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Wavelet analysis based damage localization in steel frames with bolted connections

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Abstract. This paper describes an application of wavelet analysis for damage detection of a steel frame structure with bolted connections. The wavelet coefficients of the acceleration response for the healthy and loosened connection structure were calculated at each measurement point. The difference of the wavelet coefficients of the response of the healthy and loosened connection structure is selected as an indicator of the damage. At each node of structure the norm of the difference of the wavelet coefficients matrix is then calculated. The point for which the norm has the higher value is a candidate for location of the damage. The above procedure was experimentally verified on a laboratory-scale 2-meter-long steel frame. The structure consists of 11 steel beams forming a four-bay frame, which is subjected to impact loads using a modal hammer. The accelerations are measured at 20 different locations on the frame, including joints and beam elements. Two states of the structure are considered: healthy and damaged one. The damage is introduced by means of loosening two out of three bolts at one of the frame connections. Calculating the norm of the difference of the wavelet coefficients matrix at each node the higher value was found to be at the same location where the bolts were loosened. The presented experiment showed the effectiveness of the wavelet approach to damage detection of frame structures assembled using bolted connections.

Keywords: complex bolted lap connection; frame structure; wavelet analysis; damage detection

1. Introduction

Damage in buildings may be caused by excessive earthquake excitation, severe environmental conditions, degradation of the material's properties, fatigue, cumulative crack growth, etc observing during their service life. In civil structures often the existence and the location of the damage can be determined through visual inspection. However, in some cases, visual inspection rmmay not be feasible. To ensure structural safety and low maintenance cost, structural health monitoring, (SHM), is an efficient strategy to monitor system performance and make

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corresponding maintenance decisions.

Damage detection includes the determination of the existence, severity, location of damage and prediction of the remaining service life. One main group of methods for damage detection is modal analysis methods, which are based on the fact that the change in structural properties causes a variation in the modal parameters (natural frequencies, damping ratios and mode shapes). Many analytical and experimental studies have been conducted to establish analytical correlations between damage severity and modal parameters. Kirmser (1944), investigated the relationship between natural frequencies and the introduction of a crack in an iron beam. A literature review on methods of damage detection using vibration signals for structural and mechanical systems was provided by Fan and Qiao (2011). Other work based on changes in modal parameters is in the work of Humar, Bagchi *et al.* (2006). Ciambella, Vestroni *et al.* (2011) investigated damage localization and assessment based on eigenfrequencies and eigenvectors curvatures.

More innovative methods like neural network approaches can also be used for damage detection. Wu, Ghabossi *et al.* (1992), trained a neural network to recognize the behaviour of an undamaged structure as well as the behaviour of a structure with various possible damage states. When the trained network is subjected to the measurements of the structural response, it is able to detect any existing damage. Masri, Nakamura *et al.* (1996), trained a neural network with measurements from a healthy structure and this trained network was fed comparable vibration measurements from the same structure under different episodes of response in order to monitor the health of structure. Vanik and Beck (1997) and Chandrashekhar and Ganguli (2009), used fuzzy logic for determination of the damage location. Friswell and Mottershead (1995), used a combination of sensors and an analytical model of the structure for the damage detection. Yun *et al.* (2009) used genetic algorithms for their damage detection approach. Papadimitriou and Ntotsios (2009) updated the parameters of the model that is related to damage so that the dynamic characteristics of the model corresponded to the sensor measurements. Sakelariou and Fasois (2006), introduced a stochastic output error for damage detection and assessment (location and quantification) in structures under earthquake excitation. Chatzi, Smyth *et al.* (2010), propose a methodology for the on-line identification of non-linear hysteretic systems where the parameters of the system are unknown and also the nature of the analytical model describing the system is not clearly established. Dertimanis and Chatzi (2014) investigate a hybrid optimization algorithm to the state–space parameter estimation problem. The hybrid algorithm was designed in a way that takes advantage of its deterministic and stochastic counterparts, combining fast local convergence and increased reliability in the search of the global optimum.

Casciati's work (2006) with Lyapunov exponents of a non-linear time series for health monitoring that is capable of localizing the damage. De Roeck and Reynders (2009), presented a number of innovations to extend the borders of what is realistically feasible with current system identification and damage detection methods.

Another tool for measuring damage detection is wavelet-based damage detection method, where the work of Staszewski (1998), is more representative. Newland (1993), used wavelets for vibration analysis. He applied a wavelet analysis to study the vibration of buildings caused by underground trains, road traffic and earthquake excitations. Taha, Noureldin *et al.* (2006), presented a view of wavelet transformation and its technologies. They discussed specific needs of health monitoring addressed by wavelet transformation. Kim and Melhem (2004), provide a review of the research that has been conducted on damage detection by wavelet analysis. Hou, Noori *et al.* (2000), proposed a wavelet-based approach for structural damage detection. Their model consisted of multiple breakable springs that may suffer either irreversible damage when the response exceeds a limit value or the cumulative number of cycles of motion exceeds the fatigue life. In any case, occurrence of damage and the time when it occurs can be clearly determined in the details of the wavelet decomposition of these data. Alonso, Noori *et al.* (2004) used orthogonal wavelet decomposition for identifying the stiffness loss in a single degree of freedom spring-mass-damper system. Their work shows that pseudo-alias effects caused by the orthogonal wavelet decomposition (OWD), affect damage detectability. Rucka and Wilde (2010) use neuro-wavelet technique in order to detect damage in beam, plate and shell structures, their results were also validated with experiments. Hera and Hou (2004), applied wavelet analysis for detection and locating the damage. They found that structural damage due to the sudden breakage of structural brace elements can be detected by spikes in the wavelet details. In the work of Khatam, Golafshani *et al*. (2007) wavelet analysis is used for damage identification in beams subjected in harmonic loading. The damaged region can be determined by the spatial distribution pattern of the observed spikes. Soyoz and Feng (2007) work theoretical and experimental for damage detection of bridge structures. Noh, Lignos *et al.* (2011, 2012), introduced three wavelet-based damage-sensitive features (DSFs) which are defined as functions of wavelet energies at particular frequencies and specific times. These DSFs can be used to diagnose structural damage.

Adaptive-scale damage detection strategy for plate structures based on wavelet finite element model was developed by He and Zhu (2015). Law, Zhu *et al*. (2013) worked on statistical damage classification method based on wavelet packet analysis. Liu *et al*. (2015), developed a structural time-varying damage detection method using synchrosqueezing wavelet transform. Yang, Xia *et al.* (2014) used discrete Wavelet transform for time-varying physical parameter identification of shear type structures. Finally Fan and Qiao (2013) proposed a novel transmissibility concept based on wavelet transform for structural damage detection.

Output only modal identification and structural damage detection of multi-degree of freedom of linear time variant systems based on time-frequency techniques such as short-time Fourier transform, empirical mode decomposition, and wavelets was developed by Nagarajaiah and Basu, (2009). With regard to identification procedures based on modal analysis methods a recent work that are able to deliver full exact reconstruction of multiple cracks in beam like structures is done by Caddemi and Caliò (2014). In this work a full nonlinear inverse problem is solved in terms of position and severity of an arbitrary number of cracks by frequency and mode shape measurements.

A wavelet based, distortion energy approach is presented in the work of Bukkapatnam, Nichols *et al.* (2005), as a method, for quantifying and locating the damage to structural systems. Goggins, Broderick *et al.* (2007) used a wavelet-based equivalent linearization technique to determine the temporal variations in frame stiffness that occurs due to brace yielding and buckling. Lima, Amiri *et al.* (2012) use wavelet analysis for damage detection of non-linear structures.

In most of the studies, damage deal with damage scenarios in the form of reduced beam cross sections Świercz, Kołakowski *et al.* (2007), introduction of cracks Caddemi and Morassi (2013), reduced plate thickness or plate cracks Krawczuk (2004). In reality, however, many structural failures start from damages which occur at connections. One of examples is the loosening of one or more bolts in bolted lap joints. The work on damage identification of bolted connections in a steel frame by Yang, Xia *et al.* (2014) is a representative study. The method of artificial neural networks was adopted for damage detection in truss bridge joints, Mehrjoo, Khaji *et al.* (2008). The influence of joint stiffness on global modes of structures was presented in the work of Blachowski and Gutkowski (2010). A damage detection approach to bolted flange joints in pipelines was presented by Razi, Esmaeel *et al*. (2013).

In literature the damage in structure is introduced by removing of one element or by reducing the section of the inertial properties. In this paper the structural damage is introduced by losing the bolts of the connections of the frame. This type of damage describes in more realistic way the real situation of civil structures. The data to be analyzing were obtained by experimental laboratory-scale steel frame. Then a computational technique for damage localization is developed based on wavelet analysis. According to this technique the norm of difference of the wavelet coefficients matrix of the response of the healthy and loosen connection structure is calculated. The point where the norm has the higher value is an indicator of the location of damage. From the wavelet techniques for damage detection in literature none use the norm of wavelet coefficients at a given measurement location as an indicator of damage stage. Furthermore, the above procedure is very easy to be applied in engineering practice. The above procedure is an indication of detecting the existence and location of the damage without providing information regarding the severity of the corresponding damage. The severity of damage is not addressed since it would require the same input force in both tests something that was not done in our experiment.

2. Continuous and discrete wavelet analysis

Wavelet analysis provides a powerful tool to characterize local features of a signal. Unlike the Fourier transform, where the function used as the basis of decomposition is always a sinusoidal wave, other basis functions can be selected for the wavelet shape according to the features of the signal. The basis function in wavelet analysis is defined by two parameters: scale and translation. These properties lead to a multi-resolution representation for non-stationary signals.

The continuous wavelet transform of a signal $f(t)$ is defined as

$$f(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t) \bar{\Psi}\left(\frac{t-b}{a}\right) dt \tag{1}$$

where a,b are the scale and translation parameters respectively and $\bar{\Psi}$ denotes the complex conjugate of Ψ . The functions $\Psi(t,a,b)$ are called wavelets. They are dilated and translated versions of the mother wavelet $\Psi(t)$, Newland (1993).

By discrediting the parameters a and b , a discrete version of the wavelet transform (DWT) is obtained Newland (1993). The procedure becomes more efficient if dyadic values of a and b are used, i.e.

$$a = 2^j \quad b = 2^j k \quad j,k \in Z \tag{2}$$

where Z is a set of positive integers. The corresponding discretized wavelets $\Psi_{j,k}$ are defined as

$$\Psi_{j,k}(t) = 2^{-j/2} \Psi\left(2^{-j} t - k\right) \tag{3}$$

where $\Psi_{j,k}$ forms an orthonormal base. In the discrete wavelet analysis, the signal can be represented by its approximations and details. The signal is passed through a series of high pass filters, which relate to details, to analyze the high frequencies, and through a series of low-pass filters, which relate to approximations, in order to analyze the low frequencies. The detail at level j is defined as

$$D_j = \sum_{k \in Z} a_{j,k} \Psi_{j,k}(t) \tag{4}$$

where $a_{j,k}$ is defined as

$$a_{j,k} = \int_{-\infty}^{\infty} f(t) \bar{\Psi}_{j,k}(t) dt \tag{5}$$

and the approximation at level J is defined as

$$A_J = \sum_{j > J} D_j \tag{6}$$

Finally, the signal $f(t)$ can be represented by

$$f(t) = A_J + \sum_{j \le J} D_j \tag{7}$$

The discrete wavelet transform (DWT) can be very useful for on-line health monitoring of structures, since it can efficiently detect the time of a frequency change caused by stiffness degradation.

3. Damage detection in steel frame structures using wavelets coefficients

A damage detection procedure should ask to the following three aspects:

1. The existence of damage
2. The location of damage
3. The severity of damage

The proposed procedure replies to the first and the second aspect.

In order to detect the existence and the location of the damage the following steps should be done.

Firstly, the output response signal is acquired from each elopement of steel structure using the sensors of the monitoring system which is installed to the structure. Based on the response signal a wavelet analysis is performed and the wavelets coefficients matrix, $f(a,b)$ is calculated according to Eq.(1).

Two states of the structure is considered the healthy stage were the connections of the structure is as they are constructed and the damage stage where some connections are loosed. Acquiring signals for each element from the two stages, the wavelet coefficients matrices $f_{healthy}(a,b)$ and $f_{damaged}(a,b)$ are calculated for healthy and damage stage respectively.

The wavelet coefficients calculated at the same element for healthy and damaged structure are plotted in plan view or in 3D view as it shown in Fig. 1. If two corresponding plots (for healthy and damage stage) for at least one element are different then this is an indicator that damage exists between the two stages. If two corresponding plots (for healthy and damage stage) for all elements of the structure are similar then a conclusion that no damage exist between the two stages can be drawn.

In order to identify the location of the damage the norm of the difference of the wavelet coefficients matrices $f_{i,healthy}(a,b)$ and $f_{i,damaged}(a,b)$ for each element, i, is calculated according to Eq.(8). The element with the higher norm is joint with the loosen connection.

$$\left\|\Delta f_i(a,b)\right\| = \left\| f_{i,damaged}(a,b) - f_{i,healthy}(a,b)\right\|, \quad i = 1,2,3,\ldots, \text{Number of elements} \tag{8}$$

Three norms were applied in the difference of the wavelet coefficients matrices $\left[\Delta f_i(a,b)\right]_{mxn}$ those are the first, the second and the infinity norm. The definitions of these norms are shown in Eqs. (9)-(11).

The first and the infinity norm can be computed as

$$\left\|\Delta f(a,b)\right\|_1 = \max_{1\le j\le n} \sum_{i=1}^{m} \left|\Delta f_{ij}\right| \tag{9}$$

which is simply the maximum absolute column sum of the matrix.

$$\left\|\Delta f(a,b)\right\|_\infty = \max_{1\le i\le m} \sum_{j=1}^{n} \left|\Delta f_{ij}\right| \tag{10}$$

which is simply the maximum absolute row sum of the matrix

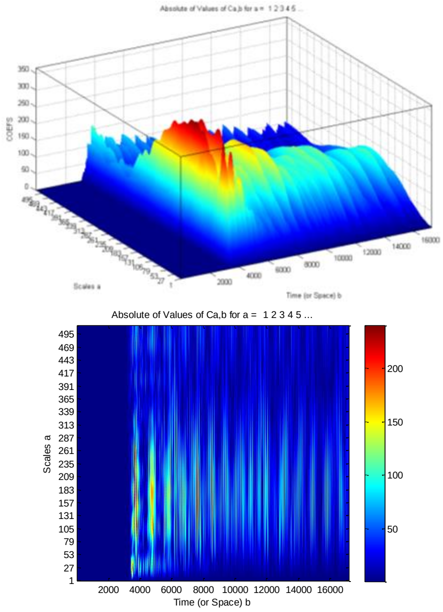
While the second norm of a matrix $\left[\Delta f_i(a,b)\right]_{mxn}$ is the largest singular value, σ_{max}, of $\left[\Delta f_i(a,b)\right]_{mxn}$ i.e., the square root of the largest eigenvalue, λmax, of the positive-semi-definite matrix $\left[\Delta f_i(a,b)\right]_{mxn}^{*} \left[\Delta f_i(a,b)\right]_{mxn}$

$$\left\|\Delta f(a,b)\right\|_2 = \sqrt{\lambda_{\max}\left(\Delta f(a,b)^{*} \Delta f(a,b)\right)} = \sigma_{\max}\left(\Delta f(a,b)\right) \tag{11}$$

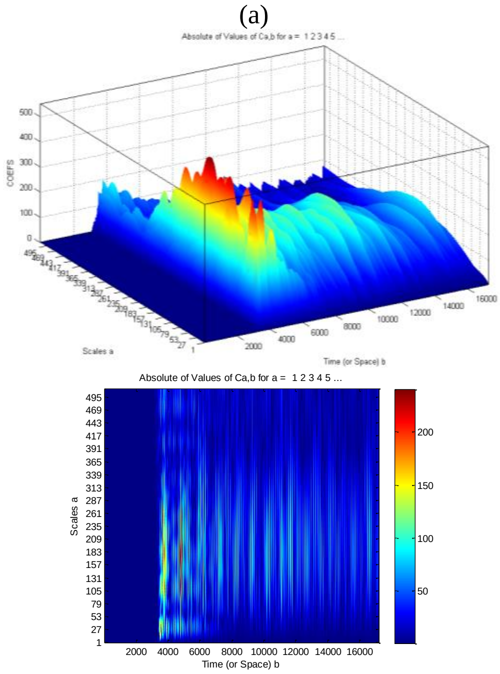
where $\left[\Delta f_i(a,b)\right]_{mxn}^{*}$ denotes the conjugate transpose of $\left[\Delta f_i(a,b)\right]_{mxn}$.

4. Experimental setup

For the purpose of experimental verification of the proposed procedure, a simple frame structure which is used in the work of Blachowski, Swiercz *et al*. (2015) is shown in Fig. 2(a). The frame structure consists of four square bays, each 0.51 m high and wide. Each bay is composed of steel elements of equal length with a rectangular cross-section of 8 by 80 mm. The total number of elements is 11 and the total length of the structure is 2.04 m. The structure is supported at the outermost nodes, preventing both translational and rotational displacements. The connections between elements are realized by means of rigid connector elements (nodes) and allen bolts (6 mm diameter), which are shown in Figs. 2(c) and 2(b), respectively. Each such connection is designed to use 3 bolts screwed into threaded holes in the elements. The original structure (without modification) is referred here as the healthy structure.



(a)



(b)

Fig. 1 3D and plan view plots of continuous wavelet coefficients of (a) the healthy structure, $f_{healthy}(a,b)$ and (b) the damaged structure $f_{damaged}(a,b)$

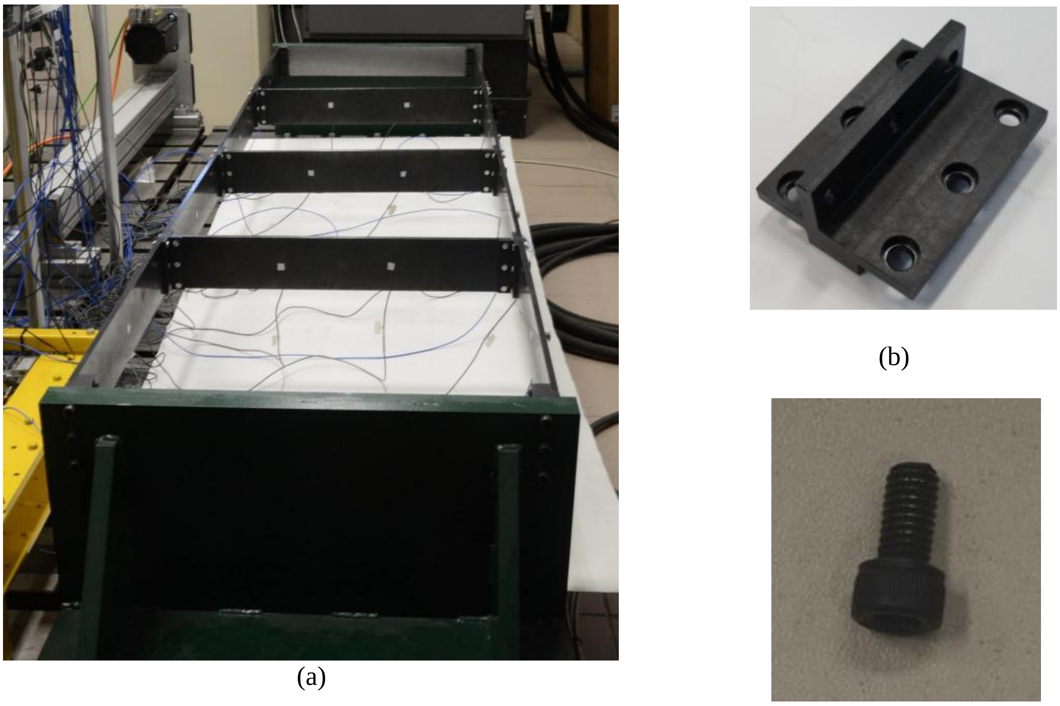

(a) (b) (c)

Fig. 2 The examined frame structure, (a), an allen bolt used for the element-node connections, (b) and a connector element, (c)

A modification is introduced to node 5 and element 10 (see Fig. 3). The modification of the connection consists in removing 2 bolts (bottom and upper), leaving the middle one. In contrast to the healthy structure, we will use here the notions of damaged structure. The structural vibrations were induced using a modal hammer with an embedded force sensor. The applied medium tip allowed for covering the frequency range of up to 1 kHz.

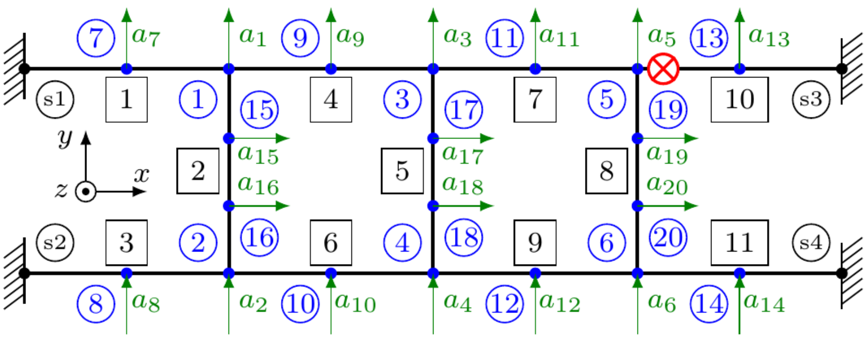


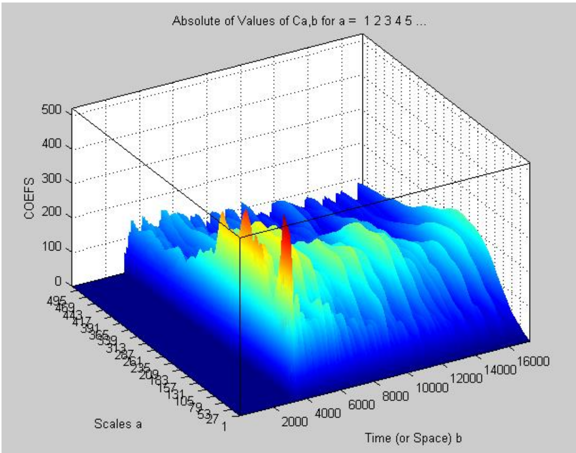
Fig. 3 Scheme of the tested frame structure. Notation: s1-s4 fixed nodal points, 1-20 (in circles) nodal points, 1-11 (in boxes) element numbers, a1-a20 measured accelerations at nodal points. The location of the loosened bolted connection in shown in red

Vibrations of the healthy and damaged structure were measured using 20 single axis accelerometers arranged as presented in Fig. 3. Analytical details about the monitoring system and collection and manipulation of the output data can be found in the work of Blachowski et al (2015).

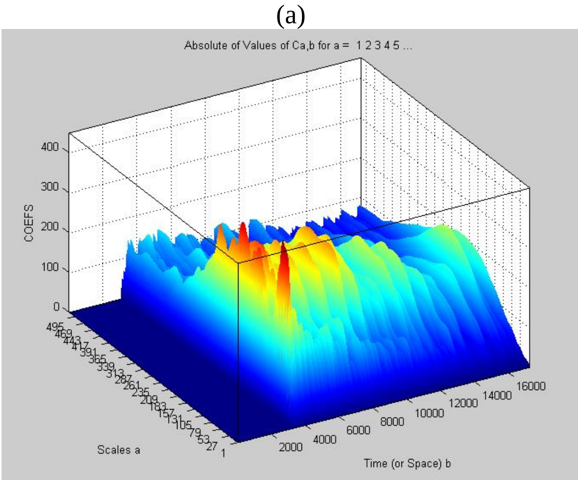
5. Results of damage localization using proposed method

The wavelets coefficients for healthy and damaged structure at measured accelerations nodal points, a_1 to a_{20} were calculated. The 'haar' wavelet family name was used for the analysis. The 'haar' wavelet has orthogonal and biorthogonal properties, is suitable for continues and discrete wavelet analysis, is symmetrical and is like first order daubechies wavelet.

The wavelets coefficients for healthy and damaged structure at nodal point a_{13} which is locate at element 10 that is near to the joint 5 where the bolts were loosed are presented in Fig. 4.

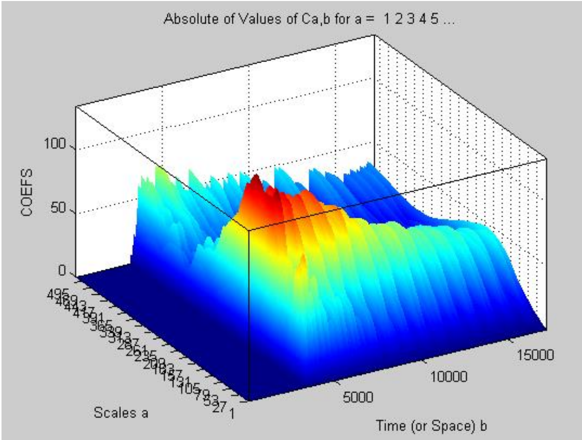


(a)

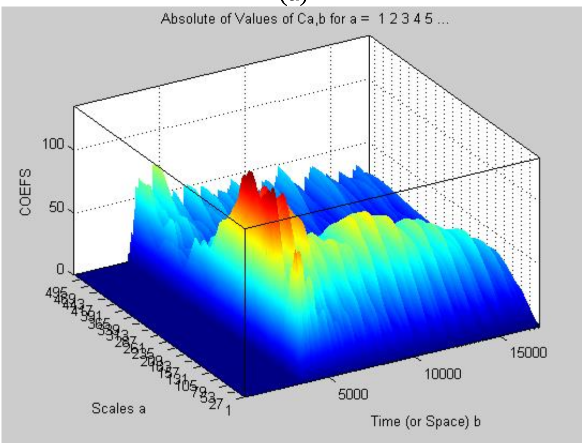


(b)

Fig. 4 Wavelets coefficients for healthy, (a) and damaged and (b) structure nodal at point a_{13}



(a)



(b)

Fig. 5 Wavelets coefficients for healthy, (a) and damaged and (b) structure at nodal point a_1

From the above picture it is seen the difference in wavelet coefficients for healthy and damaged structure, is a good indicator of the damage. It worth to note, that the difference in wavelet coefficients at elements far away from joint 5 is less as it can be seen in Figs. 5 and 6. In that figure the wavelet coefficients for healthy and damaged structure at measured acceleration point 1, (joint 1), are shown.

The difference in wavelet coefficients shows if the damage in the structures is present or not. The location of the damage can be identified as follows: First, norm of the matrix, which is a difference of wavelet coefficients matrices for healthy and damage structure at each measurement point is calculated. Then, for the points, which are placed in the middle of the elements (not at the connections), the damage can be identified for those locations which have the highest value of the norm. The second norm of difference wavelet coefficients matrix at each measurement point is presented in graph in Fig. 6. It can be seen in that figure that the measurement point no. 13 has the highest norm. As can be seen from Fig. 3 this point belongs to the structural element 10 where the loosening of bolts in the connection took place.

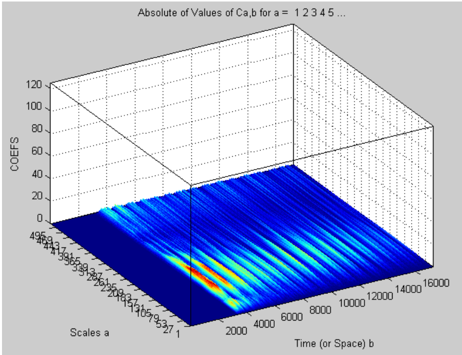
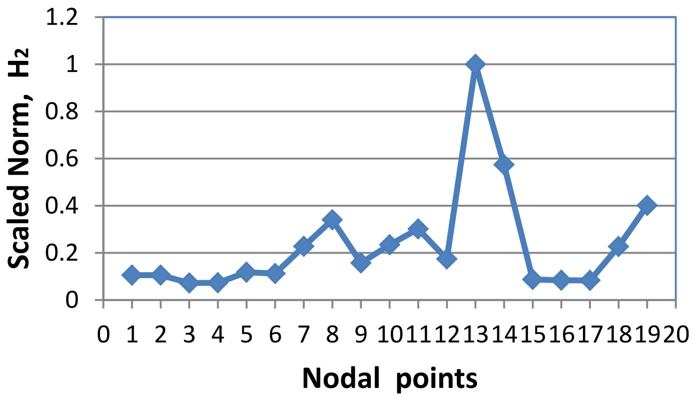
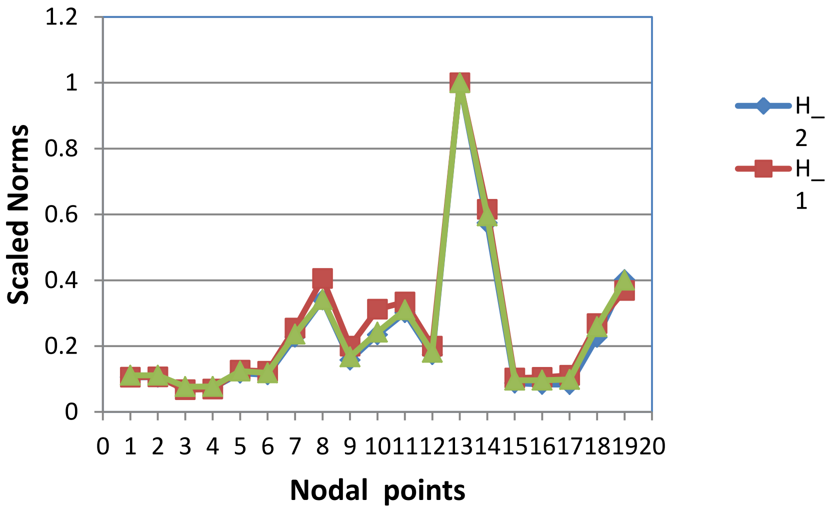


Fig. 6 Difference of wavelets coefficients for healthy and damaged structure at nodal point a_1



(a)



(b)

Fig. 7 Scaled norms of wavelets coefficients

6. Conclusions

A numerical procedure for detection and localization of the damage due to loosened bolts, has been presented. According to the proposed method the wavelet coefficients matrix of the response at each point of measurements for healthy and damaged structure is calculated. The difference of these matrices at the same point of the structure is an indicator that the damage has occurred in the structure. The norm of the difference matrix at each point in middle of the elements is then calculated. For the point, which is located near to the damaged connection, the norm has the highest value. From the experimental results it was shown that the above method can be effectively used to identify the damage location due to loosened bolts in the steel connections.

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