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An effective online delay estimation method based on a simplified physical system model for real-time hybrid simulation

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Abstract. Real-Time Hybrid Simulation (RTHS) is a novel approach conceived to evaluate dynamic responses of structures with parts of a structure physically tested and the remainder parts numerically modelled. In RTHS, delay estimation is often a precondition of compensation; nonetheless, system delay may vary during testing. Consequently, it is sometimes necessary to measure delay online. Along these lines, this paper proposes an online delay estimation method using least-squares algorithm based on a simplified physical system model, i.e., a pure delay multiplied by a gain reflecting amplitude errors of physical system control. Advantages and disadvantages of different delay estimation methods based on this simplified model are firstly discussed. Subsequently, it introduces the least-squares algorithm in order to render the estimator based on Taylor series more practical yet effective. As a result, relevant parameter choice results to be quite easy. Finally in order to verify performance of the proposed method, numerical simulations and RTHS with a buckling-restrained brace specimen are carried out. Relevant results show that the proposed technique is endowed with good convergence speed and accuracy, even when measurement noises and amplitude errors of actuator control are present.

Keywords: real-time hybrid simulation; delay compensation; online delay estimation; least-squares algorithm

1. Introduction

Recently due to cost-effectiveness and uniqueness, much worldwide attention has been paid to Real-Time Hybrid Simulations (RTHS) for the evaluation of dynamic performance of structures with complicated and/or rate-dependent components or substructures (Nakashima *et al*. 1992, Blakeborough *et al*. 2001, Jung and Shing 2006, Bursi and Wagg 2008, Carrion *et al*. 2008, Wu *et*

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al. 2009, Bursi *et al*. 2010, Wu *et al*. 2011). This method consists of physical simulations of complicated and/or rate-dependent parts of structures and numerical simulations of the well-understood remainder parts. In order to obtain reliable test results, high performance of integration schemes and loading control are required; hence, these subjects represent hot research topics. Accurate loading control, as one challenge of RTHS, allows two substructures to be compatible, requiring proper delay treatment and transfer system control. Since first identified in 1999 (Horiuchi *et al*. 1999), delay estimation and compensation have been research focuses in this field owing to negative influence of system delay on test results. Delay herein is defined as the time needed from displacement command being sent to actuator and for actuator reaching required position (after Darby *et al*. 2002). It is resulted from actuator dynamics and hence inherently inevitable. Horiuchi *et al*. (1999) investigated the effect of delay in RTHS and showed that delay equivalently adds negative damping to a structure, when a stiffness-based specimen is taken as the physical substructure. If negative damping exceeds the actual damping of a structure, experimental results will be affected by loss of stability. Delay differential equations were also used to study the influence of delay based on a continuous system model; moreover, critical delay above which the system becomes unstable was identified (Wallace *et al*. 2005a).In addition, spectral analysis techniques were also applied to study delay influence on stability of time-stepping algorithms for RTHS (Wu *et al*. 2013).

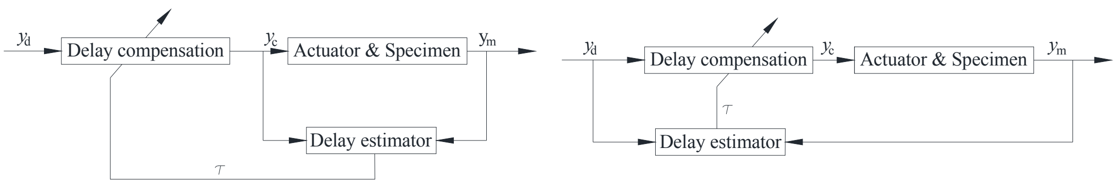
Delay compensation for RTHS was widely investigated in literature, and numerous compensation schemes were proposed and proved effective. Horiuchi *et al*. (1999) proposed a polynomial extrapolation approach based on delay information and desired displacements in previous steps; it is extensively applied owing to its simplicity. In order to smooth actuator movement, interpolation was introduced to the compensation by Nakashima and Masaoka (1999) and Darby *et al*. (2001), respectively. Wallace *et al*. (2005b) used polynomial fitting incorporated in a least-squares algorithm, to filter out the displacement measurement noise as well as to compensate for delay. Different from methods based on mathematical fitting of a polynomial, another type of methods based on kinematics assumptions was proven more favourable. These methods include that based on linearly predicted acceleration (Horiuchi and Konno 2001) and onthe explicit Newmark method (Ahmadizadeh *et al*. 2008), respectively. Delay actually implies phase lag between desired and measured displacements, so many control strategies, such as phase-lead network (Zhao *et al*. 2003), feedforward control (Jung *et al*. 2006), inverse control (Chen *et al*. 2009) and outer loop control (Bonnet *et al*. 2007b) were employed to reduce or compensate for this lag, and hence, to partially achieve objective of delay compensation. A phase lag compensation method based on an ARMAX model and Pseudo Linear Regression with forgetting factor was conceived and validated by Nguyen and Dorka (2008). Also adaptive delay compensation techniques based on varying system delay was deeply investigated by Wallace *et al*. (2005b), Nguyen *et al*. (2011), Chen *et al*. (2012) and Chae *et al*. (2013), among others.

Those compensation schemes developed by Horiuchi *et al*. (1999, 2001), Wallace *et al*. (2005b) and Ahmadizadeh *et al*. (2008), among others, require explicit delay information. Further studies showed that system delay varies according to specimen stiffness (Darby *et al*. 2001) and to other possible causes, such as command frequencies and amplitude and adaptive controllers. Therefore in order to obtain reliable experimental results, online delay estimation is deemed to be essential for complicated and large scale problems. Some methods to measure delay online were developed by Darby *et al*. (2002), Ahmadizadeh *et al*. (2008) and other researchers. Unfortunately, owing to some limitations as demonstrated in numerical simulations and actual tests later in this paper, these methods seem not to be adequate for wide applications to RTHS. In this paper, delay estimation methods based on a simplified physical system model, i.e., a pure delay model multiplied by a gain reflecting control amplitude errors, are investigated; and in particular, in order to improve its performance, the least-squares method is introduced to a Taylor series based law. As a result, any compensation method based on displacement prediction can be applied together with the proposed estimator.

The remainder of the paper is organized as follows. Section 2 briefly introduces the framework of RTHS with online delay estimation. Section 3 presents and comments on delay estimation methods based on a simplified physical system model according to both Newton's method and Taylor series. Along that line, a new method based on Taylor series together with the least-squares algorithm is proposed in Section 4. Successively, numerical simulations on the ability of the proposed method to estimate time-invariant and time-varying delays are presented in Section 5;in addition, numerical comparisons of three delay estimation schemes for RTHS with a second order actuator model are shown. Section 6 describes RTHS with a buckling-restrained brace as physical substructure that be haves both in the linear and in the nonlinear range. Finally, brief conclusions are drawn in Section 7.

2. Framework of RTHS with online delay estimation

In RTHS, delay can be measured with commanded and measured displacements, or with desired and measured displacements; hence as shown in Fig. 1, two different frameworks can be conceived for RTHS. Clearly in both cases, the same response prediction for delay compensation can be evaluated; the actual difference is related to the delay estimators, which determine how to obtain the system delay according to displacement time histories. In the first framework, the measured delay is the actual system delay between commanded and measured displacements; in the second framework, delay between desired and measured signals is measured, which is what we try to minimize via a delay compensation scheme. Therefore in the second case, the measured delay depends on delay compensation; this implies that the resulting estimated delay is dependent both on the physical system and on the compensation scheme. Conversely in the first case, both delay estimation and compensation are uncoupled; consequently, it is easier to measure system delay. In summary, in order to propose a practical and effective method, this paper relies on the first framework.



(a) Based on estimator with commanded and measured displacements

(b) Based on estimator with desired and measured displacements

Fig. 1 Framework of RTHS with online delay estimators

3. Delay estimators based on simplified physical system model

A delay-compensated system is schematically depicted in Fig. 2(a). Firstly, the command y_{c} is generated by applying a compensation scheme to the desired (analysed) displacement y_{d}. In order to smoothly drive the physical system, interpolation is frequently carried out but not included in the figure. Then at the specified instant, the achieved displacement y_{m} of the physical system is measured. For simplicity, it is assumed that both the actuator and the specimen share the same displacement response, i.e., y_{m}. As shown in Fig. 2(b), the delay compensation process and the physical system are modelled by a pure delay multiplied by a gain reflecting amplitude errors of actuator control; this model was justified after Bonnet (2006). In Fig. 2(b), τ_{c} denotes the estimated delay while τ_{a} the actual delay of the actuator. Therefore, we can formulate relationships between various displacements as followsEquation Chapter 1 Section 1(1)

$$y_{\mathrm{c}}(t_i-\tau_{\mathrm{c}})=k_{\mathrm{c}}y_{\mathrm{d}}(t_i) \tag{1}$$

$$y_{\mathrm{m}}(t_i)=k_{\mathrm{a}}y_{\mathrm{c}}(t_i-\tau_{\mathrm{a}}) \tag{2}$$

where gains k_{a} and k_{c} are referred to as amplitude factors, i indicates the i-th sampling. Eq. (2) shows that the measured displacement $y_{\mathrm{m}}(t_i)$ is dependent on time and system delay. Inserting (1) into (2) yields

$$y_{\mathrm{m}}(t_i)=k_{\mathrm{a}}k_{\mathrm{c}}y_{\mathrm{d}}(t_i+\tau_{\mathrm{c}}-\tau_{\mathrm{a}}) \tag{3}$$

Evidently, if $\tau_{\mathrm{c}}=\tau_{\mathrm{a}}$ and $k_{\mathrm{c}}=1/k_{\mathrm{a}}$, perfect compensation is achieved, which entails neither phase lag nor phase lead between desired and measured displacements.

3.1 Use of Newton's method

3.1.1 Delay estimation between commanded and measured displacements

Eq. (2) can be regarded as a nonlinear equation with unknown τ_{a} and constant $y_{\mathrm{m}}(t_i)$. Therefore, Newton’s method (Isaacson and Keller 1994) can be applied to obtain the unknown delay, i.e.

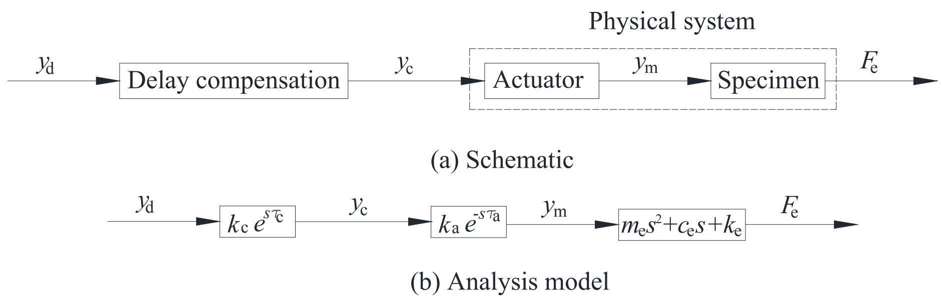


Fig. 2 Schematic and analysis model of a delay-compensated system

$$\tau_{\mathrm{a},i}^{(j+1)}=\tau_{\mathrm{a},i}^{(j)}-\frac{y_{\mathrm{m}}(t_i)-k_{\mathrm{a}}y_{\mathrm{c}}(t_i-\tau_{\mathrm{a},i}^{(j)})}{k_{\mathrm{a}}\dot{y}_{\mathrm{c}}(t_i-\tau_{\mathrm{a},i}^{(j)})} \tag{4}$$

where $y_{\mathrm{c}}(t_i-\tau_{\mathrm{a},i}^{(j)})$ and $\dot{y}_{\mathrm{c}}(t_i-\tau_{\mathrm{a},i}^{(j)})$ denote commanded displacement and velocity at the instant $(t_i-\tau_{\mathrm{a},i}^{(j)})$. They are available since $\tau_{\mathrm{a},i}^{(j)}$ is theoretically greater than zero. In this formula, *i* and *j* indicate the *i*-th sampling and *j*-th iteration, respectively.

For a common nonlinear equation, Eq. (4) can be applied until convergence is achieved. However, in real-time tests, the iteration may be not convergent because of the limited time range before the estimated delay is required for compensation in the subsequent step. Taking into account of these, real-time iteration, which conducts single iteration at one time step instead of many iterations until convergence is achieved, is a more realistic solution. This method was utilized for nonlinear optimization in optimal feedback control (Diehl *et al.* 2005). It is expressed as

$$\tau_{\mathrm{a},i+1}=\tau_{\mathrm{a},i}-\frac{y_{\mathrm{m}}(t_i)-k_{\mathrm{a}}y_{\mathrm{c}}(t_i-\tau_{\mathrm{a},i})}{k_{\mathrm{a}}\dot{y}_{\mathrm{c}}(t_i-\tau_{\mathrm{a},i})} \tag{5}$$

Wang (2012) provided a sufficient condition for convergence of this iteration. Although convergence might be guaranteed, problems in Section 3.3 require careful treatment prior to its application in RTHS.

3.1.2 Delay estimation between desired and measured displacements

The purpose of delay estimation is to compensate for system delay. So the delay can be used as an intermediate variable in a closed-loop delay compensation scheme. Apparently, the delay should be adapted in order to match measured and desired displacement as close as possible. In order to investigate the theoretic foundation of the estimator for system delay, one can consider the system delay τ_{a} as an parameter herein rather than a variable determined by iteration in the last subsection. Note that τ_{a} is not explicitly required to estimate τ_{c} in the actual test; in fact τ_{a} is inherently included in the actual actuator system and the adaptation of τ_{c} is needed to match the measured track with the desired one. If the dynamics can be represented by pure delay, then for a perfect compensation, τ_{c} must be equal to the system delay τ_{a}. Hence, the estimated delay τ_{c} should be chosen to match the realized or measured displacement with the desired or calculated one; therefore, τ_{c} should satisfy

$$y_{\mathrm{m}}(t_i)=k_{\mathrm{a}}k_{\mathrm{c}}y_{\mathrm{d}}(t_i+\tau_{\mathrm{c}}-\tau_{\mathrm{a}})=y_{\mathrm{d}}(t_i) \tag{6}$$

Like Eq. (5), τ_{c} could be obtained through real-time iteration as

$$\tau_{\mathrm{c},i+1}=\tau_{\mathrm{c},i}+\frac{y_{\mathrm{d}}(t_i)-y_{\mathrm{m}}(t_i)}{\dot{y}_{\mathrm{m}}(t_i)} \tag{7}$$

Since τ_{a} is not known, for an implementation y_{m} cannot be replaced with $k_{\mathrm{a}}k_{\mathrm{c}}y_{\mathrm{d}}$.

It is worth while to point out that Eq. (7) is similar to those proposed by Ahmadizadeh *et al.* (2008), which read

$$\tau_{c,i} = \tau_{c,i-1} + 2G\Delta t \frac{y^{a}_{d,i} - y^{a}_{m,i}}{y_{m,i} - y_{m,i-2}} \tag{8}$$

$$y^{a}_{d,i} = \frac{y_{d,i} + y_{d,i-1} + y_{d,i-2}}{3} \qquad y^{a}_{m,i} = \frac{y_{m,i} + y_{m,i-1} + y_{m,i-2}}{3} \tag{9}$$

where G is a learning gain and the superscript a denotes the average of displacements in the last three steps. Rearranging Eq. (8) yields

$$\tau_{c,i} = \tau_{c,i-1} + G \frac{y^{a}_{d,i} - y^{a}_{m,i}}{v^{a}_{m,i-1}} \tag{10}$$

with the average velocity

$$v^{a}_{m,i} = \frac{y_{m,i} - y_{m,i-2}}{2\Delta t} \tag{11}$$

Eq. (7) will be reduced to Eq. (10) if a learning gain is introduced and Eq. (11) is applied to evaluate the velocity $\dot{y}_{m}(t_i)$.

3.2 Use of Taylor series

If $y_{c}(t_i - \tau_{a})$ in Eq. (2) is expanded with Taylor series, we obtain

$$y_{m}(t_i) = k_{a} y_{c}(t_i) - k_{a} \dot{y}_{c}(t_i) \times \tau_{a} + k_{a} \frac{\ddot{y}_{c}(t_i)}{2!} \times \tau_{a}^{2} - k_{a} \frac{\dddot{y}_{c}(t_i)}{3!} \times \tau_{a}^{3} + \cdots \tag{12}$$

Thus, $y_{m}(t_i)$ can be approximated by the first two terms on the right-hand side of Eq. (12), i.e.

$$y_{m}(t_i) = k_{a} y_{c}(t_i) - k_{a} \dot{y}_{c}(t_i) \times \tau_{a} \tag{13}$$

Rearranging Eq. (13) gives

$$\tau_{a} = \frac{k_{a} y_{c}(t_i) - y_{m}(t_i)}{k_{a} \dot{y}_{c}(t_i)} \tag{14}$$

which indicates that the delay can be approximately evaluated once measured displacement, commanded displacement and velocity, and amplitude gain k_{a} of the physical system are available.

As a by-product, Eq. (13) indicates the effect of delay in RTHS. If the physical substructure is a stiffness-based specimen, the restoring force is

$$F(t_i) = k_{e} y_{m}(t_i) = k_{e} k_{a} y_{c}(t_i) - k_{e} k_{a} \dot{y}_{c}(t_i) \tau_{a} \tag{15}$$

where k_{e} is the stiffness of the specimen. If $k_{a} = 1$, Eq. (15) means that the physical substructure constitutes of a spring and a viscous damper with a negative damping coefficient $-k_e\tau_a$. This conclusion is in agreement with Horiuchi *et al*. (1999) and Wallace *et al*. (2005a).

3.3 Pros and cons of analysed estimators

Newton's method is second-order convergent when the derivative of the function with respect to the variable is not equal to zero. Hence, delay estimators based on Newton's method are favourable for its rapid convergence. The estimator based on Taylor series is attractive for its simplicity of understanding and implementation. However, there are some problems to be resolved prior to their applications to RTHS. Pros and cons of these schemes are summarized as follows:

- Eqs. (5) and (14) are favourable when amplitude errors owing to control are negligible, namely, $k_a \approx 1$. Otherwise, though it is difficult, k_a should be firstly estimated;
- Eq. (6) will not be satisfied for the case $k_a k_c \neq 1$; then, this condition limits application of Eq. (7);
- Velocity values are required in Eqs. (7) and (14). Hence they should be either numerically evaluated or physically measured during testing; thus, noise effects should be reduced for velocity estimates;
- In order to avoid sharp increments of estimated delay, velocity values close or equal to zero at the denominator of previous formulae should be cautiously considered. Any minor error, e.g., measurement noise, could cause larger estimation errors or even instability in these cases.
- The relationship proposed in Eq.(14) is first-order accurate, and this represents a shortcoming. However, it may be acceptable, since actual delay in RTHS is of the order of 0.01s, meaning that estimated errors are of the order of 0.0001s;

In order to improve the accuracy of Eq. (14), Padè approximation (Chi *et al*. 2010) rather than Taylor series suggested in this paper might be a good choice to expand pure delay. This approximation reads

$$e^{-\tau s} \approx \frac{2-\tau s}{2+\tau s} \tag{16}$$

in which *s* is the Laplace variable. Combining Eqs. (2) and (16) results in the following formula,

$$y_m(t_i) + \dot{y}_m(t_i) \times \frac{\tau_a}{2} = k_a y_c(t_i) - k_a \dot{y}_c(t_i) \times \frac{\tau_a}{2} \tag{17}$$

which entails that the achieved velocity is required. Therefore, if velocity of the physical substructure is available, the proposal of this paper can be employed with this approximation.

From these comments, one can see that these estimators perform well in RTHS when special treatments are considered. Therefore, we propose to apply the least-squares algorithm for the estimator based on Taylor series in the following sections (Wang *et al*. 2009).

4. Delay estimation based on Taylor series and least-squares algorithm

Eq. (14) implies that the nonlinear relationship between commanded and measured displacements is linearized at t_i. Therefore, this relationship can be represented by a series of this kind of linearized equation at different time instants, though both the delay and the amplitude error may vary with time in RTHS. If system parameters change slowly, in order to estimate system delay, online estimation approaches for lineartime-invariant systems can be applied. In Eq. (2), $\dot{y}_{\mathrm{c}}(t_i)$ can be approximately expressed by a backward difference, i.e.,

$$\dot{y}_{\mathrm{c}}(t_i) = \frac{y_{\mathrm{c}}(t_i) - y_{\mathrm{c}}(t_{i-1})}{\Delta t} \tag{18}$$

Note that $\dot{y}_{\mathrm{c}}(t_i)$ can also be replaced with predicted velocities obtained via extrapolation polynomials on desired velocities. Substituting Eq. (18) into Eq. (13), one attains

$$y_{\mathrm{m}}(t_i) = \mathbf{\Psi}_i^T \hat{\mathbf{\theta}} \tag{19}$$

with

$$\mathbf{\Psi}_i^T = \begin{bmatrix} y_{\mathrm{c}}(t_i) & y_{\mathrm{c}}(t_{i-1}) \end{bmatrix} \quad \hat{\mathbf{\theta}}^T = \begin{bmatrix} \theta_1 & \theta_2 \end{bmatrix} = \begin{bmatrix} k_{\mathrm{a}} - \dfrac{k_{\mathrm{a}}\tau_{\mathrm{a}}}{\Delta t} & \dfrac{k_{\mathrm{a}}\tau_{\mathrm{a}}}{\Delta t} \end{bmatrix} \tag{20}$$

If there are two groups of data, the gain k_{a} and the delay τ_{a} can be solved from the above linear equations, namely

$$k_{\mathrm{a}} = \theta_1 + \theta_2 \quad \tau_{\mathrm{a}} = \frac{\Delta t \theta_2}{\theta_1 + \theta_2} \tag{21}$$

In view of displacement measurement errors in RTHS, it is advisable to apply the least-squares algorithm to estimate these parameters. In essence, the gain k_{a} and the delay τ_{a} in RTHS are time-varyingas aforementioned; as a result, the recursive least-squares algorithm with a forgetting factor (Söderström and Stoica 1989) is a favourable candidate. Different from the standard recursive least-squares algorithm which is equivalent to Kalman filter, this method is performed with weighted data and requires an initial guess to start up the process. It is suitable to online estimation for time-varying parameters because of its small storage size and low calculation efforts as well as its weighted data. The recursive formulae of the method are

$$\hat{\mathbf{\theta}}_i = \hat{\mathbf{\theta}}_{i-1} + \frac{\mathbf{P}_{i-1}\mathbf{\Psi}_i}{\mathbf{\Psi}_i^{\mathrm{T}}\mathbf{P}_{i-1}\mathbf{\Psi}_i + \rho}[y_{\mathrm{m}}(t_i) - \mathbf{\Psi}_i^{\mathrm{T}}\hat{\mathbf{\theta}}_{i-1}] \tag{22}$$

$$\mathbf{P}_i = \frac{1}{\rho}[\mathbf{P}_{i-1} - \frac{\mathbf{P}_{i-1}\mathbf{\Psi}_i\mathbf{\Psi}_i^{\mathrm{T}}\mathbf{P}_{i-1}}{\mathbf{\Psi}_i^{\mathrm{T}}\mathbf{P}_{i-1}\mathbf{\Psi}_i + \rho}] \tag{23}$$

where ρ is the forgetting factor, $0<\rho\leq 1$. The greater the forgetting factor ρ is, the greater effect on the current estimated delay the previous data have. When $\rho=1$, the algorithm degenerates to the recursive least-squares algorithm. In applications, the value of ρ is usually set as $0.95<\rho\leq 1$. As to the initial values for the recursive procedure, the standard least-squares algorithm is recommended, which provides

$$\mathbf{P}_p = (\mathbf{\Phi}^{\mathrm{T}}\mathbf{\Phi})^{-1} \quad \hat{\mathbf{\theta}}_p = \left(\mathbf{\Phi}^{\mathrm{T}}\mathbf{\Phi}\right)^{-1}\mathbf{\Phi}^{\mathrm{T}}\mathbf{Y} \tag{24}$$

with

$$\mathbf{\Phi} = \begin{bmatrix} \mathbf{\Psi}_1 & \mathbf{\Psi}_2 & \cdots & \mathbf{\Psi}_p \end{bmatrix}^{\mathrm{T}} \qquad \mathbf{Y} = \begin{bmatrix} y_{\mathrm{m}}(t_1) & y_{\mathrm{m}}(t_2) & \cdots & y_{\mathrm{m}}(t_p) \end{bmatrix}^{\mathrm{T}} \tag{25}$$

where p>2 means the group number of data that are employed for the standard least-squares method, to provide an initial guess for the recursive least-squares method with forgetting factor. The value of p is dependent on the signal-noise-ratio of measured displacement. A smaller p can be used for less contaminated measured displacements, while a larger p is required for more noisy data. Before the p-th step, the system delay estimated prior to tests has to be applied to delay compensation, in that the online estimated one is not available. Since the standard least-squares algorithm is utilized to estimate the initial value for the recursive procedure, the first estimation obtains the current delay. Even though the desired displacement is small and the measured displacement is contaminated by measurement noises in real tests, the method can rapidly and accurately converge since the estimated value is originated from over-determined systems. If the matrix form is expanded, it needs not to calculate the inverse matrix and the calculation efforts are limited. In this sense, the proposed method is expected to be favorable. Note that the estimated value converges to a first-order approximation of the actual delay.

5. Numerical simulations

This section presents two types of numerical simulations: i) estimation of time-invariant and time-varying delays with the delay estimator proposed in the previous section; ii)simulations of RTHS based on a second-order actuator model in conjunction with three estimators (Wang *et al.* 2009).

5.1 Delay estimation based on the proposed estimator

5.1.1 Time-invariant delay

Suppose that the physical system can be simplified to a pure delay element with the dead time 10ms. The actuator command is a sinusoidal wave with the amplitude 1mm and the frequency 1Hz.A white noise is taken into account and signal-to-noise ratio is 30dB.Themethod proposed herein was applied with the time interval 5 ms and p = 20. The estimated delay is illustrated in Fig.3. One can observe that the estimated delay of the standard least-squares algorithm is about 90% of the final value. The following estimation is based on this result and therefore, the method exhibits wonderful convergence speed. In addition, the estimated delay varies around the actual delay and the oscillation is affected by the forgetting factor. A larger factor renders the oscillation amplitude smaller. In summary, the proposed method exhibits good convergence speed and accuracy for constant system delay even though the method is based on an approximate expression, i.e., Eq. (13) and measurement noises are considered.

5.1.2 Time-varying delay

Herein we assume that the delay can be formulated as

$$\tau_{\mathrm{a}} = 0.015 + 0.008 \times \sin(0.2\pi t) \tag{26}$$

In addition, the proportional gain k_a = 1.1 is adopted to simulate amplitude errors of physical system control. All other parameters and conditions not especially stated here are employed with the same values as those in the last subsection. The estimated delay with different parameters is depicted in Fig. 4. Clearly, the closer to unity the forgetting factor is, the smoother the estimated delay history is. As an extreme case, it approaches the constant 0.015 when the forgetting factor equals 1. In view of the accuracy and the oscillation, ρ= 0.98 may be a better choice. This simulation shows that the proposed method can trace the delay change even the noise and amplitude control errors exist in the system. In addition, it is easy to choose the suitable forgetting factor for the scheme: it is smaller than unity and suggested greater than 0.95, varying a little according to noise and the change speed of the delay. The ability of amplitude error estimation is another feature of this method. Even though it is not used in this paper, the estimated gain can be applied to improve control performance of the loading system.

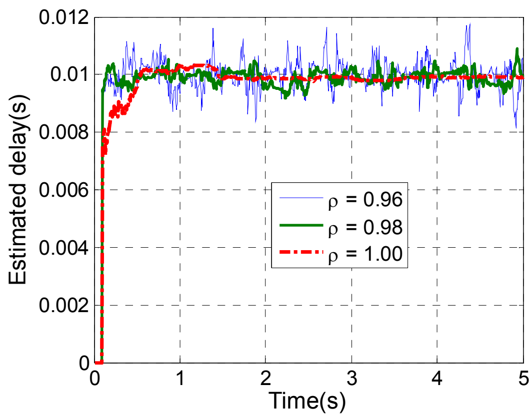


Fig. 3 Time histories of estimated delay for a time-invariant delay

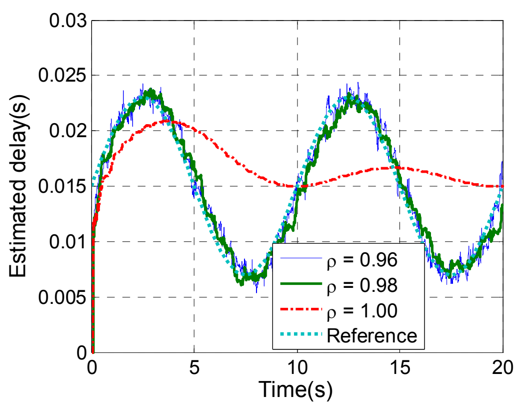


Fig. 4 Time histories of estimated delay for a time-varying delay

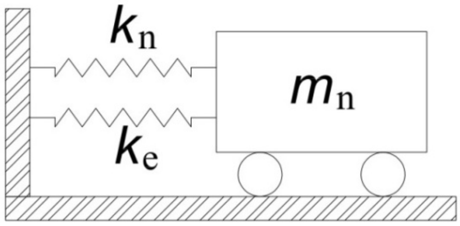


Fig. 5 Schematic of the emulated structure in RTHS

5.2 Numerical simulation of RTHS with online delay estimation

Numerical simulations of RTHS on a linear SDOF system as shown in Fig. 5 are carried out in this subsection. The structural parameters are chosen in such a way that the natural period of the emulated structure is 0.5s and damping ratio 5%. The Tab as earthquake recorded in Iran in 1978, with PGA equal to 0.852g was utilized to excite the structure (Ahmadizadeh *et al*. 2008). In the analysis, mass and damping are simulated in the numerical substructure whilst the spring is modelled as a specimen (in the figure $k_n = 0$), as schematically depicted in Fig. 5. In addition, the actuator is modelled by a second-order system, namely,

$$T_{\mathrm{A}}(s) = \frac{\omega_{\mathrm{A}}^2 e^{-\tau s}}{s^2 + 2\xi_{\mathrm{A}}\omega_{\mathrm{A}} s + \omega_{\mathrm{A}}^2} \tag{27}$$

in which ω_{A} and ξ_{A} denote the circular frequency and equivalent damping ratio, respectively; τ and s indicate the dead time of the system and the Laplace variable, respectively. In the simulation, ω_{A} = 100rad/s, ξ_{A} = 0.80 and τ= 0 are set. According to the fact that the delay is identical to the ratio of phase lag with respect to the corresponding frequency, the delay corresponding to the structural natural frequency, about 16.01 ms, is viewed as the reference in the following simulation. Moreover, the Central Difference Method is used to evaluate the response of the structure with the time interval 10 ms. Delay of the actuator is compensated for by means of the polynomial extrapolation proposed by Bonnet *et al*. (2007a), expressed as

$$\begin{aligned} y_{\mathrm{c}}(t_{i+1}) = {} & \left(1 + \frac{11}{6}\eta + \eta^2 + \frac{1}{6}\eta^3\right)d_i - \left(3\eta + \frac{5}{2}\eta^2 + \frac{1}{2}\eta^3\right)d_{i-1} \\ & + \left(\frac{3}{2}\eta + 2\eta^2 + \frac{1}{2}\eta^3\right)d_{i-2} - \left(\frac{1}{3}\eta + \frac{1}{2}\eta^2 + \frac{1}{6}\eta^3\right)d_{i-3} \end{aligned} \tag{28}$$

with

$$\eta = \frac{\tau_{\mathrm{a}}}{\Delta t} \tag{29}$$

where τ_{a} means the online estimated delay.

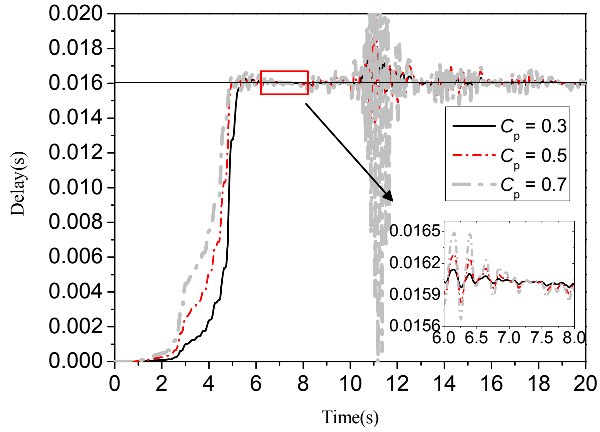


Fig. 6 Time histories of the estimated delay with Darby's method (after Darby *et al*. 2002)

Time histories of the estimated delay with Darby's method (Darby *et al*. 2002) are shown in Fig. 6.From the figure, larger parameters can cause fast convergence speeds and larger oscillations; this trend is in agreement with that of Ahmadizadeh *et al*. (2008). Nonetheless, in the first five seconds, the estimated values are much smaller than the reference, whatever the parameter is. This is due to the fact that if the relative position errors (defined as the discrepancy between the desired and measured displacements) are small, the method responds slowly. The estimated value may oscillate dramatically or even be unstable at the peak of the relative position errors if the parameter is too large. Therefore, the parameter value is limited by this peak. For the sake of stability, only smaller parameters are feasible even though it may mean slower response speeds. Meanwhile, time histories of the estimated delay of Ahmadizadeh's method (Ahmadizadeh *et al*. 2008) are illustrated in Fig. 7. The estimated values are smoother when the parameter is smaller. However, sharp increments are observed if increasing the parameters, which can be contributed to smaller velocities as the denominator in the expression, see Eq. (8).

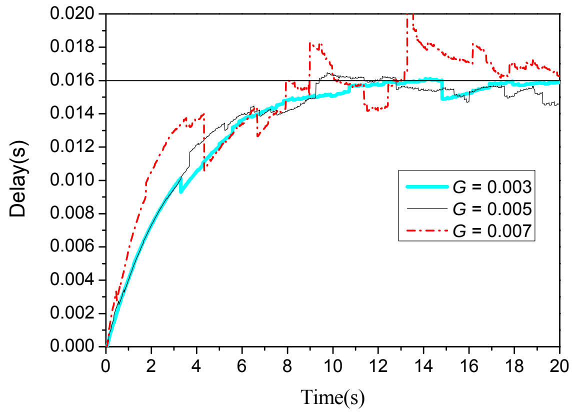


Fig. 7 Time histories of the estimated delay with Ahmadizadeh's method (after Ahmadizadeh *et al.* 2008)

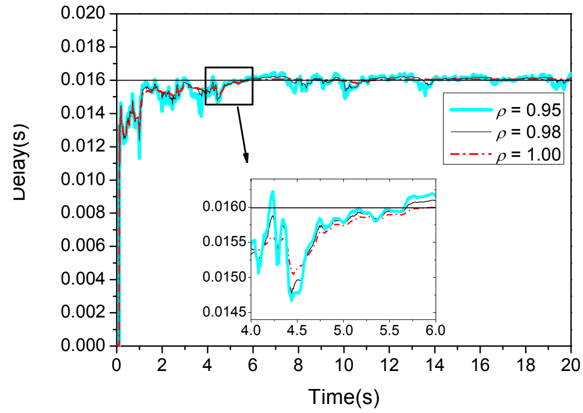


Fig. 8 Time histories of the estimated delay with the proposed estimator

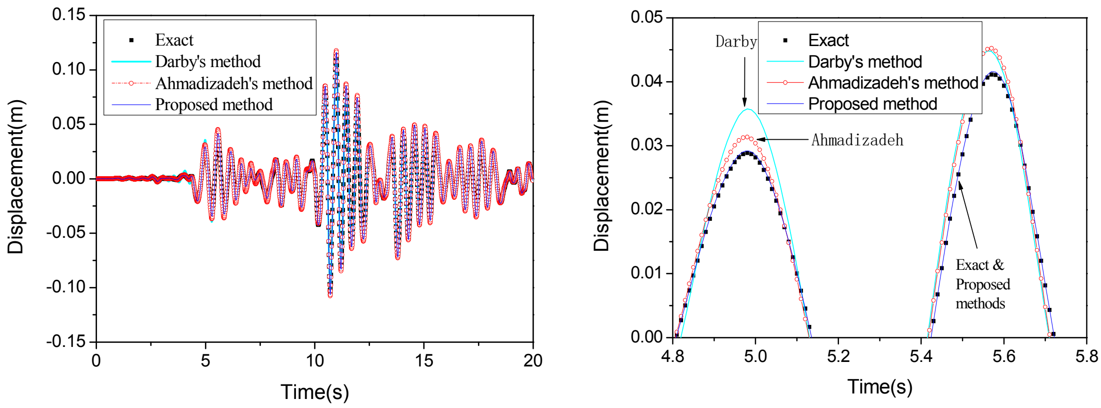


Fig. 9 Time histories of desired displacements in RHTS with different methods

Time histories of estimated delay provided by the proposed estimator with forgetting factors 0.95, 0.98 and 1.00 are plotted in Fig. 8. Obviously, smaller forgetting factors result in larger oscillations of estimated values. When ρ = 1.00, the method is favourable in terms of accuracy and response speed; this is in agreement with results in Subsection 5.1. However, RTHS combining online delay estimation and delay compensation are taken into account in these simulations.

Time histories of desired displacement responses with three delay estimation methods and parameters C_p = 0.3, G = 0.003 and ρ = 1.00 are plotted in Fig. 9. For Darby's method, the error in the first five seconds is large with respect to the exact response followed by a smaller error in the next 15 seconds. Ahmadizadeh's method causes larger response amplitudes owing to smaller estimated delay. Conversely, the displacement responses provided by the proposed method match better the exact results.

From the aforementioned simulations, we can draw the following conclusions:

- Darby's method converges slowly to the exact response owing to feasible but smaller parameters determined by the peak of relative position errors;

- Ahmadizadeh's method exhibits sharp increments of the estimated delay even in the case without noise;
- The proposed method exhibits some advantages, such as the suitability to online estimation, the treatment of noise-contaminated data and amplitude errors, the ease to determine unknown parameters, and favourable convergence speed and accuracy.

6. Validation tests

Validation tests were carried out at the Mechanical and Structural Testing Center of the Harbin Institute of Technology. The schematic diagram of the overall emulated structure is shown in Fig. 5. A Buckling-Restrained Brace (BRB) after Li (2007), was regarded as the experimental substructure. A picture of the experimental substructure installed on the MTS servo-hydraulic actuator is illustrated in Fig. 10. This actuator is characterized by a dynamic loading capacity of 2500 kN and a stroke of 300 mm. In the tests, computation of the time-discretized equation of motion, delay estimation and delay compensation were performed in Calculation Editor of the control system of MTS servo-hydraulic actuator, i.e., Flex Test GT (MTS) (MTS System Corporation 2001). The Calculation Editor provides an easy way to online process signals by programming.

The initial stiffness of the physical substructure was found to be 144×10^6 N/m from previous tests while the stiffness of the numerical substructure was chosen as half of that, i.e., $k_n=72\times10^6$ N/m. The mass of the system was chosen in such a way that the natural frequency of the overall elastic structure was 1 Hz for all test cases. The damping of the numerical substructure was assumed to be zero. The Central Difference Method was used to solve the equation of motion with a time interval of 4.902 ms. In view of the system delay of about 18ms, which was about three times the time interval value, the third-order polynomial extrapolation was carried out with structural responses at only even or odd steps. Therefore, Eqs. (28) and (29) are replaced with

Fig. 10 Actualtest set-up for RTHS

$$y_{\mathrm{c}}(t_i+\tau_a)=\left(1+\frac{11}{6}\eta+\eta^2+\frac{1}{6}\eta^3\right)d_i-\left(3\eta+\frac{5}{2}\eta^2+\frac{1}{2}\eta^3\right)d_{i-2} \\ +\left(\frac{3}{2}\eta+2\eta^2+\frac{1}{2}\eta^3\right)d_{i-4}-\left(\frac{1}{3}\eta+\frac{1}{2}\eta^2+\frac{1}{6}\eta^3\right)d_{i-6} \tag{30}$$

and

$$\eta=\frac{\tau_{\mathrm{a}}}{2\Delta t} \tag{31}$$

As a result, structural responses at a larger time span were applied for displacement prediction; thus, the influence of higher-frequency components in structural displacement on predicted displacement can be suppressed.

6.1 Linear case

In order to ensure linear behavior of the specimen, the PGA of El Centro earthquake (NS 1940) was tuned to 0.0163g. RTHS with different delay estimation methods were performed. For Ahmadizadeh's method (Ahmadizadeh *et al*. 2008), similar results to those in Section 4.2 of this paper were obtained. For the sake of brevity, only test results with the proposed estimator and Darby's method are discussed herein.

Fig. 11 compares estimated delays provided by Darby's method with different parameters. It takes around four seconds for the estimated delay to reach the final value for the first time when C_{p}=0.01 is utilized, while the rising time is about two seconds with C_{p}=0.04. Meanwhile, the oscillation amplitude of the estimated delay increases when the parameter increases. In agreement with those in Section 4.2, it is inconsistent for Darby's method to increase convergence speed while suppressing the oscillation amplitude of the estimated delay. According to Eq. (7), the estimated delay should be related to the relative position error $(y_{\mathrm{d}}(t_i)-y_{\mathrm{m}}(t_i))$ and achieved velocity $(\dot{y}_{\mathrm{m}}(t_i))$. However, the velocity information is not fully applied in Darby's method (Darby *et al*. 2002). As a constant, the parameter C_{p} can be an optimal parameter only for a specific velocity. When it is optimal for a maximum structural velocity in a test, it results in a relative slow response speed for small velocity; when it is a reasonable parameter for a small structural velocity, it introduces estimated delay oscillation for a maximum velocity. This also reveals the reason why the parameters adopted in previous numerical simulations and in Darby *et al*. (2002) are different from those here. Different maximum velocity responses and different compromises require different parameter values. Therefore, online tuning is often necessary for Darby's method.

Fig. 12 shows the desired displacements in two tests and the numerically predicted response of the structure. In the figure, the solid line matches the simulated response better than the dashed line, which indicates that the estimated delay with C_{p}=0.04 is more accurate. It is worth noting that although the oscillation does not induce instability, we are not confident that the test is stable when the parameter keeps increasing. An algorithm that is endowed with rapid convergence speed, limited oscillations of estimated delay and easy parameter choice is desired.

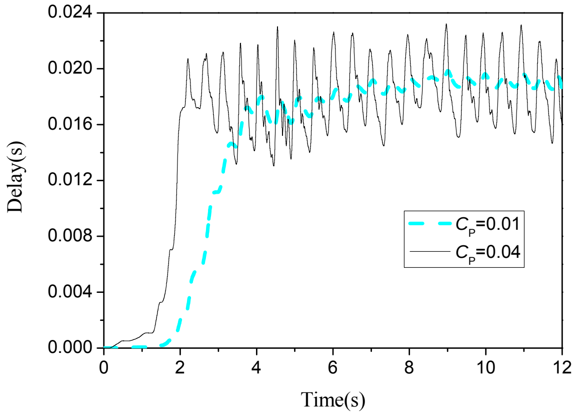


Fig. 11 Time histories of the estimated delay with Darby's method in linear RTHS

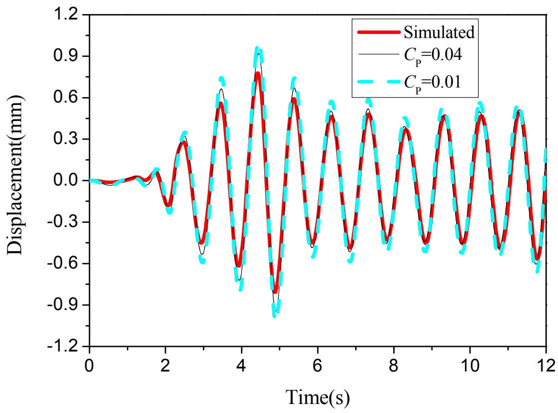


Fig. 12 Time histories of displacement with Darby's method in linear RTHS

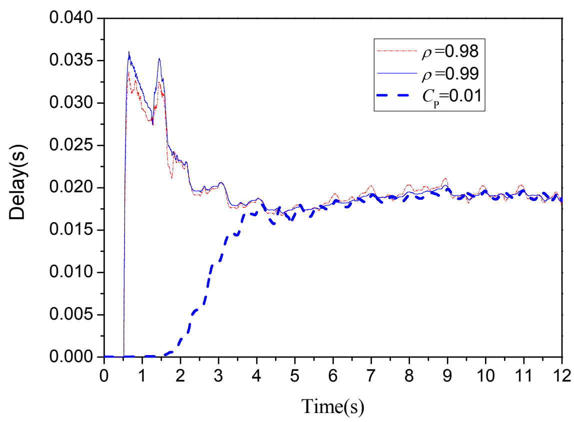


Fig. 13 Time histories of estimated delay with the proposed method in linear RTHS

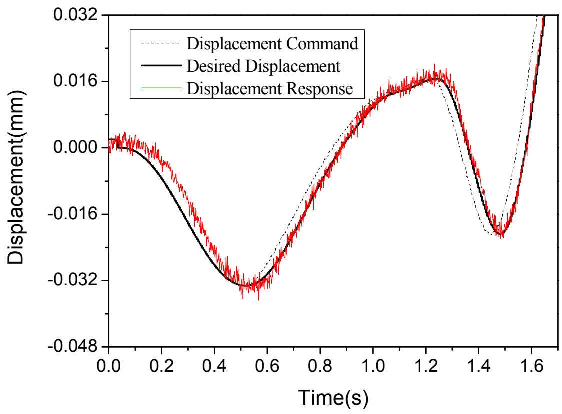


Fig. 14 Time histories of displacement responses with the proposed method in linear RTHS

Fig. 13 presents the estimated delay (Wu and Wang 2014) of the proposed method with both the forgetting factor ρ=0.98 and ρ=0.99 compared with that provided by Darby's method. Evidently, the two test results based on the proposed method provide similar delay histories and give estimated values consistent to Darby's method after the first four seconds. In the first four seconds, the estimated values by the proposed method are greater than those provided by Darby's method. However, as shown in Fig. 14, the measured displacement matches well the desired one after the first 0.5s; this implies that the delay is not over-compensated for in the beginning of the test. Note that in the first 0.5s, the system delay is not compensated for and hence, the displacement command and the desired displacement are identical to each other. Consequently, the estimated delay of Darby's method is less than the actual delay. In fact, it is likely that the system delay is greater in this stage since the loading system has to switch from the static state to movement. In addition, the tested displacement provided in the simulation with ρ=0.98matches the predicted displacement well (Wang 2012).

6.2 Nonlinear case

In this section, the PGA is tuned to 0.122 g. Fig. 15 shows the estimated delay histories provided by Darby's method; they exhibit oscillations. The hysteretic relationship of the specimen corresponding to the test with C_p = 0.01 is shown in Fig. 16; it is characterized by a maximum displacement of about 4 mm and a maximum restoring force of about 400 kN, respectively.

Fig.17 plots the estimated delay histories in RTHS with the proposed estimation method. In order to investigate the repeatability of the testing method, each test was carried out twice. This figure illustrates response characteristics similar to those described above. However, the delay oscillation is smaller. In fact, as aforementioned, the proposed method measures the system delay with both the displacement command and displacement response; therefore, the delay could be measured offline. This is not possible with both Ahmadizadeh's and Darby's methods. The estimated delay histories with different parameters are plotted in Fig. 18. A careful reader can observe that the offline estimated-delay histories with ρ = 0.95 and ρ = 0.98 are almost identical to the online measured result in the second test with ρ = 0.98. This indicates that the smoother estimated delay cannot be attributed to a larger forgetting factor, and hence the delay in the nonlinear tests may not change greatly. Actually, the MTS loading facility has a loading capacity of 2500kN, about 6.3 times the maximum restoring force, and thus in the test, the nonlinearity of the actuator is not apparent. In addition, the tangent stiffness of the specimen did not change too much during the tests; therefore, the delay did not greatly vary during simulations. Moreover, Fig. 18 indicates that the parameter ρ = 0.98 is a good candidate for common tests; this implies that the parameter choice for the proposed method is easy and the overall method quite practical.

In addition, RTHS with the proposed method exhibits better relative displacement errors in the first two seconds. This can be ascribed to the proper-compensated delay in this time range; in the subsequent time range, two test results are similar, which may result from hysteretic damping. More detailed results can be found in Wang (2012).

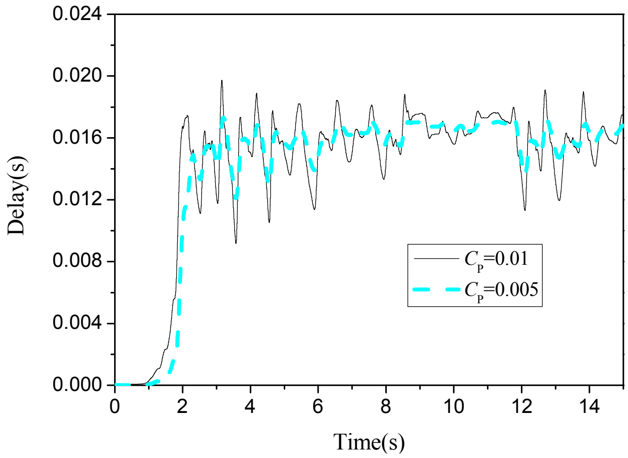


Fig. 15 Time histories of the estimated delay with Darby's method in nonlinear RTHS

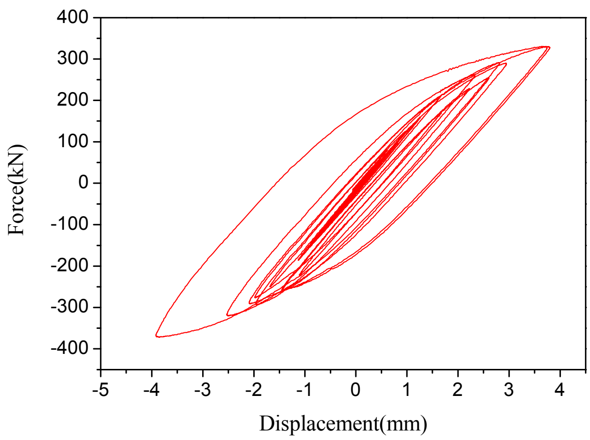


Fig. 16 Hysteretic response of the specimen

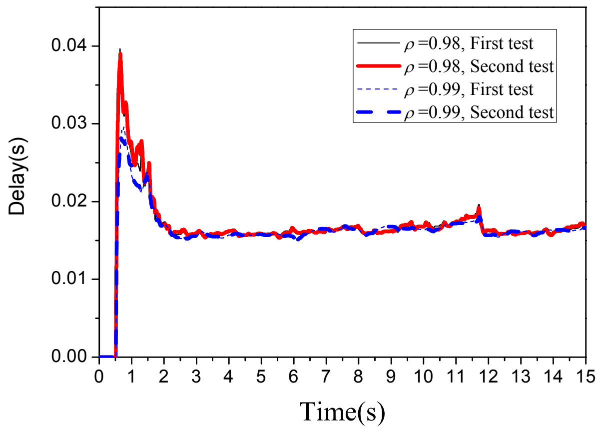


Fig. 17 Time histories of the estimated delay with the proposed method in nonlinear RTHS

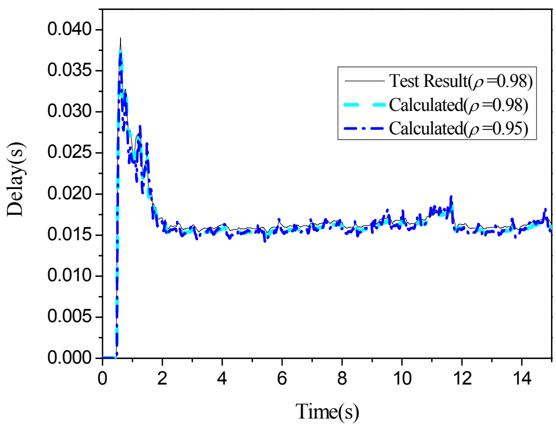


Fig. 18 Comparisons of online and offline estimated delays

7. Conclusions

In this paper, delay estimation methods based on a simplified physical system model, i.e. a pure delay model multiplied by a gain reflecting control amplitude errors, were suggested; and in order to improve its performance, the least-squares method was introduced with a Taylor series-based law. In order to verify the performance of the proposed method, both numerical simulations and RTHS with a buckling-restrained brace specimen were carried out. Relevant results showed that the proposed delay compensation technique is endowed with good convergence speed and accuracy, even when measurement noises and amplitude errors of actuator control are present. Also the choice of method parameters proves to be simple. As a result, any compensation method based on displacement prediction can be applied together with the proposed estimator. Conversely, the parameter choice for Darby's method is difficult and the estimated delay sometimes converges slowly. Ahmadizadeh's method sometimes exhibits sharp increments of the estimated delay because of noises and/or very small denominator in the delay expression.

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