

Active tendon control of suspension bridges

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(Received September 26, 2015, Revised April 21, 2016, Accepted May 4, 2016)

Abstract. The paper first reviews the theory of active tendon control with decentralized Integral Force Feedback (IFF) and collocated displacement actuator and force sensor; a formal proof of the formula giving the maximum achievable damping is provided for the first time. Next, the potential of the control strategy for the control of suspension bridges with active stay cables is evaluated on a numerical model of an existing footbridge; several configurations are investigated where the active cables connect the pylon to the deck or the deck to the catenary. The analysis confirms that it is possible to provide a set of targeted modes with a considerable amount of damping, reaching $\xi = 15\%$. Finally, the control strategy is demonstrated experimentally on a laboratory mock-up equipped with four control stay cables equipped with piezoelectric actuators. The experimental results confirm the excellent performance and robustness of the control system and the very good agreement with the predictions.

Keywords: suspension bridge; active control; collocated control; integral force feedback

1. Introduction

Suspension bridges and cable-stayed bridges are widely used in infrastructures, because they are elegant and they allow very long spans. However, they are subjected to all sorts of complicated dynamic phenomena ranging from wind or traffic induced vibration to flutter instability (e.g., Tacoma Bridge). The problem is difficult, in particular because of the highly nonlinear behavior of cable structures, responsible for such phenomena as parametric excitation when some tuning conditions are satisfied (Nayfeh and Mook 2012, Costa *et al.* 1996, Lilien and Costa 1994). Footbridges are very sensitive to pedestrian and jogger induced vibrations. It is generally admitted that the over sensitivity to dynamic excitation of cable bridges is associated with the very low structural damping in the global bridge modes (often below 1%), and even less in the cable modes

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(Pacheco *et al.* 1993). The classical way of attenuating the global modes is the use of tuned-mass-dampers (at least one by critical mode), e.g., (Caetano *et al.* 2010, Tubino and Piccardo 2015, Bortoluzzi *et al.* 2015). More elaborate strategies may also be considered (see Casciati *et al.* 2014 for a review); integrating sensors and actuators in structures is now becoming a reality (Liu *et al.* 2005). The active control of cable structures has also been considered; the application of active tendons to flutter control was considered numerically by Yang and Giannopoulos (1979a,b) and experimental studies were pioneered by Fujino and co-workers (Warnitchai *et al.* 1993, Fujino *et al.* 1993, 1994). All these studies were performed with non-collocated actuator-sensor configurations; this did not lead to any trouble in Yang's numerical study since a perfect knowledge and a linear system was assumed, but Fujino's experimental results revealed that, even for relatively simple systems, instabilities tend to occur when the cable-structure interaction is large.

Cable-structures are much easier to control if *collocated* actuator-sensor pairs are used, because this produces alternating poles and zeros in the open-loop transfer function of every channel of the control system (Cannon and Rosenthal 1984, Preumont 2011), reducing drastically the spillover and other problems associated with the high frequency dynamics. This property was successfully exploited in several studies demonstrating the active damping of cable-stayed bridges (Achkire and Preumont 1996, Achkire *et al.* 1998, Bossens and Preumont 2001) and guyed space trusses (Preumont and Achkire 1997, Preumont and Bossens 2000, Preumont *et al.* 2000). All these studies use a decentralized control strategy based on the Integral Force Feedback (IFF) family (Preumont *et al.* 1992); the control strategy exhibits very good performance and robustness, and the control design is based on clear physical parameters such as natural frequencies and modal strain energy; the method is summarized below.

A large scale experiment on a mock-up representative of a *cable-stayed bridge* (Bossens and Preumont 2001) confirmed the results obtained on smaller test beds, but it pointed out a significant drawback: the stay cables must carry the control loads, but also the dead loads, which are substantially larger than the control loads and complicate significantly the design of the active tendons. However, it was shown by Auperin and Dumoulin (2000) that active damping of *suspension bridges* could be achieved with a very small number of stay cables equipped with active tendons (Fig. 1), without the drawback just described for the cable-stayed bridges of having to carry a substantial part of the dead loads. This idea was the motivation for the present study.

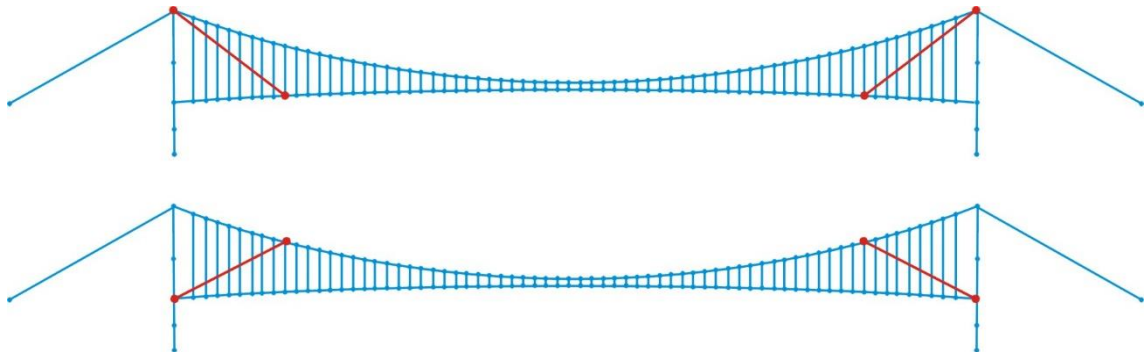


Fig. 1 Configurations for active tendon control of suspension bridges (the active control cables are in red)

2. Decentralized active damping of a cable-structure

Consider the cable-structure system similar to that of Fig. 2, where a passive structure is connected to a set of active cables operated with active tendons. In the example shown, the passive structure consists of a vertical truss structure and there are 3 active cables and 3 active tendons. Each active tendon consists of a displacement actuator (e.g. piezoelectric) co-linear with a force sensor. T_i is the tension in the active cable i , measured by the sensor integrated in the active tendon, and δ_i is the free extension of the actuator, the variable used to control the system. k_i is the axial stiffness of the cable and the active tendon, jointly. We assume that the dynamics of the active cables can be neglected and that their interaction with the structure is restricted to the tension T_i . Assuming a classical finite element formulation, the equation governing the dynamic response of the system is

$$M \ddot{x} + K x = -B T + f \quad (1)$$

where x is the vector of global coordinates of the finite element model, M and K are respectively the mass and stiffness matrices of the passive structure (including a linear model of the passive cables, if any, but excluding the active cables); the structural damping is neglected to simplify the presentation. The right hand side represents the external forces applied to the system; f is the vector of external disturbances such as gravity and wind loads (expressed in global coordinates), $T = (T_1, \dots, T_i, \dots)^T$ is the vector of tension in the active cables and B is the influence matrix of the cable forces, projecting the cable forces in the global coordinate system (the columns of B contain the direction cosines of the various active cables); B depends on the topology of the active cable network.

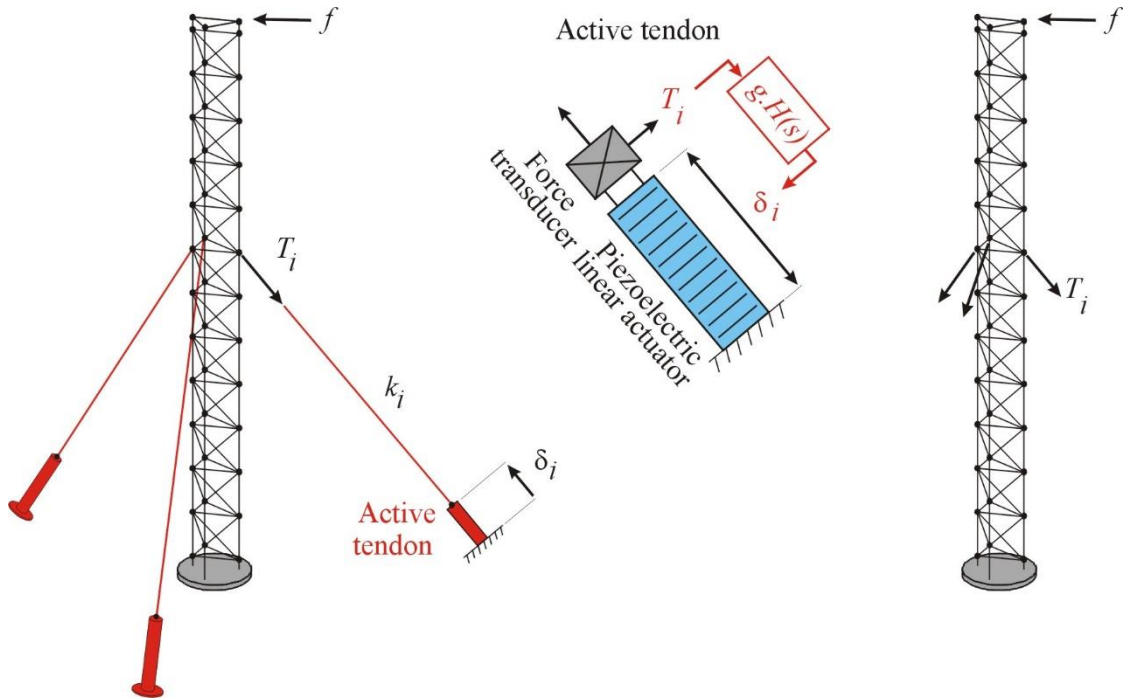


Fig. 2 Left: Cable-structure system with active tendons. Center: Active tendon. Right: Passive structure

If we neglect the cable dynamics, the active cables behave like (massless) bars. If $\delta = (\delta_1, \dots, \delta_i, \dots)^T$ is the vector of (free) active displacements of the active tendons acting along the cables, the tension in the cables are given by

$$T = K_c(B^T x - \delta) \quad (2)$$

where $K_c = \text{diag}(k_i)$ is the stiffness matrix of the cables, $B^T x$ are the relative displacements of the end points of the cables projected along the chord lines. This equation expresses that the tension in the cable is associated with the elastic extension of the cable. Combining Eqs. (1) and (2), we get

$$M\ddot{x} + (K + BK_c B^T)x = BK_c \delta + f \quad (3)$$

This equation indicates that $K + BK_c B^T$ is the stiffness matrix of the structure including all the guy cables (passive + active). Next, we assume that all the active cables are controlled according to the decentralized force feedback law

$$\delta = gh(s).K_c^{-1}T \quad (4)$$

where $gh(s)$ is the scalar control law applied to all control channels* (note that $K_c^{-1}T$ represents the elastic extension of the active cables). Combining Eqs. (2)-(4), the closed-loop equation is

$$[Ms^2 + K + \frac{1}{1+gh(s)}BK_c B^T]x = f \quad (5)$$

It is readily observed that the open-loop poles, solutions of the characteristic equation for $g = 0$, satisfy

$$[Ms^2 + K + BK_c B^T]x = 0 \quad (6)$$

(the solutions are the eigenvalues of the structure with all cables), while the transmission zeros, solutions of Eq. (5) for $g \rightarrow \infty$, satisfy

$$[Ms^2 + K]x = 0 \quad (7)$$

which is the eigenvalue problem for the open-loop structure where the active cables have been removed (they can be computed very easily).

2.1 Control law

If an Integral Force Feedback (IFF) controller is used, $h(s) = s^{-1}$, the closed-loop equation becomes

$$[Ms^2 + K + \frac{s}{s+g}BK_c B^T]x = f \quad (8)$$

which indicates that the closed-loop static stiffness matrix is

$$\lim_{s \rightarrow 0} [Ms^2 + K + \frac{s}{s+g}BK_c B^T] = K \quad (9)$$

This means that the active cables do not contribute to the static stiffness and this may be

* s is the Laplace variable.

problematic in some applications. However, if the control is slightly changed into[†]

$$gh(s) = \frac{gs}{(s+\beta)^2} \quad (10)$$

where β is small and positive (the influence of β will be discussed later), the closed-loop equation becomes

$$[Ms^2 + K + \frac{(s+\beta)^2}{gs+(s+\beta)^2}BK_cB^T]x = f \quad (11)$$

and the closed-loop static stiffness matrix becomes

$$\lim_{s \rightarrow 0} [Ms^2 + K + \frac{(s+\beta)^2}{gs+(s+\beta)^2}BK_cB^T] = K + BK_cB^T \quad (12)$$

which indicates that the active cables have a full contribution to the static stiffness.

2.2 Modal behavior

Next, let us project the characteristic equation on the normal modes of the structure with all the cables, $x = \Phi z$, which are normalized according to $\Phi^T M \Phi = 1$. According to the orthogonality condition of the normal modes

$$\Phi^T (K + BK_cB^T) \Phi = \Omega^2 = \text{diag}(\Omega_i^2) \quad (13)$$

where Ω_i are the natural frequencies of the complete structure. In order to derive a simple and powerful result about the way each mode evolves with g , let us assume that the mode shapes are little changed by the active cables, so that we can write

$$\Phi^T K \Phi = \omega^2 = \text{diag}(\omega_i^2) \quad (14)$$

where ω_i are the natural frequencies of the structure where the active cables have been removed. It follows that the *fraction of modal strain energy* is given by

$$\nu_i = \frac{\phi_i^T BK_cB^T \phi_i}{\phi_i^T (K + BK_cB^T) \phi_i} = \frac{\Omega_i^2 - \omega_i^2}{\Omega_i^2} \quad (15)$$

Considering the IFF controller, the closed-loop characteristic Eq. (8) can be projected into modal coordinates, leading to

$$(s^2 + \Omega_i^2) - \frac{g}{g+s} (\Omega_i^2 - \omega_i^2) = 0$$

or

$$1 + g \frac{s^2 + \omega_i^2}{s(s^2 + \Omega_i^2)} = 0 \quad (16)$$

This result indicates that the closed-loop poles can be predicted by performing two modal analyses (Fig. 3), one with all the cables, leading to the open-loop poles $\pm j\Omega_i$, and one with only the passive cables, leading to the open-loop zeros $\pm j\omega_i$, and drawing the independent root loci of

[†]We will refer to this as *Beta controller* in what follows.

Eq. (16). The maximum modal damping is given by

$$\xi_i^{\max} = \frac{\Omega_i - \omega_i}{2\omega_i} \quad (17)$$

and it is achieved for $g = \Omega_i \sqrt{\Omega_i / \omega_i}$. This analytical result was established many years ago by the senior author and coworkers using a symbolic calculation software; a much simpler proof is provided in Appendix.

Eq. (17) relates directly the maximum achievable modal damping with the spacing between the pole Ω_i and the zero ω_i , which is essentially controlled by the fraction of modal strain energy in the active cables, as expressed by Eq. (15).

The foregoing results are very easy to use in design. Although they are based on several assumptions (namely that the dynamics of the active cables can be neglected, the passive cables behave linearly and that the mode shapes are unchanged), they are in good agreement with experiments (Preumont and Achkire 1997, Preumont et al. 2000).

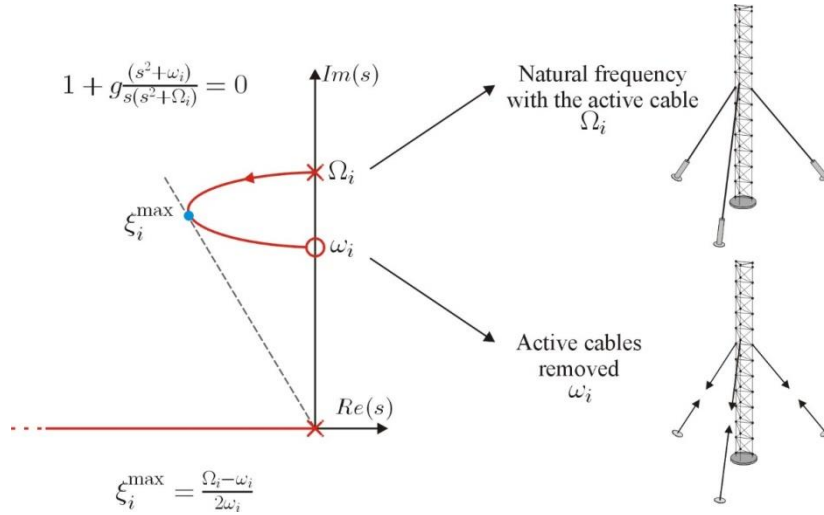


Fig. 3 Root locus of the closed-loop poles with an IFF controller. The system is unconditionally stable

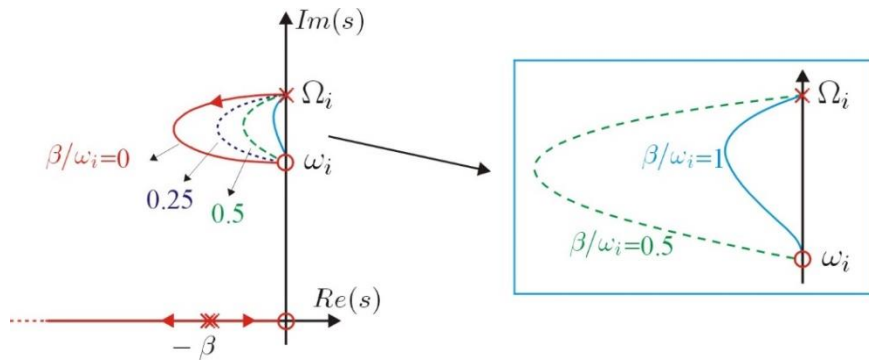


Fig. 4 Root locus of the closed-loop poles with the Beta controller $gs/(s + \beta)^2$, for various values of the ratio β/ω_i . The IFF controller corresponds to $\beta = 0$. The locus is always stable for $\beta < \omega_i$; for $\beta = \omega_i$ it is tangent to the imaginary axis at the zero $\pm j\omega_i$.

If, instead of the IFF controller, the Beta controller is used, the closed-loop characteristic equation projected into modal coordinates reads

$$(s^2 + \Omega_i^2) - \frac{gs}{gs + (s + \beta)^2} (\Omega_i^2 - \omega_i^2) = 0$$

or

$$1 + g \frac{s(s^2 + \omega_i^2)}{(s + \beta)^2 (s^2 + \Omega_i^2)} = 0 \quad (18)$$

Thus, as compared to the IFF controller, the pole at the origin has been replaced by a zero at the origin and a pair of poles at $-\beta$ on the real axis. The effect of this change on the root locus is shown in Fig. 4. When $\beta = 0$, there is a pole-zero cancellation and the control is reduced to the IFF. As β increases, the root locus has two branches on the real axis, starting from $s = -\beta$ in opposite directions; one of the closed-loop poles remains trapped between 0 and $-\beta$; the loops still go from $\pm j\Omega_i$ to $\pm j\omega_i$, but they tend to be smaller, leading to less active damping; this is the price to pay for recovering the static stiffness of the active cables. Analyzing the root locus in detail, one can show that the system is unconditionally stable provided that $\beta < \omega_1$.

The reader will notice the similarity of some of the foregoing results with well-known results for ideal passive viscous dampers (root locus going from the natural frequency without damping cables to natural frequency with blocked damping cables as the damper constant increases). However, passive viscous damping cannot be implemented with cables, because they cannot support compression loads.

3. Application to the Seriate footbridge

3.1 Model

A model of the Seriate footbridge (Figs. 5 and 6) was used to evaluate the active control strategy to suspension bridges. The footbridge, located in the North of Italy near the city of Bergamo, has been reported to exhibit excessive vibrations induced by the passage of pedestrians. The survey carried out by Prof. C. Gentile (2014) revealed that the passage of 8 walking pedestrians is inducing a vertical acceleration of 1.8 m/s^2 and the vertical acceleration induced by 4 joggers reaches 4 m/s^2 . These values are far beyond those recommended by the European HiVoSS guidelines (Van Nimmen *et al.* 2014). The third and fourth bending modes, respectively at 2.17 Hz (modal damping $\xi_3 = 1.48\%$) and 2.86 Hz ($\xi_4 = 1.5\%$) were identified as the critical modes within the pedestrian excitation range^{*} and will be the target of the active control system.

The bridge has a span of 64m, the deck weights 40T, the main steel cables (catenary) have a diameter of 60 mm and the 2×21 hangers have a diameter of 16 mm and a mean tension force of 15.3 kN according to the data sheet; the columns are articulated at the base and connected at the top; the main cables holding the column have an axial load of 425 kN according to data sheet. In the SAMCEF model, the deck is modeled with finite elements of beams with bending stiffness and mass matching those of the deck, the main cables are modeled with bars (one element between two

^{*}Typical pedestrian excitation range: walking: 1.6-2.4 Hz, running: 2.0-3.5 Hz, jumping: 1.8-3.4 Hz.

hangers) following a parabola (approximation of the catenary) and the hangers are also modeled with bars (a single element per hanger). The initial shape is taken from the bridge geometry (with some minor simplifications such as the columns have been assumed of equal length) and the prestress in the hangers is achieved by applying a thermal field until the appropriate value is reached. This model was able to capture quite well the natural frequencies and the mode shapes measured on the actual bridge (Gentile 2014). A simplified 2D model was also developed, which was also well representative of the bending behavior of the bridge (Table 1).



Fig. 5 View of the Seriate footbridge

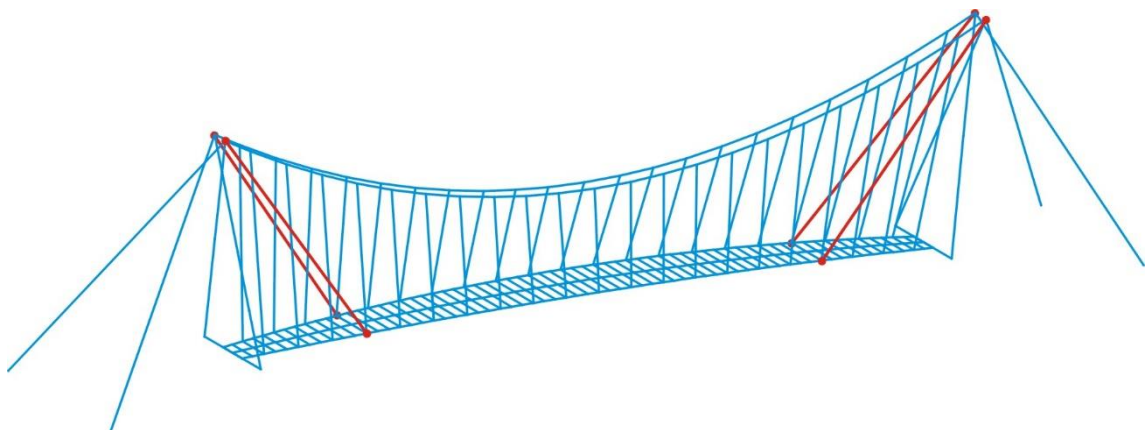
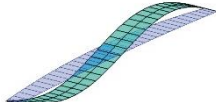
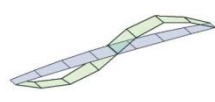
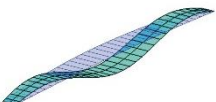
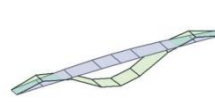
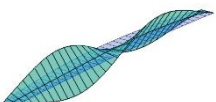
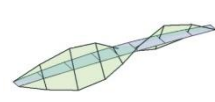
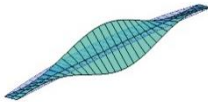
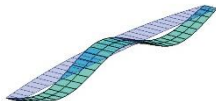
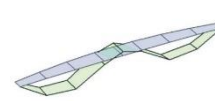
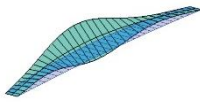
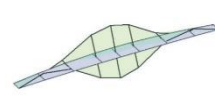
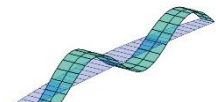
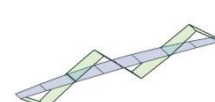


Fig. 6 3D model of the Seriate footbridge, with in red a possible configuration of 4 active stay cables

Table 1 Natural frequencies and mode shapes of the Seriate footbridge, comparison of the 3D model and 2D model with experiments (Gentile 2014). The two critical modes are **3B** and **4B**

Mode #	2D model ω_i (Hz)	3D model ω_i (Hz)	Experimental ω_i (Hz)	Numerical mode shape	Experimental mode shape
1 st B	1.03	1.02	1.03 $\xi_1 = 2.77\%$		
2 nd B	1.39	1.48	1.48 $\xi_2 = 1.34\%$		
1 st T	/	1.79	1.92		
2 nd T	/	2.1	1.94		
3 rd B	2.22	2.20	2.17 $\xi_3 = 1.48\%$		
3 rd T	/	2.65	2.75		
4 th B	2.81	2.78	2.86 $\xi_4 = 1.50\%$		

3.2 Active damping

In this study, we will restrict ourselves to an active control configuration involving 4 symmetrically located active steel cables with a diameter of 10 mm; the control system will consist of four independent identical loops with the same gain g . We first consider the situation where the active cables are attached to the pylons; the position of the attachment points with the deck is taken as a parameter, restricting ourselves to the positions where the hangers are attached (Fig. 7(a)). Because of the small size of the active cables, all the configurations considered in Fig. 7 leave the mode shapes almost unchanged (in agreement with the assumption made in the theory).

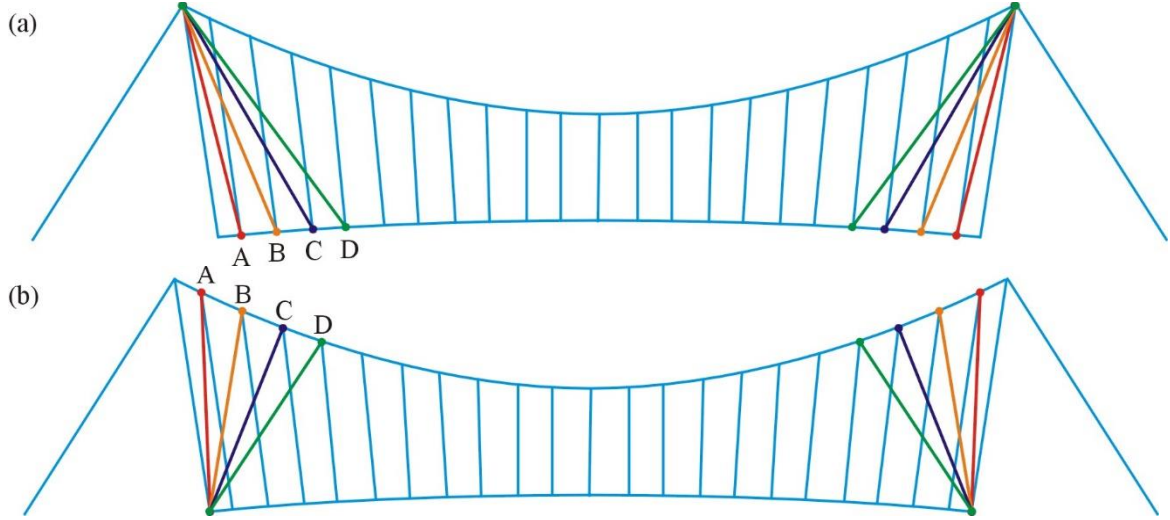


Fig. 7 Active control configurations of the Seriate footbridge. (a) Active cable attached to the pylon and (b) Active control attached to the catenary

According to the foregoing section, the closed loop poles follow closely the root locus (16) and the maximum damping ratio which can be achieved on one mode is given by Eq. (17). Table 2 shows the values that can be achieved for the various positions of the active cables investigated. Position C and D are clearly very good positions for the targeted modes (3rd and 4th bending modes), with damping ratios between 7% and 10%. Note that this is achieved with active cables with a diameter of 10 mm only.

Table 2 Active control cables attached to the pylon. Natural frequencies with (Ω_i) and without (ω_i) active cables and maximum achievable damping ratio ξ_i for the various modes and the various positions of the active cables shown in Fig. 7(a). The critical modes are in bold

Mode #	Position A			Position B		Position C		Position D	
	ω_i (Hz)	Ω_i (Hz)	$\xi_i^{\max}$ (%)	Ω_i (Hz)	$\xi_i^{\max}$ (%)	Ω_i (Hz)	$\xi_i^{\max}$ (%)	Ω_i (Hz)	$\xi_i^{\max}$ (%)
1 st B	1.02	1.07	2.2	1.22	9.8	1.38	17.5	1.53	24.7
2 nd B	1.48	1.49	0.6	1.54	2.1	1.55	2.5	1.53	1.8
1 st T	1.79	1.81	0.6	1.91	3.3	2.04	6.9	2.12	9.0
2 nd T	2.10	2.10	0.2	2.13	1.5	2.13	0.8	2.18	2.0
3rd B	2.2	2.23	0.7	2.36	3.6	2.54	7.7	2.64	10.0
3 rd T	2.65	2.65	0.0	2.65	0.0	2.65	0.0	2.65	0.0
4th B	2.78	2.85	1.26	3.13	6.3	3.13	9.6	3.17	7.1
4 th T	3.26	3.28	1.7	3.37	1.7	3.52	3.9	3.66	6.1

Table 3 Active control cables attached to the catenary. Natural frequencies with (Ω_i) and without (ω_i) active cables and maximum achievable damping ratio ξ_i for the various modes and the various positions of the active cables shown in Fig. 7(b)

Mode #	Position A			Position B		Position C		Position D	
	ω_i (Hz)	Ω_i (Hz)	$\xi_i^{\max}$ (%)	Ω_i (Hz)	$\xi_i^{\max}$ (%)	Ω_i (Hz)	$\xi_i^{\max}$ (%)	Ω_i (Hz)	$\xi_i^{\max}$ (%)
1 st B	1.02	1.06	1.6	1.21	9.4	1.40	18.5	1.58	27.2
2 nd B	1.48	1.50	0.6	1.56	2.9	1.59	4	1.58	3.4
1 st T	1.79	1.81	0.5	1.93	3.7	2.12	9.1	2.36	15.7
2 nd T	2.10	2.11	0.3	2.16	1.5	2.18	1.9	2.16	1.5
3rd B	2.2	2.21	0.3	2.30	2.4	2.52	5.1	2.90	15.9
3 rd T	2.65	2.65	0.0	2.66	0.0	2.66	0.1	2.66	0.1
4th B	2.78	2.83	0.9	3.09	5.7	3.63	15.4	3.59	14.7
4 th T	3.26	3.27	0.1	3.35	1.3	3.54	4.3	3.81	8.3

Next, we consider the situation where the active cables connect the deck at the foot of the pylon to the catenary (Fig. 7(b)). Table 3 shows the key numbers for the various positions corresponding to the attachment point of the hangers on the catenary. We note that, for position D, the performances are even better than for the previous configuration, reaching 15% for both the 3rd and 4th bending modes.

The performance of the control system expected on the basis of the previous discussion are excellent. However, although Eq. (17) has been verified experimentally on several occasions (Preumont 2011), one can always argue that the control system design is based on linear models which ignore all nonlinear aspects of cable structures and that robustness issues could eventually hamper the practical use of this technology. In order to investigate this, a laboratory mock-up has been built and tested.

4. Laboratory mock-up

The laboratory mock-up (Fig. 8) consists of two articulated towers of 0.62 m distant of 2.2 m; the deck is free to rotate at both ends and is attached to the catenary by two rows of 10 hangers. The catenary consists of a steel cable with a diameter of 1mm and the hangers are made of steel cables of 0.5 mm; the tension in the catenary and in the hangers can be adjusted with screws. The tension T_0 in a hanger is measured indirectly from its natural frequency f according to the string formula:

$$f = \frac{1}{2L} \sqrt{\frac{T_0}{\rho A}} \quad (19)$$

f being measured by a non contact custom made laser sensor (Achkire and Preumont 1998). In this way, it was possible to distribute the tension in the hangers uniformly. Two types of active cables have been tested, one steel cable similar to the hangers, with a diameter of 0.5 mm, and one

made of dyneema with a diameter of 0.2 mm; only the results obtained with the steel cables are reported in this paper. The selected configuration uses active cables between the deck and the pylon (Fig. 7(a)) rather than the one with active cables attached to the catenary which performs better, because it is closer to classical configurations in current use [e.g., Albert Bridge, London (1873), or Bosphorus-3 (under construction)], and therefore easier to accept by the bridge community.

Fig. 9 shows a close view of the active tendon; it consists of a APA-50s piezoelectric actuator from CEDRAT with a stroke of $52\ \mu\text{m}$ collocated with a B&K 8200 force sensor connected with a Nexus charge amplifier (the charge amplifier acts as a second-order high-pass filter with a corner frequency adjustable between 0.1 and 1 Hz). A small magnet is attached to the deck and a voice coil is used to apply a disturbance to the structure (band-limited white noise); all vibration tests have been performed with amplitudes typically of one thousands of the span.

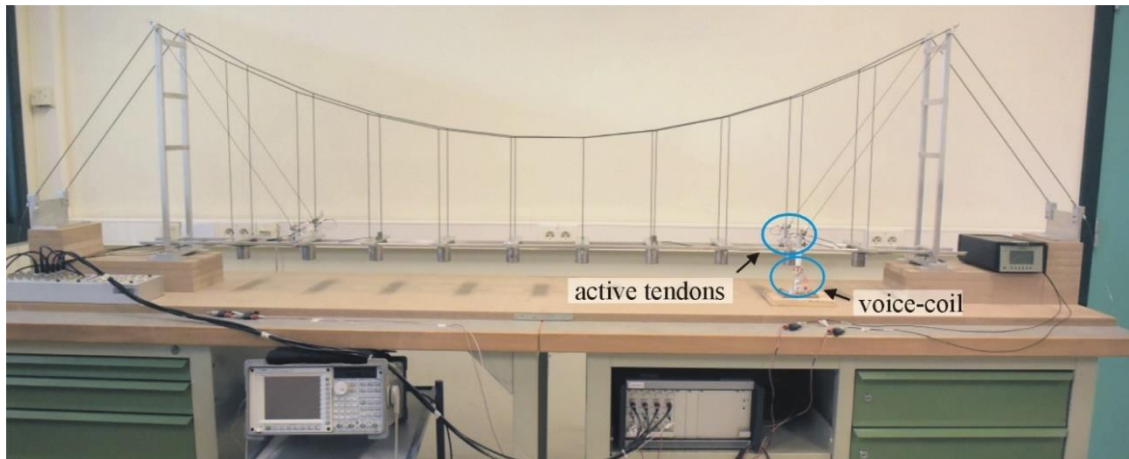


Fig. 8 Laboratory mock-up equipped with 4 active cables connecting the pylon to the deck

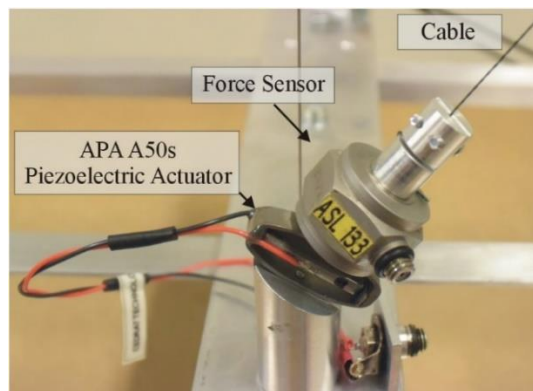


Fig. 9 Detail of the active tendon

Table 4 Laboratory demonstrator without control cables, comparison between the numerical and experimental mode shapes and natural frequencies

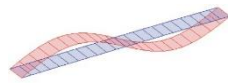
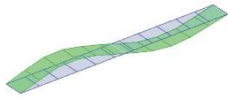
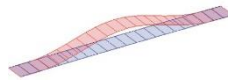
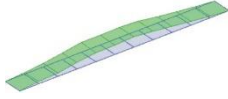
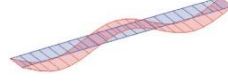
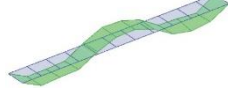
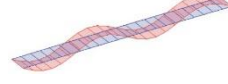
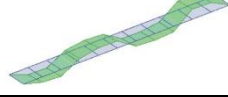
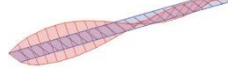
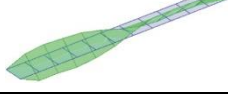
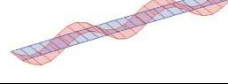
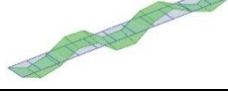
Mode #	Numerical ω_i (Hz)	Experimental ω_i (Hz)	Numerical mode shape	Experimental mode shape
1 st B	4.84	4.81		
2 nd B	7.68	5.59		
3 rd B	11.33	10.82		
4 th B	17.93	18.25		
1 st T	19.12	21.75		
5 th B	28.01	28.84		

Table 5 Laboratory demonstrator with four steel control cables of 0.5 mm. Comparison between numerical and experimental mode shapes and natural frequencies

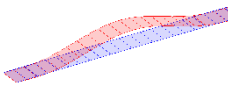
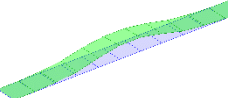
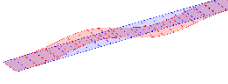
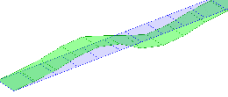
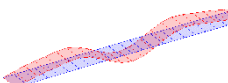
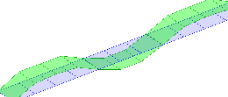
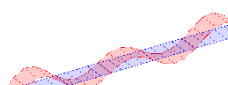
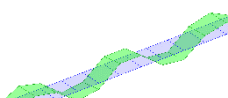
Mode #	Numerical Ω_i (Hz)	Experimental Ω_i (Hz)	Numerical mode shape	Experimental mode shape
1 st B	7.7	6.0		
2 nd B	12.0	12.1		
3 rd B	21.1	20.2		
4 th B	29.3	28.8		

Table 4 compares the experimental natural frequencies with the model predictions for the bridge without the active cables; the agreement is fairly good, except for the natural frequency of the second bending mode which is overestimated by the model; the measured structural damping ratios range between 0.8% and 1%. Table 5 compares the experiments with the numerical predictions for the bridge with the active stay cables (without control); some changes in the order of the modes are observed: the first bending mode has the shape of the second mode of the bridge without active cables; the second mode has a shape similar to the first mode without active cables. Fig. 10 shows the open-loop transfer functions T/δ of one of the four individual control loops (with the three other control cables passive) for two values of the tension in the control cable corresponding to a natural frequency of the control cable of respectively 40 Hz and 60 Hz; the curves exhibit alternating poles and zeros even above the natural frequency of the local mode of the control cable. According to Eq. (17), the distance between the poles Ω_i and the zeros ω_i is a measure of the controllability of the various modes (with a single loop). The very good agreement between the curves for the four loops (not shown) is representative of the good symmetry of the experimental set-up (Sangiovanni and Voltan 2015).

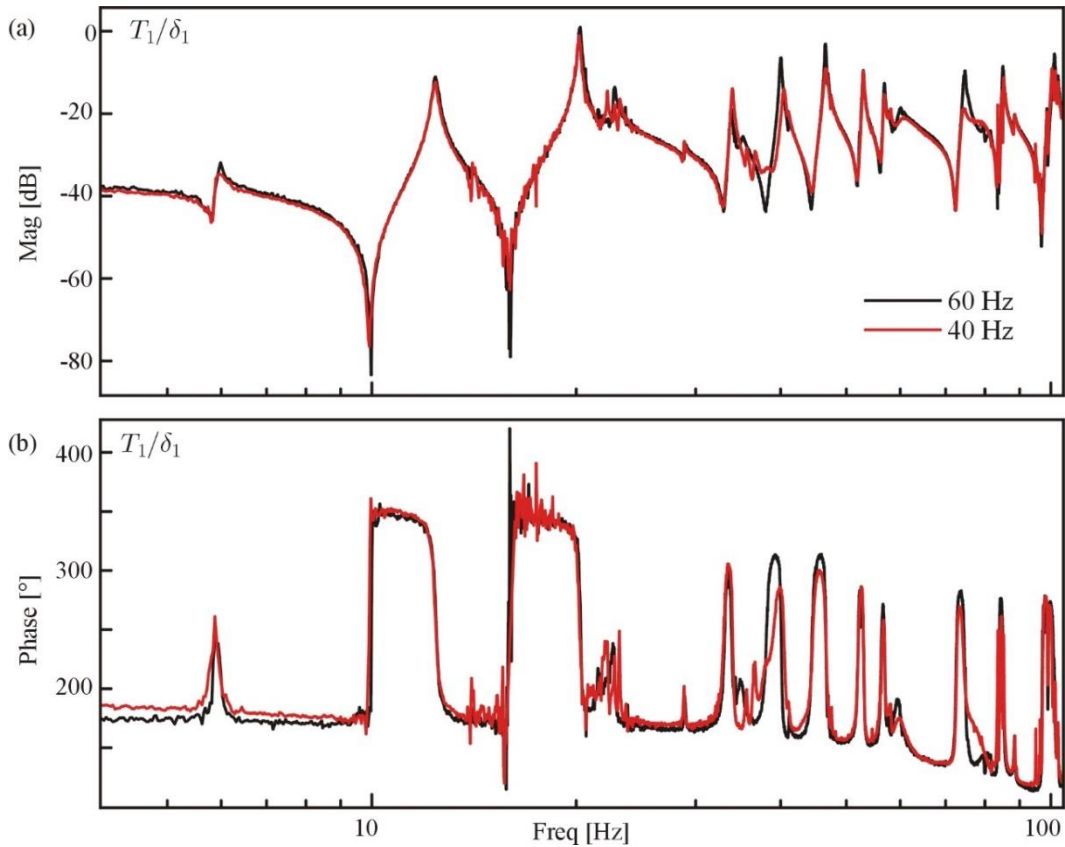


Fig. 10 Experimental open-loop transfer functions $G(\omega) = T/\delta$ of one control loop for two values of the tension in the control cable corresponding to a natural frequency of the control cable of respectively 40 Hz and 60 Hz

5. Active damping with one loop

The control law is the Integral Force Feedback (IFF) $H(s) = g/s$, with minor modification at low frequency because of the charge amplifier. The loop gain $GH(\omega)$ is shown in Fig. 11 (the pure IFF is in dotted line).

Fig. 12 shows the displacement response of the deck to a force disturbance applied to the deck by the voice coil, for various values of the gain g , when only one control loop is in operation. Fig. 12(a) shows the experimental Frequency Response Function (FRF) $R(\omega) = z/f$ between the force f applied to the deck by the voice coil and the deck displacement z , for various gains; Fig. 12.b shows the cumulative RMS defined as

$$\sigma(\omega) = [\int_{\omega}^{\infty} |R(v)|^2 dv]^{1/2} \quad (20)$$

(this form assumes a white noise input f). The steps in the diagram indicate how much each mode contributes to the RMS response. Increasing values of the gain lead to increasing values of the control force. Fig. 13 shows the influence of the control gain on the overall RMS value of the response and the RMS value of the control input, measured by the voltage v applied to the piezoelectric actuator. One sees that for small gains, the response reduces quickly, but for values larger than $g = 300$, no further reduction is achieved in the response while the control input increases steadily; this diagram allows to make a trade-off between performance and control cost. Fig. 14 shows the root locus reconstruction for various values of the gain, $g = 0, 160, 300, 650, 800, 950, 1200$, and the comparison with the root locus prediction of Eq. (16); the part of the locus in the vicinity of the real axis corresponds to the charge amplifier.

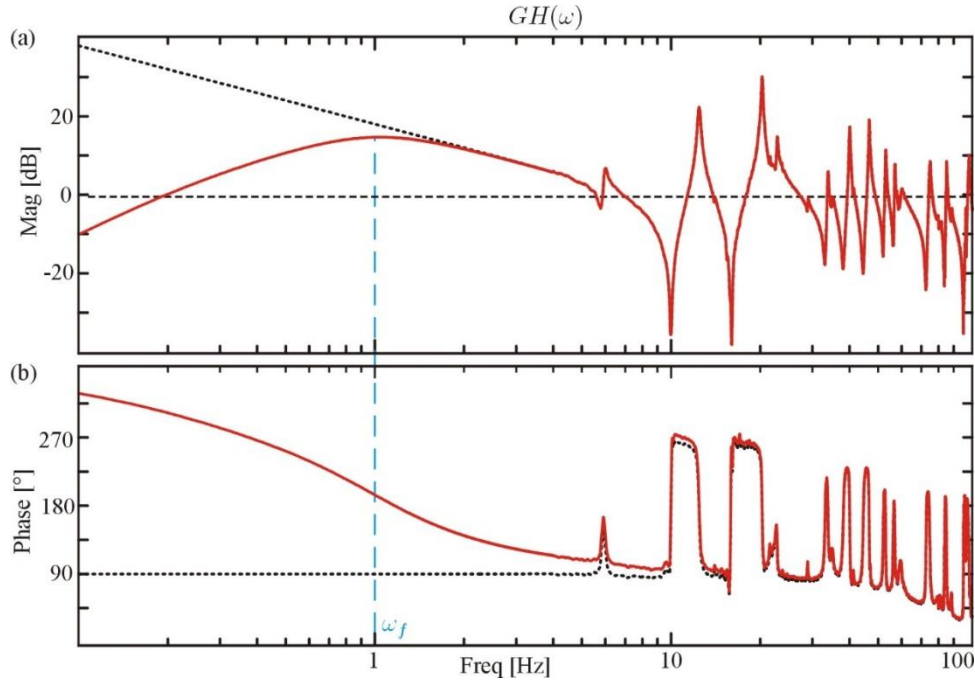


Fig. 11 Loop gain $GH(\omega)$ of one control loop including the controller and the charge amplifier (IFF is in dotted line)

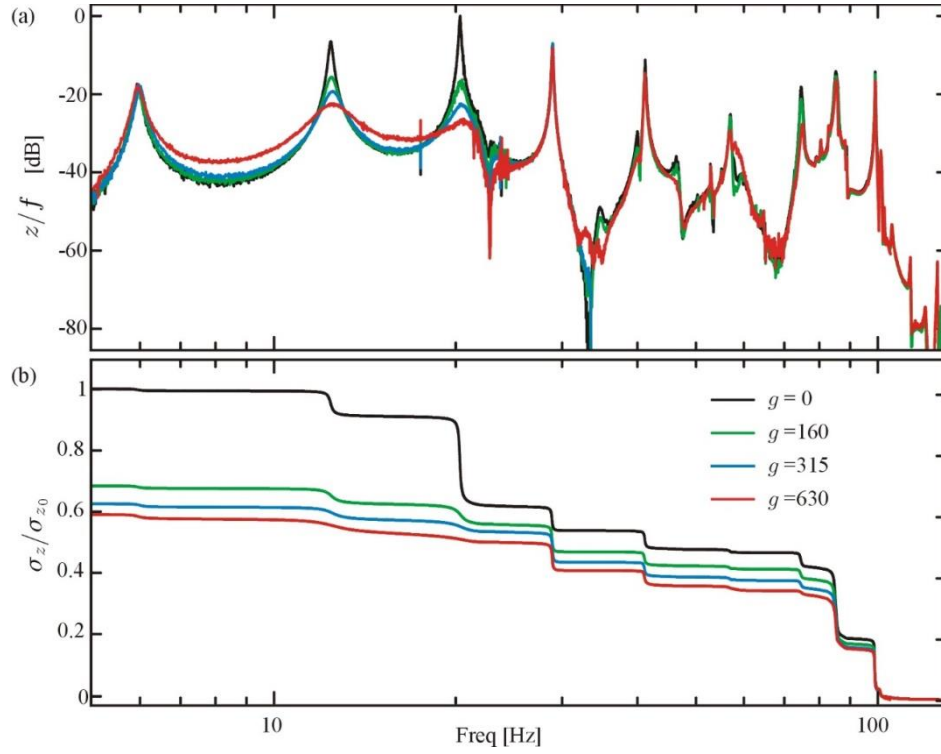


Fig. 12 Response to disturbance z/f with one loop of control, for various values of the gain g . (a) Experimental FRF and (b) Cumulative RMS $\sigma(\omega)$, normalized to its value when $g = 0$

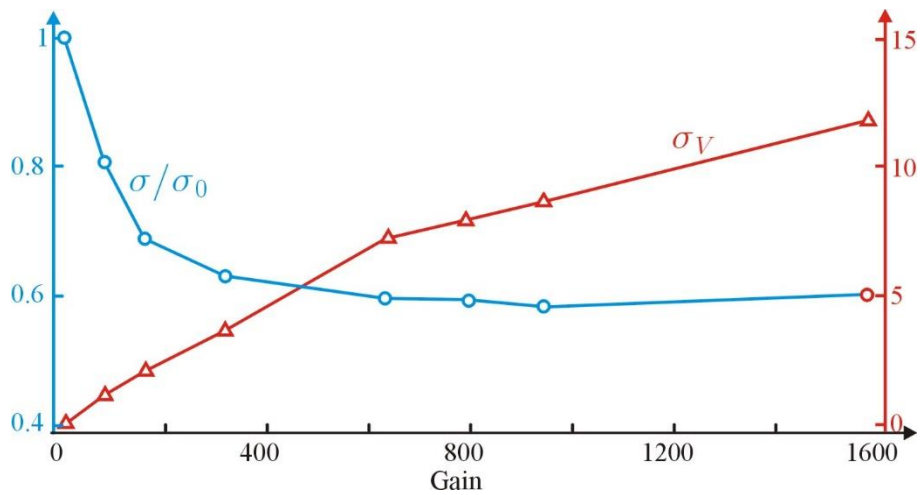


Fig. 13 Response with one control loop. Evolution of the RMS value of the deck displacement z (normalized to the uncontrolled response) and the actuator input v as a function of the control gain

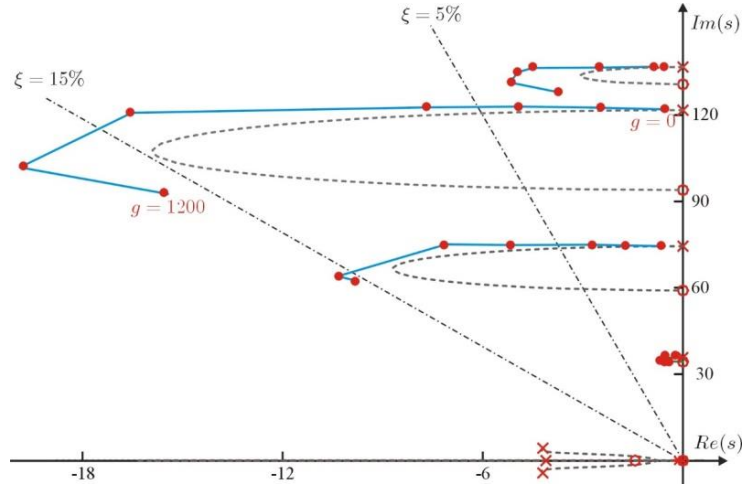


Fig. 14 Response with one control loop. Root locus reconstruction for various values of the gain: $g = 0, 160, 300, 650, 800, 950, 1200$ and comparison with the root locus of Eq.(16). Only the upper half of the root locus is shown

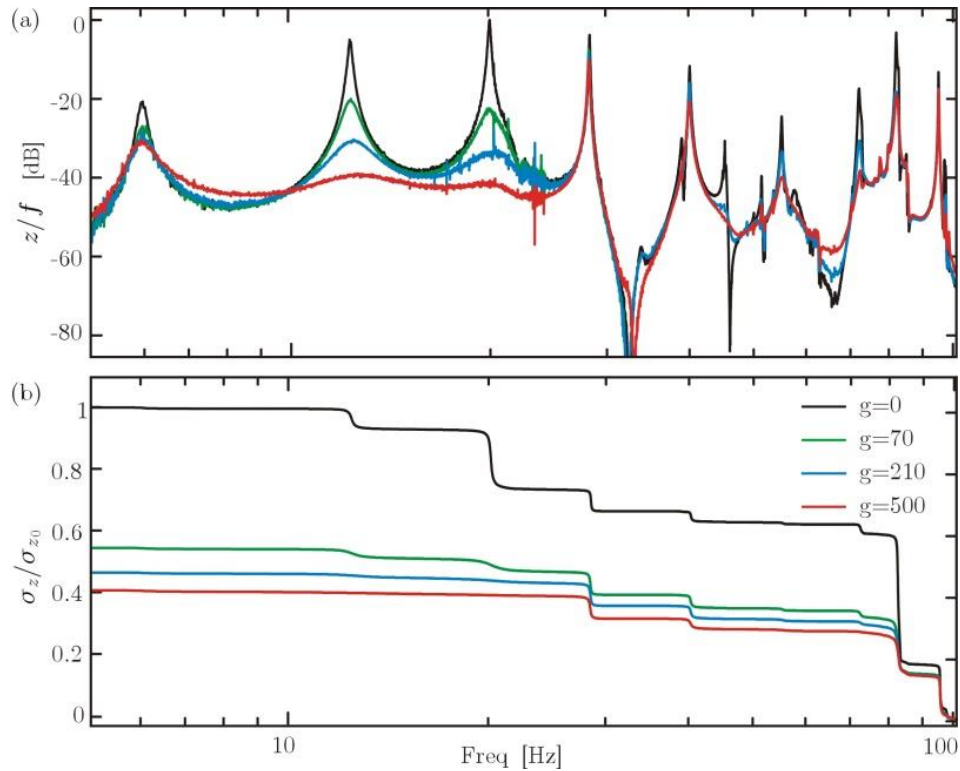


Fig. 15 Decentralized control with 4 independent control loops. Response to disturbance z/f , for various values of the gain g . (a) Experimental FRF and (b) Cumulative RMS $\sigma(\omega)$, normalized to its value when $g = 0$. No spillover is observed in the high frequency modes

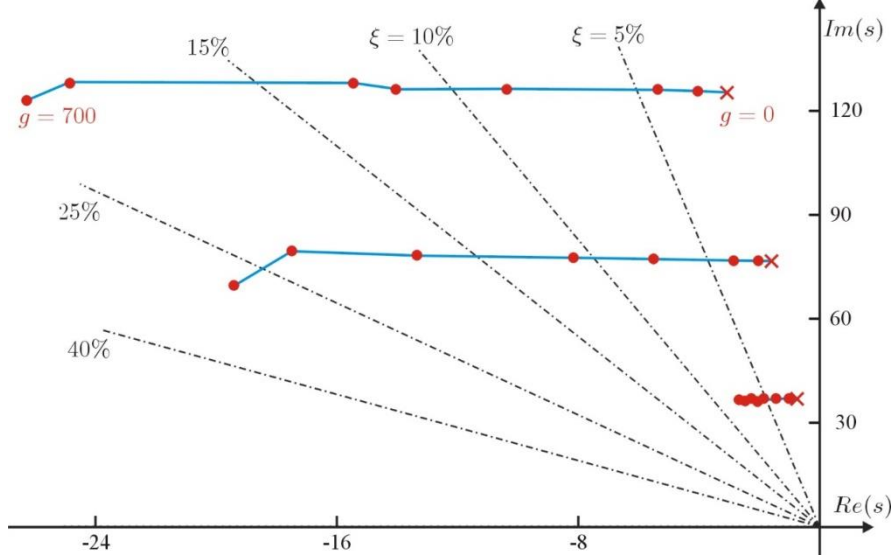


Fig. 16 Decentralized control with 4 independent loops. Root locus reconstruction for various values of the gain: $g = 50, 70, 100, 210, 300, 500, 600, 700$

6. Decentralized control with 4 loops

Next, a decentralized active damping has been implemented with four independent loops using the same gain. Fig. 15 shows the same information as in Fig. 12, with 4 channels of control, and Fig. 16 shows the root locus reconstruction of the closed-loop poles for various values of the gain: $g = 50, 70, 100, 210, 300, 500, 600, 700$. Observe in Fig. 15 that spillover is totally absent. Finally, regarding the quality of the model and the ability of the fairly simple linear bridge model to capture properly the closed-loop response, Fig. 17 compares the FRF z/f obtained experimentally with those obtained with the numerical model (the absolute values of the gain are irrelevant here, because the experimental loop gain includes many items such as charge amplifier gain, current amplifier gain, etc... which do not appear in the numerical model).

7. Conclusions

This paper explores the feasibility of active damping of suspension bridges with the addition of stay cables controlled with active tendons.

The first part of the paper reviews the theory of active tendon control with decentralized Integral Force Feedback (IFF) and collocated displacement actuator and force sensor; a formal proof of the formula giving the maximum achievable damping is provided for the first time.

The second part of the paper evaluates the potential of the control strategy on a numerical model of an existing footbridge; several configurations have been investigated where the active cables connect the pylon to the deck or the deck to the catenary. The analysis confirmed that it is possible to provide a set of targeted modes with a considerable amount of damping, reaching

$\xi = 15\%$. In the last part of the paper, the control strategy is demonstrated experimentally on a laboratory mock-up equipped with four control stay cables. The experimental results confirm the excellent performance and robustness of the control system and the very good agreement with the predictions. The linear bridge model is sufficient to capture properly the open-loop and the closed-loop response of the bridge for the range of amplitudes investigated. No spillover has been observed.

The next logical steps towards the application of the idea to large suspension bridges should be: (i) to further investigate the nonlinearities of the bridge response for larger amplitudes, to identify the limits of stability, if any, and (ii) to conduct a full scale experiment on a footbridge (including the actuator design).

Acknowledgments

The authors wish to thank Prof. Carmelo Gentile from the civil engineering department of the Politecnico di Milano for providing the data of the Seriate footbridge and Prof. Mihaita Horodincu from the Technical University "Gheorghe Asachi" Iasi, Romania, for his help in the construction of the bridge mock-up.

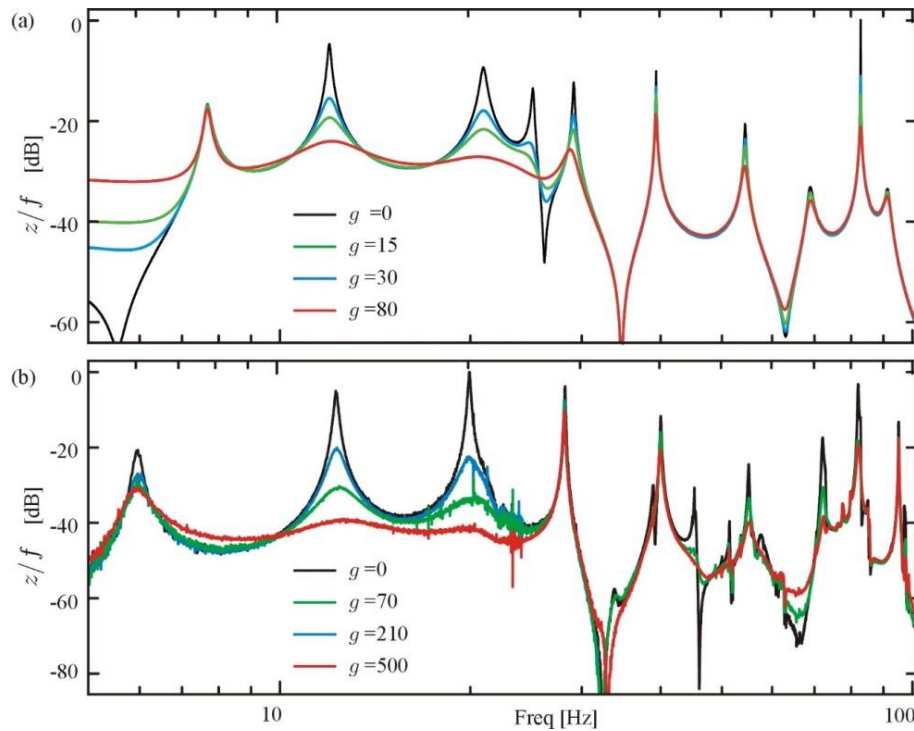


Fig. 17 Decentralized control with 4 independent control loops. Response to disturbance z/f , for various values of the gain g . Comparison between numerical predictions and experimental results (only the relative values of g matter). (a) Model and (b) Experiment

References

- Achkire, Y. and Preumont, A. (1996), "Active tendon control of cable-stayed bridges", *Earthq. Eng. Struct. D.*, **25**(6), 585-597.
- Achkire, Y. and Preumont, A. (1998), "Optical measurement of cable and string vibration", *J. Shock Vib.*, **5**(3), 171-179.
- Achkire, Y., Bossens, F. and Preumont, A. (1998), "Active damping and flutter control of cable-stayed bridges", *J. Wind Eng. Ind. Aerod.*, **74**, 913-921.
- Auperin, M. and Dumoulin, C. (2000), "Structural control: Point of view of a civil engineering company in the field of cable-supported structures", *Proceedings of the 3rd International Workshop on Structural Control*, Paris, France, July.
- Bortoluzzi, D., Casciati, S., Elia, L. and Faravelli, L. (2015), "Design of a TMD solution to mitigate wind-induced local vibrations in an existing timber footbridge", *Smart Struct. Syst.*, **16**(3), 459-478.
- Bossens, F. and Preumont, A. (2001), "Active tendon control of cable-stayed bridges: a large-scale demonstration", *Earthq. Eng. Struct. D.*, **30**(7), 961-979.
- Caetano, E., Cunha, A., Moutinho, C. and Magalhães, F. (2010), "Studies for controlling human-induced vibration of the Pedro e Inês footbridge, Portugal. Part 2: Implementation of tuned mass dampers", *Eng. Struct.*, **32**(4), 1082-1091.
- Cannon, R.H. and Rosenthal, D.E. (1984), "Experiments in control of flexible structures with noncollocated sensors and actuators", *J. Guid. Control Dynam.*, **7**(5), 546-553.
- Casciati, S., Chassiakos, A.G. and Masri, S.F. (2014), "Toward a paradigm for civil structural control", *Smart Struct. Syst.*, **14**(5), 981-1004.
- Costa, A.P.D., Martins, J.A.C., Branco, F. and Lilien, J.L. (1996), "Oscillations of bridge stay cables induced by periodic motions of deck and/or towers", *J. Eng. Mech. - ASCE*, **122**(7), 613-622.
- Fujino, Y. and Susumpow, T. (1994), "An experimental study on active control of in-plane cable vibration by axial support motion", *Earthq. Eng. Struct. D.*, **23**(12), 1283-1297.
- Fujino, Y., Warnitchai, P. and Pacheco, B.M. (1993), "Active stiffness control of cable vibration", *J. Appl. Mech. - ASME*, **60**(4), 948-953.
- Gentile, C. (2014), *Personal communication*, Politecnico di Milano, Civil Engineering Department.
- Lilien, J.L. and Costa, A.P.D. (1994), "Vibration amplitudes caused by parametric excitation of cable stayed structures", *J. Sound Vib.*, **174**(1), 69-90.
- Liu, S.C., Tomizuka, M. and Ulsoy, A.G. (2005), "Challenges and opportunities in the engineering of intelligent systems", *Smart Struct. Syst.*, **1**(1), 1-12.
- Nayfeh, A.H. and Mook, D.T. (1979), *Non Linear Oscillations*, Wiley, New York, NY, USA.
- Pacheco, B.M., Fujino, Y. and Sulekh, A. (1993), "Estimation curve for modal damping in stay cables with viscous damper", *J. Struct. Eng. - ASCE*, **119**(6), 1961-1979.
- Preumont, A. (2011), *Vibration Control of Active Structures, An Introduction*, (3rd Ed.), Springer.
- Preumont, A. and Achkire, Y. (1997), "Active damping of structures with guy cables", *J. Guid. Control Dynam.*, **20**(2), 320-326.
- Preumont, A. and Bossens, F. (2000), "Active tendon control of vibration of truss structures: Theory and experiments", *J. Intel. Mat. Syst. Str.*, **11**(2), 91-99.
- Preumont, A., Achkire, Y. and Bossens, F. (2000), "Active tendon control of large trusses", *AIAA J.*, **38**(3), 493-498.
- Preumont, A., Dufour, J.P. and Malekian, C. (1992), "Active damping by a local force feedback with piezoelectric actuators", *J. Guid. Control Dynam.*, **15**(2), 390-395.
- Sangiovanni, A. and Voltan, M. (2015), "Active tendon control of suspension bridges", MSc Thesis, Politecnico di Milano, Department of Mechanical Engineering, Milano.
- Tubino, F. and Piccardo, G. (2014), "Tuned mass damper optimization for the mitigation of human-induced vibrations of pedestrian bridges", *Meccanica*, **50**(3), 809-824.

- Van Nimmen, K., Lombaert, G., De Roeck, G. and Van den Broeck, P. (2014), "Vibration serviceability of footbridges: Evaluation of the current codes of practice", *Eng. Struct.*, **59**, 448-461.
- Warnitchai, P., Fujino, Y., Pacheco, B.M. and Agret, R. (1993), "An experimental study on active tendon control of cable-stayed bridges", *Earthq. Eng. Struct. D.*, **22**(2), 93-111.
- Yang, J.N. and Giannopoulos, F. (1979a), "Active control and stability of cable-stayed bridge", *J. Eng. Mech. Div.*, **105**(4), 677-694.
- Yang, J.N. and Giannopoulos, F. (1979b), "Active control of two-cable-stayed bridge", *J. Eng. Mech. Div.*, **105**(5), 795-810.

Appendix: Proof of Equation (17)

The characteristic equation corresponding to Eq. (16) reads

$$s^3 + gs^2 + \Omega_i^2 s + g\omega_i^2 = 0 \quad (\text{A.1})$$

The root locus (locus of the solutions of the characteristic equation when g varies from 0 to ∞ , Fig. 3) has one branch on the negative real axis (say in $-a$) and two branches corresponding to a complex conjugate pair at $-\xi\omega \pm j\omega\sqrt{1-\xi^2}$. This leads to the characteristic equation

$$(s + a)(s^2 + 2\xi\omega s + \omega^2) = 0 \quad (\text{A.2})$$

where a , ξ and ω depend on the gain g . Observe that the frequency ω decreases monotonously from Ω_i to ω_i . Matching the coefficients of the two foregoing equations, one gets the three identities

$$a\omega^2 = g\omega_i^2, \quad 2a\xi\omega + \omega^2 = \Omega_i^2, \quad a + 2\xi\omega = g$$

We seek the maximum value of ξ and the corresponding value of the gain g . From the first equality, $a = g\omega_i^2/\omega^2$; substituting in the other two equalities

$$2g\omega_i^2\xi/\omega + \omega^2 = \Omega_i^2, \quad g\omega_i^2/\omega^2 + 2\xi\omega = g$$

From the second of these equalities, one finds

$$\xi = \frac{g}{2\omega} \left(1 - \frac{\omega_i^2}{\omega^2}\right) \quad (\text{A.3})$$

and substituting into the first one,

$$g^2 = \left(\frac{\Omega_i^2 - \omega^2}{\omega^2 - \omega_i^2}\right) \frac{\omega^4}{\omega_i^2} \quad (\text{A.4})$$

Back substituting into Eq. (A.3), one finds the relationship between ξ and ω along the root locus:

$$\xi = \frac{[(\Omega_i^2 - \omega^2)(\omega^2 - \omega_i^2)]^{1/2}}{2\omega_i\omega} \quad (\text{A.5})$$

This expression may be regarded as $\xi(\omega)$ (recall that ω is monotonously decreasing function of g). Solving the equation $d\xi/d\omega = 0$, one easily finds

$$\omega = (\Omega_i\omega_i)^{1/2} \quad (\text{A.6})$$

and, substituting in Eq. (A.4)

$$g = \Omega_i(\Omega_i/\omega_i)^{1/2} \quad (\text{A.7})$$

and from Eq.(A.3), the maximum damping ratio is

$$\xi = \frac{\Omega_i - \omega_i}{2\omega_i} \quad (\text{A.8})$$

which is the desired equation. Additionally, one finds $a = (\Omega_i\omega_i)^{1/2}$.