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Damage detection in structural beam elements using hybrid neuro fuzzy systems

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Abstract. A damage detection algorithm based on neuro fuzzy hybrid system is presented in this study for location and severity predictions of cracks in beam-like structures. A combination of eigenfrequencies and rotation deviation curves are utilized as input to the soft computing technique. Both single and multiple damage cases are considered. Theoretical expressions leading to modal properties of damaged beam elements are provided. The beam formulation is based on Euler-Bernoulli theory. The cracked section of beam is simulated employing discrete spring model whose compliance is computed from stress intensity factors of fracture mechanics. A hybrid neuro fuzzy technique is utilized to solve the inverse problem of crack identification. Two different neuro fuzzy systems including grid partitioning (GP) and subtractive clustering (SC) are investigated for the highlighted problem. Several error metrics are utilized for evaluating the accuracy of the hybrid algorithms. The study is the first in terms of 1) using the two models of neuro fuzzy systems in crack detection and 2) considering multiple damages in beam elements employing the fused neuro fuzzy procedures. At the end of the study, the developed hybrid models are tested by utilizing the noise-contaminated data. Considering the robustness of the models, they can be employed as damage identification algorithms in health monitoring of beam-like structures.

Keywords: neuro fuzzy system; grid partitioning; subtractive clustering; beam; damage detection

1. Introduction

Engineering structures deteriorate due to their steady usage over time. This process may be commenced or even accelerated on the score of ambient effects, accidents or adverse loading conditions such as earthquakes and storms, etc. The presence of damage in a structure will eventually lead to poor performance or failure of the system and may result in loss of human lives and loss of resources. To avoid this, detection of damage at its onset, if possible, is essential. Infrastructure health conditioning and monitoring, hence, have attracted the attention of scientific and engineering communities and have been an active area of research and development for the last several decades (Fan and Qiao 2011). Consequently, a broad range of damage detection methodologies have been developed during this period.

Broadly speaking, there are a couple of ways in which health monitoring and damage detection

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methodologies can be classified. The first categorization is done according to detection capability of damage detection procedures: local techniques and global techniques. Local damage diagnosis methods generally require that the vicinity of any damage be realized a priori (Saadat *et al*. 2007), and that the location of damage being inspected be easily accessible (Lauwagie *et al*. 2002). Subject to such limitations, these methods can detect damage on or near the surface of a structure. This cannot be guaranteed for most cases in civil, mechanical or aerospace engineering. In an effort to overcome these difficulties dynamic-based damage detection methods as global identification have been developed. Global techniques attempt to simultaneously asses the condition of the whole structure (Sohn *et al*. 2004). The second categorization is based on the extent of prior knowledge required by identification techniques: model-based and non-model-based or feature techniques. The former assumes that a detailed numerical model of the structure is available for damage detection; while the latter relies on experimentally obtained response data from structures. Non-model-based or feature based approaches typically seek to identify damage from changes in structural vibrational properties. These approaches determine structural changes by using some damage features, without the need of a detailed simulation of the structure. Examples of features that are extracted from measured responses and utilized in non-model based studies are natural frequencies (Garesci *et al*. 2006), mode shapes (Kim *et al*. 2006), mode shape derivatives (Hamey *et al*. 2004), stiffness matrix (Shi *et al*. 2000), flexibility matrix (Choi *et al*. 2008), modal strain energy (Li *et al*. 2006), etc. Readers interested in more details of vibration-based features and detection algorithms are referred to the review studies by Carden and Fanning (2004) and Fan and Qiao (2011).

The development of vibration-based structural damage detection as global methods can further be divided into traditional and modern types (Yan *et al*. 2007). The traditional type refers to detection method for structural damage only utilizing dynamic characteristics of structures such as natural frequency, modal damping, modal strain energy or modal shapes, etc. It has several disadvantages: 1) it is more dependent on experimentally measured modal properties of structures. 2) When this type of identification is employed, it is hard to establish a universal methodology for various structures encountered in engineering applications. Also, 3) the traditional type of damage detection is often insensitive to small damages. The modern-type of vibration-based structural fault detection, also called as intelligent damage diagnosis, is based on online measured structural vibration responses to detect damage. Its advantages can be listed as: 1) its dependency on experiments is less than that of traditional type. Response data measured at few points in a structure would be enough for fault identification purposes. 2) A universal methodology can conveniently be established within modern-type detection that will be applicable to any structures. 3) By selection and extraction of better dynamic characteristics of a structure, smaller damage can be diagnosed. For these reasons, the development of damage detection technique using modern-type has accelerated in the last decade. Among the modern-type methods for structural damage identification, the representative ones include wavelet analysis, genetic algorithm (GA), artificial neural network (ANN), fuzzy logic system, swarm intelligence techniques, and hybrid techniques that are formed by combining two or more of the former techniques. These methods predominantly take modern signal processing technique and artificial intelligence as analysis tools. The combination of artificial intelligence and signal processing is important in order to develop and extract a more reliable and sensitive damage feature index (Oruganti *et al*. 2009).

Vibration-based damage detection is an inverse problem (Dackermann 2010), where damage properties such as severity and location are determined from changes in structural dynamic properties (Nanda *et al*. 2012). A unique solution oftentimes does not exist for an inverse problem, particularly when the relationship between the vibration data identified by modal testing and those computed from analytical model are very complex and only limited data are available. This makes the inverse problem too difficult to be solved by conventional algorithms. However, the black box mapping between damage properties and dynamic changes in structures can be resolved by recent computational intelligence methods. Therefore, these methods have been gaining much more attention as compared to traditional techniques and many attempts have been made in recent years to utilize artificial intelligence techniques (Thatoi *et al*. 2013). An overview of these techniques confined only to structural damage detection area is presented in the following. It should be noted that the artificial intelligence techniques have been employed in numerous applications of science and engineering problems and, hence, the literature review supplied herein is by no means a comprehensive survey of the studies of soft computing techniques.

In general, the inverse problem of determining damage parameters is a pattern recognition problem, where changes in dynamic properties of a structure are accounted for certain characteristics of damage. ANN is one of the effective tools in pattern recognition (Bakhary 2008). It also serves in function approximation for structural damage detection. ANN can display substantial tolerance of noisy, partially incomplete and partially faulty data, which is particularly useful for damage identification of large engineering structures where in-situ measured data are expected to be incomplete and corrupted with noise. The first successful application of ANN in damage detection of civil structures was performed by Wu *et al*. (1992). Since then, the use of ANN has considerably increased. To present to the reader an overview of the damage detection methods utilizing ANN in a compact and clear fashion, it would be convenient to categorize them in two ways. The first categorization can be done considering the type of vibration-based parameters as input to ANN: neural networks trained with natural frequencies, modal shapes and their derivatives (Tsou and Shen 1994, Sahin and Shenoi 2003, Gonzales and Zapico 2008, among others), neural networks trained with frequency response functions (Zang and Imregun 2001a, Fang *et al*. 2005, Saeed and George 2011), neural networks trained with time domain data (Su and Ye 2004, Kao and Hung 2005). The second categorization can be based on type of structure the ANN is applied to: simple numerical or laboratory structures (Barai and Pandey 1995, Chang *et al*. 2000, Zapico *et al*. 2003) and complex numerical or real structures (Ni *et al*. 2000, Zang and Imregun 2001b, Vinayak *et al*. 2008). It is deduced that a large number of publications exists. However, the area of dynamic-based damage identification employing ANN is still an active field of research and there are several problems that need to be resolved by researchers: optimization of input parameters, network design and network training scheme.

Fuzzy logic is another artificial intelligence method for modeling sophisticated systems where indeterminacy and imprecision can be important (Pawar and Ganguli 2005) and can be utilized for solving the inverse problem of damage detection. Fuzzy systems handle uncertainty directly by using linguistics reasoning which is more robust to uncertainty than pure numerical reasoning (Zadeh 1996). Recent examples of the use of fuzzy logic systems for developing damage detection systems are discussed next. A generalized methodology for structural fault detection using fuzzy logic is presented by Sawyer and Rao (2000). The numerical examples demonstrated the performance advantages of the fuzzy logic-based system in noisy or uncertain conditions. Ganguli (2001) demonstrated the use of fuzzy logic for damage detection and isolation in the presence of noise for a cantilever beam that resembled a helicopter rotor blade. Dempsey and Afjeh (2004) developed a diagnostic tool for detecting damage to spur gears using fuzzy logic with two different measurement technologies: wear debris analysis and vibration. From the results, it was observed that the use of two measurement technologies together improved the detection of pitting damage on spur gears. The fuzzy logic-based methods are also applied to civil engineering structures in order to develop structural health monitoring systems. Soh and Bhalla (2005) proposed a fuzzy probability damage model based on the extracted equivalent stiffness for the non-destructive evaluation of concrete, covering both strength prediction and damage assessment, using the electro-mechanical impedance technique. Zhao and Chen (2002) presented a fuzzy rule-based inference system for bridge damage diagnosis and prediction. This aimed to supply bridge designers with valuable information about the impacts of design factors on bridge deterioration.

In these traditional fuzzy systems, it is difficult to perform the fuzzy partitioning of the input and output spaces and to establish the fuzzy rules, which may require a time consuming trial-and-error process (Zio and Gola 2006). Moreover, as fuzzy logic systems do not have the capability of learning from data, it is difficult to develop a fuzzy system when a large number of discrete data is available as a knowledge base (Pawar and Ganguli 2011). As for the ANN, it lacks flexibility, human interaction or knowledge representation. However, these characteristics are clearly present in fuzzy logic systems. The active research, hence, has been going through a phase in which these soft computing methods are combined. The combination enables one to exploit the advantages of the simple learning procedures and computational power of ANN and the high-level, human-like thinking and reasoning of fuzzy systems (Du and Er 2004). The resulting hybrid system is called neuro fuzzy method and offers an appealingly powerful framework for tackling pattern recognition problems (Marseguerra *et al*. 2004, Czogaa *et al*. 2000). Neuro fuzzy technique was first proposed by Jang (1993) and, since then, has been applied for solving various engineering problems where the traditional techniques do not supply an easy and accurate solution (Vieira *et al*. 2004). The following presents a literature overview of hybrid neuro fuzzy systems pertinent to damage detection field. Ramu and Johnson (1995) and Nyongesa *et al*. (2001) utilized fuzzy logic for improving the performance of neural network for damage identification of composite structures. The effectiveness of adaptive network-based fuzzy inference system along with continuous evolutionary algorithm was investigated for crack detection by Shim and Suh (2002). The parameters of crack on clamped-free beam were estimated within 3% error. Zio and Gola (2009) and Lei *et al*. (2008) developed a neuro fuzzy technique for fault diagnosis in rotating machinery. Fuzzy inference system has also been successfully applied for fault diagnosis in induction motor (Altug *et al*. 1999, Ye *et al*. 2006, Tran *et al*. 2009) and railway track circuits (Chen *et al*. 2008). In a recent study by Saeed *et al*. (2012), a modular neuro fuzzy approach was adopted for crack identification in curvilinear beams based on changes in natural frequencies and frequency response functions. It should be noted that, apart from the fused neural fuzzy systems, there are other hybrid artificial intelligence systems such as neural genetic, genetic fuzzy and genetic neural fuzzy, and also optimization algorithm methods such as particle swarm optimization, ant colony optimization, bee colony optimization approaches being employed for detections of cracks in structural elements. The interested reader is referred to relevant studies for more information.

The implication from the overview above is that very few researchers have studied the hybrid neuro fuzzy systems in solving the inverse problem of determining the crack parameters in structural beam elements. Moreover, to the authors' knowledge, identification of multiple damage cases has not been investigated previously using the fused method. These two factors motivate the study herein. It is well known that beam elements are one of the most commonly used structural components in various engineering applications and subjected to a wide variety of static and dynamic loads. Cracks may generate in beam-like structures due to such loads. If undetected at an early state, they may lead to catastrophic failure. A model-based approach is developed in this study for crack identification using both global (e.g., natural frequencies) and local (e.g., mode shape rotation deviation values) modal properties. Knowing the effect of crack on stiffness, the structural element is modeled using Euler-Bernoulli beam theory. The boundary conditions are utilized along with the crack compatibility requirements to obtain the characteristic equation of damaged element, which subsequently yields the eigenfrequencies and modal shape rotation functions. The evaluation between the modal characteristics and crack parameters is carried out using the hybrid neuro fuzzy system which combines the ANNs adaptive capability and the fuzzy logic qualitative approach. The ability of two different neuro fuzzy inference systems including grid partitioning (GP) and subtractive clustering (SC) in damage detection is examined. The applicability of these methods for damage detection has also not been investigated beforehand. This sets the third motivation for this study. The study ends by testing the robustness of the developed systems in the presence of ambient noise.

2. Theoretical formulation

2.1 Damaged beam modeling

Beams are commonly used load bearing elements in various engineering systems (e.g., steel structures, industrial machinery, bridges, etc.) and are subjected to static and dynamic loads. Due to these loadings and environmental effects, they may experience cracks, which thereafter drastically reduce the life cycle of structural systems. A crack in a beam element introduces additional flexibility, which can be utilized along with prevailing boundary conditions to formulate the characteristic equation of vibration to obtain dynamic properties of damaged element. The current section aims at the development of such a formulation as follows.

The problem considered here is a cantilever beam with multiple cracks along its length and is shown schematically in Fig. 1. There are three main approaches to the modeling of cracks in beam-type structures reported in the literature: local stiffness reduction, discrete spring models, and complex models in two or three dimensions (Friswell and Penny 2002). The first two approaches are a gross simplification of the crack dynamics and the last approach, although it permits a detailed and accurate model of a damaged structure, is a complicated and computationally intensive approach for modeling simple structures like beams (Sinha *et al*. 2002). The formulation of each of these models has not completely satisfactory and some basic aspects have not been clarified yet (Caddemi and Morassi 2013). Despite this fact, they have been immensely utilized in both forward and inverse problem of damage detection for beam elements with single or multiple damages. For this reason, discrete spring model is used in this study to simulate the cracked section in favor of simplicity of mechanical model of crack. Fig. 1(b) shows the crack as a massless elastic rotational spring. The effects of discontinuities in axial and transverse displacements are considered to be negligible compared with that of bending slope. It is assumed that the cracks are perpendicular to beam surface and always remain open. The cracked beam is divided into sub-beams, as shown in Fig. 1(b), connected by the rotational springs at the cracked sections whose bending compliances (C_i's) are determined from the empirical expressions of stress intensity factors from fracture mechanics (Dimaragonas 1996).

The Cartesian coordinates are chosen such that x coordinate is along the beam axis, and y and z are along the principal axes of beam cross-section. The cracks are located at distances of x_1 and x_2 such that $0 < x_1 < x_2 < L$, where L is the beam length. The governing equation for the transverse vibration of a uniform Euler-Bernoulli beam can be formulated (Weaver *et al*. 1990) as

$$\frac{\partial^4 v(x,t)}{\partial x^4} + \frac{\bar{m}}{K_b}\frac{\partial^2 v(x,t)}{\partial t^2} = 0 \tag{1}$$

where K_b=EI is the flexural stiffness, E is the Young's modulus of elasticity, I is the area moment of inertia, $\bar{m} = \rho A$ is the mass per unit length, ρ is the mass density, and A is the area of beam cross-section. Assuming the variation of transverse deflection of the beam at any location to be

$$v(x,t) = X(x)e^{i\omega t} \tag{2}$$

Eq. (1) becomes

$$\frac{d^4 X(x)}{dx^4} - \frac{\bar{m}\omega^2}{K_b} X(x) = 0 \tag{3}$$

In Eq. (2), $i = \sqrt{-1}$, ω and $X(x)$ are the circular frequency and mode-shape function of the beam respectively. The general form of the solution for Eq. (3) is given by

$$X(x) = D_1 S_1(x/L) + D_2 S_2(x/L) + D_3 S_3(x/L) + D_4 S_4(x/L) \tag{4}$$

where D_j (j=1,2,3,4)'s are constants to be determined from the boundary conditions and S_j's are the linearly independent solutions given by

$$S_1(\chi) = \sin(k\chi),\ S_2(\chi) = \cos(k\chi),\ S_3(\chi) = \sinh(k\chi),\ \text{and}\ S_4(\chi) = \cosh(k\chi) \tag{5}$$

where $\chi = x/L$ is the non-dimensional length parameter and $k = \sqrt[4]{\bar{m}\omega^2 L^4 / K_b}$. To simplify the analysis of vibration of the Euler-Bernoulli beam, the linearly independent fundamental solutions denoted by $\bar{S}_j(\chi)$ (j=1,2,3,4) are constructed based on linearly independent solutions. This is done by satisfying the normalization condition at the origin of axes (see for details Li 2001). Based on the $\bar{S}_j$'s, the mode-shape of the first segment ($0 \le \chi \le \chi_1$) can be formulated as

$$X_1(\chi) = X(0)\bar{S}_1(\chi) + \phi(0)L\bar{S}_2(\chi) - \frac{M(0)}{K_b}L^2\bar{S}_3(\chi) - \frac{V(0)}{K_b}L^3\bar{S}_4(\chi) \tag{6}$$

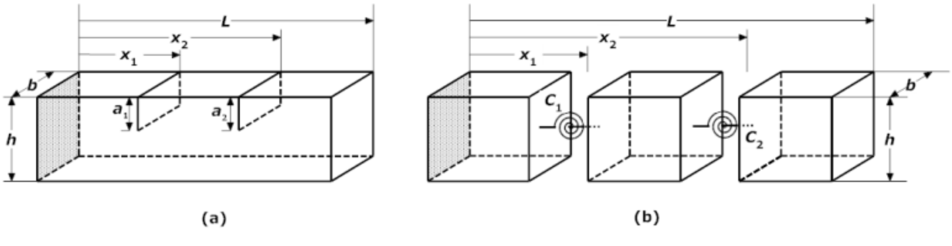

Fig. 1 (a) Cantilever beam with two cracks and (b) beam modeled as sub-beams and cracks as rotational spring elements where $X(0)$, $\phi(0)$, $M(0)$, and $V(0)$ are the displacement, rotation, moment, and shearing force of the beam at $\chi = 0$, respectively. Enforcing the continuity of the force and displacement fields across the crack and making use of Eq. (6), one obtains the mode-shape function for the section ($\chi_1 \le \chi \le \chi_2$) after the first crack as

$$X_2(\chi) = X_1(\chi) + C_1 X_1''(\chi_1)\bar{S}_2(\chi-\chi_1)H(\chi-\chi_1) \tag{7}$$

in which $H(\cdot)$ is the Heaviside function, which assumes 0 for $\chi < \chi_1$ and 1 for $\chi \ge \chi_1$ and C_1 is the compliance of the first crack. The characteristic equation of the beam having a single crack is readily established by imposing boundary conditions on the above equation. This is outlined next for a clamped-free beam. Since the beam is fixed at the left support, the compatibility requirements at $\chi = 0$ are

$$X(0) = 0,\ \phi(0) = 0 \tag{8}$$

The mode shape of the first segment ($0 \le \chi \le \chi_1$) becomes then

$$X_1(\chi) = -\frac{M(0)}{K_b}L^2\bar{S}_3(\chi) - \frac{V(0)}{K_b}L^3\bar{S}_4(\chi) \tag{9}$$

The boundary conditions of the right end are

$$X_2''(1) = 0 \text{ and } X_2'''(1) = 0 \tag{10}$$

Utilizing Eqs. (7), (9) and (10) leads to the following two equations for a fixed-free beam having a single crack

$$\left[-\frac{L^2}{K_b}\bar{S}_3''(1) - \frac{C_1L^2}{K_b}\bar{S}_3''(\chi_1)\bar{S}_2''(1-\chi_1)\right]M(0) + \left[-\frac{L^3}{K_b}\bar{S}_4''(1) - \frac{C_1L^2}{K_b}\bar{S}_4''(\chi_1)\bar{S}_2''(1-\chi_1)\right]V(0) = 0 \tag{11}$$

and

$$\left[-\frac{L^2}{K_b}\bar{S}_3'''(1) - \frac{C_1L^2}{K_b}\bar{S}_3''(\chi_1)\bar{S}_2'''(1-\chi_1)\right]M(0) + \left[-\frac{L^3}{K_b}\bar{S}_4'''(1) - \frac{C_1L^2}{K_b}\bar{S}_4''(\chi_1)\bar{S}_2'''(1-\chi_1)\right]V(0) = 0 \tag{12}$$

In the case of two cracks, the above equations turn into

$$\begin{aligned}&\left[-\frac{L^2}{K_b}\bar{S}_3''(1) - \frac{L^2}{K_b}\sum_{i=1}^{2}C_i\bar{S}_3''(\chi_i)\bar{S}_2''(1-\chi_i) - \frac{C_1C_2L^2}{K_b}\bar{S}_3''(\chi_1)\bar{S}_2''(\chi_2-\chi_1)\bar{S}_2''(1-\chi_2)\right]M(0)\\ &+\\ &\left[-\frac{L^3}{K_b}\bar{S}_4''(1) - \frac{L^3}{K_b}\sum_{i=1}^{2}C_i\bar{S}_4''(\chi_i)\bar{S}_2''(1-\chi_i) - \frac{C_1C_2L^3}{K_b}\bar{S}_4''(\chi_1)\bar{S}_2''(\chi_2-\chi_1)\bar{S}_2''(1-\chi_2)\right]V(0) = 0\end{aligned} \tag{13}$$

and

$$\left[-\frac{L^2}{K_b}\bar{S}_3'''(1) - \frac{L^2}{K_b}\sum_{i=1}^{2} C_i \bar{S}_3''(\chi_i)\bar{S}_2'''(1-\chi_i) - \frac{C_1C_2L^2}{K_b}\bar{S}_3''(\chi_1)\bar{S}_2''(\chi_2-\chi_1)\bar{S}_2'''(1-\chi_2) \right] M(0)$$
$$+ \tag{14}$$
$$\left[-\frac{L^3}{K_b}\bar{S}_4'''(1) - \frac{L^3}{K_b}\sum_{i=1}^{2} C_i \bar{S}_4''(\chi_i)\bar{S}_2'''(1-\chi_i) - \frac{C_1C_2L^3}{K_b}\bar{S}_4''(\chi_1)\bar{S}_2''(\chi_2-\chi_1)\bar{S}_2'''(1-\chi_2) \right] V(0) = 0$$

The frequency equation can be implemented by setting the second-order determinant that consists of the coefficients of $M(0)$ and $V(0)$ equal to zero. The calculated value of ω_i is substituted back into these equations with one of $M(0)$ and $V(0)$ set equal to 1 or any other value in order to obtain *i*th mode shape. Once the modal functions are determined, their time derivatives can readily be obtained. The first derivative of the mode shape yields the mode shape rotation, from which the mode shape rotation deviation curve can be readily computed. In the model-based predictions of crack parameters using the hybrid neuro fuzzy approach, these frequencies and mode shape rotation values are utilized.

2.2 Theoretical data simulation

The training set of the hybrid system is built up from the analytical solutions of Eqs. (11) and (12) for the beam with one crack, and Eqs. (13) and (14) for the beam with two cracks. Assuming different variations of crack location along the beam length and crack extent along the beam thickness, a number of damage scenarios are created. For the damaged beam having a single crack, the crack location x_1 is varied from 0.05L to 1.0L in an increment of 0.05 and crack depth a_1 is varied from 0.05h to 0.60h in an increment of 0.05. This generates 912 damage circumstances. An additional crack is introduced to the faulty beam element so as to obtain a beam with two damages. Both the spatial distributions and extents of the two cracks are altered to produce the input data for the neuro fuzzy system. This arrangement of crack parameters yields 2206 damage scenarios.

Of the dynamic properties of damaged structural beam elements determined using the procedure of the previous section, the natural frequency is the first input parameter selected for the inverse problem. This quantity has been shown to be a sensitive parameter and to have a good measuring accuracy. In order to ascertain the necessary input data and to determine uniquely the number of vibrational frequencies to be utilized in the development of neuro fuzzy systems, the effect of severity and location of damage on the natural frequencies should be investigated. Fig. 2 shows the frequencies of the first four modes of vibration (ω_{ic}) of cracked beams normalized by those of uncracked beams (ω_{io}). The figure presents the normalized eigenfrequencies versus crack location (x_1/L) for five different crack severities (a_1=0.025, 0.10, 0.20, 0.30 and 0.50h). It is observed that as the crack height increases, the change in the normalized frequencies also increases. This effect is most apparent in the fundamental mode. However, as the crack approaches the free end, the influence of crack start to diminish. The remaining three frequencies show much sharper variations with respect to crack position. It is noted that the crack has no influence on the frequencies at specific nodal points regardless of the crack extent. Fig. 3 displays the vibrational frequencies of damaged Euler-Bernoulli beam having two cracks. The location and severity of the first crack are fixed at x_1=0.35L and a_1=0.225h while those of the second crack are varied. Comparing to Fig. 2, one observes that this figure shows that the variations of eigenfrequencies of the beam having two cracks are similar, except that the influence of crack extent on this beam is higher, as expected. For all considered simulations of damaged beams with one and two cracks, it may be deduced that different vibration modes are influenced to different extents due to crack depth, depending on the spatial distribution of damage. Also, the different crack parameters leading to the same normalized frequencies make the inverse problem ill-conditioned. This in turn renders the development of the hybrid system harder. Therefore, an additional parameter is needed to improve the predictions of the system in solving the inverse problem.

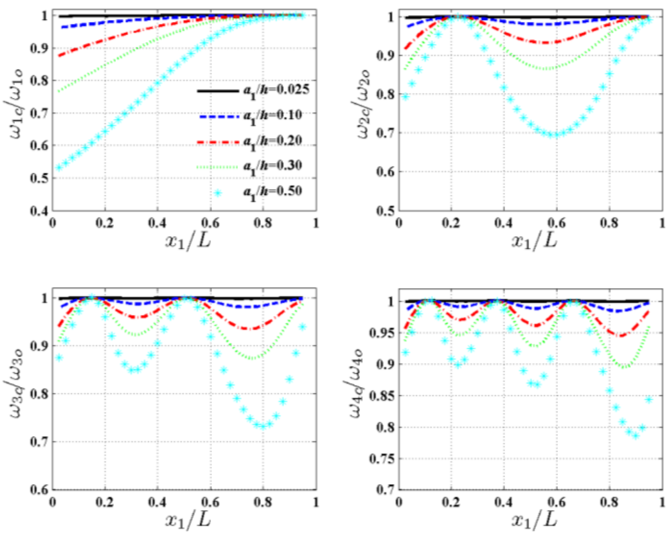

Fig. 2 Normalized natural frequencies of the first four modes versus normalized crack locations for the beam with one crack

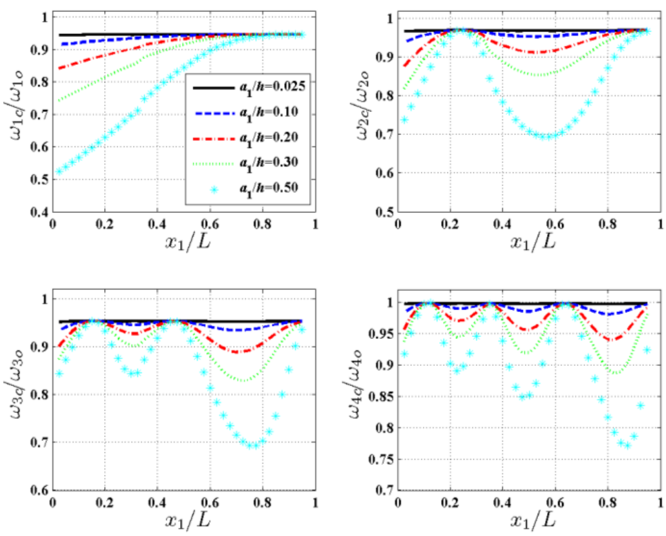

Fig. 3 Normalized natural frequencies of the first four modes versus normalized crack locations for the beam with two cracks

The square of the frequency ratio, frequency variation ratio, mode shape, mode shape derivatives, frequency response function, etc. have been employed in the past. This study uses mode shape rotation deviation curve as an additional parameter. It is inspired from the amplitude deviation curve introduced by Babu and Sekhar (2008) who studied the diagnostics of cracked rotors at operational conditions. Since the study herein considers only the free oscillation of damaged beam elements, amplitude deviation curve is defined in a slightly different way to generate mode shape rotation deviation curve. This definition has been exploited by the authors (Aydin and Kisi 2015) and has been shown potential for use in defect detection. Mode shape rotation deviation curve, similar to classical displacement mode shape, varies along the beam length for a given crack location and depth. Therefore, the maximum value of the curve corresponding to each mode of vibration is extracted as an input for damage detection scheme. It would of interest to see the variation of maxima of rotation deviation curves with respect to crack location and severity. Fig. 4 shows these variations for the first four modes of free oscillation. As the crack severity increases, the maxima of deviation curves also increase. Unlike the natural frequencies, the rotation deviation curve of the first mode is the least affected. The influence of crack depth on this mode is higher when the crack is in the proximity of the fixed end. The sharper shifts of maxima of deviation curves are observed for higher modes. In the case of two cracks, the maximum values of deviation curves are determined to be much close to those values in the case of one crack. For this reason, and considering that the beam has now two cracks, the second maxima of the rotation deviation curves are computed. Fig. 5 shows these values. It is observed that for the fundamental mode, the higher the crack depth, the lower the maxima. For the remaining modes, the variation of the second maxima with respect to crack extent depends on the crack position.

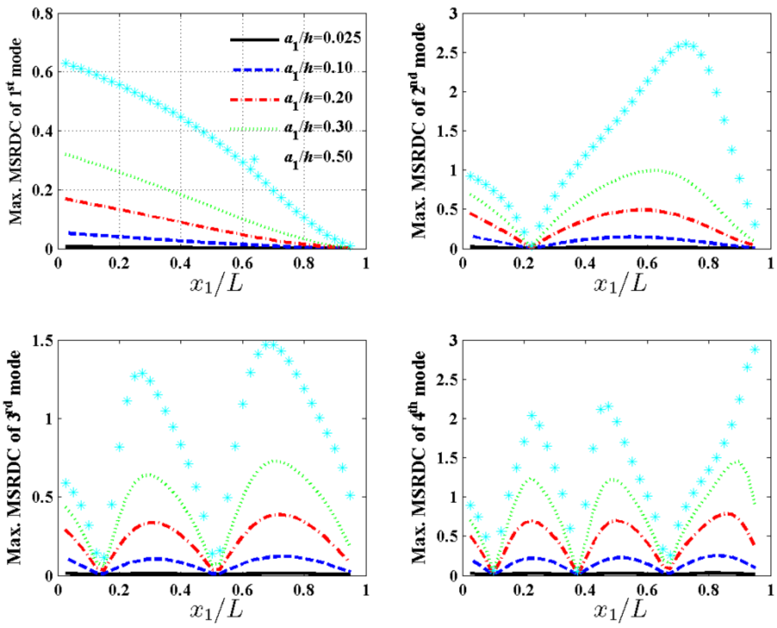

Fig. 4 Maxima of mode shape rotation deviation curves of the first four modes versus normalized crack locations for the beam with one crack

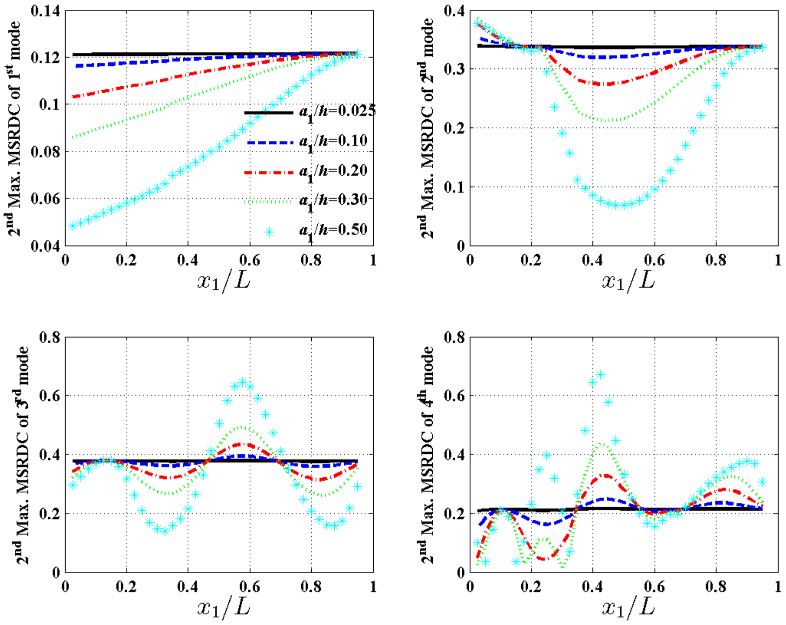

Fig. 5 The second maxima of mode shape rotation deviation curves of the first four modes versus normalized crack locations for the beam with two cracks

Combination of these peak deviation quantities with natural frequencies constitutes the input data to the neuro fuzzy system. The aim of the study is to utilize as few vibration-based analysis data as possible and at the same time to get a high accuracy in the estimates of the inverse problem. Therefore, only those values corresponding to the first four modes of vibration are employed as input to the fused artificial intelligence algorithm.

3. Adaptive neuro-fuzzy inference system

Adaptive Neuro-Fuzzy Inference System (ANFIS) is first introduced by Jang (1993). It is a universal approximator and capable of approaching any real continuous function on a compact set to any degree of accuracy (Jang *et al*. 1997). The ANFIS system used in this study is functionally equivalent to the Sugeno first-order fuzzy model (Jang *et al*. 1997, Drake 2000). Below, the hybrid learning algorithm composed of gradient descent and least-squares method is explained.

As an example, assume a fuzzy inference system with two inputs x and y and one output z. The first-order Sugeno fuzzy model, a typical rule set with two fuzzy If-Then rules can be expressed as

Rule 1: If x is A_1 and y is B_1, then $$z_1 = p_1 x + q_1 y + r_1 \tag{15}$$

Rule 2: If x is A_2 and y is B_2, then $$z_2 = p_2 x + q_2 y + r_2 \tag{16}$$

The resulting Sugeno fuzzy reasoning system is illustrated in Fig. 6. In this figure, the output z is the weighted average of the individual rule outputs and is itself a crisp value. Fig. 7 shows the equivalent ANFIS architecture. Same layer's nodes have similar functions. The node functions are described next. The output of the *i*th node in layer *l* is denoted as $O_{l,i}$.

Layer 1: Every node *i* in this layer is an adaptive node with node function

$O_{l,i} = \mu A_i(x)$, for i = 1, 2, or
$O_{l,i} = \mu B_{i-2}(y)$, for i = 3, 4

where x (or y) is the input to the *i*th node and A_i (or B_{i-2}) is a linguistic label (such as ‘‘low’’ or ‘‘high’’) associated with this node. Stated in words, $O_{l,i}$ is the membership grade of a fuzzy set A (= A_1, A_2, B_1, or B_2) and it expresses the degree to which the given input x (or y) satisfies the quantifier A. The membership functions for A and B are usually defined by generalized bell functions, e.g.

$$\mu A_i(x) = \frac{1}{1+\left[\left(x-c_i\right)/a_i\right]^{2b_i}} \tag{17}$$

where $\{a_i, b_i, c_i\}$ is the parameter set. As the values of these parameters vary, the bell-shaped function changes accordingly, thus, exhibiting various forms of membership functions on linguistic label A_i. Strictly speaking, any continuous and piecewise differentiable functions such as Gaussian membership functions that are frequently utilized are also qualified candidates for node functions in this layer (Jang 1993). Parameters in this layer are referred to as premise parameters. The outputs of this layer are the membership values of the premise part.

Layer 2: This layer consists of the nodes labeled Π which multiply incoming signals and sending the product out. For instance,

$$O_{2,i} = w_i = \mu A_i(x)\mu B_i(y), \quad i = 1, 2. \tag{18}$$

Each node output represents the firing strength of a rule.

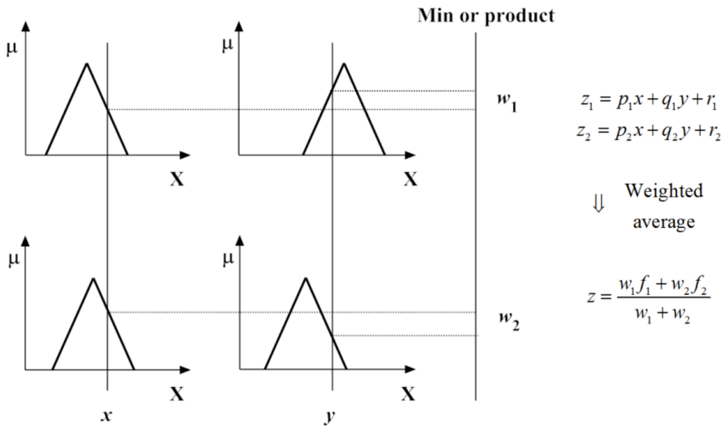

Fig. 6 Two-input first-order Sugeno fuzzy model with two rules (Kisi, 2006)

Layer 3: In this layer, the nodes labeled N calculates the ratio of the *i*th rule's firing strength to the sum of all rules' firing strengths

$$O_{3,i} = \bar{w}_i = \frac{w_i}{w_1 + w_2}, \quad i = 1, 2. \tag{19}$$

The outputs of this layer are called normalized firing strengths.

Layer 4: This layer's nodes are adaptive with node functions

$$O_{4,i} = \bar{w}_i z_i = \bar{w}_i \left(p_i x + q_i y + r_i \right) \tag{20}$$

where $\bar{w}_i$ is the output of layer 3, and $\{p_i, q_i, r_i\}$ are the parameter set. Parameters of this layer are referred to as consequent parameters.

Layer 5: This layer's single fixed node labeled Σ computes the final output as the summation of all incoming signals

$$O_{5,i} = \sum_{i=1} \bar{w}_i z_i = \frac{\sum_i w_i z_i}{\sum_i w_i} \tag{21}$$

Thus, an adaptive network is functionally equivalent to a Sugeno first-order fuzzy inference system.

The learning rule specifies how the premise parameters (see layer 1) and consequent parameters (see layer 4) should be updated to minimize a prescribed error measure *E*. The error measure is a mathematical expression that measures the difference between the networks actual output and the desired output, such as the squared error. The steepest descent method is used as the basic learning rule of the adaptive network. In this method the gradient is derived by repeated application of the chain rule. Calculation of the gradient in a network structure requires use of the ordered derivative denoted as ∂^+ as opposed to the ordinary partial derivative ∂. This technique is called the back propagation rule (Jang 1993, Drake 2000). The core of this learning rule involves how to recursively obtain a gradient vector in which each element is defined as the derivative of an error measure with respect to a parameter (Haykin 1998). The update formula for simple steepest descent for the generic parameter $\propto$ is

$$\Delta\alpha = -\eta \frac{\partial^+ E}{\partial \alpha} \tag{22}$$

where η is the learning rate.

While the back propagation learning rule can be used to identify the parameters in an adaptive network, this method is slow to converge. The hybrid learning algorithm (Jang 1993), which combines back propagation and the least-squares method can be used to rapidly train and adapt the equivalent fuzzy inference system. It can be seen from the Fig. 7 that if the premise parameters are fixed, the overall output can be given as a linear combination of the consequent parameters. The output *z* can be written as

$$\begin{aligned} z &= \frac{w_1}{w_1+w_2} z_1 + \frac{w_2}{w_1+w_2} z_2 \\ &= \bar{w}_1 \left(p_1 x + q_1 y + r_1 \right) + \bar{w}_2 \left(p_2 x + q_2 y + r_2 \right) \\ &= \left(\bar{w}_1 x \right) p_1 + \left(\bar{w}_1 y \right) q_1 + \left(\bar{w}_1 \right) r_1 + \left(\bar{w}_2 x \right) p_2 + \left(\bar{w}_2 y \right) q_2 + \left(\bar{w}_2 \right) r_2 \end{aligned} \tag{23}$$

which is linear in the consequent parameters p_1, q_1, r_1, p_2, q_2, and r_2. Then we have

S = set of total parameters,
S_1 = set of premise (nonlinear) parameters,
S_2 = set of consequent (linear) parameters.

Given the values of S_1, we can plug P training data into Eq. (23) and obtain the matrix equation

$$A\theta = y \tag{24}$$

where θ is an unknown vector whose elements are parameters in S_2, the set of consequent (linear) parameters.

Then the set S_2 of consequent parameters can be identified with the standard least-squares estimator (LSE):

$$\theta^* = \left(A^T A \right)^{-1} A^T y \tag{25}$$

where A^T is the transpose of A and $(A^T A)^{-1} A^T$ is the pseudo-inverse of A if $A^T A$ is nonsingular. The recursive least-square estimator (RLS) can also be used to calculate θ^* (Jang 1993).

Back propagation and LSE can now be combined to update the parameters of the adaptive network. For hybrid learning applied in batch mode (off-line learning), each epoch is composed of a forward pass and a backward pass as summarized in Table 1.

In the forward pass of the hybrid learning algorithm, node outputs go forward until the final layer (layer 4 in Fig. 7) and the consequent parameters are identified by the least-squares method. In the backward pass, the error signals propagate backward and the premise parameters are updated by gradient descent. More information for ANFIS can be found in Jang (1993).

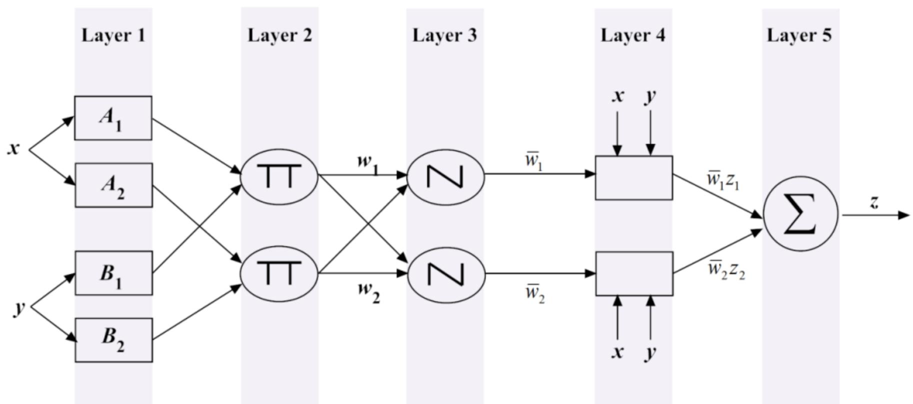

Fig. 7 Equivalent ANFIS architecture (Kisi 2006)

Table 1 Hybrid learning procedure of ANFIS for one epoch (Kisi 2006)

	Forward Pass
Layer 1	Set premise parameters of the membership functions, arbitrarily (e.g., $\{a_i, b_i, c_i\}$ and compute the membership values (e.g., $\mu A_i(x)$, $\mu B_i(y)$).
Layer 2	Compute the firing strength of the i_{th} rule by multiplying the membership values obtained in layer 1 (e.g., $w_i = \mu A_i(x) \times \mu B_i(x)$).
Layer 3	Compute the normalized firing strengths by calculating the ratio of the *i*th rule's firing strength to the sum of all rules' firing strengths (e.g., $\overline{w}_i = w_i / \sum_{i=1}^{2} w_i$)
Layer 4	Obtain the weighted node function by multiplying the normalized firing strengths with the node function for each rule (e.g., $\overline{w}_i z_i = \overline{w}_i (p_i x + q_i y + r_i)$, here the p_i, q_i and r_i are the consequent parameters).
Layer 5	Obtain the matrix equation $A \cdot \theta = y$ using the Eq. (23). The θ is an unknown vector comprising the consequent parameters. Determine the consequent parameters with the standard least-squares estimator (LSE) given by the Eq. (25).
	Backward Pass
Layer 1	Determine the premise parameters using the update formula of simple gradient descent given by Eq. (22).

In the present study two different ANFIS methods, Grid Partition (GP) and Subtractive Clustering (SC) were used. GP divides the input space into rectangular subspaces using a number of local fuzzy regions by axis-paralleled partition based on predefined number of membership functions and their types in each dimension. Least square method is used for calculating fuzzy sets' parameters. During constructing the fuzzy rules, consequent parameters in the output membership function are set to zero. Then, the parameters are identified and refined by using ANFIS. The detailed information about GP and its combination with ANFIS (ANFIS-GP) can be found in Abonyi *et al*. (1999). Increasing the number of input variables exponentially increases the number of fuzzy rules. For example, if we have *m* input variables and *n* membership functions for each input variable, the total number of fuzzy rules is equal to m^n (Wei *et al*. 2007). SC is an extension of mountain clustering method proposed by Yager and Filev (1994) in which each data point (not a grid point) is considered as a center for potential cluster center (Chiu 1994). Using this method, the number of effective “grid points” to be evaluated equals to the number of data points, independent of the dimension of the problem. Another advantage of SC method is that it eliminates the need to specify a grid resolution, in which tradeoffs between accuracy and computational complexity must be considered. The SC method also extends the criterion of the mountain method for accepting and rejecting cluster centers. The detailed information about GP and its combination with ANFIS (ANFIS-SC) can be found in Wei *et al*. (2007) and Cobaner (2011).

4. Results

4.1 Application of neuro fuzzy models to noise-free data

This section presents the design and predictions of ANFIS models for damage identification in structural beam elements. The model is based on Euler-Bernoulli beam theory and only support conditions of cantilever beam are considered. Cantilever beam-type components represent the response of various structures in civil, mechanical and microelectromechanical systems. The inverse problem of determining the extent and position of cracks are investigated in the present study and beams with one and two cracks are considered. Two different program codes in MATLAB including Fuzzy Logic toolboxes are written for the ANFIS-GP and ANFIS-SC simulations. Various types of fuzzy membership functions are used for the ANFIS-GP simulations. Mean absolute relative error (MARE), root mean square error (RMSE) and determination coefficient (R^2) criteria are utilized for evaluating the accuracy of each model. The R^2 indicates the degree to which two variables are linearly related. MARE and RMSE show different types of information about the predictive capabilities of the model. The RMSE measures the goodness-of-fit relevant to high values whereas the MARE yields a more balanced perspective of the goodness-of-fit at moderate values (Karunanithi *et al*. 1994). Although both MARE and RMSE measure the average magnitude of the error in the predictive model, MARE is a linear score, meaning that all the individual differences are weighted equally in the average, whereas RMSE is a quadratic scoring rule. The optimal model should have the minimal RMSE and MARE, and an R^2 should be close to 1. The MARE, RMSE and R^2 statistics can be expressed as

$$\mathrm{MARE} = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{Y_O - Y_M}{Y_O}\right|100 \tag{26}$$

$$\mathrm{RMSE} = \frac{1}{N}\sqrt{\sum_{i=1}^{N}\left(Y_O - Y_M\right)^2} \tag{27}$$

$$\mathrm{R}^2 = \left(\frac{\sum_{i=1}^{N}\left(Y_O - \bar{Y}_O\right)\sum_{i=1}^{N}\left(Y_M - \bar{Y}_M\right)}{\sqrt{\sum_{i=1}^{N}\left(Y_O - \bar{Y}_O\right)^2 \sum_{i=1}^{N}\left(Y_M - \bar{Y}_M\right)^2}}\right)^2 \tag{28}$$

where N is the number of observations, Y_o is the observed datum, Y_M is the corresponding simulated datum, $\overline{Y}_O$ and $\overline{Y}_M$ are mean of the observed and simulated values.

In the first part of the study, the crack parameters are tried to be determined for the Euler-Bernoulli beam having a single crack. The natural frequencies and the maxima of mode shape rotation deviation curves of the first four modes and the spatial location of the maximum of mode shape rotation deviation curve of the fundamental mode of vibration are used as inputs to the ANFIS models to predict crack location and crack depth. Optimum parameters of the neuro fuzzy models are obtained by minimizing the objective function (RMSE between calculated and observed values) in test period. The final architectures of the ANFIS models are determined by trial and error process for each model. In order to develop ANFIS-SC, determining the proper cluster radius is critical. The cluster radius shows the range of influence of a cluster when one assumes the data space as a unit hypercube. Radius values vary between 0 and 1. Assigning a small cluster radius will yield many small clusters in the data, (with many rules resulting many membership functions and parameters) and specifying a large cluster radius will produce a few large clusters in the data, (with fewer rules resulting fewer membership functions and parameters). The test results of the optimal neuro fuzzy models in crack estimation are shown in Table 2. The model structures are given in the second column of the table. Here, (2,gaussmf) indicates an ANFIS-GP model having 2 Gaussian membership functions for each input and (2, 0.95) reveals an ANFIS-SC model having 2 clusters and radius value of 0.95. It is clear from the table that the ANFIS-SC model has the lowest MARE (0.15), RMSE (0.0003) and the highest R^2 (0.9999) values in the prediction of crack location. Table 1 also shows that the ANFIS-SC model whose inputs have 2 clusters and radius value of 0.80 performs better than the ANFIS-GP model in the prediction of crack depth. The estimates of the optimal ANFIS-GP and ANFIS-SC models in test period are shown in Figs. 8 and 9. These figures present the best predictions of the ANFIS structures for the crack location and depth. These predictions are obtained using the previously constituted input data. The fit line equations and R^2 values of each model are also provided in these figures. It is observed from the graphs that the estimates of the both ANFIS models closely follow the corresponding calculated crack location and depth values and ANFIS-SC model performs slightly better than the ANFIS–GP models.

Table 2 Test results of the ANFIS-GP and ANFIS-SC models in crack estimation (beam with 1 crack)

Model	**Model Structure**	**MARE**	**RMSE**	**R^2**
			Crack location	
ANFIS-GP	(2,gaussmf)	3.25	0.0055	0.9996
ANFIS-SC	(2, 0.95)	0.15	0.0003	0.9999
			Crack depth	
ANFIS-GP	(2,gaussmf)	9.62	0.0179	0.9902
ANFIS-SC	(3, 0.80)	8.71	0.0137	0.9940

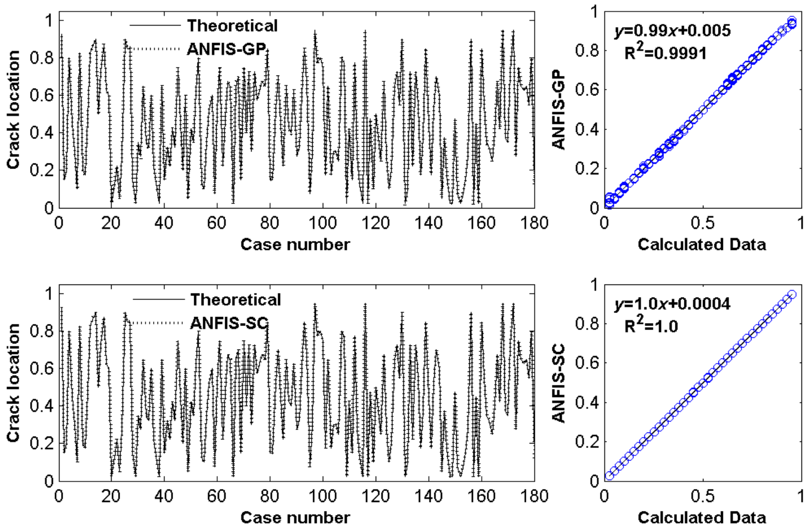

Fig. 8 Calculated and modeled crack location by ANFIS-GP and ANFIS-SC models in test phase

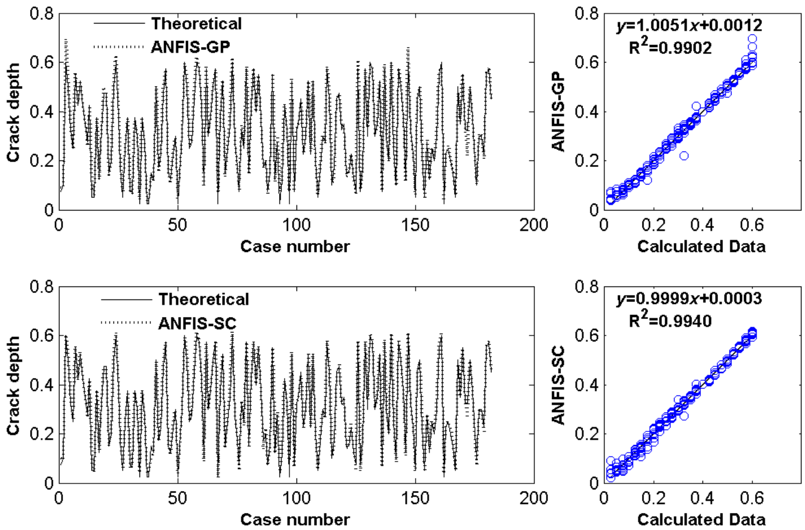

Fig. 9 Calculated and modeled crack depth by ANFIS-GP and ANFIS-SC models in test phase

In the second part of the study, four ANFIS-GP and ANFIS-SC models are developed using ten inputs to predict location and depth of the two cracks. Table 3 compares the test results of the optimal ANFIS models in the estimation of two cracks. The table indicates that the ANFIS-SC model comprising 4 cluster (radius=0.95) performs better than the ANFIS-GP model whose inputs have 2 Gaussian membership functions in the location prediction of both of the cracks. The ANFIS-SC model has a better performance than the ANFIS-GP model for estimating the depth of the first crack while the both models show almost same accuracy for estimating the depth of the second crack. The estimates of the optimal ANFIS-GP and ANFIS-SC models in test period are shown in Figs. 10 and 11 for the first crack. It is clearly seen from the fit line equations (assume that the equation is $y=ax+b$) given in scatterplots that the coefficients a and b of the ANFIS-SC models are respectively closer to the 1 and 0 with higher R^2 values than those of the ANFIS-GP models.

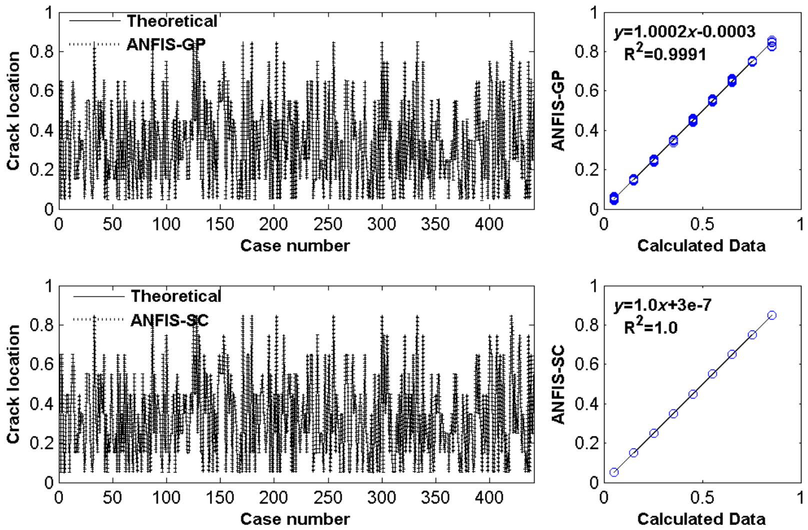

Fig. 10 Calculated and modeled location of the first crack by ANFIS models in test phase

Table 3 Test results of the ANFIS-GP and ANFIS-SC models in crack estimation (beam with two cracks)

Model	**Model Structure**	**MARE**	**RMSE**	**R^2**		**MARE**	**RMSE**	**R^2**
				Crack 1				
			Location			***Depth***		
ANFIS-GP	(2,gaussmf)	1.61	0.0040	0.9997	(2,gaussmf)	5.98	0.0115	0.9968
ANFIS-SC	(4, 0.95)	0.11	0.0003	0.9999	(5,0.85)	5.53	0.0113	0.9969
				Crack 2				
			Location			***Depth***		
ANFIS-GP	(2,gaussmf)	0.71	0.0051	0.9994	(2,gaussmf)	19.6	0.0401	0.9595
ANFIS-SC	(4, 0.95)	0.03	0.0002	0.9999	(9,0.70)	19.4	0.0404	0.9586

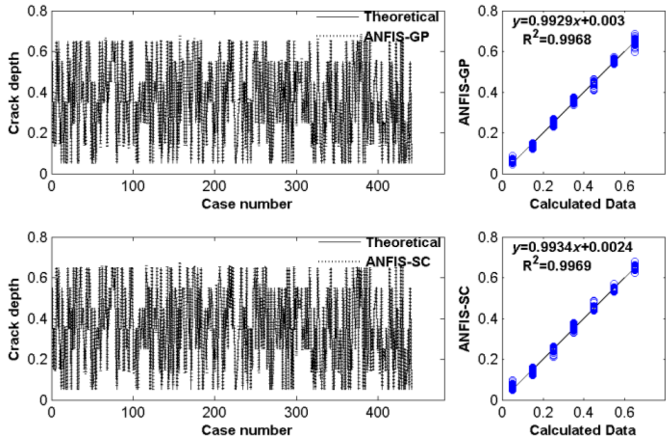

Fig. 11 Calculated and modeled depth of the first crack by ANFIS models in test phase

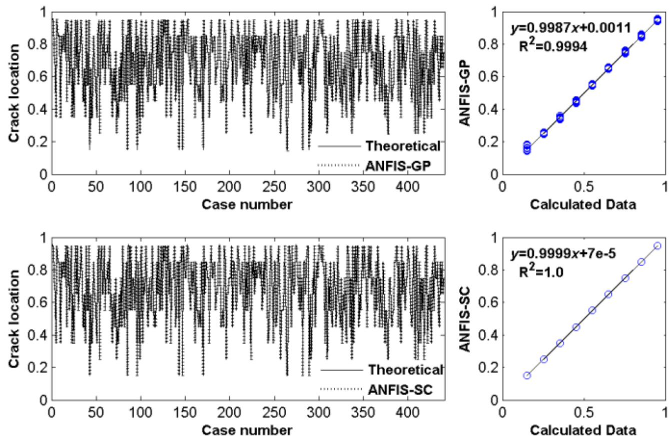

Fig. 12 Calculated and modeled location of the second crack by ANFIS models in test phase

Figs. 12 and 13 illustrate the location and depth estimates of the second crack. It is seen from the figures that the estimates of the both ANFIS models closely follow the corresponding calculated values, specifically the crack locations. It is realized from these figures that the both model estimates are more scattered for crack depths than those of the crack locations. This is also valid for the case of single crack. This is most likely due to the ill-posed nature of the inverse problem wherein more than one crack positions and severities produce the same modal properties of beam elements. Another observation from the figures is that the better estimates are obtained in the case of one crack compared to the case of multiple cracks. The reason for this is that a higher chance of the same natural frequencies and mode shape deviation curves from a different set of crack parameters exist now in the case of two cracks.

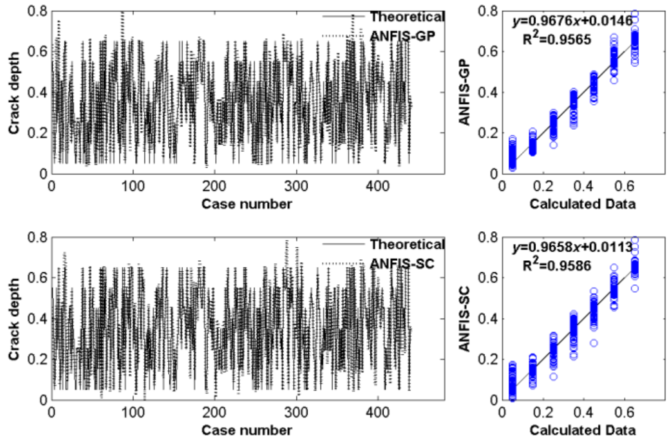

Fig. 13 Calculated and modeled depth of the second crack by ANFIS models in test phase

4.2 Application of neuro fuzzy models to noise-contaminated data

In order to assess the robustness of the optimal neuro fuzzy models developed in the previous section, they need to be tested against noisy data. In this way, a better generalization during the training ANFIS models and simulation of the uncertainties of ambient vibration and environmental effects would be performed. Artificial random noise with a normal distribution having a zero mean and a variance and standard deviation of one is added to the raw input data according to the following equation

$$data_{\text{with noise},\, n} = data_{\text{noise-free},\, n} \left(1 + SF \cdot y\right) \tag{29}$$

where *data* are un-normalized natural frequencies or mode shape rotation deviation curves, *SF* is the scale factor (or noise level), *y* is the randomly generated noise vector and *n* is the number of mode under consideration. The random noise is multiplied by a frequency modulation function to take into account the dependency of noise on frequency values (Wang and Chuang 2004). The noise level introduced into the frequency and modal vectors hence increases with increasing frequency values. It is generally assumed that the variations in frequency measurements due to environmental conditions can be as high as 5% (Fan and Qiao 2011). Therefore, three different noise levels, i.e., 1, 3 and 5 are considered in the study. Noise levels of 1 and 3% may be argued to be small whereas 5% may be viewed as somewhat high level of contamination.

The optimal neuro fuzzy models that have provided the lowest error metrics in noise-free cases are tested herein. Table 4 presents the error estimates of the optimal ANFIS-SC models for the beam having a single crack. It is seen that the both models of crack location and severity perform reasonably well at all levels of noise. Table 5 shows the performance of the optimal ANFIS-SC models, each having different cluster and radius, in the presence of noise for the beam having two cracks. It is observed that the model predictions are worse than those in the case of single crack. Although the performance of the optimal models can be acceptable for 5% noise ratio, the performance is expected to further deteriorate for higher noise ratios. The reason for this is attributed to the fact that the membership functions are determined based on clustering and the number of fuzzy rules is equal to that of membership functions. Hence, the ANFIS-SC is sensitive to higher levels of noise. Another observation from Tables 4 and 5 is that the developed models predict the crack location better than the crack extent in both one and two crack cases at all noise levels. This may be due to non-uniqueness of the relation between the crack parameters and vibration data.

Table 4 Error metrics of the neuro fuzzy systems in crack parameters using noisy data (beam with 1 crack)

	Noise Level	**MARE**	**RMSE**	**R^2**
	1%	0.6821	0.0044	0.9997
Crack location	3%	1.9023	0.0129	0.9978
	5%	3.2458	0.0228	0.9932
	1%	12.902	0.0191	0.9885
Crack height	3%	24.053	0.0289	0.9739
	5%	25.044	0.0358	0.9598

Table 5 Error metrics of the neuro fuzzy systems in crack parameters using noisy data (beam with 2 cracks)

	Noise Level	**MARE**	**RMSE**	**R^2**
	1%	0.7538	0.0038	0.9997
Location of 1st Crack	3%	2.0253	0.0105	0.9978
	5%	3.6177	0.0173	0.9939
	1%	5.7832	0.0154	0.9942
Height of 1st Crack	3%	9.9033	0.0315	0.9764
	5%	13.1297	0.0529	0.9349
	1%	0.7765	0.0069	0.9990
Location of 2nd Crack	3%	2.2386	0.0197	0.9915
	5%	4.0705	0.0338	0.9760
	1%	21.4581	0.0470	0.9442
Height of 2nd Crack	3%	24.3069	0.0561	0.9232
	5%	30.9971	0.0724	0.8748

5. Conclusions

This study examines the relevance of fused neuro fuzzy systems for damage identification in structural beam elements. Due to common usage in various fields and low sensitivity of modal properties to damage, boundary conditions of cantilever beam are considered. Beams with single and multiple cracks are studied in the inverse problem of determining the crack parameters-crack spatial distribution and severity. Two different codes are prepared for the hybrid models of ANFIS-GP and ANFIS-SC. Various types of membership functions are evaluated in the ANFIS-GP simulations while the number and radius of cluster are tried to be determined in the ANFIS-SC simulations. Optimum parameters of these simulations are determined by minimizing the objective function in test periods of the fused systems. It is shown that the ANFIS-SC model having 2 clusters and radius value of 0.95 for the crack location and that model having 3 clusters and radius value of 0.80 for the crack extent yield the lowest error estimates for the beam with a single crack. In the case of beam element having two cracks, the ANFIS-SC model containing 4 clusters with a radius of 0.95 performs better that the ANFIS-GP models in the location prediction of both of the cracks. The ANFIS-SC model has also a better performance than the ANFIS-GP for predicting the depth of the first crack while the two models have close accuracy for estimating the depth of the second crack. It is observed from the figures showing the performance of ANFIS-GP and ANFIS-SC models and from the quantitative values of error metrics that the model predictions of crack depth are somewhat poorer than those of crack position. This may be attributed to the ill-conditioned property of the inverse problem studied. Another reason is that the crack extent is not as significant as crack spatial distribution on the natural frequencies and mode shape rotation deviation values. The robustness of these optimal neuro fuzzy models is tested against noise-contaminated data. For this purpose, a random noise with zero mean and unit variance and standard deviation is added to the raw data. In order to consider the frequency dependency of noise, the random noise vector is scaled by a frequency modulation function in time domain. Although the performance of the optimal models degrades with augmenting noise ratios, the accuracy is rather reasonable for a noise level of 5% anticipated in real-life conditions.

Sound as the results of this study may appear, the reader is cautioned for two points: 1) the crack on beam element is modeled as an elastic spring, which is an apparent simplification of crack dynamics. Since the formulation of this model has not yet been proven theoretically and some aspects have not been completely clarified, one should be careful in utilizing the localized massless spring model particularly when there are a number of cracks that are closely spaced on beam elements. 2) The developed hybrid models need to be verified employing the experimentally obtained response data. As future work, the analysis of the two neuro fuzzy systems can be extended for localization and identification of crack in more complex beam or plate structures. The current research is carried out based on the Euler-Bernoulli beam-like structures but it can also be applied for Timoshenko beam-like structures.

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