Smart Structures and Systems, *Vol. 17, No. 6 (2016) 1017-1030*
DOI: http://dx.doi.org/10.12989/sss.2016.17.6.1017

Operational modal analysis of reinforced concrete bridges using autoregressive model

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(Received January 22, 2016, Revised April 27, 2016, Accepted April 29, 2016)

Abstract. This study focuses on the system identification of reinforced concrete bridges using vector autoregressive model (VAR). First, the time series output response from a bridge establishes the autoregressive (AR) models. AR models are one of the most accurate methods for stationary time series. Burg's algorithm estimates the autoregressive coefficients (ARCs) at p-lag by reducing the sum of the forward and the backward errors. The computed ARCs are assembled in the state system matrix and the eigen-system realization algorithm (ERA) computes: the eigenvector matrix that contains the vectors of the mode shapes, and the eigenvalue matrix that contains the associated natural frequencies. By taking advantage of the characteristic of the AR model with ERA (ARMERA), civil engineering can address problems related to damage detection. Operational modal analysis using ARMERA is applied to three experiments. One experiment is coupled with an artificial neural network algorithm and it can detect damage locations and extension. The neural network uses a specific number of ARCs as input and multiple submatrix scaling factors of the structural stiffness matrix as output to represent the damage.

Keywords: system identification; autoregressive model; Burg's algorithm; eigen-system realization algorithm

1. Introduction

Bridges are exposed to different external loads such as: wind, earthquake, temperature change, and various traffic loads. These loads influence the structural system and over time cause change in it. The system will suffer different forms of degradation, such as: material degradation, partial fracture within the elements, and behavioral changes of one or more structural elements. All kinds of local changes in its elements are considered as damage in the structure because they can cause change in the overall behavior, parting from the design concept, and severe destruction of the whole system according to the extension of the damage (Ye *et al.* 2012). Because the actual external loads are uncertain, the structural system is treated as an output only system. To avoid either personal or material loss of the society, many researchers in civil engineering are involved in system identification (SI) and structural health monitoring (SHM) (Ye *et al*. 2013).

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http://www.techno-press.org/?journal=sss&subpage=8 ISSN: 1738-1584 (Print), 1738-1991 (Online)

Modal analysis is one of the most successful branches of SHM and SI. Modal analysis studies the changes in the modal parameters of a structure: natural frequencies, mode shapes, mass participation factor, and damping ratio to understand the changes within it (Hearn and Testa 1991). Modal analysis includes many methods to address and to solve this problem. Two popular methods are: the frequency domain decomposition (FDD) (Brincker *et al*. 2000), which treats the problem in frequency domain, and the time domain based identification (Mohanty and Rixen 2004). Nowadays, FDD is the popular because it is straightforward and efficient to identify the modal parameters. However, it has limitations when it operates on stationary time series data set because it relies on confined observations. Furthermore, when a windows technique is used the resulting modal parameters are the average of the time frame analyzed and sudden changes cannot be easily identified. Instead, time domain based modal analysis solves this data processing problem because it takes into account the evolution of stationary and non-stationary time-series (Priestley 1981).

Autoregressive (AR) model is a stochastic differential equation and it is one of the most accurate methods to represent a time series output data set through parameters (Omenzetter *et al*. 2003). The strength of the AR model is the presence of parameters that change over time, the Autoregressive coefficients (ARCs). The ARCs are used in the eigen-system realization algorithm (ERA) to obtain the modal parameters of the structure (Vu *et al.* 2011). The variation of modal response of a structure between two different time intervals implies that some changes has occurred within the structure, such as: variation of material properties, ground settlements, deformations, or damage in one or more structural elements. In addition, modal analysis through the ARCs can identify small sudden changes within the structure. Furthermore, because both the data length and the lag of an AR model are flexible the user is not restricted to a specific range size and number of coefficients, i.e. model order.

In this study, modal analysis is done using the AR model and the ERA model together (ARMERA). ARMERA is used to detect the damage location within the structural system up to the sub-element level. The evolution of the ARCs according to a predefined time step enables the computation of practical modal outcomes for the target structure. Furthermore, the presence of a large number of ARCs enables the use of optimization algorithms on a large parametric controlled model, such as: neural network algorithm, genetic algorithm, response surface methodology, and Monte Carlo optimization. Though this study focuses on system identification using ARMERA, damage detection methodologies using ARCs are one of the great challenges within civil engineering problems.

For the application of AR model to real structures, time series output data from three experiments is used as the input to the model and ERA is conducted on each set of data. The first experiment was conducted with a single remote sensing vibrometer (RSV) on a single span reinforced concrete bridge to build a single output AR model. Because the ARMERA applied to such experiment can only identifies a single DOF system it does not guarantee detailed sampling of the modal information. Instead, it can do so when it is applied to multiple outputs of multiple sensors, like in the second experiment, which includes four sensors nodes deployed on the same reinforced concrete bridge, with a total of ten sensors installed. Finally, the AR analysis with the ERA of full bridge scale sensing system is carried out to obtain comprehensive modal information of long span reinforced concrete bridge. To validate the result of the ARMERA methodology, the modal outcome of frequency domain decomposition is compared with the result of the method given in this study.

2. Method

In operational modal analysis ARMERA needs the time series of the output data that is obtained from the target structure. Because bridges are too large to install sensors on each element a limited number of channels are available. However, with a sufficient coverage the capability of the system of sensors can produces a suitable output data. The multiple time series are aggregated together to form the AR model. AR model is a Markov process of a stochastic differential equations, each equation had its own number of ARCs. The ARCs compose the state matrix used in the eigenvalue decomposition that extracts the modal parameters searched.

2.1 System identification through ARMERA

2.1.1 AR model

The AR model, time series representation of random process, successfully calculates the current stochastic value at time t based on the previous values at time t-i. In this study, the influence of the input excitation of the target structure is ignored and the AR model only take into account the output, either acceleration and/or velocity.

The output only time series AR model with p number of prior physical quantities is defined as Eq. (1) (Brockwell and Davis 2002).

$$x(t) = \sum_{i=1}^{p} \phi_{ii} x(t-i) \tag{1}$$

where, $x(t)$ is the value of the time series at current time t, p is the lag and the model order, ϕ_{ii} is the ARCs of size $m\times p$, and $x(t-i)$ is the observation of the values of the time series data of the i-th previous steps.

In this study, the main interest within AR model is ϕ_{ii} which are ARCs for constructing the state matrix. Both Yule-Walker's algorithm and Burg's algorithm are regarded as the most effective method for the estimation of the ARCs. In this study, Burg's algorithm is used for calculating ARCs because it proved to be more accurate than Yule-Walker algorithm.

2.1.2 Burg's algorithm

In the output only AR model shown in Eq. (1), Burg's algorithm minimizes the prediction errors of both the forward and the backward steps and the ARCs [ϕ_{11}, ..., ϕ_{pp}] are computed by Eqs. (2)-(7) (Brockwell and Davis 2002).

$$u_0(t) = v_0(t) = x_{n+1-t} \tag{2}$$

$$u_i(t) = u_{i-1}(t-1) - \phi_{ii} v_{i-1}(t) \tag{3}$$

$$v_i(t) = v_{i-1}(t) - \phi_{ii} u_{i-1}(t-1) \tag{4}$$

where, $u_i(t)$ and $v_i(t)$ are respectively the forward predictions on the error and the backward prediction error.

$$d(1) = \sum_{t=2}^{n} (u_0{}^2(t-1) + v_0{}^2(t)) \tag{5}$$

$$\phi_{ii} = \frac{2}{d(i)} \sum_{t=i+1}^{n} v_{i-1}(t) u_{i-1}(t-1) \tag{6}$$

where, ϕ_{ii} is Burg's estimation for the partial autocorrelation functions.

$$d(i+1) = (1-\phi_{ii}^{2}) d(i) - v_i^2(i+1) - u_i^2(n) \tag{7}$$

After computing the forward and the backward predictions using Eqs. (2)-(7), the ARCs at p-lag value are obtained. If the statistical properties of the modal properties at different time steps are similar, the time series data set is regarded as stationary and ARCs also are similar regardless of the scope of the data.

2.1.3 Modal parameters identification using ERA

After computing ARCs using Burg's algorithm, the system matrix of the AR model is assembled as in Eq. (8). (Neumaier and Schneider 2001)

$$\Pi = \begin{pmatrix} \phi_{11} & \phi_{22} & \cdots & \phi_{p-1p-1} & \phi_{pp} \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{pmatrix}_{p\times p} \tag{8}$$

The eigenvalue decomposition of the system matrix gives the modal parameters as the matrix of eigenvalues and matrix of the eigenvectors (Juang and Pappa 1985).

$$\Pi = L \begin{pmatrix} \pi_1 & 0 & \cdots & 0 \\ 0 & \pi_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \pi_p \end{pmatrix} L^{-1} \tag{9}$$

The matrix of the eigenvectors $\boldsymbol{L}$ at Eq. (9) had a size of $p\times p$ and it contains the mode shapes of the p eigenvalue.

<u>*Natural frequencies*</u>

$$f_i = \frac{\sqrt{\mathrm{Re}^2(\lambda_i) + \mathrm{Im}^2(\lambda_i)}}{2\pi}; \text{where } \lambda = \frac{\ln(\pi_i)}{T_s} \tag{10}$$

<u>*Mode shapes*</u>

$$\Psi = (\Psi_1 \quad \Psi_2 \quad \cdots \quad \Psi_p) = (1 \quad 0 \quad \cdots \quad 0) L \tag{11}$$

<u>*Damping ratios*</u>

$$\zeta_i = -\frac{\mathrm{Re}(\lambda_i)}{2\pi f_i} \tag{12}$$

2.1.4 Vector autoregressive model

$$\begin{aligned} x_1(t) &= \sum_{i=1}^{p} \phi_1^{(ii)} x_1(t-i), \\ x_2(t) &= \sum_{i=1}^{p} \phi_2^{(ii)} x_2(t-i), \\ &\vdots \\ x_m(t) &= \sum_{i=1}^{p} \phi_m^{(ii)} x_m(t-i) \end{aligned} \tag{13}$$

With multiple autoregressive model of p lag, the m numbers of the ARCs sets for each AR model $[\Phi_1\ \Phi_2\ \cdots\ \Phi_m]$ are computed by Burg's algorithm. Now the system state matrix of m number of AR model is assembled with m sets of ARCs as Eq. (14). (Neumaier and Schneider 2001)

$$\Pi = \begin{pmatrix} \Phi^{(1)}_{m\times m} & \Phi^{(2)}_{m\times m} & \cdots & \Phi^{(\mathrm{p}-1)}_{m\times m} & \Phi^{(\mathrm{p})}_{m\times m} \\ I_{m\times m} & 0 & \cdots & 0 & 0 \\ 0 & I_{m\times m} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I_{m\times m} & 0 \end{pmatrix}_{(m\times p)\times(m\times p)}$$
$$\text{where, } \Phi^{(i)} = \begin{pmatrix} \phi_1^{(ii)} & 0 & \cdots & 0 \\ 0 & \phi_2^{(ii)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \phi_m^{(ii)} \end{pmatrix}_{m\times m} \tag{14}$$

2.1.5 Time domain variation

If a structure is not damaged, its stiffness and damping ratio do not change overtime, and the ARCs, which are the parameters representing the time series output data, are stationary. Instead, if the structure is damaged or is subject to degradation, different *p-lag* number of ARCs, which are generated by Burg's algorithm, have different values. This difference can be used to estimate whether damage is present, its extent, and location. In a stationary time series process, each ARCs obtained from the stationary time-series data set has its own mean and standard deviation, which are constant over time. The standard deviation takes into account the stiffness change within the structure due to temperature change, daily changes and seasonal changes. A change in the mean or standard deviation of the ARCs means that something happened to the structure, such as: degradation of the materials, change in the boundary conditions, or damage in one or more structural elements.

Direct comparison between the ARC of the intact and current structure cannot identify what exactly happened. It is probable that something happened but the location and type of the damage cannot be determined only by comparing ARCs or their statistical properties. Even though the focus of the paper is bounded on the system identification using ARMERA, a more sophisticated algorithm is required to identify the source of the property changes.

2.2 Neural network algorithm

ARMERA is regarded as one of the most accurate methods to calculate modal parameters with time series output data. ARMERA can be used to solve structural engineering problems including damage detection algorithm, which is one of the great challenging topic within civil engineering field, because it has the advantage of stationary time series data sequences and abundant number of coefficients, i.e., ARCs. While many algorithms exist to estimate the damage location, neural network (Lee *et al.* 2011) is deemed an efficient algorithm to perform this task due to the recent advances in its performance, computational efficiency, and computational power. Artificial neural network, which is inspired to biological neural networks, leads the research in damage identification techniques for civil structures. The *p-lag* number of ARCs forms the input layer of the artificial neural network algorithm. The algorithm has several hidden layer, the exact number varies with the complexity of the structure under investigation, and in the output the submatrix scaling factors (SSF) are identified (Yun and Bahng 2000).

These tentative SSFs are used to calibrate the stiffness matrix of a finite element model, which represents the real structure. The result of the modal analysis of the model are compared with the modal properties of the experimental ERA to compute the errors within the neural network during the training period. The next step is to iterate different output data set inside the neural network algorithm to obtain updated SSFs. This process ends when the modal assurance criteria (MAC) is satisfied (Allemang 2003). The MAC value correlates the experimental mode shapes and numerical mode shapes and computes whether the updated numerical model is a well fit to the actual structural model. The MAC of the *i*-th mode is described as Eq. (15)

$$MAC_i = \frac{\left| \left[\overline{\Psi_i^{(e)}} \right]^T \Psi_i^{(n)} \right|}{\sqrt{\left[\overline{\Psi_i^{(e)}} \right]^T \overline{\Psi_i^{(e)}}} \sqrt{\left[\overline{\Psi_i^{(n)}} \right]^T \overline{\Psi_i^{(n)}}}} \tag{15}$$

where, Ψ_i^e and Ψ_i^n are the *i*-th complex mode shape vectors of the experimental model and the numerical model respectively.

The MAC value calculated after operating forward propagation of neural network is used as the criteria to update the parameters of the hidden layers in the neural network.

2.2.1 Submatrix scaling factor

The submatrix considered in this study is not the element stiffness matrix but a further level below. A SSFs does not multiply the element stiffness matrix but one of the submatrix representing a specific force-displacement relationship. The substructural identification method takes into account the force-type based submatrix division: the axial force submatrix $\boldsymbol{K}^{\boldsymbol{N}}$, the two shear-bending moment submatrices $\boldsymbol{K}^{\boldsymbol{My,Vz}}$ and $\boldsymbol{K}^{\boldsymbol{Mz,Vy}}$, and the torsional moment submatrix $\boldsymbol{K}^{\boldsymbol{T}}$. For a 2D frame structure this matrixes assembled as follow, Eq. (16)

$$K_j = K_j^N + K_j^{V_y,M_z} + K_j^T \tag{16}$$

The global stiffness matrix of a 3D frame structure, which is assembled from the element stiffness matrices after a coordinate transformation, is divided by the force level submatrices Eq. (17)

$$\left[K\right]_{6\cdot n\times 6\cdot n} = \sum_{j=1}^{q}\sum_{i=1}^{4} s_{j,i}\left[K_{j,i}\right]_{12\times 12}^{t=0} \tag{17}$$

where q is the number of elements, n is the number of nodes in the system.

The stiffness matrix obtained represents the updated model and the SSFs, are computed by the neural network algorithm.

2.2.2 Modal analysis using script based finite element analysis tool

A finite element model (FEM) based on Ansys parametric design language (APDL) (Yun 2004) and the parametric tool command language (tcl) of Opensees (McKenna 2011) are used for the study. The two FEA software are used due to their parametric characteristics that make them suitable to be used with the neural network algorithm by exchanging the ARCs, the modal parameters, and the SSFs within a single iterative framework. The advance in computational power enables a rapid computation of both the FEM and the ARMERA. After a series of repeated forward and backward propagation inside the neural network algorithm and FEA using the SSF obtained, the trained neural network algorithm can detect the possible location and type of the damages, when the statistical parameters of the ARCs change.

3. Experiments

Three experiments were performed on two different bridges; the first test bed is a short span reinforced concrete bridge and the second one is a long span reinforced concrete cable stayed bridge. The work done on the bridge is to obtain time series output data either in acceleration or velocity response at designated locations. The input excitation or any kind of external loading on the bridges was ignored when the AR models were built. The acquired temporal data are used in the output only AR models and Burg's algorithm on multiple AR models computes the ARCs for each *p-lag* numerical model. The ERA of these ARCs gives the experimental modal parameters: natural frequencies, mode shapes and damping ratio.

3.1 The Short span reinforced concrete bridge

The first test-bed is a reinforced concrete bridge on the UNIST campus shown in Fig. 1. The bridge is a simple bridge with a single span of 60m, and the traffic on the bridge is exclusively pedestrian. The arch shaped yellow steel is attached along the bridge but it is only esthetic purpose of the campus and does not affect structural behavior of the bridge system.

3.1.1 Remote sensing vibrometer

The first experiment on the bridge was done with a remote sensing vibrometer (RSV). The experimental setup is shown in Fig. 2. The purpose of the experiment was to obtain the temporal data set from the bridge structure so that the ARMERA could be structured for a single output system.

The RSV's data were acquired with a 480 Hz of sampling frequency and the laser head was 165 m from the bridge. The experiment was repeated 3 times and each data set is 5 minutes long. The output response is the time domain velocity data of a single point on the bridge; the yellow steel arc bridge was the target because of its good reflectivity.

The time series velocity data of the bridge were used to build the AR model of the output only single temporal data to extract the ARCs using Burg's algorithm. The algorithm was written in Matlab and computed with different *p-lag* number of ARCs. The ARCs were assembled in the system state matrix, Eq. (14). The eigenvalue decomposition of the state matrix computed the modal parameters in the form of the mode shape vector and diagonal eigenvalue matrix including information of natural frequency. The Fig. 3 shows the identified frequencies of the bridge based on different *p-lag*, i.e. model order, from 40 to 80. Because the output response puts into the algorithm is just a single velocity data, the ARMERA gives one distinct frequency result, which is 7.39 Hz.

Fig. 1 Short reinforced concrete bridge

Fig. 2 Experimental setting up

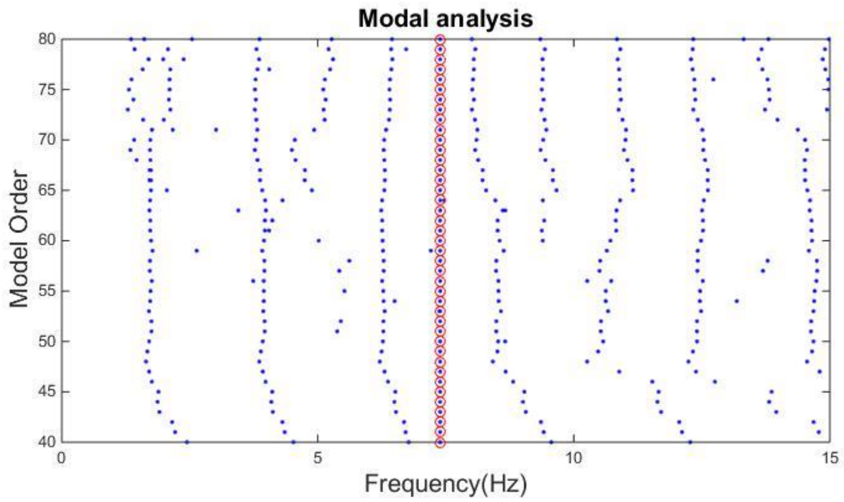

Fig. 3 Natural frequency of the bridge regarding RSV

The Fig. 4 shows the behavior of 5 different ARCs for 60,000 data sets, each data set is 1,000 steps long. The ARCs from ϕ_{11} to ϕ_{55} have different values, they are different parameters but each ARC has its own stationary pattern with its mean and its standard deviation, as shown in Table 1. Although, the flow has sudden alteration section occasionally, it doesn't mean that the bridge has been damaged. The plunge states indicate that the system is subject to some disturbance, such as pedestrian traffic on the bridge.

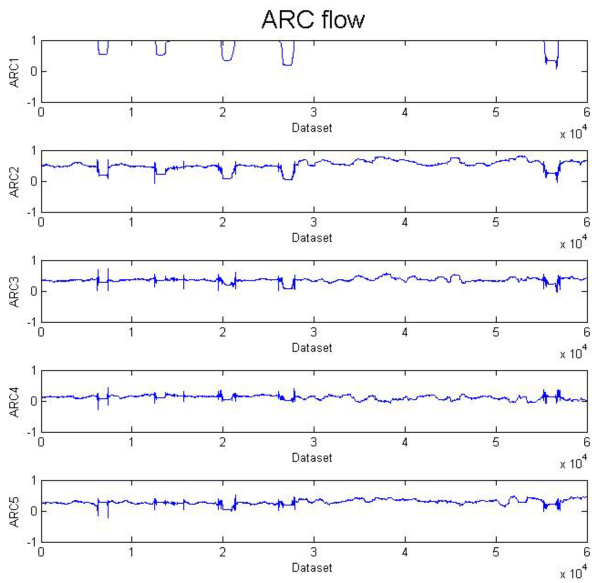

Fig. 4 ARC flow regarding RSV

Table 1 Mean and standard deviation of ARCs

	ARC1	ARC2	ARC3	ARC4	ARC5
Mean	0.9363	0.5194	0.3565	0.1121	0.2899
Standard deviation	0.1848	0.1526	0.0706	0.0660	0.0756

3.1.2 The Wi-Fi sensor system experiment

The second experiment on the reinforced concrete bridge was conducted using 4 sensors node. The purpose of the second experiment was to build ARMERA for vector autoregressive (VAR) model, including not only natural frequencies but Also mode shapes. The sensing system for SHM was derived from the wireless sensing technology of the DuraMote system (Torbol *et al*. 2013). Wireless sensing network was composed by: a base station, four Roocas, the data aggregators, and four gophers, the sensor units. Each sensors unit was equipped with 3 accelerometer to cover all axes. The topography of the network is shown in Fig. 5.

The four vertical acceleration data are used to build four AR model for the vertical mode, and Burg's algorithm gives *p-lag* number of ARCs for each data set. These ARCs compose the state matrix of the VAR model and the eigenvalue decomposition of the matrix computes the modal properties.

The Fig. 6 shows the natural frequencies of the vertical modes of the bridge for the model orders from 53 to 80. When a sufficient number of sensors is used more accurate and complete modal results are obtained. In comparison with the result of the FDD (Brincker *et al.* 2000) shown as Fig. 7, the result of ARMERA methodology is considered to be within the purview of FDD result and it is even more lapidary than the result of FDD.

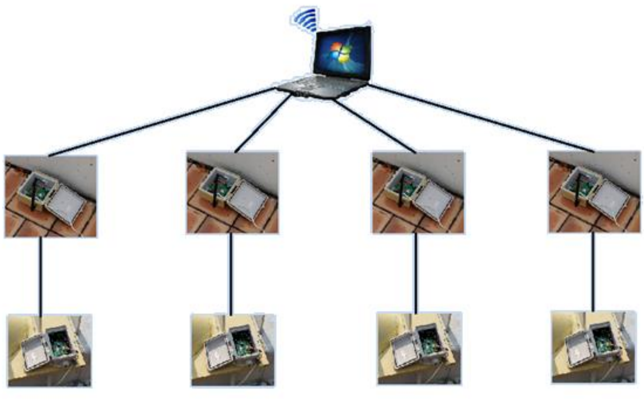

Fig. 5 Sensing network system

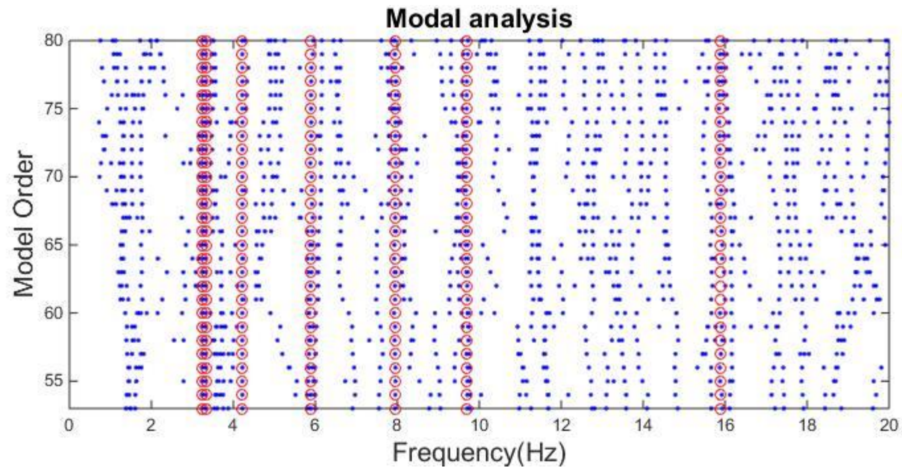

Fig. 6 Natural frequency result from ARMERA

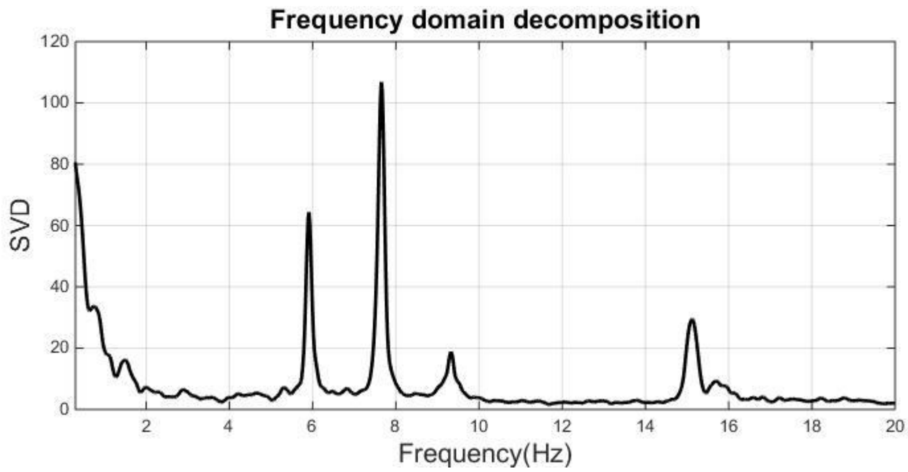

Fig. 7 Natural frequency result from frequency domain decomposition

3.2 Long span reinforced concrete bridge

The purpose of the third experiment is to apply ARMERA to a long span reinforced concrete bridges. The experiment was done on a cable stayed bridge. The length of main span of the bridge is 270 m and 6 sensors node were placed on the deck. Since this case contains a large number of output signals, the VAR is more complex than the previous experiment. The ARMERA result of the third experiment is shown in Fig. 8. When compared with the results of FDD, the frequencies from ARMERA are close to the values from FDD.

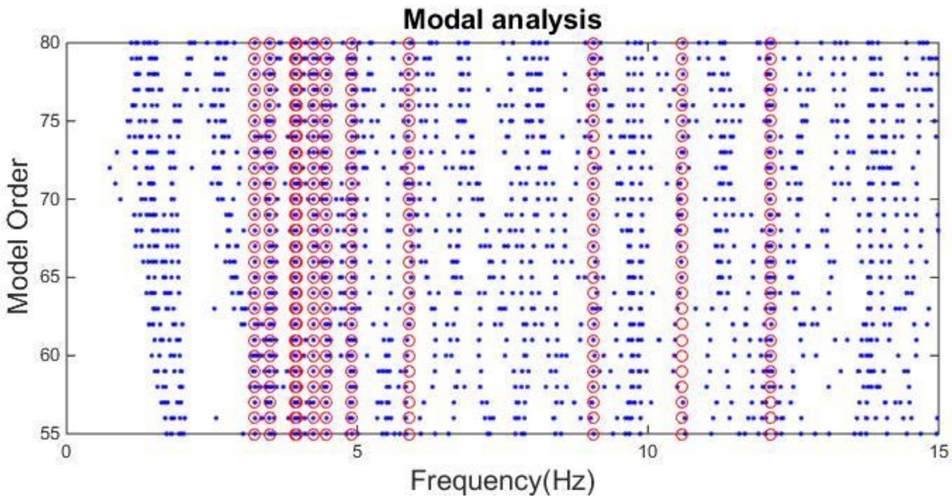

Fig. 8 Natural frequency result from ARMERA

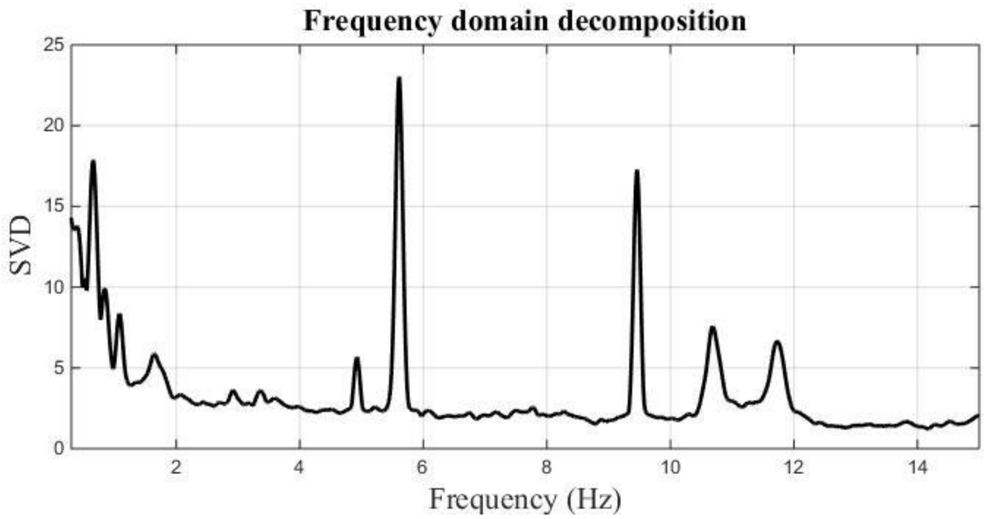

Fig. 9 Natural frequency result from frequency domain decomposition

4. Conclusions

In this study, ARMERA is used for the experimental modal analysis to compute both the modal parameters and the autoregressive coefficients, ARCs. The simple eigenvalue decomposition of a large system matrix composed by ARCs gives accurate modal information. Even when a small number of the data set is used, ARMERA is sensitive enough that it can compute the modal information in the relevant data range. The stationary properties of ARCs computed by Burg's algorithm can be used to detect changes within the structure itself overtime. If a large number of ARCs is used the algorithm is sensitive enough to assess the condition of the system numerically.

An additional algorithm is necessary to convert the numerical estimation in physical parameters that can be understood; in this study artificial neural network is used. Due to this advantage it is possible to use this method to solve several civil engineering problems like damage detection. Though the scope of the study is only the modal analysis through the ARMERA, the damage detection using ARMERA with a model updating algorithm is the future challenge. While the ARM is well established, coding the model updating algorithm and reaching good performance with it will be a challenge.

In the future, because singular value decomposition on a large state matrix takes intensive computational effort and because training a large neural network also takes intensive computational effort, parallel programming on GPGPU using CUDA or OpenCL will be used to address this issues.

Acknowledgments

This research was supported by a grant (13SCIPA01) from Smart Civil Infrastructure Research Program funded by Ministry of Land, Infrastructure and Transport (MOLIT) of Korea government and Korea Agency for Infrastructure Technology Advancement (KAIA).

References

Allemang, R.J. (2003), "The modal assurance criterion - Twenty years of use and abuse", *J. Sound Vib.*, **37**(8):,14-23.

Brincker, R., Zhang, L.M. and Andersen, P. (2000), "Modal identification from ambient responses using frequency domain decomposition", *Imac-Xviii: A Conference on Structural Dynamics, Vols 1 and 2, Proceedings*, **4062**, 625-630.

Brockwell, P. J. and Davis, R.A. (2002), *Introduction to time series and forecasting*, Springer. New York.

Hearn, G. and Testa, R. B. (1991), "Modal-analysis for damage detection in structures", *J. Struct. Eng. - ASCE*, **117**(10), 3042-3063.

Juang, J.N. and Pappa, R.S. (1985), "An eigensystem realization-algorithm for modal parameter-identification and model-reduction", *J. Guid. Control Dynam.*, **8**(5), 620-627.

Lee, H., Grosse, R., Ranganath, R. and Ng, A.Y. (2011), "Unsupervised learning of hierarchical representations with convolutional deep belief networks", *Communications of the Acm*, **54**(10), 95-103.

McKenna, F. (2011), "OpenSees: A framework for earthquake engineering simulation", *Comput. Sci. Eng.*, **13**(4), 58-66.

Mohanty, P. and Rixen, D.J. (2004), "A modified Ibrahim time domain algorithm for operational modal analysis including harmonic excitation", *J. Sound Vib.*, **275**(1-2), 375-390.

Neumaier, A. and Schneider, T. (2001), "Estimation of parameters and eigenmodes of multivariate autoregressive models", *Acm T. Math. Software*, **27**(1), 27-57.

Omenzetter, P., Brownjohn, J.M.W. and Moyo, P. (2003), "Application of time series analysis for bridge health monitoring", *Struct. Health Monit. Intell. Infrast.*, **1-2,** 1073-1080.

Priestley, M. B. (1981), *Spectral analysis and time series*, Academic Press. London ; New York.

Torbol, M., Kim, S. and Shinozuka, M. (2013), "Long term monitoring of a cable stayed bridge using

DuraMote", *Smart Struct. Syst*., **11**(5), 453-476.
Vu, V.H., Thomas, M., Lakis, A.A. and Marcouiller, L. (2011), "Operational modal analysis by updating autoregressive model", *Mech. Syst. Signal Pr*., **25**(3), 1028-1044.
Ye, X.W., Ni, Y.Q., Wai, T.T., Wong, K.Y., Zhang, X.M. and Xu, F. (2013), "A vision-based system for dynamic displacement measurement of long-span bridges: algorithm and verification", *Smart Struct. Syst*., **12**(3-4), 363-379.
Ye, X. W., Ni, Y.Q., Wong, K.Y. and Ko, J.M. (2012), "Statistical analysis of stress spectra for fatigue life assessment of steel bridges with structural health monitoring data", *Eng. Struct*., **45**, 166-176.
Yun, C.B. and Bahng, E.Y. (2000), "Substructural identification using neural networks", *Comput. Struct*., **77**(1), 41-52.
Yun, Y.A.N. (2004), "Design of structure optimization with APDL", *J. East China Jiaotong Univ*., **21**(4), 52-55.