nonlinear analysis

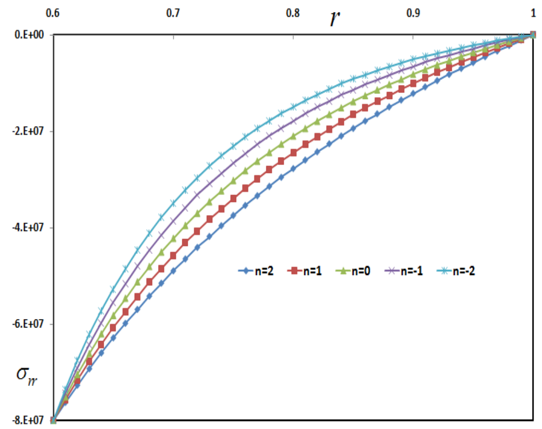

Fig. 5 The radial distribution of radial stress for different values of non-homogeneous index based on a linear analysis

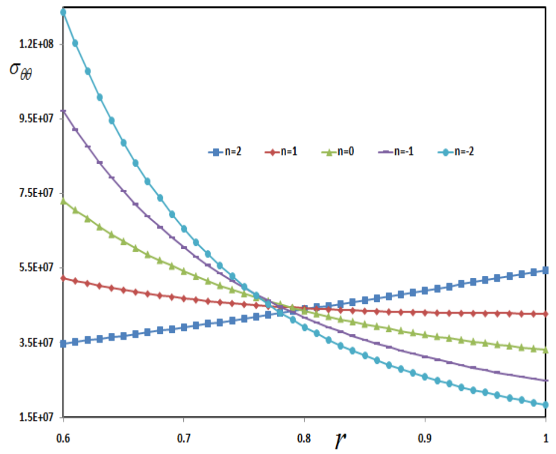

Fig. 6 The radial distribution of circumferential stress for different values of non-homogeneous index based on a linear analysis

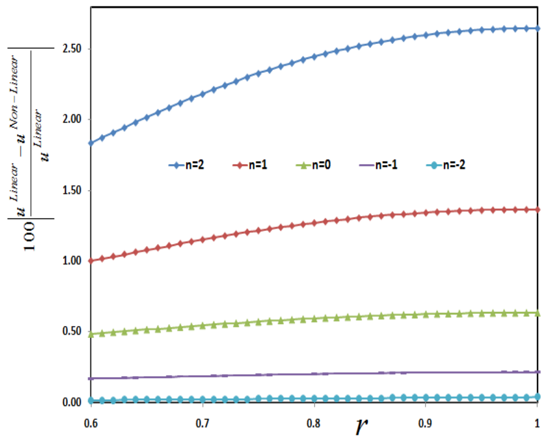

Fig. 7 The improvement of radial displacement for different values of non-homogeneities using a nonlinear analysis

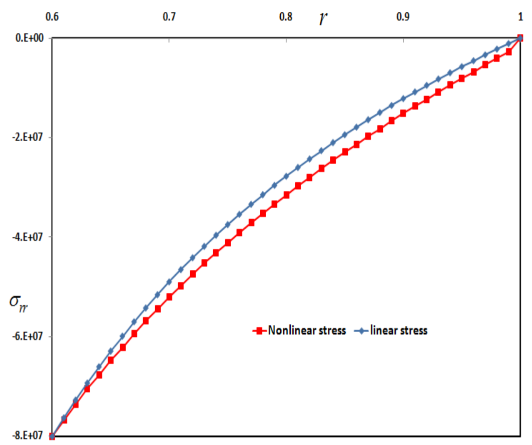

Fig. 8 The radial distribution of radial stress for different values of non-homogeneous index based on the linear and nonlinear analyses

We can conclude that with increase of non-homogeneous index, the displacement and strain are increased and consequently the radial stress is increased.

In order to better presentation and discussion on the difference between linear and nonlinear results, a figure showing the percentage of difference between linear and nonlinear results is required. Shown in Figure 7 is the percentage of improvement of radial displacement using a nonlinear analysis rather than a linear analysis. The obtained results indicate that with increasing the non-homogenous index, the percentage of difference increases. Maximum improvement can be obtained for $n = 2$.

Shown in Fig. 8 is the comparison between linear and nonlinear radial stresses along the radial direction for $n = 2$.

5. Conclusions

Nonlinear analysis of a functionally graded spherical shell under thermomechanical loading has been investigated in this paper. Geometric nonlinearity has been considered in strain-displacement relations based on the finite strain tensor. Material properties have been graded along the radial direction using the power function. Most important results of this research can be expressed as follows:

1. Investigation on the radial displacement for different values of non-homogenous index indicates that with increasing the non-homogeneous index, radial displacement increases. This increasing is due to decreasing the stiffness of material.
2. Consideration of a nonlinear analysis and evaluation of the improvement of the results using the nonlinear analysis gives important conclusions. Investigation on the percentage of difference between linear and nonlinear shows that with increasing the values of non-homogenous index, the percentage of difference increases.
3. The presented methodology in this paper has capability to evaluate the responses of different one and multi fields nonlinear differential equations.

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