

A two-stage damage detection method for truss structures using a modal residual vector based indicator and differential evolution algorithm

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Abstract. A two-stage method for damage detection in truss systems is proposed. In the first stage, a modal residual vector based indicator (MRVBI) is introduced to locate the potentially damaged elements and reduce the damage variables of a truss structure. Then, in the second stage, a differential evolution (DE) based optimization method is implemented to find the actual site and extent of damage in the structure. In order to assess the efficiency of the proposed damage detection method, two numerical examples including a 2D-truss and 3D-truss are considered. Simulation results reveal the high performance of the method for accurately identifying the damage location and severity of trusses with considering the measurement noise.

Keywords: damage detection; truss structure; modal residual vector; damage indicator; differential evolution

1. Introduction

Damage in structural components may lead to catastrophic failure of a structure. Therefore, developing effective methods for damage identification has become a topic of great importance in the structural engineering. Using these methods the local damage of the structure can be detected and after rehabilitating the damage, the functional age of the structure will increase. Mathematically, the problem of structural damage identification is a highly non-linear problem and special methods need to be employed to properly solve it. In recent years, many methods have been introduced to detect the site and severity of damage in the structural systems (Messina and Williams 1998, Wang *et al.* 2001, Bakhtiari-Nejad *et al.* 2005, Koh and Dyke 2007, Begambre and Laier 2009). One type of the methods employs the optimization algorithms for detecting the multiple structural damages. Although, the use of optimization algorithms can provide a robust tool for damage detection, however, they impose much computational effort to the process due to a great number of damage variables. In order to decline the computational effort related to the optimization process, some worthwhile techniques can be considered. A valuable method is reducing the dimension of optimization problem by excluding the healthy elements of the structure at the first step and then applying the optimization algorithm to the reduced problem for accurately

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determining the location and severity of damaged elements.

During the last years, the use of two-stage damage detection methods has significantly attracted the attention of many researchers. A two-level approach incorporating the genetic algorithm for damage localization and the standard eigensensitivity method for damage quantification was proposed by Friswell *et al.* (1998). Au *et al.* (2003) described a two-stage procedure for detecting structural damage based on the elemental energy quotient difference for damage localization and a micro-genetic algorithm for damage quantification using incomplete and noisy modal test data. Some other researchers used the term hybrid technique for a two-stage method. As, He and Hwang (2007) proposed a hybrid technique consisting of two strategies based on a grey relation analysis for excluding the impossible damage locations as the first step and a real-parameter genetic algorithm combined with simulated annealing and adaptive mechanisms for finding the actual damage as the second step. Guo and Li (2009) presented a two-stage method of determining the location and extent of multiple structural damages by using information fusion technique and genetic algorithm. A damage detection methodology consisting of two main stages, including an adaptive neuro-fuzzy inference system (ANFIS) to determine potentially damage elements in the first stage and a particle swarm optimization (PSO) algorithm to accurately identify the damage location and extent as the second stage has been proposed by Fallahian and Seyedpoor (2010). Also, in another study, Seyedpoor (2012) determined the location and severity of structural damage using a two-stage damage detection method by applying modal strain energy based index (MSEBI) and PSO. The use of two-step methods for detecting cracks in beam like structures based on wavelet transform was made by Xiang and Liang (2012a) and Xiang *et al.* (2012). In another study, Xiang and Liang (2012b) presented a two-step approach for detecting multiple damages in thin plates by focusing on 2D-wavelet transform to the mode shape for localizing damage and employing the PSO algorithm for determining damage severity. A damage detection technique combining proper orthogonal decomposition (POD) with self-adaptive differential evolution algorithm considering environmental variability and measurement noise has been presented by Rao *et al.* (2012). A hybrid particle swarm–Nelder–Mead (PS–NM) algorithm has been proposed by Baghmisheh *et al.* (2012) for estimating the crack location and depth in cantilever beams. The hybrid PS–NM was made up of a modified particle swarm optimization algorithm (PSO), aimed to identify the most promising areas, and a Nelder–Mead simplex algorithm (NM) for performing a local search within these areas. A two-step approach based on mode shape curvature and response sensitivity analysis for crack identification in beam structures has been presented by Lu *et al.* (2013). A two-stage method of identifying and quantifying structural damage based on the grey system theory and imperialist competitive algorithm has also been introduced by Zare Hosseinzadeh *et al.* (2013). A hybrid stochastic/deterministic optimization algorithm (P-NMA) has been presented by Miguel *et al.* (2013) to solve the target optimization problem of vibration-based damage detection. The main goal of the stochastic part was to provide a starting point close to the global solution for deterministic part aiming to perform a local search. The authors showed that the performance of the proposed optimization scheme for damage assessment are more accurate and needs a lower computational cost than the GA, HS and PSO algorithms. Two-step damage detection methods based on employing wavelet transform for damage localization and support vector machine for damage quantification were also introduced by Xiang *et al.* (2013) and (2014). Wang *et al.* (2014) defined a damage index-strain statistical moment, and formulated the fourth strain statistical moment (FSSM) of beam-type structures to locate the damage and then employed the model updating method based on the least square algorithm to assess their damage severity.

In this study, a two-stage method of determining the location and severity of multiple structural

damages is proposed. In the first step, the location of damage is determined using a modal residual vector based indicator (MRVBI) and in the second step, the differential evolution (DE) algorithm as a robust optimization solver is considered to determine the extent of damaged elements discovered in the first stage as the high potentially damaged elements. Numerical results including two and three dimension truss structures show the efficiency of the proposed method for accurately identifying the location and severity of multiple damage cases of truss systems.

2. The proposed modal residual vector based indicator

In the past decade, much progress has been made in developing analytical methods for vibration-based damage detection. Damage detection based on modal residual vectors (Mares and Surace 1996; Guidelines for Structural Health Monitoring, 2001) is one type of these methods. This method relies on the measurement of both the frequencies and mode shapes of a damaged structure. In this study, a modal residual vector based indicator (MRVBI) is proposed to locate the damaged element of truss structures. In order to formulate the proposed index, consider the eigenvalue equation of a structure as:

$$(\mathbf{K} - \omega_i^2 \mathbf{M})\Phi_i = 0 \quad (1)$$

where \mathbf{K} and \mathbf{M} are the stiffness and mass matrices of the structure, respectively; Φ_i is the i th mode shape and ω_i is the associated frequency of the structure.

Damage in the structure may change both the stiffness and mass matrices, altering the frequencies as well as the mode shapes. Assuming that damage does not change the mass matrix, the eigenvalue equation for the damaged structure can be written as

$$(\mathbf{K}_d - \omega_{di}^2 \mathbf{M})\Phi_{di} = 0 \quad (2)$$

where \mathbf{K}_d is the stiffness matrix of the damaged structure, Φ_{di} is the i th mode shape of the damaged structure and ω_{di} is the associated frequency.

When damage occurs in the structure, the stiffness matrix of the damaged structure \mathbf{K}_d can be represented as (Mares and Surace 1996)

$$\mathbf{K}_d = \sum_{j=1}^{ne} (1 - \alpha_j) \mathbf{K}_j^e = \mathbf{K} - \sum_{j=1}^{ne} \alpha_j \mathbf{K}_j^e \quad (3)$$

where \mathbf{K}_j^e is the stiffness matrix of j th element from ne total elements of the structure and α_j is a reduction factor representing here for reducing the stiffness. The values of the parameter α_j fall in the range [0 1]. The value of zero for a particular element indicates that the element is undamaged while the value of unity represents that the element is fully damaged.

By substituting Eq. (3) into Eq. (2), the expression for a modal residual vector for the i th mode of the structure can be provided as (Mares and Surace 1996, Guidelines for Structural Health Monitoring, 2001)

$$R_i = (K - \omega_{di}^2 M) \Phi_{di} = \sum_{j=1}^{ne} \alpha_j K_j^e \Phi_{di} \quad (4)$$

Matrix $\delta K = \sum_{j=1}^{ne} \alpha_j K_j^e$ will have non-zero components corresponding to only those degrees of freedom (DOFs) that are associated to a damaged element ($\alpha_j \neq 0$). Correspondingly, the non-zero components in R_i will also lie along the same DOFs, and the connectivity relation between the elements and DOFs, can provide the determination of damage location.

In order to consider nm modes of the structure for damage localization, an absolute sum of the modal residual (ASMR) vectors given by the left term of Eq. (4) can be used as

$$ASMR = \sum_{i=1}^{nm} |R_i| \quad (5)$$

For identifying the damaged elements, an indicator is defined here using the corresponding values of $ASMR$ for each element as

$$Index_j = (d_j^T d_j)^4, \quad j = 1, 2, \dots, ne \quad (6)$$

where d_j is a vector containing the components of $ASMR$ corresponding to the nodal DOFs of j th element. The size of this vector for 2D-truss and 3D-truss is 4×1 and 6×1 , respectively.

Assuming that the set of the damage indices $Index_j (j = 1, 2, \dots, ne)$ represents a sample population of a normally distributed random variable, a normalized damage indicator can be defined as follows

$$Index_j^n = \frac{[Index_j - \text{mean}(Index)]}{\text{std}(Index)}, \quad j = 1, 2, \dots, ne \quad (7)$$

where $\text{mean}(Index)$ and $\text{std}(Index)$ represent the mean and standard deviation of the vector of damage indices, respectively.

In order to obtain a more accurate damage extent for an element, the damage indicator of Eq. (7) is further scaled to introduce a modal residual vector based indicator (MRVBI) as

$$MRVBI_j = \frac{Index_j^n}{\|Index^n\|}, \quad j = 1, 2, \dots, ne \quad (8)$$

where $\|\cdot\|$ symbolizes the magnitude of a vector.

The indicator $MRVBI_j (j = 1, 2, \dots, ne)$ can now be utilized to locate the potentially damaged elements of truss structures. According to Eq. (8), any $MRVBI_j > \varepsilon$ can be considered as a predictor representing the j th element is damaged where $\varepsilon > 0$ is named a damage border. Moreover, the $MRVBI_j \cong 0$ indicates that the element is intact.

It can be observed that, for evaluating the proposed indicator, the mode shapes at all DOFs are

required to be measured and it is not a realistic assumption for operational damage detection. However, for actual use of the suggested method, it is not needed to measure the full set of mode shapes. The mode shapes of the damaged structure in partial degrees of freedom can be first measured, and then the incomplete mode shapes are expanded to match with all degrees of freedom of the structure by a common technique such as a method described by Au *et al.* (2003).

3. Damage detection based on differential evolution

The damage detection problem can be interpreted to find a set of damage variables minimizing/maximizing a correlation index between response data of a structure before and after damage (Seyedpoor 2011, Nobahari and Seyedpoor 2011). Therefore, the problem can be transformed into a standard optimization problem (Golizadeh and Barati 2014) as

$$\begin{aligned} \text{Find : } & X^T = \{x_1, x_2, \dots, x_n\} \\ \text{Minimize : } & W(X) \\ \text{Subject to : } & X^l \leq X \leq X^u \end{aligned} \quad (9)$$

where $X^T = \{x_1, x_2, \dots, x_n\}$ is a damage variable vector containing the locations and sizes of n unknown damages; X^l and X^u are the lower and upper bounds of the damage vector. Also, W is an objective function that should be minimized.

In many researches, various correlation indices between response data of damaged and analytical structures have been chosen as the objective function for optimization. In this study, an efficient correlation based index introduced by Nobahari and Seyedpoor (2011) employing n_f natural frequencies of a structure is used as an objective function

$$W(X) = -\frac{1}{2} \left[\frac{|\Delta F^T \cdot \delta F(X)|^2}{(\Delta F^T \cdot \Delta F)(\delta F^T(X) \cdot \delta F(X))} + \frac{1}{n_f} \sum_{i=1}^{n_f} \frac{\min(f_i(X), f_{di})}{\max(f_i(X), f_{di})} \right] \quad (10)$$

where ΔF and $\delta F(X)$ are the changes of frequency vectors of damaged structure and an analytical model with respect to the frequency vector of healthy structure. Also, $f_i(X)$ and f_{di} are the i th component of analytical frequency vector $F(X)$ and damaged frequency vector F_d of the structure, respectively.

The selection of an efficient algorithm for solving the optimization based damage detection problem is a critical issue, because the damage identification problem has many local solutions. Therefore, achieving the global optimum needing fewer structural analyses without trapping into local optima must be the main characteristic of the algorithm. In this study, the differential evolution (DE) is employed to properly solve the damage detection problem. The framework of DE is similar to a standard GA, however, the classical crossover and mutation operators in GA have been replaced by alternative operators and consequently came up to a suitable differential operator. The DE can be implemented very easily and requires a minimal parameter tuning. The main steps of the algorithm can be explained as (Storn and Price 1997, Seyedpoor *et al.* 2015):

Step 1) Initialization: the initial parameters, constants and initial population are identified. Like other evolutionary algorithms, DE starts to search from an initial population. The initial population is generated randomly in the search space as

$$X^l \leq X_i \leq X^u \quad , \quad i = 1, 2, \dots, np \quad (11)$$

where X^l and X^u are the lower and upper vectors of an optimization variable vector, respectively. Also, np represents the population size that must be at least 4.

Step 2) Mutation: for a given vector X_i ($i = 1, 2, \dots, np$), a basic mutant vector is defined as

$$V_i = X_{r_1} + mf.(X_{r_2} - X_{r_3}) \quad , \quad r_1 \neq r_2 \neq r_3 \neq i \quad (12)$$

where the three different indices r_1, r_2 and $r_3 \in \{1, 2, \dots, np\}$ are randomly chosen to be different from index i . Also, $mf \in [0, 2]$ is the mutation factor which controls the amplification of the differential variation $(X_{r_2} - X_{r_3})$.

In this study to speed up the optimization process, instead of the basic mutation scheme, the mutation scheme DE/best/1 is used as

$$V_i = X_{\text{best}} + mf.(X_{r_1} - X_{r_2}) \quad (13)$$

where X_{best} is the best vector of the current population

Step 3) Crossover: in order to increase the diversity of the perturbed parameter vector, crossover is introduced by producing the trial vectors U_i ($i = 1, 2, \dots, np$) as

$$u_{ji} = \begin{cases} v_{ji} & \text{if } (\text{rand}_{ji} \leq cf \quad \text{or} \quad j = \text{irnd}_i) \\ x_{ji} & \text{if } (\text{rand}_{ji} > cf \quad \text{and} \quad j \neq \text{irnd}_i) \end{cases} \quad , \quad j = 1, 2, \dots, n \quad (14)$$

where rand_{ji} is a uniformly random number $\in [0, 1]$; cf is the crossover factor $\in [0, 1]$ and irnd_i is a random integer $\in \{1, 2, \dots, n\}$ which ensures that U_i gets at least one parameter from V_i .

Step 4) Selection: for final selection, the trial vector U_i and target vector X_i are compared. If the vector U_i yields a smaller objective function value than X_i , then X_i is set to U_i ; otherwise, the old value X_i is retained.

Step 5) Convergence: in this step, solution convergence is controlled. If the solution is converged, the optimization will be stopped otherwise returns to step 2.

4. Numerical examples

In order to show the performance of the proposed method for identifying the damage, two illustrative test examples are considered. The first example is a 47-bar planar truss and the second one is a 36-bar spatial truss. In both the examples, a proportional error generating by a uniformly random number is applied to the natural frequencies and mode shapes of damaged structure in order to consider the measurement noise.

4.1 Forty seven-bar planar truss

The 47-bar planar power line tower (Nouri Shirazi *et al.* 2014) shown in Fig. 1, is considered to show the robustness of the proposed method. The structure has forty-seven members and twenty-two nodes. All members are made of steel, and the material density and modulus of elasticity are 0.3 lb/in.³ and 30,000 ksi, respectively. A damage variable in the structure is defined here via a relative reduction in the elasticity modulus of individual bars. Therefore, the problem originally has 47 damage variables. Four different damage cases given in Table 1 are induced in the structure and the proposed method is tested with considering noise. For considering the measurement noise, the frequencies and mode shapes of the damaged structure are randomly polluted here by a standard error of 0.15% and 3%, respectively.

4.1.1 Locating the damaged elements using MRVBI

In order to consider the stochastic nature of the damage localization method due to considering noise, 100 independent sample runs are made for each damage case and the mean of the indicator given by Eq. (8) is considered as a damage index. The potentially damaged elements of the truss for various damage cases when considering 5 to 8 modes of the structure are given in Table 2. For this, the damage border is set to $\varepsilon = 0.10$. It means that those elements whose indices exceed 0.10 are selected as suspected damaged elements.

As can be seen in the table, for accurately locating the damaged elements, 6, 5, 6 and 7 modes of the structure are needed for damage cases 1 to 4, respectively. Therefore, it can be concluded that for properly locating all damage cases, at least 7 mode shapes of the structure are required to be considered. Figs. 2(a)-2(d) show damage localization charts for various damage cases considering 7 vibration modes, where the mean of MRVBI has been depicted versus the element number of the structure. As can be seen, the damaged elements identified are similar to those listed in Table 2. It is apparent that the proposed indicator MRVBI can decrease the damage variables of the structure from 47 variables to 5, 3, 4 and 6 ones for damage cases 1 to 4, respectively.

4.1.2 Quantifying the damage using DE

The DE is now employed to solve the reduced damage detection problem to determine the damage extent. The initial parameters of DE, including the population size (np), mutation factor (mf), and crossover factor (cf) are set to 20, 0.6 and 0.3 respectively. The maximum number of iterations for optimization is also set to 1000. The optimization process will be stopped as the objective function is smaller than -0.999 or does not change significantly after 100 successive iterations.

Table 1 Four different damage cases induced in 47-bar planar truss

Case 1		Case 2		Case 3		Case 4	
Element number	Damage ratio	Element number	Damage ratio	Element number	Damage ratio	Element number	Damage ratio
10	0.30	30	0.30	10	0.30	40	0.30
-	-	-	-	30	0.30	41	0.20

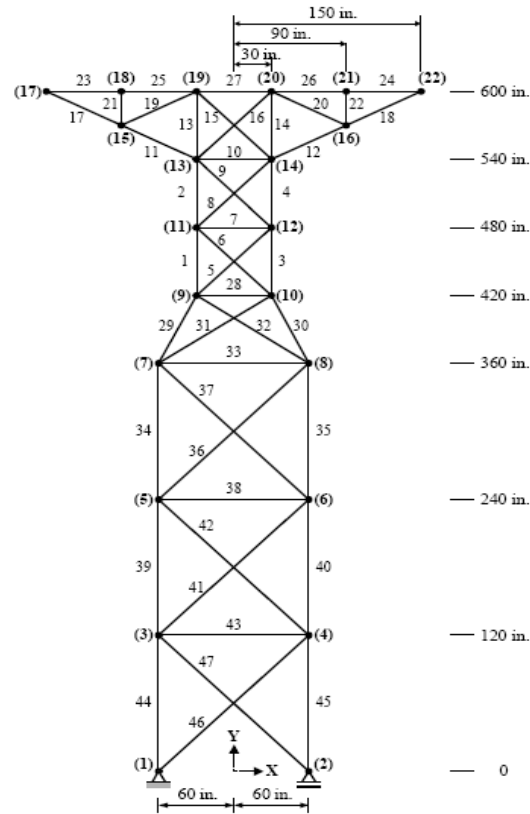


Fig. 1 The 47-bar planar power line tower

Table 2 Potentially damaged elements of 2D-truss identified by MRVBI with different modes

Damage cases	Actually damaged elements	5 modes	6 modes	7 modes	8 modes
1	10	11, 12, 19, 20, 21, 22	10, 11, 12, 21, 22	10, 11, 12, 21, 22	10, 11, 12
2	30	11, 12, 19, 20, 21, 22, 30	11, 19, 21, 22, 30	22, 28, 30	30
3	10 and 30	11, 12, 19, 20, 21, 22, 30	10, 11, 12, 21, 22, 30	10, 11, 12, 30	10, 30
4	40 and 41	11, 12, 19, 20, 21, 22	11, 12, 19, 20, 21, 22	12, 20, 21, 22, 40, 41	11, 12, 21, 22, 40, 41

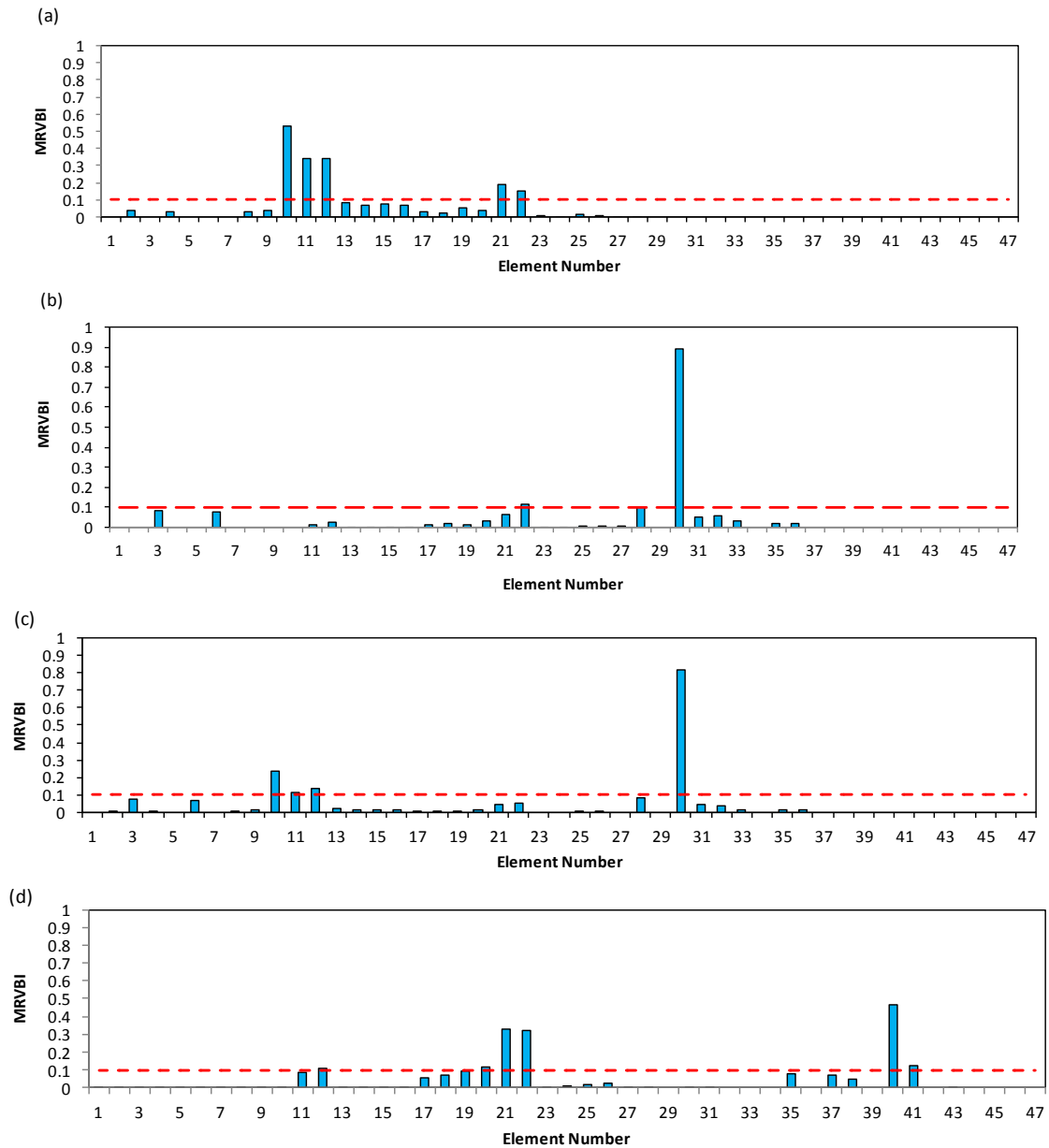


Fig. 2 Damage localization charts for 47-bar planar truss considering 7 vibrating modes (a) damage case 1, (b) damage case 2, (c) damage case 3 and (d) damage case 4

The damage identification results for cases 1 to 4 obtained by DE considering 7 modes are shown in Figs. 3(a)-3(d), respectively. It is observed that the optimization achieves to the site and extent of actual damage truthfully. It should be noted that the optimization process for cases 1 to 4 converges to actual damage after about 117, 109, 161 and 139 iterations (2360, 2200, 3240 and 2800 modal analyses), respectively. The simulation results of various damage cases reveal the effectiveness of the two-stage method proposed here for properly determining the damage site and extent.

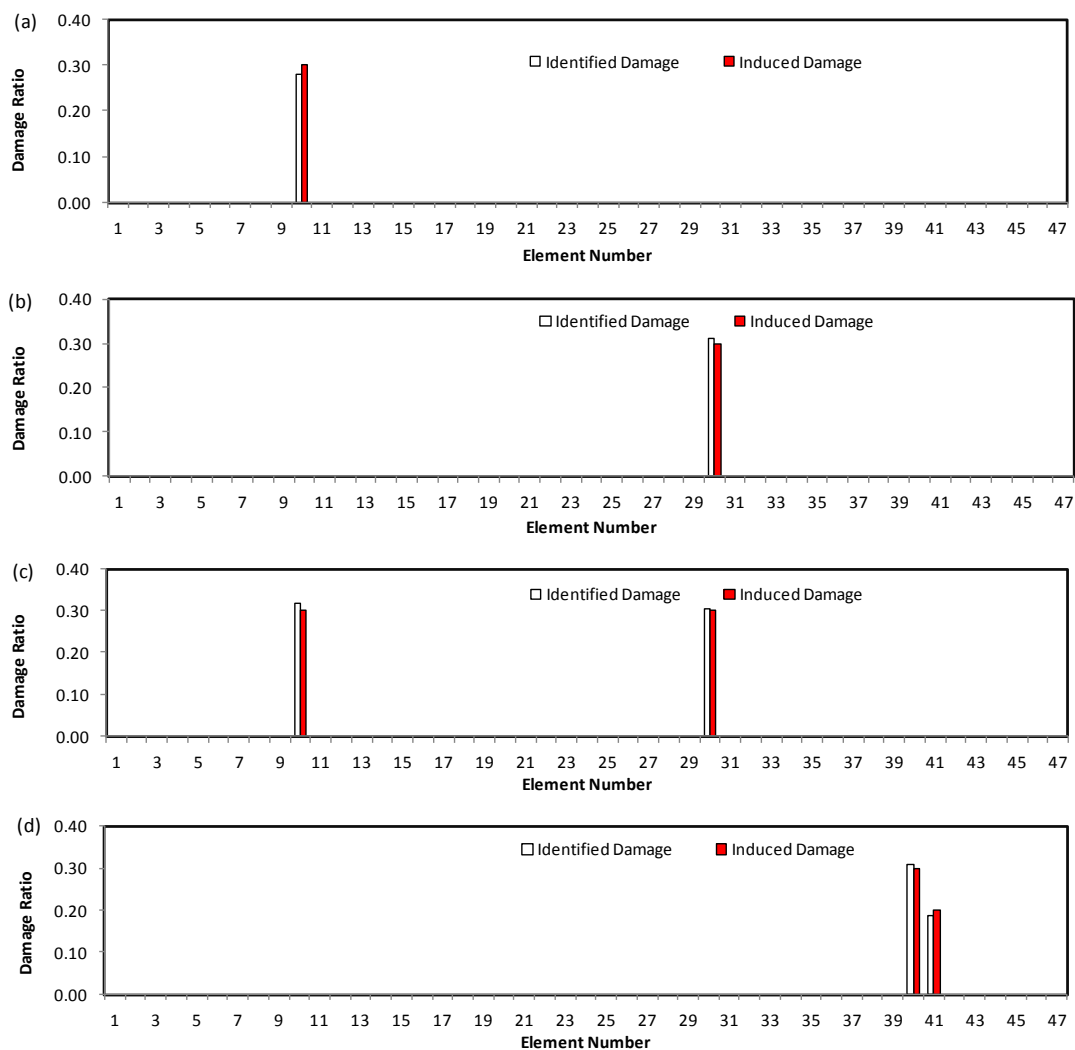


Fig. 3 The final damage identification for 47-bar planar truss considering 7 vibrating modes (a) damage case 1, (b) damage case 2, (c) damage case 3 and (d) damage case 4

4.2 Thirty six-bar spatial truss

In order to assess the performance of the proposed method for damage detection of space trusses, a 36-bar spatial truss shown in Fig. 4 is considered as the second example. The structure is fixed at the ground and consists of 36 steel tubular members that comprise 12 leg members, 12 horizontal members and 12 diagonal brace members in vertical planes. All members have a uniform outer diameter 17.8 cm and wall thickness 0.89 cm. The heights of all three stories are 9.14 m, and the side lengths of the bottom base and top floor are 12.19×10.97 m and 4.88×3.66 m, respectively. The material properties of the steel tubular members are: elastic modulus $E=210$ GPa/m² and mass density $\rho=7850$ kg/m³. Three different damage cases given in Table 3 are induced in the structure and the proposed method is tested with considering 0.15% and 3% noises for the frequencies and mode shapes of the damaged structure, respectively.

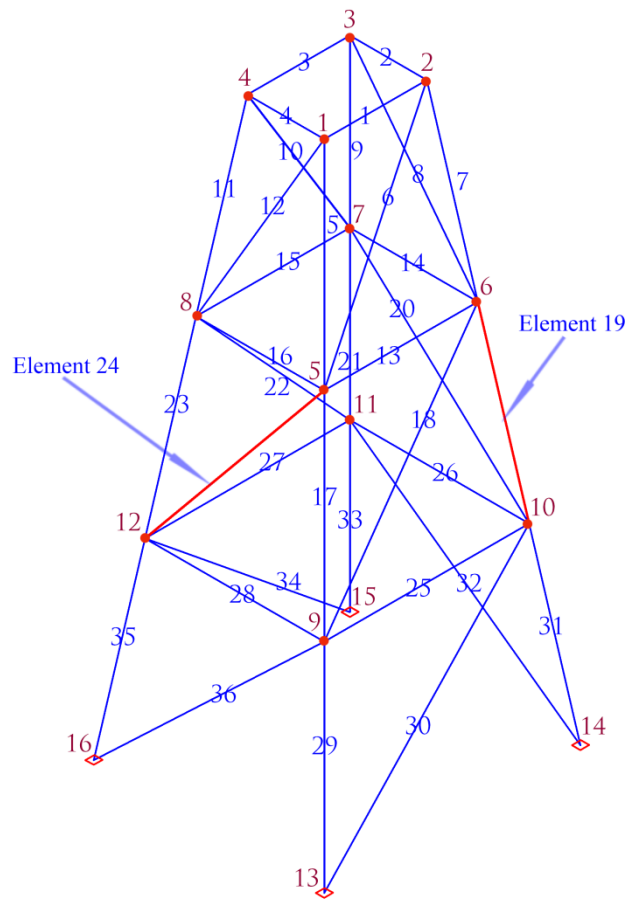


Fig. 4 The 36-bar spatial truss

Table 3 Three different damage cases induced in 36-bar spatial truss

Case 1		Case 2		Case 3	
Element number	Damage ratio	Element number	Damage ratio	Element number	Damage ratio
19	0.25	24	0.25	19	0.25
-	-	-	-	24	0.25

Table 4 Potentially damaged elements of 3D-truss identified by MRVBI with different modes

Damage cases	Actually damaged elements	9 modes	10 modes	11 modes	12 modes
1	19	1, 2, 3, 4, 7, 8, 19	2, 7, 8, 19	7, 8, 19	7, 8, 19
2	24	1, 2, 3, 4, 5, 6, 24	4, 5, 6, 24	5, 6, 24	5, 6, 24
3	19 and 24	1, 2, 3, 4, 7, 8, 13, 19	4, 5, 6, 13, 24	5, 6, 7, 8, 13, 19, 24	5, 6, 7, 8, 13, 19, 24

The potentially damaged elements of the truss obtained by MRVBI for various damage cases when considering 9 to 12 modes of the structure are listed in Table 4. For finding the suspected elements, the damage border has also been set to $\varepsilon = 0.10$. As can be seen in the table, for precisely locating the damaged elements 9, 9 and 11 modes of the structure are needed for damage cases 1 to 3, respectively. Therefore, it can be concluded that for properly locating all damage cases, at least 11 mode shapes of the structure are required to be considered.

Figs. 5(a)-5(c) show damage localization charts of various damage cases considering 11 vibration modes. As can be observed, the damaged elements identified are similar to those listed in Table 4. It is revealed that the damage indicator can reduce the damage variables of the structure from 36 variables to 3, 3 and 7 ones for cases 1 to 3, respectively. The DE can now be employed to solve the reduced damage detection problem in order to accurately determine the location and severity of actual damage. The optimization parameters are set to as the first example.

The damage identification results for cases 1 to 3 obtained by DE considering 11 modes are shown in Figs. 6(a)-6(c), respectively. It is observed that the optimization achieves to the site and extent of actual damage truthfully. It should be noted that the optimization process for cases 1 to 3 converges to actual damage after about 11, 7 and 29 iterations (240, 160 and 600 modal analyses), respectively. The final results of different damage cases reveal the efficiency of the two-stage method proposed here for determining the damage site and extent.

5. Conclusions

A two-stage method has been proposed to properly determine the location and severity of multiple damage cases in truss structures. In the first stage, the concept of modal residual vector has been used to introduce an efficient damage indicator named here as MRVBI to find the potentially damaged element and reduce the damage variables of the structure. In the second stage, a differential evolution (DE) is used to solve the reduced damage detection problem for finding the actual site and extent of damage in the structure. Two test examples including a 2D-truss and 3D-truss have been considered to demonstrate the performance of the proposed damage detection method with considering measurement noise. The numerical results show that the MRVBI can effectively find the potentially damaged elements of truss structures and reduce a great number of damage variables to much fewer ones while requiring the first few vibration modes of a structure. Moreover, it has also been revealed that the DE can successfully solve the reduced damage detection for achieving the actual site and extent of damage induced. As a final result, it can be concluded that the combination of MRVBI and DE can be used as a robust tool for properly identifying the damage when the noise effect is considered.

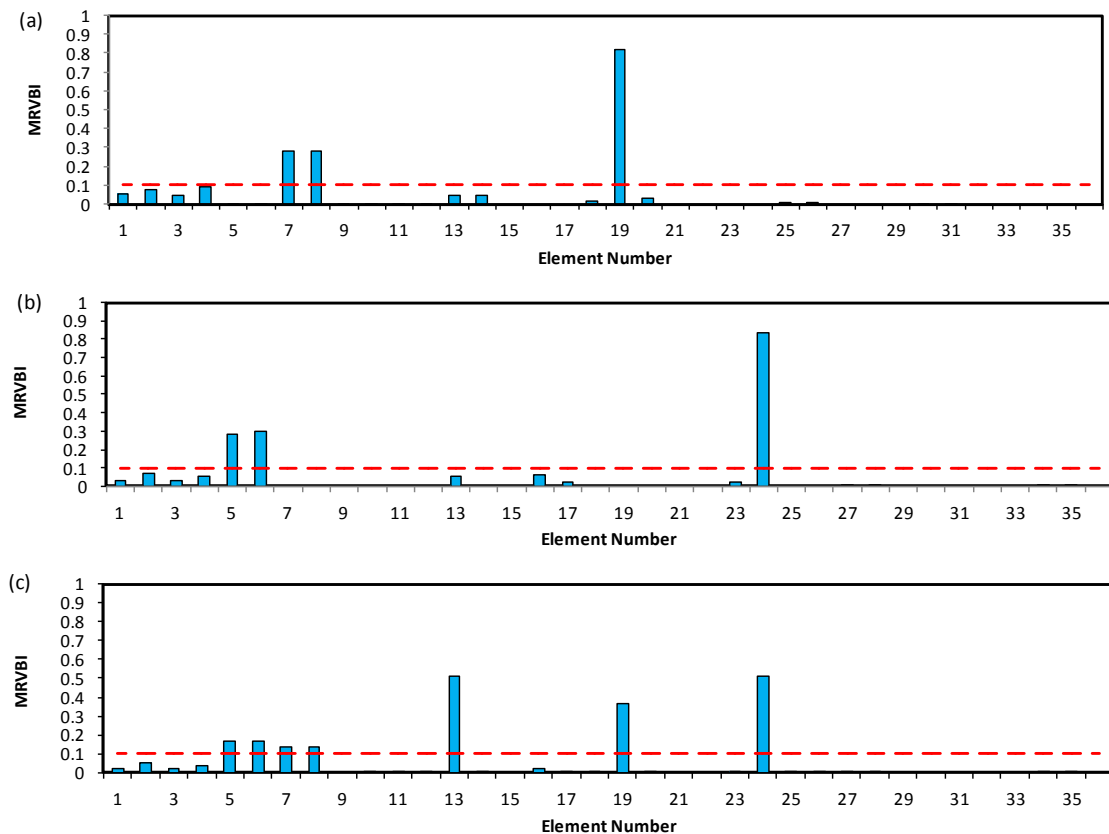


Fig. 5 Damage localization charts for 36-bar spatial truss considering 11 vibrating modes (a) damage case 1, (b) damage case 2 and (c) damage case 3

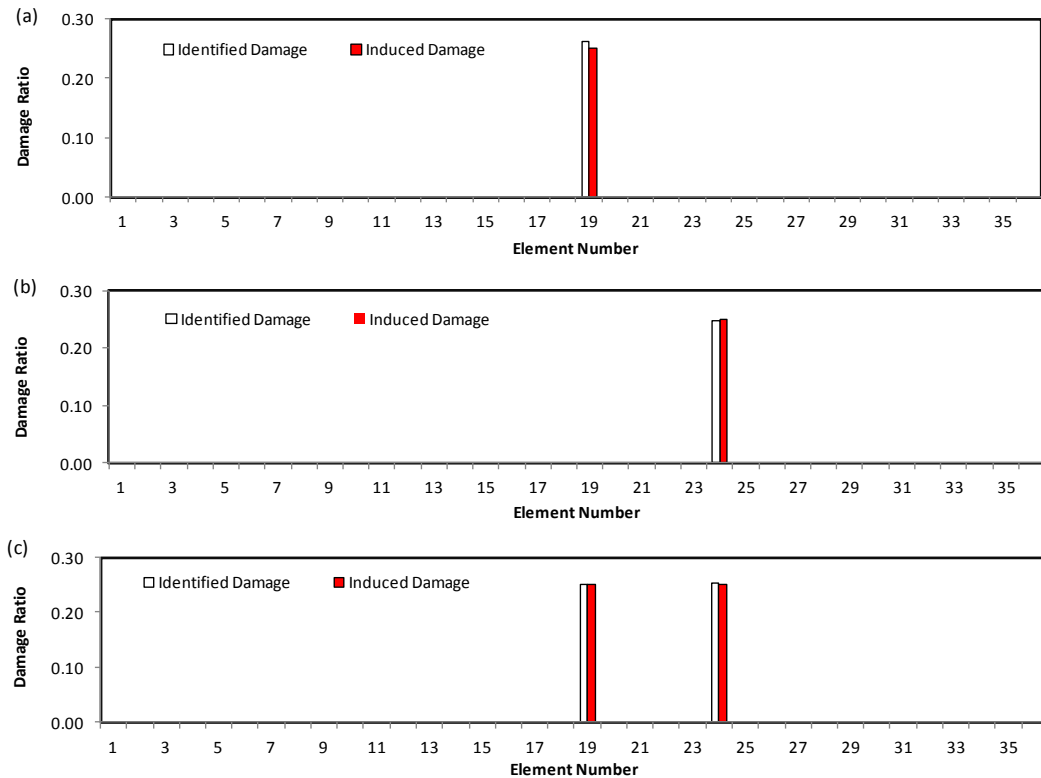


Fig. 6 The final damage identification for 36-bar spatial truss considering 11 vibrating modes (a) damage case 1, (b) damage case 2 and (c) damage case 3

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