Smart Structures and Systems, *Vol. 18, No. 3 (2016) 569-584*
DOI: http://dx.doi.org/10.12989/sss.2016.18.3.569

Stochastic modelling and optimum inspection and maintenance strategy for fatigue affected steel bridge members

Tian-Li Huang[1,2], Hao Zhou[1], Hua-Peng Chen[*1,2] and Wei-Xin Ren[1]

[1]*School of Civil Engineering, Central South University, Changsha, Hunan Province, 410075, China*
[2]*Department of Engineering Science, University of Greenwich, Chatham Maritime, Kent, ME4 4TB, UK*

(Received September 14, 2015, Revised January 30, 2016, Accepted March 20, 2016)

Abstract. This paper presents a method for stochastic modelling of fatigue crack growth and optimising inspection and maintenance strategy for the structural members of steel bridges. The fatigue crack evolution is considered as a stochastic process with uncertainties, and the Gamma process is adopted to simulate the propagation of fatigue crack in steel bridge members. From the stochastic modelling for fatigue crack growth, the probability of failure caused by fatigue is predicted over the service life of steel bridge members. The remaining fatigue life of steel bridge members is determined by comparing the fatigue crack length with its predetermined threshold. Furthermore, the probability of detection is adopted to consider the uncertainties in detecting fatigue crack by using existing damage detection techniques. A multi-objective optimisation problem is proposed and solved by a genetic algorithm to determine the optimised inspection and maintenance strategy for the fatigue affected steel bridge members. The optimised strategy is achieved by minimizing the life-cycle cost, including the inspection, maintenance and failure costs, and maximizing the service life after necessary intervention. The number of intervention during the service life is also taken into account to investigate the relationship between the service life and the cost for maintenance. The results from numerical examples show that the proposed method can provide a useful approach for cost-effective inspection and maintenance strategy for fatigue affected steel bridges.

Keywords: steel bridge; fatigue crack; maintenance strategy; Gamma process; life-cycle cost analysis; genetic algorithm

1. Introduction

Steel bridges are deteriorating with time due to the daily cyclic loading caused by traffic and/or aggressive environmental conditions. The performance deterioration can adversely affect structural safety and decrease the service life of steel bridges. Fatigue cracks, common damage due to cyclic loading, can exist in structural components of steel bridges, such as the welded joints of cover plate, the web gap between the transverse stiffeners and the tension flanges, the welded rib-to-deck details in decks (Fisher 1984, Fisher and Roy 2011, 2015, Tang 2011, Mertz 2012). If these fatigue cracks cannot be detected and repaired in time, it may result in the functional failure or even catastrophic collapse of steel bridges, e.g. the collapse of the I-35W Bridge in Minnesota in 2007 caused by fatigue induced member fracture (Deng *et al.* 2015).

∗Corresponding author, Professor, E-mail: h.chen@gre.ac.uk

http://www.techno-press.org/?journal=sss&subpage=8 ISSN: 1738-1584 (Print), 1738-1991 (Online)

In order to investigate the influence of fatigue cracks on structural behaviour, some deterministic models have been proposed to provide the relationship between fatigue crack growth and cyclic loads, for example, the Paris-Erdogen model and the Forman model (Paris and Erdogan 1963, Forman *et al.* 1967). However, experimental results for fatigue crack growth in metals show that there exists a significant scatter due to the uncertainties in fatigue crack propagation (Virkler *et al.* 1979, Ghonem and Dore 1987). The probabilistic approaches were then adopted to take the parameters in the Paris-Erdogen, Forman or other deterministic models as random variables. Currently, the statistical data, such as mean, coefficient of variation and probability density functions, are typically assumed or determined using limited fatigue experimental results (Kwon and Frangopol 2011, Soliman *et al.* 2013). In order to consider the uncertainties and predict fatigue crack evolution, the stochastic models may be more appropriate (Sobczyk and Spencer 1992). During the past two decades, many stochastic methods have been proposed to address the stochastic modelling of crack growth, e.g., Markov-chain models (Spencer *et al*. 1989, Chiquet *et al.* 2009), modified deterministic models with multiplicative randomness function (Yang and Manning 1996, Wu and Ni 2004, Casciati *et al*. 2007, Beck and de Santana Gomes 2013) and Gamma process (Guida and Penta 2015, Chen and Alani 2012, 2013).

Inspection actions on steel bridge members can provide information about the location and length of fatigue cracks. Based on the results of the inspections, maintenance such as repair and retrofit may be performed to delay the propagation or reduce the length of fatigue cracks. It is critical to maintain structural reliability level related to fatigue and to extend the service life of the steel bridges by using proper inspection and maintenance schedules. In practice, the inspection and maintenance schedules for steel bridges are normally made at the start of the bridge construction. The conflict between the resources available and the actual inspection and maintenance needs often exists in making decision, i.e., extensive necessary maintenance is often required but the resources are limited, or excessive resources are scheduled for unnecessary maintenance. Therefore, there is a need to find an appropriate strategy to balance the risk of structural failure and the cost of inspection and maintenance (Jandu 2008, Frangopol 2011).

The approaches for the optimal fatigue inspection planning of steel bridges have been proposed in many studies (Lovejoy 2003, Chung *et al*. 2006, Soliman *et al.* 2013). Since maintenance actions generally follow inspections, the integrated inspection and maintenance schedule is required to make the informed decisions (Kim and Frangopol 2013, Chen and Huang 2012). Considering the uncertainties in the fatigue crack propagation and the inspection and maintenance actions, the probabilistic approaches for the optimum management for deteriorating steel bridges were developed (Kwon and Frangopol 2011, Soliman and Frangopol 2014). Monte Carlo simulations are often utilised in these approaches on the basis of the adopted probabilistic models for modelling the fatigue crack propagation, requiring significant computational efforts. Therefore, there is limited research available on optimum inspection and maintenance strategy based on the stochastic models for the fatigue crack propagation (Valdebenito and Schuëller 2010, Gomes and Beck 2014).

This paper presents a method for stochastic modelling of fatigue crack growth and optimising inspection and maintenance strategy for the structural members of steel bridges. The Gamma process is adopted to simulate the fatigue crack propagation. From the stochastic modelling for fatigue crack growth, the probability of failure caused by fatigue is predicted over the service life of steel bridges. Then, the remaining fatigue life of steel bridge members is determined by comparing the fatigue crack length with its predetermined threshold. The probability of detection is adopted to consider the uncertainties in detecting fatigue crack by using existing techniques. The effectiveness of maintenance actions on steel bridge members is measured by reducing the length of fatigue cracks. A multi-objective optimisation problem is formulated and solved by genetic algorithm (GA) to determine the optimised inspection and maintenance strategy for the fatigue affected steel bridge members by minimising the life-cycle cost, including the inspection, maintenance and failure costs, and maximizing the service life after necessary intervention. The number of intervention during the service life is also considered to investigate the relationship between the service life and the cost for maintenance. Finally, a numerical example of the fatigue damaged stiffening rib is used to illustrate the proposed method. The Pareto solutions are obtained to provide asset manager with a set of optimised inspection and maintenance strategies for steel bridges.

2. Modelling of fatigue crack growth

2.1 Fatigue crack growth

Fatigue in steel bridge members is the process of initiation and growth of cracks under the action of repeated traffic loading. A typical model for fatigue crack growth may include three phases, as indicated in Fig. 1, i.e., crack initiation phase, steady-state crack propagation phase and unstable crack growth phase. In phase I, the crack growth rate is relatively low and threshold effects are important. In phase II, the crack growth rate is steady and predictable. In phase III, the crack growth rate is relatively high and unstable. In general, phase III does not contribute significantly to the fatigue life and can be ignored (Mertz 2012). For a steel bridge composed of welded members, it is inevitable that some initial cracks or flaws would be generated in the process of either manufacturing or assembly, thus the fatigue crack initiation life can be ignored. In this study, only steady-state crack propagation phase, i.e., phase II, is taken into account for the service life estimation of the steel bridge members.

The typical crack propagation model proposed by Paris and Erdogan (Paris and Erdogan 1963) is used in this study. The rate of fatigue crack propagation over number of stress cycles in steel bridges (da/dN) is a function of the range of stress intensity factor (ΔK), expressed here as

$$\frac{da}{dN} = C(\Delta K)^m \ for \ \Delta K > \Delta K_{th} \tag{1}$$

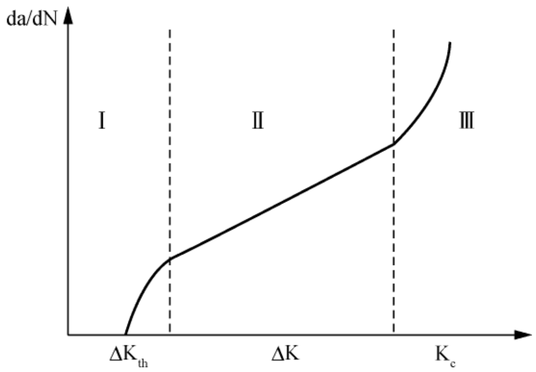


Fig. 1 Typical fatigue crack growth in metals

$$\Delta K = S_{sr} \cdot Y(a)\sqrt{\pi a} \tag{2}$$

$$N_{cycle} = \frac{1}{C \cdot S_{sr}^m} \cdot \int_{a_0}^{a_N} \frac{1}{\left(Y(a)\sqrt{\pi a}\right)^m} da \tag{3}$$

where a is the fatigue crack length; a_0 is the initial fatigue crack length; C and m are the material parameters which can be determined experimentally; ΔK is the range of stress intensity factor; ΔK_{th} is the threshold range of stress intensity factor; N_{cycle} is the cumulative number of stress cycles associated with crack length a_N; S_{sr} is the amplitude of stress; $Y(a)$ represents the symmetry function which varies with the location of the crack.

By substituting Eqs. (2) and (3) into Eq. (1) and calculating the integration, the fatigue crack length a_N can be determined as a function of the number of stress cycles N_{cycle} namely

$$a_N = \left[a_0^{(2-m)/2} + \left(\frac{2-m}{2}\right) \cdot C \cdot S_{sr}^m \cdot Y^m \cdot \pi^{m/2} \cdot N_{cycle}\right]^{\left(\frac{2}{2-m}\right)} \tag{4}$$

It is assumed here that there is no increase in the annual volume of traffic, and the annual number of stress cycles on this steel bridge member remains a constant (e.g., N_{an} cycles/year) during the whole life of the steel bridge. According to the traffic volume statistics, the fatigue service lifetime T_L (year) for N_{cycle} cycles of stress can be expressed as

$$T_L = \frac{N_{cycle}}{N_{an}} = \frac{1}{N_{an} \cdot C \cdot S_{sr}^m} \cdot \int_{a_0}^{a_N} \frac{1}{\left(Y(a)\sqrt{\pi a}\right)^m} da \tag{5}$$

2.2 Gamma process modelling of fatigue crack growth

The Gamma process is a stochastic process with independent and non-negative increments, having a Gamma distribution with an identical scale parameter. The Gamma process is appropriate for modelling gradual damage monotonically accumulating over time in a sequence of small increments, such as wear, fatigue, corrosion, crack growth, erosion, etc. The advantage of modelling the above deterioration processes by Gamma process is that the required mathematical calculations are relatively straightforward (Van Noortwijk and Frangopol 2004, Van Noortwijk 2009, Chen and Nepal 2015, Chen and Xiao 2015).

In this paper, the fatigue crack length A is assumed as a random quantity following the Gamma distribution, the probability density function (PDF) of the fatigue crack length is given as

$$f_A(a) = Ga(a|\eta, \lambda) = \frac{\lambda^\eta}{\Gamma(\eta)} a^{\eta-1} e^{-\lambda a} \cdot I_{(0,\infty)}(a) \tag{6a}$$

$$I_{(0,\infty)}(a) = \begin{cases} 1, & a \geq 0 \\ 0, & elsewhere \end{cases} \tag{6b}$$

where $\eta > 0$ and $\lambda > 0$ are the shape and scale parameters of the Gamma distribution respectively, which can be estimated by the maximum likelihood estimation (MLE) for a random quantity with continuous and positive increment. $\Gamma(\eta) = \int_{z=0}^{\infty} z^{\eta-1} e^{-z} dz$ is the gamma function for $\eta > 0$.

If the shape function $\eta(t)$ is a non-decreasing, right-continuous, and real-value function for

$t \geq 0$ with $\eta(0) \equiv 0$. The random fatigue crack length growth over time $\{A(t), t \geq 0\}$ can be considered as a Gamma process with shape function $\eta(t) > 0$ and scale parameter $\lambda > 0$, which is a continuous-time stochastic process with the following properties (Van Noortwijk 2009)

(a) For all $t_{i+1} > t_i > 0$, $A(t_{i+1}) - A(t_i) \sim Ga(\eta(t_{i+1}) - \eta(t_i), \lambda)$.

(b) $A(t)$ has independent increment.

According to the above properties, the fatigue crack length growth can be simulated using the Gamma sequential sampling (GSS) method (Avramidis *et al.* 2003). In the time period between zero and $T > 0$, a series of time, $t_0, t_1, \cdots, t_i, \cdots, t_{n-1}, t_n$, are generated, namely $t_i = i\Delta T, i = 0,1,2, \cdots n$, $t_n = n\Delta T = T$ where n is a positive integer and ΔT is a fixed time interval. The increment of fatigue crack length δ_i at each time t_i can be obtained from

$$\delta_i = a(t_i) - a(t_{i-1}),\ i \geq 1,\ \delta_0 = a(t_0) = a_0 \tag{7}$$

By using the above properties of the Gamma process, the fatigue crack length increments δ_i at each time interval follow the Gamma distribution and can be estimated from

$$\delta_i \sim Ga(\delta | \eta(t_{i+1}) - \eta(t_i), \lambda) = \frac{\lambda^{[\eta(t_{i+1}) - \eta(t_i)]}}{\Gamma(\eta(t_{i+1}) - \eta(t_i))} \delta^{[\eta(t_{i+1}) - \eta(t_i)] - 1} e^{-\lambda\delta} \tag{8}$$

Finally, the cumulative fatigue crack length $a(t_i)$ can be obtained by summing the increments for the associated time intervals, namely

$$a(t_i) = \sum \delta_i \tag{9}$$

A simple numerical example is used here to illustrate the above GSS method for fatigue crack growth. Assuming that $N_{an} = 10^6$ times per year, $a_0 = 0.5\,mm$, $C = 3.54 \times 10^{-12}$, $m = 2.54$ and $Y(a) = 1$, the simulated fatigue crack growth curve using the GSS method, together with the deterministic curve predicted by the Paris-Erdogan model in Eq. (1), is shown in Fig. 2. From the results, the simulated fatigue crack growth generally matches the prediction by the Paris-Erdogan model, although the simulated curve does not appear smooth due to uncertainties in the parameters. When the service life is less than 20 years, the rate of fatigue crack growth is slow. The rate increases sharply after the service life of over 20 years.

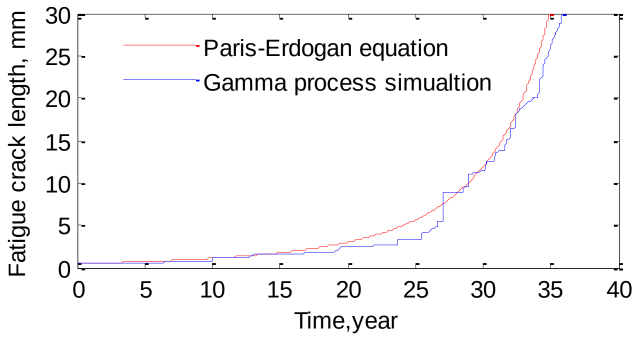


Fig. 2 Fatigue crack growth curves simulated by present Gamma sequential sampling method and predicted by the Paris-Erdogan model

3. Probability of fatigue failure

In the time-dependent reliability analysis of fatigue affected steel bridge members, the probability of structural failure (P_f) can be defined as the time when the fatigue crack length reaches its predetermined threshold a_{cr} after experiencing N_T cycles of stress. The threshold a_{cr} is determined according to the operational conditions and the allowable limit of the structure.

The probability of the fatigue affected steel bridge members to fail during their life time is given as (Chen and Alani 2012, 2013, Chen and Nepal 2015, Huang *et al*. 2014, 2015)

$$P_f = Pr\{N \geq N_T\} = Pr\{a \geq a_{cr}\} = \int_{a=a_{cr}}^{\infty} f_{A(N)}(a)da = \frac{\Gamma(\eta(N), \lambda a_{cr})}{\Gamma(\eta(N))} \tag{10}$$

where $\Gamma(\eta(N)) = \int_0^{\infty} \nu^{\eta(N)-1} e^{-\nu} d\nu$ is Gamma function for $\eta(N) = \lambda a(N)$. The incomplete Gamma function is defined as

$$\Gamma(\eta, x) = \int_{\nu=x}^{\infty} \nu^{\eta-1} e^{-\nu} d\nu \tag{11}$$

where $\eta(N) \geq 0$ and $x \geq 0$.

The probability of failure against service time is plotted for different thresholds, i.e., a_{cr}= 5 mm, 10 mm and 15 mm, respectively, as shown in Fig. 3. At the beginning of service, since the fatigue crack length grows slowly, the probability of failure is very small and close to zero. As the service time increases, fatigue crack length increases gradually until reaching the critical threshold. After the service time gets to the time corresponding to the critical fatigue crack threshold, the probability of failure increases significantly and quickly approaches unity. From the results, in the case with larger threshold of fatigue crack, the failure probability experiences faster growth before structural failure. These characteristics indicates that proper inspection and maintenance should be carried out before the crack length reaches a critical value to keep the structure safe.

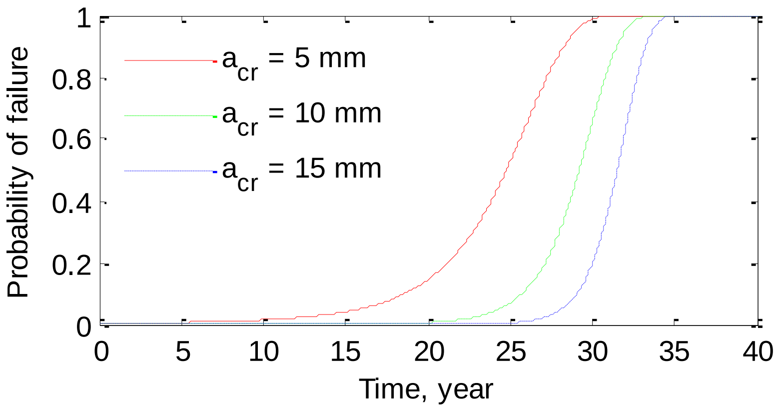


Fig. 3 Probability of failure under different critical thresholds of fatigue crack length

4. Optimum inspection and maintenance strategy

4.1 Fatigue crack inspection

There are various non-destructive evaluation (NDE) techniques available for fatigue crack inspections, which can provide the useful information such as the location and length of fatigue cracks. These typical NDE techniques for fatigue crack detection include visual inspection, ultrasonic detection, liquid permeating detection, etc. (Kwon and Frangopol 2011). During inspections, significant uncertainties may exist in the capacity and accuracy of these NDE techniques due to the limitations of the facilities, operators' experience and operational environments. Therefore the fatigue cracks with certain size may not be detected and the size of detected cracks may not be correctly measured by a specific NDE technique. The capacity of NDE techniques could be quantitatively represented here by the probability of detection (PoD) curves, which provides the relationship between the actual crack size and the probability of detection. In this study, a popular PoD model, proposed by Berens and Hovey (Berens and Hovey 1981), is adopted to represent the PoD curve as a function of crack length a_i, namely

$$PoD = \frac{\exp(\alpha+\beta\ln(a_i))}{1+\exp(\alpha+\beta\ln(a_i))} \tag{12}$$

where PoD is the probability of detection for fatigue crack; α and β are constant parameters related to different NDE techniques. The parameters for different detection techniques are listed in Table 1 (Forsyth and Fahr 1998).

The relationship between the PoD and the fatigue crack length is shown in Fig. 4. It can be seen that when the crack length is less than 0.5 mm, the PoD is close to zero, which indicates these NDE techniques almost cannot detect such a small crack. After the crack length increases to 0.8 mm, the PoD for the ultrasonic detection technique increases dramatically, reaching approximately 90% when the crack length is 1.5 mm. Comparing with the ultrasonic detection technique, the liquid permeating detection technique has a smaller value of PoD for the same crack length, which shows the liquid permeating detection technique may not be as good as the ultrasonic detection technique in terms of the PoD. It is assumed here that the fatigue cracks can be detectable when the PoD exceeds 0.5.

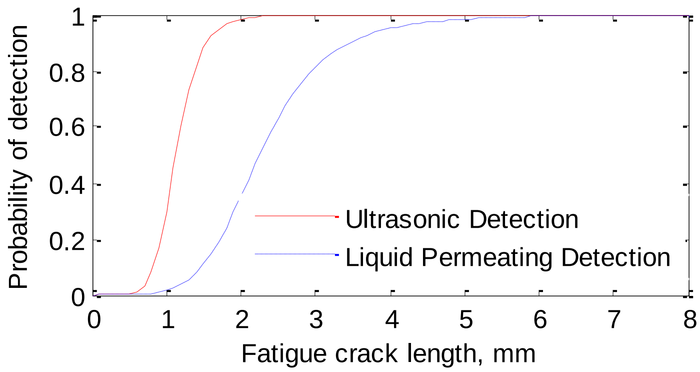


Fig. 4 Relationship between the probability of detection (PoD) and the fatigue crack length

Table 1 Constants α and β associated with different NDE techniques

Ultrasonic detection		Liquid permeating detection	
α	β	α	β
-0.87	7.1	-4.17	5.14

4.2 Influence of inspection and maintenance on service life

After inspections, the maintenance actions may be required to restore the capacity of the bridge structural members. The selection of maintenance types depends on the actual information provided by the timely inspection. In order to optimise the inspection and maintenance strategies, the influence of inspection and maintenance on the estimation of the remaining service life of steel bridge members should be investigated. Typical bridge maintenance can be classified as preventive maintenance (PM), which is performed before the members of the bridge structures are out of service, and corrective maintenance (CM), which is undertaken by replacing the damaged structural members to maintain the serviceability of the structural system. For the fatigue cracks in the welding members of steel bridges, many repair methods are available, including peening, gas tungsten arc re-melting, re-welding and bolted splices (Fisher 1984, Miki 2007, Kwon and Frangopol 2011).

In this study, the effectiveness of different types of maintenance is assumed as the reduction of fatigue crack length, namely

$$a_{maint} = k a_{ins} \tag{13}$$

where a_{ins} and a_{maint} are the initial fatigue crack length and the fatigue crack length after maintenance, respectively; k is the maintenance coefficient representing the effectiveness of maintenance, ranging from 0 to 1. A larger value of k indicates the maintenance is less effective in repair. It is assumed here that repairs may not be possible to recover the fatigue crack length to zero, and the fatigue crack propagation rate remains the same after the maintenance. The influence of inspection and maintenance on the remaining service life of steel bridge members is illustrated in Fig. 5. When no maintenance is undertaken and the fatigue crack length becomes the predetermined threshold, the member is considered to reach its service lifetime t_{life}^0. In the case with maintenance at time t_{ins}, the maintenance takes place to reduce the fatigue crack length from the initial a_{ins} to a_{maint}. After the maintenance, the fatigue crack continues increasing until reaching the predetermined threshold with the member's service lifetime t_{life}^1.

Considering the probability of detection for crack inspection, the service life after inspection and maintenance $t_{life,maint}$ can be estimated from

$$t_{life,maint} = (1 - PoD) \cdot t_{life}^0 + PoD \cdot t_{life}^1 \tag{14}$$

where t_{life}^0 is the service lifetime without any inspection and maintenance, and t_{life}^1 is the service lifetime after inspection and maintenance.

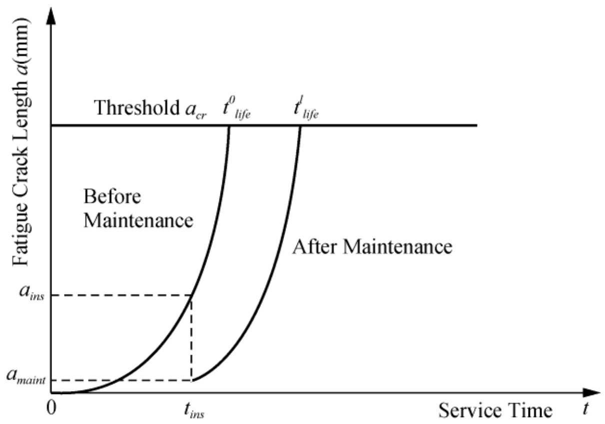


Fig. 5 Influence of inspection and maintenance on the service life of steel bridge member

4.3 Life-cycle cost analysis of Inspection and Maintenance

In this study, the life-cycle cost for the management of steel bridges in service includes the inspection cost C_{ins}, the maintenance cost C_{maint} and the failure risk cost $C_{failure}$. Here, the ultrasonic detection technique is considered to inspect the fatigue crack length and its inspection cost is assumed to be the constant C_{ins}. The maintenance cost is usually affected largely by the effectiveness of the specific maintenance methods, e.g., more effective maintenance method may require more resources. Since the maintenance is normally performed after the inspection, the influence of inspection uncertainties on the maintenance cost should be considered. The maintenance cost C_{maint} is calculated from (Kim and Frangopol 2013)

$$C_{maint} = PoD \cdot C_M \cdot (1 - 0.7k)^{r_{maint}} \tag{15}$$

where C_M is a constant; and r_{maint} is a positive integer and the probability of detection PoD is defined in Eq. (12).

The failure risk cost is generally related to the service time of the bridge, given as

$$C_{failure}(t) = \frac{t^0_{life}}{t^1_{life}+t^0_{life}} \cdot C^0_{failure} \tag{16}$$

where $C^0_{failure}$ is the loss when the bridge fails at the beginning of operation and $C_{failure}(t)$ is the loss when the bridge fails at time t.

By taking the probability of failure into account, the total failure risk cost of the bridges $C_{failure}$ is given as

$$C_{failure} = P_f(t_i)\left[PoD \cdot C_{failure}(t_i) + (1 - PoD) \cdot C_{failure}(t_j)\right] \tag{17}$$

where $C_{failure}(t_i)$ and $P_f(t_i)$ are the failure risk cost and failure probability at time t_i, respectively.

Consequently the total life-cycle cost for the management of the steel bridges in service can be given as

$$C_{total} = C_{ins} + C_{maint} + C_{failure} \tag{18}$$

4.4 Optimised inspection and maintenance strategy

During the life cycle of steel bridges affected by fatigue cracking, a series of inspection and maintenance may be needed, which requires the planned inspection and maintenance strategy to be scheduled and optimised. Different inspection and maintenance strategies may require different amounts of resources, and then improve the condition of the bridge to different levels. In this study, only single and double interventions for inspection and maintenance during the service life are considered.

The optimisation of inspection and maintenance is to find the optimised inspection time $t_{insp,i}$ and maintenance strategy which is represented by the maintenance coefficient k_i. The objectives of the optimisation problem are to maximise the service life after inspection and maintenance $t_{life,maint}$ and to minimise the total cost of inspection and maintenance C_{total}. Here, the inspection time interval is set at least one year and the maintenance coefficient k_i is set between 0.1 and 0.9 due to the actual inspection condition and repair methods. The optimisation problem can be expressed as

$$Objetives: \max\left(t_{life,maint}\right)\ and\ \min(C_{total}) \tag{19a}$$

$$Design\ Variables: T_{insp} = \left\{t_{insp,1}, \cdots, t_{insp,n}\right\};\ k = \{k_1, \cdots, k_n\} \tag{19b}$$

$$subject\ to: t_{insp,i} - t_{insp,i-1} \geq 1;\ \ 0.1 \leq k_i \leq 0.9 \tag{19c}$$

The optimisation toolbox based on genetic algorithm (GA) provided in MATLAB version R2012a (MathWorks 2012) is adopted to solve the optimisation problem for maximising the service life and minimising the maintenance cost. The details of the genetic algorithm used including parameter selections can be found in the manual for the optimisation toolbox in MATLAB.

5. Illustrative example

To demonstrate the effectiveness of the proposed optimum inspection and maintenance strategy method, a typical U-type stiffening rib in the orthotropic steel deck of long-span suspension bridges, such as the Jiangyin Yangtze River Highway Bridge (Zeng *et al.* 2013), is adopted in this study. The bridge has a typical orthotropic steel deck which has been frequently used in long-span bridges in China in the past two decades (Tang 2011). Under the action of cyclic vehicle wheel loads, the fatigue sensitive cracks may exist in the welded U-type stiffening ribs and/or the deck plate of the bridge, as shown in Fig. 6.

Parameters related to the fatigue crack growth are listed in Table 2 (Zeng *et al.* 2013), where the geometry function is assumed as unity. By using the parameters listed in Table 2, the fatigue crack growth curves for the stiffening rib, determined by the Paris-Erdogen equation and simulated by Gamma process, are shown in Fig. 7, respectively. Obviously, the fatigue crack growth of the stiffening rib has the same trend as that shown in Fig. 2.

Table 2 Fatigue parameters for a steel bridge stiffening rib

Parameter	a_0 (mm)	a_{cr} (mm)	m	C $[10^{-13}\bullet(\mathrm{N}\bullet\mathrm{mm}^{-3/2})^{-m}]$	$\Delta\delta$ (MPa)	$Y(a)$	N (10^7 /year)
Value	0.1	15	2.54	3.6	30	1.0	1.1

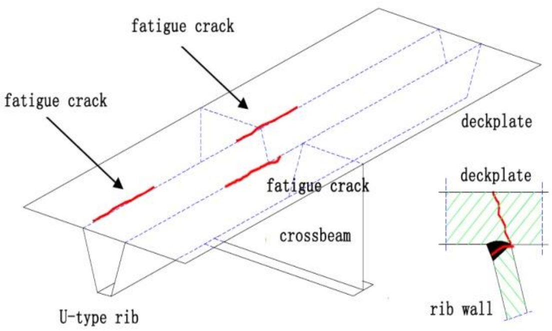


Fig. 6 Illustrative sketch of potential fatigue cracks in the orthotropic steel bridge deck

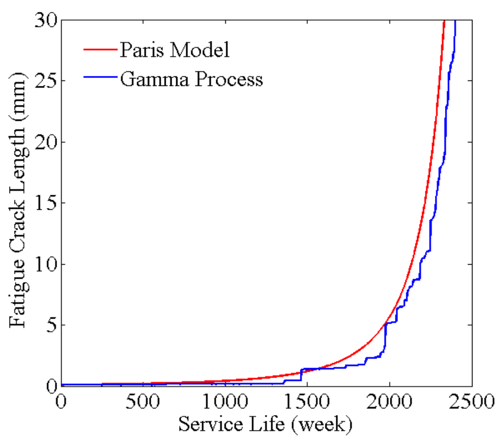


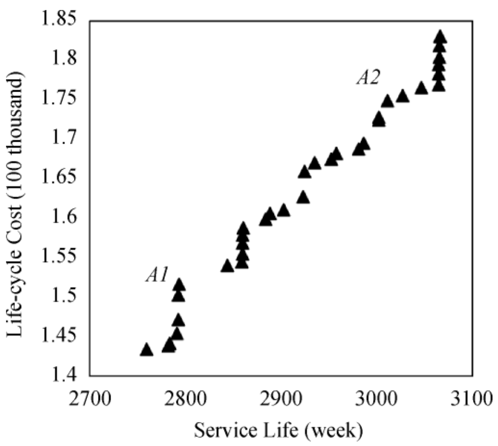
Fig. 7 Fatigue crack propagation curves of a steel bridge stiffening rib

The ultrasonic detection technique is adopted for fatigue crack inspection in this illustrative example. It is assumed that the critical threshold value of the fatigue crack length a_{cr} is 15 mm. Also, the inspection cost $\mathrm{C_{ins}}$=20,000 CNY, the maintenance cost $\mathrm{C_{M}}$=100,000 CNY, $\mathrm{r_{maint}}$=10 and the initial failure risk cost $\mathrm{C^{0}_{failure}}$=500,000 CNY are used in this illustrative example. The inspection time interval is set at least one year and the maintenance coefficient k_i is set between 0.1 and 0.9.

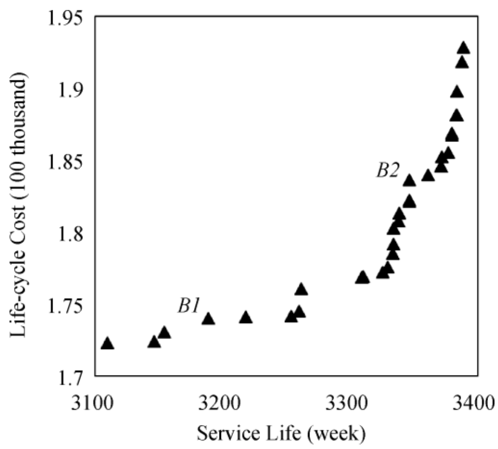
The optimum inspection and maintenance strategies for the fatigue crack in stiffing rib of the steel bridge are shown in Figs. 7(a) and 7(b), where the number of inspection is set to be 1 and 2 respectively. Every point in Figs. 7(a) and 7(b) represents a specific inspection and maintenance strategy, where both the service life after maintenance and the total cost of the inspection and maintenance are given. The values for the service life and total cost corresponding to the points A1, A2 and B1, B2 in Figs. 7(a) and 7(b) are summarised in Table 3. As expected, the required cost for the inspection and maintenance increases as the expected service life is extended.

The detailed inspection and maintenance strategy of A1 and A2 for a single intervention are shown in Fig. 9(a). For the case with strategy A1, when the bridge has been in operation for 38 years (1981 weeks), corresponding to point P1, a major structural intervention is performed. After inspection is undertaken at point P1, the length of fatigue crack is determined, and repairs that can reduce the length of fatigue crack to 0.3 time of the initial one will be carried out. In this case, the final expected service life of the bridge member will be 53.7 years and the total cost for this strategy is 151575.9 CNY. Similarly, for the case with strategy A2, the intervention is taken place after 42.5 years' service and the length of fatigue crack is assumed to be reduced to 0.13 time of the initial one. The expected service life of the bridge member is extended to 58.2 years, but the total cost for this strategy is increased to 175524.4 CNY.

The results in Fig. 9(b) shows the inspection and maintenance strategies of B1 and B2 for double interventions during the service life. For the case with strategy B1, when the bridge has been in operation for 35.2 years, corresponding to point P3, the first major structural intervention is performed, and the length of fatigue crack is estimated by inspection.

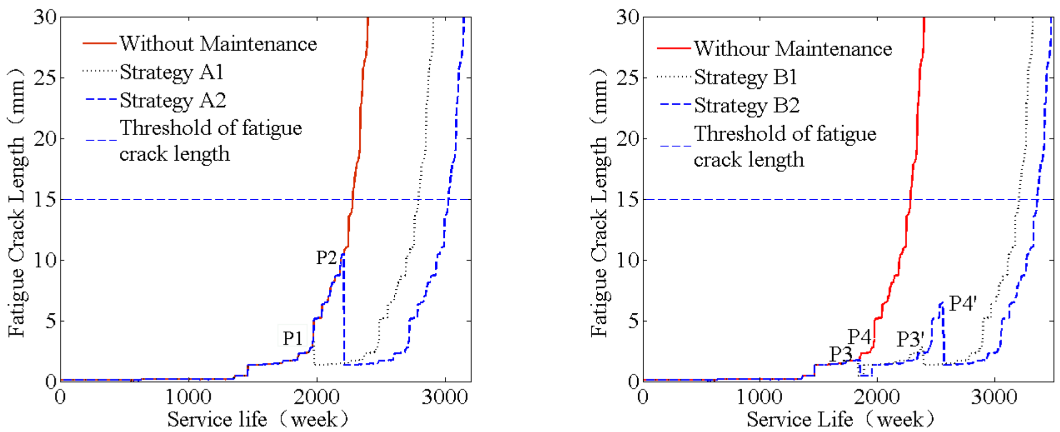


(a) single intervention



(b) double intervention

Fig. 8 Pareto optimum solution sets with single and double interventions for inspection and maintenance



(a) Strategy A1 and A2 (b) Strategy B1 and B2

Fig. 9 Inspection and maintenance strategy with various interventions for the fatigue damaged steel bridge member

Table 3 Pareto optimum sets with single and double interventions

Pareto solution	No. of inspection	Expected service life (year)	Cost of inspection and maintenance (CNY)	Inspection time (year)		Maintenance	
				$T_{insp,1}$	$T_{insp,2}$	k_1	k_2
A1	1	53.7	151575.9	38.0	—	0.30	—
A2	1	58.2	175524.4	42.5	—	0.13	—
B1	2	61.3	174042.4	35.2	46.0	0.27	0.30
B2	2	64.4	183649.2	35.6	59.3	0.25	0.21

After the inspection, the first repair, reducing the length of fatigue crack to 0.27 times of the initial one, is undertaken. The second intervention takes place when the bridge is in operation for 46.0 years, corresponding to point P3, and the second repair is conducted to reduce the length of fatigue crack to 0.30 times of the initial one. In this case, the final expected service life of the bridge member is 61.3 years, and the total cost for this strategy will be 174042.4.9 CNY. Similarly, for the case with strategy B2, the first and second interventions take place after 35.6 and 59.3 years' service, respectively, and then the length of fatigue crack is assumed to be reduced to 0.25 and 0.21 times of the initial ones, respectively. The expected service life of the bridge member extends to 64.4 years, while the total cost for this strategy is increased to 183649.2 CNY.

6. Conclusions

This study presents a method for estimating the remaining service life of the structural members of fatigue damaged steel bridges, where the Gamma process is employed to model the propagation of the fatigue crack with uncertainties. The probability of failure, when the fatigue crack length reaches a predefined allowable limit, can be estimated on the basis of the stochastic deterioration modelling. An optimisation method based on the time-dependent reliability analysis, and the genetic algorithm is proposed to determine the cost-effective inspection and maintenance strategy.

On the basis of the results from the numerical example involving the stiffening rib of a steel bridge, the following conclusions can be drawn: 1) the Gamma process can simulate the propagation of fatigue crack with uncertainties, thus the deterioration process is well represented by the stochastic modelling, and then the remaining service life can be correctly predicted. 2) The proposed optimum method based on the stochastic modelling and the genetic algorithm can give a better balance between the remaining service life and the total cost for structural maintenance. 3) The Pareto solutions can provide asset managers with a set of inspection and maintenance strategies, assisting them in scheduling inspection and maintenance with a scientific basis. 4) The proposed method will be more cost-effective for repairing the structural members experienced fatigue cracking with better bearing capacity recovery, when the interventions are conducted at the early stage of the fatigue crack propagation. In the further studies, the parameters used in stochastic simulations should be evaluated from the real measurements for bridge structures, and the increase of traffic volume over time for the bridge should be considered in the proposed method.

Acknowledgments

The research described in this paper was financially supported by the UK Royal Academy of Engineering Newton Fund (Reference NRCP/1415/14), the Natural Science Foundation of China (Grant Nos. 51478472 and 50708113), the Natural Science Foundation of Hunan Province, China (Grant No. 2015JJ2176) and the China Scholarship Council (CSC No. 201506375059). The results and opinions expressed in this paper are those of the authors only and they do not necessarily represent those of the sponsors.

References

Avramidis, A.N., L'Ecuyer, P. and Tremblay, P.A. (2003), "Efficient simulation of gamma and variance-gamma processes", *Proceedings of the 2003 Winter Simulation Conference*, Piscataway, NJ, USA.

Beck, A.T. and de Santana Gomes, W.J. (2013), "Stochastic fracture mechanics using polynomial chaos", *Probabilist. Eng. Mech.*, **34**, 26-39.

Berens, A. and Hovey, P. (1981), "Evaluation of NDE reliability Characterization", *Report No AFWAL-TR-81-4160*, Vol. 1. University of Dayton Research Institute.

Casciati, F., Colombi, P. and Faravelli, L. (2007), "Inherent variability of an experimental crack growth curve", *Struct. Saf.*, **29**(1), 66-76.

Chen, H.P. and Alani, A.M. (2012), "Reliability and optimised maintenance for sea defences", *Proc. ICE-Maritime Eng.*, **165**(2), 51-64.

Chen, H.P. and Alani, A.M. (2013), "Optimized maintenance strategy for concrete structures affected by cracking due to reinforcement corrosion", *ACI Struct. J.*, **110**(2), 229-238.

Chen, H.P. and Huang, T.L. (2012), "Updating finite element model using dynamic perturbation method and

regularization algorithm", *Smart Struct. Syst.*, **10**(4-5), 427-442.
Chen, H.P. and Nepal, J. (2015), "Stochastic modelling and lifecycle performance assessment of bond strength of corroded reinforcement in concrete", *Struct. Eng. Mech.*, **54**(2), 319-336.
Chen, H.P. and Xiao, N. (2015), "Symptom-based reliability analyses and performance assessment of corroded reinforced concrete structures", *Struct. Eng. Mech.*, **53**(6), 1183-1200.
Chiquet, J., Limnios, N. and Eid, M. (2009), "Piecewise deterministic Markov processes applied to fatigue crack growth modeling", *J. Stat. Plan. Infer*., **139**(5), 1657-1667.
Chung, H.Y., Manuel, L. and Frank, K.H. (2006), "Optimal inspection scheduling of steel bridges using nondestructive testing techniques", *J. Bridge Eng. -ASCE*, **11**(3), 305-319.
Deng, L., Wang, W. and Yu, Y. (2015), "State-of-the-art review on the causes and mechanisms of bridge collapse", *J. Perform. Constr. Fac.*, 04015005.
Fisher, J.W. (1984), *Fatigue and fracture in steel bridges: case studies*, John Wiley, New York.
Fisher, J.W. and Roy, S. (2011), "Fatigue of steel bridge infrastructure", *Struct. Infrastruct. Eng.*, **7**(7-8), 457-475.
Fisher, J.W. and Roy, S. (2015), "Fatigue damage in steel bridges and extending their life", *Adv. Steel Constr.*, **11**(3), 250-268.
Forman, R.G., Kearney, V.E. and Engle, R.M. (1967), "Numerical analysis of crack propagation in cyclic-loaded structures", *J. Basic Eng.*, **89**(3), 459-464.
Forsyth, D.S. and Fahr, A. (1998), "An evaluation of probability of detection statistics", *RTO AVT Workshop on Airframe Inspection Reliability under Field/Depot Conditions*, RTO MP-10, Brussels, Belgium.
Frangopol, D.M. (2011), "Life-cycle performance, management, and optimisation of structural systems under uncertainty: accomplishments and challenges", *Struct. Infrastruct. Eng.*, **7**(6), 389-413.
Ghonem, H. and Dore, S. (1987), "Experimental study of the constant-probability crack growth curves under constant amplitude loading", *Eng. Fract. Mech.*, **27**(1), 1-25.
Gomes, W.J. and Beck, A.T. (2014), "Optimal inspection planning and repair under random crack propagation", *Eng. Struct.*, **69**, 285-296.
Guida, M. and Penta, F. (2015), "A gamma process model for the analysis of fatigue crack growth data", *Eng. Fract. Mech.*, **142**, 21-49.
Huang, T.L., Zhou, H., Ren, W.X. and Chen, H.P. (2014), "Gamma process modelling and repair planning of fatigue damaged of steel bridge members", *Proceedings of the 12th International Conference on Computational Structures Technology (CST 2014)*, Naples, Italy.
Huang, T.L., Zhou, H., Wang, C., Ren, W.X. and Chen, H.P. (2015), "Optimization inspection and maintenance strategy for corrosive reinforced concrete girder bridges based on Gamma process", *J. Central South Univ. (Natural Science)*, **46**(5),1851-1861. (In Chinese)
Jandu, A.S. (2008), "Inspection and maintenance of highway structures in England", *Pro. ICE, Bridge Eng.*, **161**(3), 111-114.
Kim, S. and Frangopol, D.M. (2013), "Generalized probabilistic framework for optimum inspection and maintenance planning", *J. Struct. Eng. - ASCE*, **139**(3), 435-447.
Kwon, K. and Frangopol, D.M. (2011), "Bridge fatigue assessment and management using reliability-based crack growth and probability of detection models", *Probabilist. Eng. Mech.*, **26**(3), 471-480.
Lovejoy, S.C. (2003), "Determining appropriate fatigue inspection intervals for steel bridge members", *J. Bridge Eng. - ASCE*, **8**(2), 66-72.
MathWorks. (2012), "Optimization toolbox Version 6.2 user's guide", MathWorks, Natick, MA.
Mertz, D. (2012), "*Steel Bridge Design Handbook: Design for Fatigue*", Report No. FHWA-IF-12-052-Vol. 12.
Miki, C. (2007), "Retrofitting engineering for steel bridge structures", International Institute of Welding.
Paris, P.C. and Erdogan, F. (1963), "A critical analysis of crack propagation laws", *J. Basic Eng.*, **85**(4), 528-534.
Sobczyk, K. and Spencer, Jr, B.F. (1992), *Random fatigue: from data to theory*, Academic Press, New York.
Soliman, M., Frangopol, D.M. and Kim, S. (2013), "Probabilistic optimum inspection planning of steel

bridges with multiple fatigue sensitive details", *Eng. Struct.*, **49**, 996-1006.

Soliman, S. and Frangopol, D. (2014), "Life-Cycle Management of Fatigue-Sensitive Structures Integrating Inspection Information", *J. Infrastruct. Syst.*, **20**(2), 04014001.

Spencer, Jr. B.F., Tang, J. and Artley, M. E. (1989), "Stochastic approach to modeling fatigue crack growth", *AIAA J.*, **27**(11), 1628-1635.

Tang, M.C. (2011), "A new concept of orthotropic steel bridge deck", *Struct. Infrastruct. Eng.*, **7**(7-8), 587-595.

Valdebenito, M.A. and Schuëller, G.I. (2010), "Design of maintenance schedules for fatigue-prone metallic components using reliability-based optimization", *Comput. Method. Appl. Mech.*, **199**(33), 2305-2318.

Van Noortwijk, J.M. (2009), "A survey of the application of Gamma processes in maintenance", *Reliab. Eng. Syst. Safe.*, **94**(1), 2-21.

Van Noortwijk, J.M. and Frangopol, D.M. (2004), "Two probabilistic life-cycle maintenance models for deteriorating civil infrastructres", *Probabilist. Eng. Mech.*, **19**(4), 345-359.

Virkler, D.A., Hillberry, B. and Goel, P.K. (1979), "The statistical nature of fatigue crack propagation", *J. Eng. Mater. Technol. - ASME*, **101**(2), 148-153.

Wu, W.F. and Ni, C.C. (2004), "Probabilistic models of fatigue crack propagation and their experimental verification", *Probabilist. Eng. Mech.*, **19**(3), 247-257.

Yang, J.N. and Manning, S.D. (1996), "A simple second order approximation for stochastic crack growth analysis", *Eng. Fract. Mech.*, **53**(5), 677-686.

Zeng, Y., Tan, H., Sun, S., Gan, L., Liu, G. and Qu, M. (2013), "Probability-based optimization method for the fatigue maintenance strategies of steel bridge welded components", *China Railway Science*, **34**(1), 29-34. (In Chinese)