Smart Structures and Systems, *Vol. 17, No. 2 (2016) 327-345*
DOI: http://dx.doi.org/10.12989/sss.2016.17.2.327

SH-wave in a piezomagnetic layer overlying an initially stressed orthotropic half-space

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(Received June 26, 2015, Revised December 21, 2015, Accepted December 25, 2015)

Abstract. The existence of SH-wave in a piezomagnetic layer overlying an initially stressed orthotropic half-space is investigated. The coupled of differential equations are solved for piezomagnetic layer overlying an orthotropic elastic half-space. The general dispersion equation has been derived for both magnetically open circuit and magnetically closed circuits under the four types of boundary conditions. In the absence of the piezomagnetic properties, initial stress and orthotropic properties of the medium, the dispersion equations reduce to classical Love equation. The SH-wave velocity has been calculated numerically for both magnetically open circuit and closed circuits. The effect of initial stress and magnetic permeability are illustrated by graphs in both the cases. The velocity of SH-wave decreases with the increment of wave number.

Keywords: piezomagnetic; orthotropic; initial stress; magnetic permeability;dispersion equations

1. Introduction

Recently, the study of propagation of surface waves in a layered media compose of magneto-elastic materials has been the area of growing attention to numerous applications in surface acoustic wave devices (SAW) such as sensors, delay lines, resonators, transducers, actuators and filters. Composite materials which are made of piezomagnetic and elastic phases show a magneto-elastic effect which is an advanced product property absent in individual constituent. Such studies play an important role in providing notable results to them which exceptionally help them to forecast the propagation order of surface waves. This clearly allows them to deal with the practical situations. Jakoby and Vellekoop (1997) presented a note on Love wave sensor devices. Du *et al.* (1996) have discussed acoustic sensors. SAW devices and surface wave sensors are highly delicate micro-acoustic implements. These sensors are generally consisting of fine layer over a substrate. Among such sensors, piezomagnetic structure is considered to be more reliable as it enables the magnetic excitation of surface waves. Vives (2008) and Wu and Chen (2003) explained the applications of these SAW devices in the field of earthquake engineering. In the point of fact, piezomagnetism is the property of the material in

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http://www.techno-press.org/?journal=sss&subpage=8 ISSN: 1738-1584 (Print), 1738-1991 (Online)

which mechanical stress accumulates in certain classes of crystalline materials by the application of applied magnetic field. These materials are uncommon and the example of such material is hematite Fe_2O_3. It is useful to take piezomagnetic materials because these materials respond to stress which results in rotations of the magnetic moments. Due to the inherent properties of piezomagnetic materials and their wide range applications in SAW devices, information storage, signal processing and many more, the study of SH-wave propagation in piezomagnetic structures is one of the prime areas of thrust in present scenario. Also, these materials show their fruitful applications in the domain of civil engineering, electrical and electronic engineering, mechanical engineering, medical engineering as well as aerospace engineering. Comprehensive knowledge about the propagation of seismic waves in layered structure is available in renowned books (Ewing *et al.* 1957, Love 1944, Achenbach 1976, Gubbins 1990).

The earth is orthotropic i.e. its mechanical properties are, in general, different along each axis. The development of initial stresses in the elastic solid half-space (the Earth) arise due to many reasons, such as gravity variations, the distinction of temperature, process of extinguish shot pinning and frosty working, moderate process of creep and different internal forces. These initial stresses of earth induce great impact on SH-waves during propagation and have great impact on the mechanical riposte of the materials. The concept of initial stresses has important significance in engineering structures, geomechanics and in the research of soft living tissues. It is therefore of great attraction to investigate the influence of initial stresses on the elastic wave propagation. Due to large applications, pre-stressed SH-waves in different media tempt researchers' interests even nowadays. Liu *et al.* (2004) investigated the influence of initial stresses on the Love wave propagation in piezoelectric layered structure. Zakharenko (2005) studied the propagation of Love waves in cubic piezoelectric crystal. Arefi and Rahimi (2012) studied the effect of normal pressure on nonlinear behavior of piezoelectric materials as a sensor. Koutsawa *et al.* (2014) discussed the piezoelectric fiber composites. Marinkovic and Marinkovic (2012) presented a note on piezoelectric actuators and sensors. Qian *et al.* (2004) developed a mathematical model to study the effect of Love wave propagation in a piezoelectric layered structure with initial stresses. Wang and Quek (2001) discussed propagation of Love waves in piezoelectric coupled solid medium. Zaitsev *et al*. (2001) discussed the propagation of acoustic waves in piezoelectric conductive and viscous plates.

SH-waves cause more destruction to the structure than that of the body waves due to its slower attenuation of the energy. Many authors have studied the propagation of SH-wave by considering dissimilar forms of asymmetry at the interface. Watanabe and Payton (2002) discussed SH- waves in a cylindrically monoclinic material with Green's function. Chattopadhyay *et al.* (2010, 2012) used Green's function technique to study propagation of SH-waves and heterogeneity on the SH-waves in viscoelastic half-spaces. Also, Chattopadhyay *et al.* (2014) discussed the influence of heterogeneity and reinforcement on propagation of a crack due to SH-waves. Gupta and Gupta (2013) studied the effect of initial stress on wave motion in an anisotropic fiber reinforced thermoelastic medium. Sahu *et al.* (2014) showed the effect of gravity on shear waves in a heterogeneous fiber-reinforced layer placed over a half-space. Kundu *et al.* (2014) analyzed SH-wave in initially stressed orthotropic homogeneous and a heterogeneous half space.

In this paper, an exact technique is used to investigate SH-waves in a piezomagnetic layer overlying an initially stressed orthotropic half-space. The analytical solutions of SH-wave velocities in the presence of tensile and compressive stress are acquired for different magnetic boundary conditions. The effect of stress parameter and magnetic permeability are shown graphically in both the cases of magnetically open and closed circuit. This work may be useful to the analysis and design of different SAW devices constructed from piezomagnetic materials.

2. Formulation of the problem

Let $'h'$ be the thickness of piezomagnetic elastic layer lying over an orthotropic half space. The interface of these two media is considered to be at $z = 0$, whereas the free surface is at $z = -h$. The z-axis is directed vertically downward and the x-axis is assumed to be in the direction of the propagation of the SH-wave with velocity c. For SH-waves, we have following displacement components in the y-axis only, assumed as

$$u_1 = 0, \quad w_1 = 0, \quad v_1 = v_1(x, z, t) \tag{1}$$

where (u_1, v_1, w_1) are the displacement components at any point $P(x, y, z)$ of the medium.

3. Solution of the problem

3.1 Solution of the piezomagnetic layer

A piezomagnetic layer is perfectly bonded over an inhomogeneous half-space as shown in Fig. 1. We consider that the piezomagnetic structure is polarized in the x-axis direction. The coupled constitutive equations for a piezomagnetic solid can be written as, Feng (2009)

$$\left.\begin{aligned} \sigma_{ij} &= c_{ij,kl} S_{kl} - e_{k,ij} H_k \\ B_i &= e_{j,kl} S_{kl} + \mu_{jk} H_k \end{aligned}\right\} \tag{2}$$

where σ_{ij} and S_{kl} are the strain and stress tensors, H_k is the magnetic potential field, B_i is the magnetic induction respectively, $c_{ij,kl}$, $e_{j,kl}$ and μ_{jk} are the elastic, piezomagnetic and magnetic permeability coefficients respectively.

Since, in the problem we consider that the SH-wave is travelling along the x-direction and the magnetic material properties of piezomagnetic layer change continuously along the z-direction. Therefore the equation of motion and the equation of magnetic displacement equilibrium (Gauss's laws of magnetism without free charges) can be given as

$$\sigma_{ij,j} = \rho \ddot{v}_{i,} \tag{3}$$

$$B_{i,i} = 0 \tag{4}$$

where ρ is the density and v_i is the mechanical displacement component in the i^{th} direction.

The magnetic potential (ψ) are given by

$$\psi = \psi(x, y, t) \tag{5}$$

The strain-displacement relation is given by

$$S_{ij} = \frac{1}{2}(v_{i.j} + v_{j,i}) \tag{6}$$

The relation between the magnetic field and the magnetic potential (Maxwell's equation) can be given as

$$B_i = -\frac{\partial \psi}{\partial x} \tag{7}$$

The Eqs. (1) and (2) can be written in terms of rectangular form for typically isotropic piezomagnetic layer can be expressed as

$$\left.\begin{aligned}
\sigma_x &= c_{11}S_x + c_{12}S_y + c_{13}S_z - e_{31}H_z \\
\sigma_y &= c_{12}S_x + c_{11}S_y + c_{13}S_z - e_{31}H_z \\
\sigma_z &= c_{13}S_x + c_{13}S_y + c_{33}S_z - e_{31}H_z \\
\sigma_{xz} &= \left(c_{11} - c_{12}\right)\frac{S_{xz}}{2} \\
\sigma_{yz} &= c_{44}S_{yz} - e_{15}H_z \\
\sigma_{xy} &= c_{44}S_{xy} + e_{15}H_x \\
B_x &= e_{15}S_{xy} + \mu_{11}H_x \\
B_z &= e_{15}S_{zy} + \mu_{11}H_z \\
B_y &= e_{31}S_x + e_{31}S_z + e_{31}S_y + \mu_{33}H_y
\end{aligned}\right\} \tag{8}$$

Now, put Eq. (5) into Eqs. (6) and (7), again putting the Eqs. (6) and (7), into Eq. (8), using new equations into Eq. (3) and Eq. (4), we get the basic preliminaries for the displacements and the magnetic potential as

$$\left.\begin{aligned}
c_{44}\nabla^2 v_1 + e_{15}\nabla^2 \psi = \rho \ddot{v}_1 \\
e_{15}\nabla^2 v_1 - \mu_{11}\nabla^2 \psi = 0
\end{aligned}\right\} \tag{9}$$

where v_1 and ψ are the mechanical displacement and magnetic potential function in the piezomagnetic layer and c_{44}, e_{15}, μ_{11} and ρ denote the elastic, piezomagnetic and magnetic permeability coefficient and density respectively, ∇^2 is the Laplacian operator in two dimension and t is time.

Eq. (9) can be written as

$$\nabla^2 v_1 - \frac{1}{c_1^2}\ddot{v}_1 = 0 \tag{10}$$

$$\nabla^2 \psi - \frac{1}{c_1^2}\left(\frac{e_{15}}{\mu_{11}}\right)\ddot{v}_1 = 0 \tag{11}$$

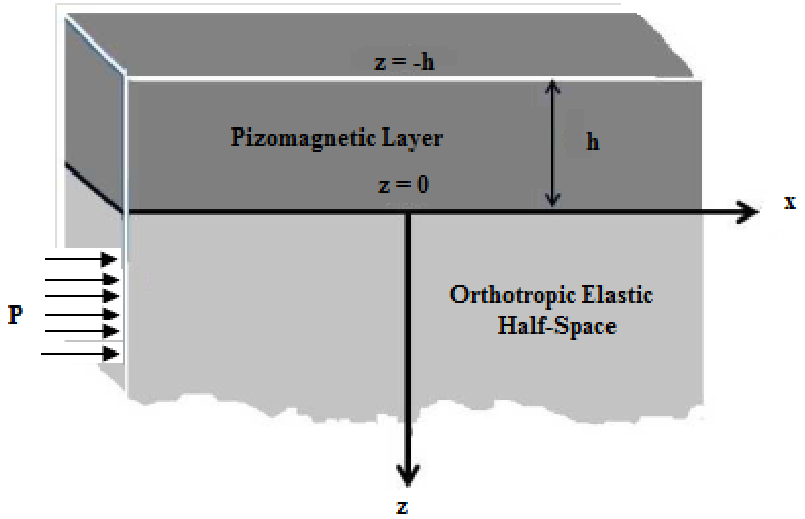

Fig. 1 Geometry of the problem

where $c_1^2 = c'_{44}/\rho$ and $c'_{44} = c_{44} + \dfrac{e_{15}^2}{\mu_{11}}$, c_1 is the shear wave velocity in the isotropic transverse piezomagnetic layer.
We may assume the solution of Eqs. (10) and (11) as

$$\left.\begin{aligned} v_1(x,y,t) &= V_1(z)e^{ik(x-ct)} \\ \psi(x,y,t) &= \psi_1(z)e^{ik(x-ct)} \end{aligned}\right\} \tag{12}$$

where $i=\sqrt{-1}$, k and c are wave number and phase velocity respectively and $V_1(z)$ be the solution of the following equation

$$\frac{d^2V_1(z)}{dz^2} + k^2\delta^2 V_1(z) = 0 \tag{13}$$

where $\delta = \left(\dfrac{c^2}{c_1^2} - 1\right)^{\frac{1}{2}}$.

and $\psi_1(z)$ be the solution of the following equation

$$\frac{d^2\psi_1(z)}{dz^2} - k^2\psi_1(z) + \frac{1}{c_1^2}\left(\frac{e_{15}}{\mu_{11}}\right)\left\{D_1\sin(k\delta z) + D_2\cos(k\delta z)\right\}k^2c^2 = 0 \tag{14}$$

The solutions of the Eqs. (10) and (11) for SH-wave travelling along *x*-axis can be written as

$$v_1(x,y,t) = \left\{D_1 \sin(k\delta z) + D_2 \cos(k\delta z)\right\} e^{ik(x-ct)} \tag{15}$$

$$\psi_1(x,y,t) = \left\{\frac{e_{15}}{\mu_{11}}\left\{D_1 \sin(k\delta z) + D_2 \cos(k\delta z)\right\} + D_3 e^{-kz} + D_4 e^{-kz}\right\} e^{ik(x-ct)} \tag{16}$$

where $D_1,\ D_2,\ D_3$ and D_4 are the arbitrary constants.

The stress component and magnetic induction of piezomagnetic layer are

$$\sigma_{yz} = \left[k\delta D_1 c'_{44}\cos(k\delta z) - k\delta D_2 c'_{44}\sin(k\delta z) + ke_{15}\left\{-D_3 e^{-kz} + D_4 e^{-kz}\right\}\right] e^{ik(x-ct)} \tag{17}$$

$$B_z = k\mu_{11}\left\{D_3 e^{-kz} - D_4 e^{-kz}\right\} e^{ik(x-ct)} \tag{18}$$

3.2 Solution of the half-space

Equation of motion for lower half-space in the presence of initial pressure can be written as Love (1944)

$$\left.\begin{aligned}
&\frac{\partial\tau_{xx}}{\partial x} + \frac{\partial\tau_{xy}}{\partial y} + \frac{\partial\tau_{xz}}{\partial z} - P\left(\frac{\partial\Omega_z}{\partial y} - \frac{\partial\Omega_y}{\partial z}\right) = \rho_2 \frac{\partial^2 u_2}{\partial t^2}\\
&\frac{\partial\tau_{yx}}{\partial x} + \frac{\partial\tau_{yy}}{\partial y} + \frac{\partial\tau_{yz}}{\partial z} - P\left(\frac{\partial\Omega_z}{\partial x}\right) = \rho_2 \frac{\partial^2 v_2}{\partial t^2}\\
&\frac{\partial\tau_{zx}}{\partial x} + \frac{\partial\tau_{zy}}{\partial y} + \frac{\partial\tau_{zz}}{\partial z} - P\left(\frac{\partial\Omega_y}{\partial x}\right) = \rho_2 \frac{\partial^2 w_2}{\partial t^2}
\end{aligned}\right\} \tag{19}$$

where $\tau_{xx}, \tau_{xy}, \tau_{xz}, \tau_{yx}, \tau_{yy}, \tau_{yz}, \tau_{zx}, \tau_{zy}$ and τ_{zz} are the incremental stress components u_2, v_2 and w_2 are the components of the displacement vector in the lower half-space, ρ_2 is the density of the lower half-space. Here, Ω_x, Ω_y and Ω_z are the rotational components in the lower half-space and P is the initial pressure in the lower half-space.
where

$$\Omega_x = \frac{1}{2}\left(\frac{\partial w_2}{\partial y} - \frac{\partial v_2}{\partial z}\right), \Omega_y = \frac{1}{2}\left(\frac{\partial u_2}{\partial z} - \frac{\partial w_2}{\partial x}\right), \Omega_z = \frac{1}{2}\left(\frac{\partial v_2}{\partial x} - \frac{\partial u_2}{\partial y}\right) \tag{20}$$

The stress–strain relations are

$$\left.\begin{aligned}
\tau_{xx} &= \mathrm{N}_{xx}e_{xx} + \mathrm{N}_{xy}e_{yy} + \mathrm{N}_{xz}e_{zz}\\
\tau_{xy} &= 2\mathrm{E}_z e_{xy}\\
\tau_{yy} &= \mathrm{N}_{yx}e_{xx} + \mathrm{N}_{yy}e_{yy} + \mathrm{N}_{yz}e_{zz}\\
\tau_{yz} &= 2\mathrm{E}_x e_{yz}\\
\tau_{zz} &= \mathrm{N}_{zx}e_{xx} + \mathrm{N}_{zy}e_{yy} + \mathrm{N}_{zz}e_{zz}\\
\tau_{zx} &= 2\mathrm{E}_y e_{zx}
\end{aligned}\right\} \tag{21}$$

where $\mathrm{N}_{xx}, \mathrm{N}_{xy}, \mathrm{N}_{xz}, \mathrm{N}_{yx}, \mathrm{N}_{yy}, \mathrm{N}_{yz}, \mathrm{N}_{zx}, \mathrm{N}_{zy}$ and N_{zz} are the incremental normal elastic coefficients, $\mathrm{E}_x, \mathrm{E}_y$ and E_z shear modulus along x, y and z axis respectively. The strain components $e_{xy}, e_{xx}, e_{yy}, e_{yz}, e_{zx}$ and e_{zz} are defined by

$$\left.\begin{aligned} & e_{xy} = \frac{1}{2}\left(\frac{\partial v_1}{\partial x} + \frac{\partial u_1}{\partial y}\right), e_{yz} = \frac{1}{2}\left(\frac{\partial w_1}{\partial y} + \frac{\partial v_1}{\partial z}\right), \\ & e_{zx} = \frac{1}{2}\left(\frac{\partial v_1}{\partial z} + \frac{\partial w_1}{\partial x}\right), \\ & e_{xx} = \left(\frac{\partial u_1}{\partial x}\right), e_{yy} = \left(\frac{\partial v_1}{\partial y}\right), e_{zz} = \left(\frac{\partial w_1}{\partial z}\right) \end{aligned}\right\} \tag{22}$$

Using Love wave conditions $u_2 = w_2 = 0, v_2 = v_2(x,z,t)$ in Eqs. (19) and (21), the equation of motion for the lower orthotropic half-space becomes

$$\left(\mathrm{E}_z - \frac{P}{2}\right)\frac{\partial^2 v_2}{\partial x^2} + \mathrm{E}_x \frac{\partial^2 v_2}{\partial z^2} = \rho_2 \frac{\partial^2 v_2}{\partial t^2} \tag{23}$$

and stress–strain relations reduces to

$$\left.\begin{aligned} & \tau_{xx} = \tau_{xy} = \tau_{xz} = \tau_{yy} = \tau_{zx} = \tau_{zy} = \tau_{zz} = 0 \\ & \tau_{yx} = 2\mathrm{E}_z e_{xy},\ \ \tau_{yz} = 2\mathrm{E}_x e_{yz} \end{aligned}\right\} \tag{24}$$

To solve Eq. (23) we take the following substitution

$$v_2 = \mathrm{U(z)} e^{ik(x-ct)} \tag{25}$$

Using Eq. (25) in Eq. (23), we get

$$\frac{\mathrm{d}^2\mathrm{U}(z)}{\mathrm{dz}^2} - \xi^2 \mathrm{U}(z) = 0 \tag{26}$$

where

$$\xi^2 = \frac{k^2}{\mathrm{E}_x}\left[\left(\mathrm{E}_z - \frac{P}{2}\right) - c^2\rho_2\right] \tag{27}$$

Therefore, the solution for the upper orthotropic half-space is given by

$$v_2 = D_5\, \mathrm{e}^{\xi z}\, e^{ik(x-ct)} \tag{28}$$

where D_5 is an arbitrary constant.

4. Boundary conditions

The propagation of SH-waves under this assumed model has the following conditions at $z = -h$ and $z = 0$.

(i) Since the upper surface of the piezomagnetic layer is stress free, therefore

$$\sigma_{yz} = 0 \text{ at } z = -h \tag{29}$$

(ii) At $z = 0$, the displacement and normal stress component are continuous and magnetic potential function is displacement free, so

$$v_1 = v_2, \quad \sigma_{yz} = \tau_{yz}, \quad \psi = 0 \quad \text{at} \quad z = 0 \tag{30}$$

(iii) Continuity condition

$$v_2 \to 0 \quad \text{as} \quad x \to +\infty \tag{31}$$

(iv) The magnetic conditions at $z = 0$ can be generalized into two ways, i.e.

(a) Magnetic closed circuit: $B_z = 0$ at $z = -h$ (32)

(b) Magnetic open circuit: $\psi = 0$ at $z = -h$ (33)

5 Dispersion relations

Using the boundary conditions (29-33), from Eqs. (15), (16), (17), (18) and (28) we get the following relations

$$D_1 \delta c'_{44} \cos(\delta kh) + D_2 \delta c'_{44} \sin(\delta kh) - \mathrm{e}_{15} D_3 e^{kh} + \mathrm{e}_{15} D_4 e^{-kh} = 0 \tag{34}$$

$$D_2 - D_5 = 0 \tag{35}$$

$$\delta c'_{44} D_1 - e_{15} D_3 + e_{15} D_4 + \mathrm{E}_x \xi D_5 = 0 \tag{36}$$

$$\left(\frac{e_{15}}{\mu_{11}}\right) D_2 + D_3 + D_4 = 0 \tag{37}$$

$$D_3 e^{kh} - D_4 e^{-kh} = 0 \tag{38}$$

$$\left(\frac{e_{15}}{\mu_{11}}\right) \{D_1 \sin(\delta kh) + D_2 \cos(\delta kh)\} + D_3 e^{kh} + D_4 e^{-kh} = 0 \tag{39}$$

5.1 Dispersion relation for case of magnetically closed circuit

Eliminating $D_i \ (i = 1, 2....5)$ from the Eqs. (34) to (38), the dispersion relation for SH waves can be obtained as

$$\begin{vmatrix} \delta c'_{44}\cos(\delta kh) & \delta c'_{44}\sin(\delta kh) & -\mathrm{e}_{15}\, e^{kh} & \mathrm{e}_{15}\, e^{-kh} & 0 \\ 0 & 1 & 0 & 0 & -1 \\ \delta c'_{44} & 0 & -\mathrm{e}_{15} & \mathrm{e}_{15} & \mathrm{E}_x\xi \\ 0 & \dfrac{e_{15}}{\mu_{11}} & 1 & 1 & 0 \\ 0 & 0 & e^{kh} & e^{-kh} & 0 \end{vmatrix} = 0 \tag{40}$$

Expanding Eq. (40), we get

$$\mathrm{E}_x\xi\cos(\delta kh)(e^{kh}+e^{-kh}) + e^{kh}\begin{bmatrix} \dfrac{e_{15}^2}{\mu_{11}}e^{-kh} - \dfrac{e_{15}^2}{\mu_{11}}\cos(\delta kh) \\ -\delta c'_{44}\sin(\delta kh) \end{bmatrix} - e^{-kh}\begin{bmatrix} \dfrac{e_{15}^2}{\mu_{11}}e^{kh} + \dfrac{e_{15}^2}{\mu_{11}}\cos(\delta kh) \\ +\delta c'_{44}\sin(\delta kh) \end{bmatrix} = 0 \tag{41}$$

which takes the form as

$$\delta c'_{44}\tan(\delta kh) - \mathrm{E}_x\xi + \frac{e_{15}^2}{\mu_{11}} = 0 \tag{42}$$

This is the dispersion equation of SH-wave in a magnetically closed circuit piezomagnetic layer over an initially stressed orthotropic half-space

5.2 Dispersion relation for case of magnetically open circuit

Eliminating $D_i \ (i = 1, 2....5)$ from the Eqs. (34) to (37) and (39), the dispersion relation for SH waves can be obtained as

$$\begin{vmatrix} \delta c'_{44}\cos(\delta kh) & \delta c'_{44}\sin(\delta kh) & -\mathrm{e}_{15}\, e^{kh} & \mathrm{e}_{15}\, e^{-kh} & 0 \\ 0 & 1 & 0 & 0 & -1 \\ \delta c'_{44} & 0 & -\mathrm{e}_{15} & \mathrm{e}_{15} & \mathrm{E}_x\xi \\ 0 & \dfrac{e_{15}}{\mu_{11}} & 1 & 1 & 0 \\ \dfrac{e_{15}}{\mu_{11}}\sin(\delta kh) & \dfrac{e_{15}}{\mu_{11}}\cos(\delta kh) & e^{kh} & e^{-kh} & 0 \end{vmatrix} = 0 \tag{43}$$

Expanding Eq. (43), we get

$$\delta c'_{44}\,\mathrm{E}_x\,\xi\cos(\delta kh)(e^{-kh}-e^{kh})+\mathrm{E}_x\,\xi\frac{e_{15}^2}{\mu_{11}}\sin(\delta kh)(e^{-kh}-e^{kh})-\delta^2 c_{44}'^2\sin(\delta kh)(e^{-kh}-e^{kh})$$
$$+\delta c'_{44}\frac{e_{15}^2}{\mu_{11}}e^{kh}(\cos(\delta kh)-e^{-kh})-\delta c'_{44}\frac{e_{15}^2}{\mu_{11}}e^{-kh}(e^{-kh}-\cos(\delta kh))$$
$$+\delta c'_{44}\frac{e_{15}^2}{\mu_{11}}\cos(\delta kh)(e^{-kh}-\cos(\delta kh))-2\delta c'_{44}\frac{e_{15}^2}{\mu_{11}}\sin^2(\delta kh)+\left(\frac{e_{15}^2}{\mu_{11}}\right)^2\sin(\delta kh)(e^{-kh}-e^{kh})$$
$$+\delta c'_{44}\frac{e_{15}^2}{\mu_{11}}\cos(\delta kh)(e^{kh}-\cos(\delta kh))=0 \tag{44}$$

On further expanding Eq. (44), we get

$$-\delta c'_{44}\,\mathrm{E}_x\,\xi\tan(\delta kh)+\mathrm{E}_x\,\xi\frac{e_{15}^2}{\mu_{11}}\tan(\delta kh)+\delta^2 c_{44}'^2\tan(\delta kh)\times\tanh(kh)+2\delta c'_{44}\frac{e_{15}^2}{\mu_{11}}$$
$$-4\delta c'_{44}\frac{e_{15}^2}{\mu_{11}}\frac{1}{\cos(\delta kh)(\mathrm{e}^{kh}+\mathrm{e}^{-kh})}+\left(\frac{e_{15}^2}{\mu_{11}}\right)^2\tan(\delta kh)\tanh(kh)=0$$

which takes the form as

$$\left[\delta^2 c_{44}'^2+\left(\frac{e_{15}^2}{\mu_{11}}\right)^2\right]\tan(\delta kh)\tanh(kh)+\mathrm{E}_x\,\xi\left[\frac{e_{15}^2}{\mu_{11}}\tan(\delta kh)-\delta c'_{44}\tanh(kh)\right]$$
$$+2\delta c'_{44}\frac{e_{15}^2}{\mu_{11}}\left(1-\frac{2}{\cos(\delta kh)\cosh(kh)}\right)=0 \tag{45}$$

This is the dispersion equation of SH-wave in a magnetically open circuit piezomagnetic layer over an initially stressed orthotropic half-space.

6 Particular cases

1. If we neglect the initial pressure in the half-space that is $P=0$ then the Eqs. (42) and (45) reduces to the equations, in both cases as

$$\tan\left(\sqrt{\left(\frac{c^2}{c_1^2}-1\right)}kh\right)=\frac{E_x\sqrt{\left(1-\frac{c^2}{c_1^2}\right)}-\frac{e_{15}^2}{\mu_{11}}}{c'_{44}\sqrt{\left(\frac{c^2}{c_2^2}-1\right)}} \tag{46}$$

and

$$\left[\delta^2 c_{44}'^2+\left(\frac{e_{15}^2}{\mu_{11}}\right)^2\right]\tan(\delta kh)\tanh(kh)\ +\mathrm{E}_x\,\varsigma\left[\frac{e_{15}^2}{\mu_{11}}\tan(\delta kh)-\delta c_{44}'\tanh(kh)\right]$$

$$+2\delta c_{44}'\frac{e_{15}^2}{\mu_{11}}\left(1-\frac{2}{\cos(\delta kh)\cosh(kh)}\right)=0 \tag{47}$$

where $\delta=\sqrt{\frac{c^2}{c_1^2}-1}$ and $\varsigma=\sqrt{\frac{k^2}{\mathrm{E}_x}\left[\mathrm{E}_z-c^2\rho_2\right]}$

Eqs. (46) and (47) are the dispersion equations of SH-wave in both magnetically closed and open circuit piezomagnetic medium respectively over orthotropic half-space.

2. If $\mathrm{E}_x\to\mathrm{E}_z\to\mu_2, P\to 0$ then the Eqs. (42) and (45) reduces to the equations, in both cases as

$$\tan\left(\sqrt{\left(\frac{c^2}{c_1^2}-1\right)}kh\right)=\frac{\mu_2\sqrt{\left(1-\frac{c^2}{c_1^2}\right)}-\frac{e_{15}^2}{\mu_{11}}}{c_{44}'\sqrt{\left(\frac{c^2}{c_2^2}-1\right)}} \tag{48}$$

and

$$\left[\delta^2 c_{44}'^2+\left(\frac{e_{15}^2}{\mu_{11}}\right)^2\right]\tan(\delta kh)\tanh(kh)+\mathrm{E}_x\,\varsigma\left[\frac{e_{15}^2}{\mu_{11}}\tan(\delta kh)-\delta c_{44}'\tanh(kh)\right]$$

$$+2\delta c_{44}'\frac{e_{15}^2}{\mu_{11}}\left(1-\frac{2}{\cos(\delta kh)\cosh(kh)}\right)=0 \tag{49}$$

where $\delta=\sqrt{\frac{c^2}{c_1^2}-1}$, $c_2=\sqrt{\frac{\mu_2}{\rho_2}}$ and $\varsigma=\sqrt{1-\frac{c^2}{c_2^2}}$

Eqs. (48) and (49) are the dispersion equations of SH-wave in both magnetically closed and open circuit piezomagnetic medium respectively over an isotropic elastic half-space.

3. Validation

If $\mathrm{E}_x\to\mathrm{E}_z\to\mu_2, P\to 0$ and $e_{15}=0$ then the dispersion Eqs. (46) and (47) yields as

$$\tan\left(\sqrt{\left(\frac{c^2}{c_1^2}-1\right)}kh\right)=\frac{\mu_2\sqrt{\left(1-\frac{c^2}{c_1^2}\right)}}{c'_{44}\sqrt{\left(\frac{c^2}{c_2^2}-1\right)}} \tag{50}$$

and

$$\tan\left(\sqrt{\left(\frac{c^2}{c_1^2}-1\right)}kh\right)=\frac{\mu_2\sqrt{\left(1-\frac{c^2}{c_1^2}\right)}}{c'_{44}\sqrt{\left(\frac{c^2}{c_2^2}-1\right)}} \tag{51}$$

Hence the dispersion equations of SH-wave in both magnetically closed and open circuit piezomagnetic medium respectively over orthotropic half-space reduces to the classical Love wave equation.

7. Numerical analysis and discussion

In order to show the effects of dimensionless stress parameter $\zeta = P/2\mathrm{E}_x$ and magnetic permeability on the propagation of SH-waves in piezomagnetic layer overlying an orthotropic half-space, numerical computation is performed for the Eqs. (42) and (45). Numerical results are presented and discussed for the propagation of SH-waves in piezomagnetic layer overlying an orthotropic half-space, which is composed of the magnetostrictive material $CoFe_2O_4$ and elastic material, the material properties are listed in Table 1 [22] and [7]. The results are presented in Figs. 2-5 for piezomagnetic layer in case of magnetically closed circuit and Figs. 6-9 in case of magnetically open circuit. Fig. 2 depicts the effect of tensile stress on dispersion curve of SH-wave against non-dimensional wave number in a piezomagnetic structure in the case of magnetically closed circuit. The curves are plotted for selected values of tensile stress $(\zeta = P/2\mathrm{E}_x > 0)$ and fixed values of magnetic permeability $\mu_{11} = 0.05\times10^{-4}\,Ns^2/C^2$. The values of tensile stress parameter $\zeta = 0.1, 0.3, 0.5, 0.7, 0.9$ are taken for dispersion curves. It is clear from this figure, with the increase of tensile stress; the speed of SH-waves decreases. Fig. 3 presents the effect of compressive stress on dispersion curve of SH-wave against non-dimensional wave number in a piezomagnetic structure in the case of magnetically closed circuit. The curves are plotted for selected values of compressive stress $(\zeta = P/2\mathrm{E}_x < 0)$ and fixed values of magnetic permeability $\mu_{11} = 0.05\times10^{-4}\,Ns^2/C^2$. The values of tensile stress parameter $\zeta = -0.1, -0.3, -0.5, -0.7, -0.9$ are taken for dispersion curves. It is clear from this figure, with the increase of compressive stress; the speed of SH-waves increases.

Table 1 Material parameters used for computation

Upper layer $CoFe_2O_4$	
c_{44}	$43\times10^{9}\ N/m^2$
μ_{11}	$0.05\times10^{-4}\, Ns^2/C^2$
e_{15}	$11.6\times10^{-9}\, C/Vm$
ρ_1	$5800\ kg/m^3$
Orthotropic Half-space	
E_x	$5.65\times10^{10}\, N/m^2$
E_z	$2.46\times10^{10}\, N/m^2$
ρ_2	$7800\ kg/m^3$

Table 2 Parameters for figures (μ_{11} in $10^{-4}\, Ns^2/C^2$)

Figure	ζ	c_1^2/c_2^2	μ_{11}
2	—	0.32	0.05
3	—	0.32	0.05
4	0.5	0.32	—
5	0	0.32	—
6	—	0.32	0.05
7	—	0.32	0.05
8	0.5	0.32	—
9	0	0.32	—

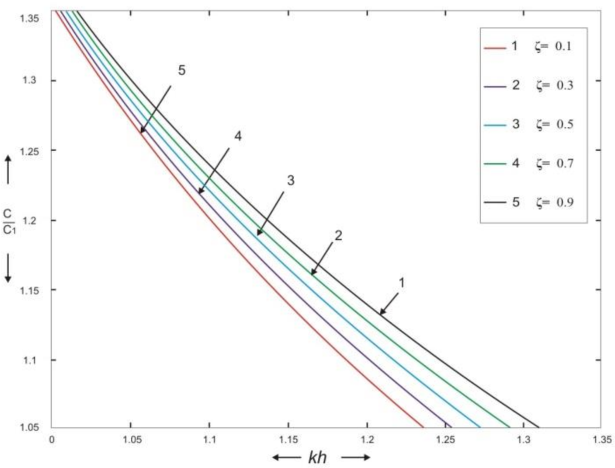

Fig. 2 Variation of phase velocity against wave number under tensile stress for magnetically closed circuit

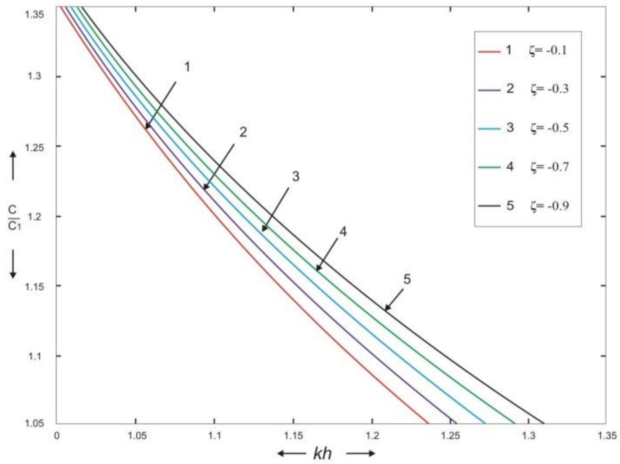

Fig. 3 Variation of phase velocity against wave number under compressional stress for magnetically closed circuit

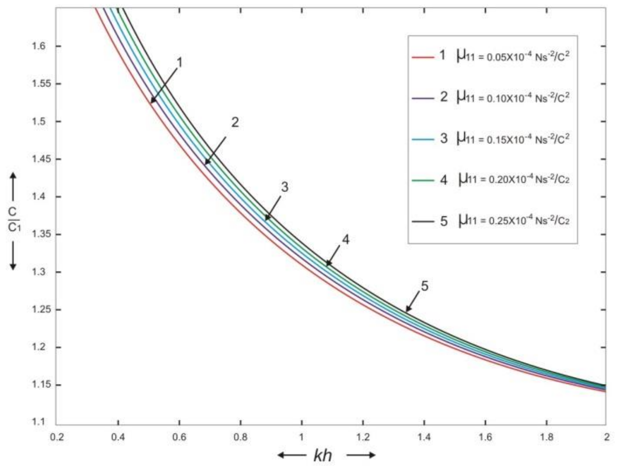

Fig. 4 Variation of phase velocity against wave number under magnetic permeability for magnetically closed circuit

Fig. 4 shows the effect magnetic permeability μ_{11} in case of magnetically open circuit under initial stress. This figure described that the phase velocity of SH-waves increases with the increase of magnetic permeability. Fig. 5 describes the effect magnetic permeability μ_{11} in case of magnetically open circuit under stress free half-space. This figure described that the phase velocity of SH-waves increases with the increase of magnetic permeability. Fig. 6 depicts the effect of tensile stress on dispersion curve of SH-wave against non-dimensional wave number in a piezomagnetic structure in the case of magnetically open circuit. The curves are plotted for selected values of tensile stress and fixed values of magnetic permeability $\mu_{11} = 0.05 \times 10^{-4} Ns^2 / C^2$. The values of tensile stress parameter $\zeta = 0.1, 0.3, 0.5, 0.7, 0.9$ are taken for dispersion curves. It is clear from this figure, with the increase of tensile stress; the speed of SH-waves decreases.

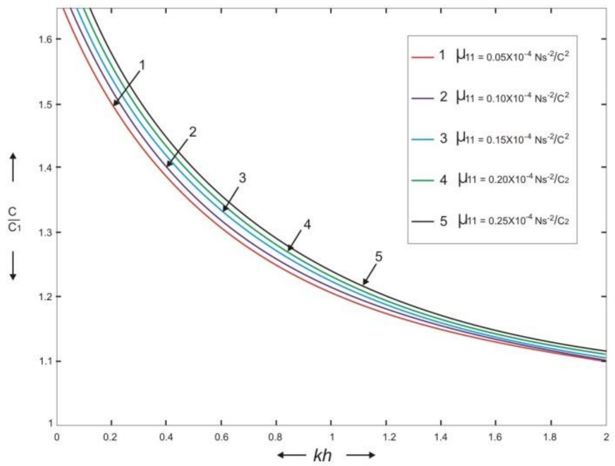

Fig. 5 Variation of phase velocity against wave number under magnetic permeability for magnetically closed circuit (stress free)

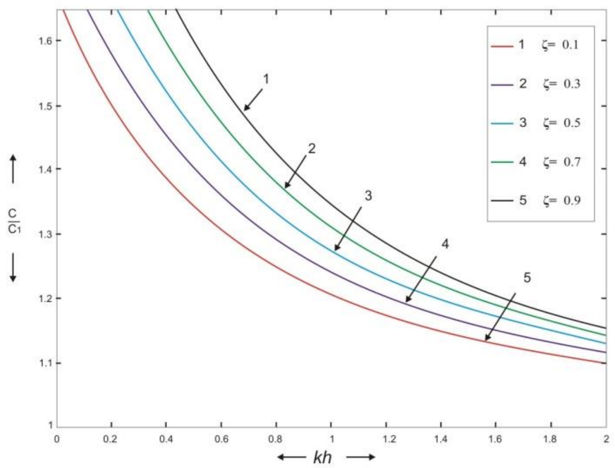

Fig. 6 Variation of phase velocity against wave number under tensile stress for magnetically open circuit

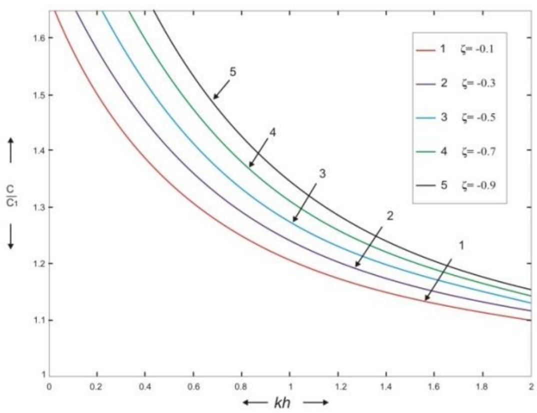

Fig. 7 Variation of phase velocity against wave number under compressional stress for magnetically open circuit

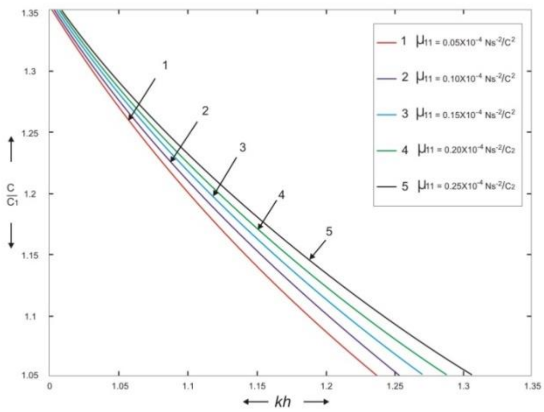

Fig. 8 Variation of phase velocity against wave number under magnetic permeability for magnetically open circuit

Fig. 7 presents the effect of compressive stress on dispersion curve of SH-wave against non-dimensional wave number in a piezomagnetic structure in the case of magnetically open circuit. The curves are plotted for selected values of compressive stress $(\zeta = P/2\mathrm{E}_x < 0)$ and fixed values of magnetic permeability $\mu_{11} = 0.05 \times 10^{-4} Ns^2/C^2$. The values of tensile stress parameter $\zeta = -0.1, -0.3, -0.5, -0.7, -0.9$ are taken for dispersion curves. It is clear from this figure, with the increase of compressive stress; the speed of SH-waves increases. Fig. 8 shows the effect magnetic permeability μ_{11} in case of magnetically closed circuit under initial stress. This figure described that the phase velocity of SH-waves increases with the increase of magnetic permeability. Fig. 9 describes the effect magnetic permeability μ_{11} in case of magnetically closed circuit under stress free half-space. This figure described that the phase velocity of SH-waves increases with the increase of magnetic permeability.

8 Conclusions

In this work, SH-wave in a piezomagnetic layer overlying an initially stressed orthotropic elastic half-space has been investigated analytically and numerically in the cases of magnetically open circuit and closed circuit separately. It has been observed that on the removal of initial stress and magnetic permeability of the layer, the derived dispersion equation reduces to Love wave dispersion equation thereby validates the solution of our problem The effect of stress parameter and magnetic permeability are shown graphically in both the cases of magnetically open and closed circuit. Finally, on the basis of result developed, the following conclusions regarding the propagation of the SH-wave in a piezomagnetic layer placed over orthotropic elastic prestressed half-space can be drawn:

(1) The phase velocities of SH-waves are remarkably influenced by stress parameter and magnetic permeability.

(2) In the case of magnetically open and closed circuit, we have observed that the phase velocity of SH-waves decreases with the increases of tensile stress parameter.

(3) The phase velocity of SH-waves increases with the increases of compressive stress parameter for both magnetically open and closed circuit.

(4) The phase velocity increases with the increase of magnetic permeability for both stress free and stressed half-space.

The presented results may be useful to understand the nature of SH-wave in a piezomagnetic and orthotropic elastic medium.

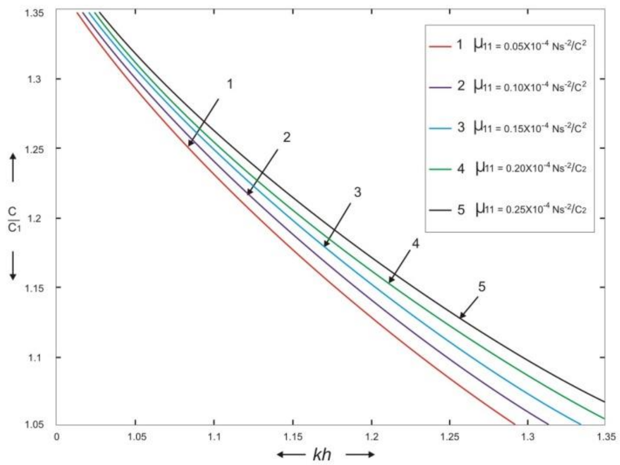

Fig. 9 Variation of phase velocity against wave number under magnetic permeability for magnetically open circuit (stress free)

Acknowledgments

The authors are thankful to unknown reviewers for their valuable comments. This research has not received any specific grant from funding agency in the public, commercial, or not-for-profit sectors.

Conflict of interest

None declared.

References

Achenbach, J.D. (1976), Wave propagation in elastic solid, New York, North Holland.

Arefi, M. and Rahimi, G.H. (2012), "Studying the nonlinear behavior of the functionally graded annular plates with piezoelectric layers as a sensor and actuator under normal pressure", *Smart Struct. Syst*., **9**(2), 127-143..

Chattopadhyay, A., Gupta, S., Sharma, V.K. and Kumari, P. (2010), "Effect of point source and heterogeneity on the propagation of SH-waves", *Int. J. Appl. Math. Mech*., **6**(9), 76-89.

Chattopadhyay, A., Gupta, S., Kumari, P. and Sharma, V.K. (2012), "Effect of point source and heterogeneity on the propagation of SH-waves in a viscoelastic layer over a viscoelastic half-space", *Acta Geophysica*, **60**(1), 119-139.

Chattopadhyay, A., Singh, A.K. and Dhua, S. (2014), "Effect of heterogeneity and reinforcement on propagation of a crack due to shear waves", *Int. J. Geomech*., 10.1061/(ASCE)GM.1943-5622.0000356, 04014013.

Du, J., Harding, G.L., Ogilvy, J.A., Dencher, P.R. and Lake, M. (1996), "A study of Love- wave acoustic sensors", *Sensor. Actuat. A - Phys*, **56**, 211-219.

Ewing, W.M., Jardetzky, W.S. and Press, F. (1957), *Elastic waves in layered media*, New York, McGraw-Hill.

Feng, W.J., Pan, E., Wang, X. and Jin, J. (2009), "Rayleigh waves in magnetoelectro-elastic half planes", *Acta. Mech*., **202**, 127-134.

Gubbins, D. (1990), *Seismology and plate tectonics*, Cambridge, Cambridge University Press.

Gupta, R.R. and Gupta, R.R. (2013), "Analysis of wave motion in an anisotropic initially stressed fiber reinforced thermoelastic medium", *Earthq. Struct*., **4**(1), 1-10.

Jakoby, B. and Vellekoop, M.J. (1997), "Properties of Love waves: applications in sensors", *Smart Mater. Struct*., **6**, 668-679.

Koutsawa, Y., Tiem, S., Giunta, G. and Belouettar, S. (2014), "Effective electromechanical coupling coefficient of adaptive structures with integrated multi-functional piezoelectric structural fiber composites", *Smart Struct. Syst*., **13**(4), 501-515.

Kundu, S., Manna, S. and Gupta, S. (2014), "Propagation of SH-wave in an initially stressed orthotropic medium sandwiched by a homogeneous and a heterogeneous semi-infinite media", *Math. Meth. Appl. Sci*., DOI: 10.1002/mma.3203

Li, X.Y., Wang, Z.K. and Huang, S.H., (2004), "Love waves in functionally graded piezoelectric materials", *Int. J. Solids Struct*., **41**, 7309-7328.

Love, A E.H. (1944), *A treatise on mathematical theory of elasticity*, New York, Dover Publications.

Marinkovic, D. and Marinkovic, Z. (2012), "On FEM modeling of piezoelectric actuators and sensors for thin-walled structures", *Smart Struct. Syst*., **9**(5), 411-426.

Qian, Z., Jin, F. and Wang, Z. (2004), "Love waves propagation in a piezoelectric layered structure with

initial stresses", *Acta Mech.*, 171, 41-57.
Sahu, S.A., Saroj, P.K. and Paswan, B. (2014), "Shear waves in a heterogeneous fiber-reinforced layer over a half-space under gravity", *Int. J. Geomech*., **10**.1061/(ASCE)GM.1943-5622.0000404.
Wang, Q. and Quek, S.T. (2001), "Love waves in piezoelectric coupled solid media", *Smart Mater Struct*., **10**, 380-388.
Watanabe, K. and Payton, R.G. (2002), "Green's function for SH waves in a cylindrically monoclinic material", *J. Mech. Phys. Solids*, **50**, 2425-2439.
Wu, T.T. and Chen, Y.Y. (2003), "Surface acoustic waves in layered piezoelectric media and its applications to the analyses of SAW devices", *Chinese J. Mech. Eng. – Series A*, **19**, 207-214.
Vives, A.A. (2008), Piezoelectric transducer and applications. Berlin, Springer.
Zakharenko, A. (2005), "A Love-type waves in layered systems consisting of two cubic piezoelectric crystals", *J Sound Vib*., **285**, 877-886.
Zaitsev, B.D., Kuznetsova, I.E., Joshi, S.G. and Borodina, I.A. (2001), "Acoustic waves in piezoelectric plates bordered with viscous and conductive liquid", *Ultrasonics*, **39**(1), 45-50.

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