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Reliability analysis of repairable k-out-n system from time response under several times stochastic shocks

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Abstract. The model of unit dynamic reliability of repairable k/n (G) system with unit strength degradation under repeated random shocks has been developed according to the stress-strength interference theory. The unit failure number is obtained based on the unit failure probability which can be computed from the unit dynamic reliability. Then, the transfer probability function of the repairable k/n (G) system is given by its Markov property. Once the transfer probability function has been obtained, the probability density matrix and the steady-state probabilities of the system can be retrieved. Finally, the dynamic reliability of the repairable k/n (G) system is obtained by solving the differential equations. It is illustrated that the proposed method is practicable, feasible and gives reasonable prediction which conforms to the engineering practice.

Keywords: random shocks; repairable; k/n (G) system; dynamic; reliability

1. Introduction

A consecutive k-out-of-n (G) system is composed of n units such that the system works if and only if at least k units work. Obviously, if $k=1$, the system is a parallel system whereas if k=n, the system is a series system. A consecutive k-out-of-n (G) system is a common type of system. It can be found in a variety of engineering systems, such as aircraft engine, power plant generator, etc.

The reliability of engineering structures is an important indicator to evaluate their structural performance. Numerous system reliability prediction problems have been studied (Scheuer 1988, Roy and Dasgupta 2001, Yao and Zhao 2005, Sun and Shi 2004, Fang *et al*. 2013) and the reliability of the k/n (G) system has been received much attention too (Zhang and Lam1998, Utkin 2004, Xie 2004). The reliability of the system was studied by using the inverse fuzzy estimator and the Systems Modeling Language (Lee 2011, David *et al*. 2010). The reliability of the k/n (G) system under one load could be well used in the industry to design the mechanism (Thomas *et al*. 2013, Rezazadeh *et al*. 2012, Peng 2010). Dynamic reliability of failure dependence k/n (G) system has been studied in the article (Fang *et al*. 2013) using the stress-strength interference

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theory. In addition, reliability of the 1/2 (G) system and k/n (G) system have also been analyzed and computed in the articles (Lewis 2001, Liu 1998) based on the assumption of independence of failure events, but large error could be produced by using these methods. The availability and queuing repair issues of the k/n (G) system were studied from a mathematical point of view in the articles (Xu *et al*. 2004, Chen 2010). It was shown the k/n (G) repairable system is an important system in the engineering that the k/n (G) repairable system was simulated by using the restart method (Jose 2010). Nevertheless, these studies did not involve the k/n (G) system under repeated random shocks and strength degradation with time. Based on the relevant literature, a new dynamic reliability model of repairable k/n (G) system under repeated random shocks has been developed, and thus the reliability of the system can be conveniently and accurately predicted from time response.

2. Unit reliability analysis of k/n (G) system under repeated random shocks

In this study, the external shocks that act on every unit of the k/n (G) system during its service period are random variables. The units of the system are not subjected to a single continuous shock, but repeated or multiple series of random shocks. Let the external random shock is denoted by s, the cumulative probability distribution function and the probability density function of s are $G(s)$ and $g(s)$, respectively. Suppose the maximum value of *m*-series of random shocks is $s_{\max}$ and it is assumed that if the unit does not fail under the maximum shock $s_{\max}$, the unit will not fail also under the repeated (*m*-series) random shocks. In other words, fatigue failure is not considered in this study. Hence, the reliability of the system under repeated random shocks is equivalent to its reliability under the maximum random shock. For this reason, from a conservative point of view, the system reliability under the maximum shock $s_{\max}$ can be used to evaluate the system reliability under the repeated random shocks. That is to say, $s_{\max}$ is used as equivalent shock to predict the system reliability. The cumulative distribution of the shock under *m*-series of random shocks which is equivalent to the maximum shock can be written as follows

$$F(s_{\max}) = [G(s_{\max})]^m \tag{1}$$

where $G(s_{\max})$ is the probability distribution function of $s_{\max}$, its probability density function is $g(s_{\max})$, the mean and variance of $s_{\max}$ are $\mu(s_{\max})$ and $\sigma(s_{\max})$, respectively.

The external random shock is usually considered to obey Possion distribution with parameter $\lambda_L t$ (Lewis 2001). Thus, the probability distribution function of $s_{\max}$ at time t is shown as follows

$$\begin{aligned} P(s(t)) &= \sum_{m=0}^{+\infty} P(N(t) - N(0) = m)[G(s)]^m \\ &= \sum_{m=0}^{+\infty} \frac{(\lambda_L t)^m e^{-\lambda t}}{m!}[G(s)]^m \\ &= e^{\lambda_L t[G(s)-1]} \end{aligned} \tag{2}$$

where $N(t)$ is the number of random shock happening at time t, $N(0)$ is the number of random shock happening at time 0 and m is the number of random shock happening from time 0 to t.

Eq. (2) can be rewritten when $s_{\max}$ is substituted by *s* as follows

$$P(s(t)) = e^{\lambda_L t[G(s)-1]} \tag{3}$$

The mean $\mu(s)$ and variance $\sigma(s)$ of the $G(s)$ can be determined using the synthesis method with algebraic approach and the results are as follows

$$\mu(s(t)) = \lambda_L t(1+\mu(s)+\mu^2(s)+\sigma^2(s)) \tag{4}$$

$$\sigma(s(t)) = \lambda_L t(1+\sigma(s)+4\mu^2(s)\sigma^2(s)+2\sigma^2(s)) \tag{5}$$

On the other hand, working resistance of each unit is considered to degrade and the value decreases with time, the remaining working resistance $r(t)$ at time t can be calculated as follows (Schaff 1997)

$$r(t) = r(0)-(r(0)-s)(t/T)^c \tag{6}$$

where $r(0)$ is the initial resistance of working unit, *T* is the service period of the unit and *c* is the exponential of material degradation. Both $r(0)$ and $r(t)$ are considered as random variables in this study, s is the same as the Eq. (3).

The mean $\mu(r(t))$ and variance $\sigma(r(t))$ of the remaining resistance $r(t)$ can be determined using Eq. (6)

$$\mu(r(t)) = \mu(r(0))-[\mu(r(0))-[\mu(r(T))](t/T)^c \tag{7}$$

$$\sigma(r(t)) = (\frac{\sigma(r(0))}{\mu(r(0))}-(\frac{\sigma(r(0))}{\mu(r(0))}-\frac{\sigma(r(T))}{\mu(r(T))})(t/T)^c)\mu(r(t)) \tag{8}$$

where $\mu(r(0))$ and $\sigma(r(0))$ are mean and variance of unit initial resistance, respectively, whereas $\mu(r(T))$ and $\sigma(r(T))$ are mean and variance of unit resistance at the end of service period, respectively.

The dynamic reliability index at time t can be determined by using the stress-strength interference theory and the first order second moment method as follows

$$\beta_r(t) = \frac{\mu(r(t))-\mu(s(t))}{\sqrt{\sigma^2(r(t))+\sigma^2(s(t))}} \tag{9}$$

Finally, the dynamic reliability can be calculated as follows

$$R_r(t) = \Phi(\beta_r(t)) \tag{10}$$

where Φ is the standard normal distribution function.

3. System reliability analysis of repairable k/n (G) system under repeated random shocks

For the case where the k/n (G) system is repairable, the repair time is considered to obey exponential distribution where its parameter is λ_t. It is assumed that the repaired unit is new after it is repaired. The status of the k/n (G) at time τ is $M(\tau)$, if $\tau \geq 0$, $M(\tau)$ is a *Markov process*, its status space is Ω={0, 1, 2, …, n-k, n-k+1, …, n}, its working state space is W={0, 1, …, n-k} and its fault state is F={n-k+1, …, n}.

The state of the system is transited from *i* to *j* and is denoted by $p_{ij}(\Delta\tau)$ which can be calculated as follows

$$p_{ij}(\Delta\tau) = P(N(\tau) = j \,|\, N(\tau+\Delta\tau) = i) \quad \forall i, j \in \Omega \tag{11}$$

The probability of the number of unit failure is added from *i* to *j* under a random shock and is denoted by a_{ij} which can be calculated as follows

$$a_{ij} = C_{n-i}^{j-i}(1-P_f)^{n-j} P_f^{\,j-i} \tag{12}$$

where $P_f = 1 - R_r(t)$.

The transfer probability function of the system is obtained by using the transfer states

$$p_{00}(\Delta\tau) = 1 - \lambda_L \sum_{j=1}^{n} a_{0j}\Delta\tau + o(\Delta\tau) \tag{13a}$$

$$p_{0j}(\Delta\tau) = \lambda_L a_{0j}\Delta\tau + o(\Delta\tau),\ j = 1, 2, \cdots, n. \tag{13b}$$

$$p_{ij}(\Delta\tau) = \lambda_L a_{ij}\Delta\tau + o(\Delta\tau), i = 1, 2, \cdots n-k, i < j \leq n. \tag{13c}$$

$$p_{ii}(\Delta\tau) = 1 - (\lambda_L \sum_{j=i+1}^{n} a_{ij} + \lambda_t)\Delta\tau + o(\Delta\tau), i = 1, 2, \cdots, n-k. \tag{13d}$$

$$p_{ii-1}(\Delta\tau) = \lambda_t\Delta\tau + o(\Delta\tau), i = 1, 2, \cdots, n-k+1. \tag{13e}$$

where $o(\Delta\tau)$ is the higher order infinitesimal of $\Delta\tau$.

The elements of the probability density matrix are given as follows

$$q_{00} = -\lambda_L \sum_{j=1}^{n} a_{0j} \tag{14a}$$

$$q_{0j} = \lambda_L a_{0j},\ j = 1, 2, \cdots, n. \tag{14b}$$

$$q_{ij} = \lambda_L a_{ij}, i = 1, 2, \cdots n-k, i < j \leq n. \tag{14c}$$

$$q_{ii} = -\lambda_L \sum_{j=i+1}^{n} a_{ij} - \lambda_t, i = 1, 2, \cdots, n-k. \tag{14d}$$

$$q_{ii-1} = \lambda_t, i = 1, 2, \cdots, n-k+1. \tag{14e}$$

The steady state probability of the system is obtained by using the steady state distribution property of the *Markov chain*

$$P_1 = \frac{\lambda_L}{\lambda_t} \sum_{j=1}^{n} a_{0j} P_0 \tag{15a}$$

$$P_{j+1} = \left(\frac{\lambda_L}{\lambda_t} \sum_{l=1}^{n} a_{jl} + 1\right) P_j - \frac{\lambda_L}{\lambda_t} \sum_{i=0}^{j-1} a_{ij} P_i, \; j = 1, 2, \cdots, n-k. \tag{15b}$$

$$P_{j+1} = P_j - \frac{\lambda_L}{\lambda_t} \sum_{i=0}^{n-k} a_{ij} P_i, \; j = n-k+1, \cdots, n-1. \tag{15c}$$

All steady state probabilities of the system can be solved by using the backward substitution method and the sigma-completeness of the probability $P_0 + P_1 + \cdots + P_n = 1$.

The repairable $(n-1)/n$ (G) system is used to illustrate dynamic reliability analysis of the repairable k/n (G) system under repeated random shocks. This is because the dynamic reliability prediction process of the repairable k/n (G) system is similar to that of the repairable $(n-1)/n$ (G) system under repeated random shocks. Based on the former analysis, dynamic reliability can be obtained by solving the following differential equations

$$(Q_0^{'}(\tau), Q_1^{'}(\tau)) = (Q_0(\tau), Q_1(\tau)) \begin{pmatrix} -\lambda_L \sum_{j=1}^{n} a_{0j} & \lambda_L a_{01} \\ \lambda_t & -\lambda_L \sum_{j=2}^{n} a_{1j} - \lambda_t \end{pmatrix} \tag{16}$$

Its initial condition is shown as follows

$$(Q_0(0),\ Q_1(0)) = (1,\ 0) \tag{17}$$

$Q_0(\tau)$ and $Q_1(\tau)$ are determined using Eqs. (18) and (19), respectively

$$Q_0(\tau) = \frac{s_1 e^{s_2 \tau}}{s_1 - s_2} \tag{18}$$

$$Q_1(\tau) = \frac{s_2 e^{s_1 \tau}}{s_2 - s_1} \tag{19}$$

$$R(\tau) = Q_0(\tau) + Q_1(\tau) = \frac{1}{s_1 - s_2}(s_1 e^{s_2 \tau} - s_2 e^{s_1 \tau}) \tag{20}$$

where s_1 and s_2 are expressed as follows

$$s_1, s_2 = \frac{1}{2}[-(\lambda_L \sum_{j=1}^{n} a_{0j} + \lambda_L \sum_{j=2}^{n} a_{1j} + \lambda_t) \pm \sqrt{\lambda_L^2 (\sum_{j=1}^{n} a_{0j} - \sum_{j=2}^{n} a_{1j})^2 + 2\lambda_L \lambda_t (\sum_{j=1}^{n} a_{0j} + \sum_{j=2}^{n} a_{1j}) + \lambda_t^2}\,] \tag{21}$$

Eq. (20) is the dynamic reliability prediction model of repairable $(n-1)/n$ (G) system under repeated random shocks.

4. Examples

The repairable 3/4 (G) system is used as an example to verify the proposed dynamic reliability prediction model. The initial resistance of each unit and the external random shocks obey the normal distribution and are given as $r(0) \approx N(600, 60)MPa$ and $L \approx N(400,40)MPa$, respectively. The service life period of the system is T = 10000 hours, $\lambda_L = 1.5/h$, $\lambda_t = 1$ and $c = 4.092$.

Firstly, the dynamic reliability index of each unit can be obtained using Eq. (9) as follows

$$\beta_r(t) = \frac{200[1-(\frac{t}{10000})^{4.092}]}{\sqrt{[60-20(\frac{t}{10000})^{4.092}]^2 + 1600}} \tag{22}$$

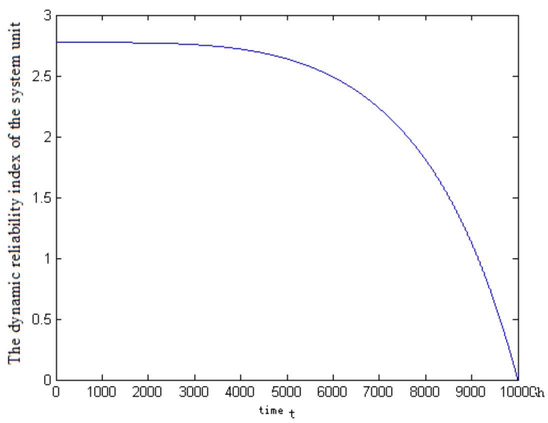

Fig. 1 The unit dynamic reliability index of 3/4 (G) repairable system

The unit dynamic reliability indices calculated from Eq. (22) are shown in Fig. 1. It is observed that the reliability of the unit is decreased drastically when the external random shocks are applied on the system at $t = 6528h$. Once $t = 6528h$ has been obtained, the dynamic reliability index, dynamic reliability and probability of failure of each unit of the system can be calculated as follows

$$\beta_r(6528) = 2.3805,$$
$$R_r(6528) = 0.9991225,$$
$$P_f(6528) = 8.775 \times 10^{-4}$$

Therefore the failure probability of the unit at $t = 6528h$ is used to compute a_{ij}. The probability of the number of unit failure is added using Eq. (12) as follows

$$a_{01} = 0.0035, \quad a_{02} = 4.6119 \times 10^{-6}, \quad a_{03} = 2.7004 \times 10^{-9}$$
$$a_{04} = 5.9291 \times 10^{-13}, \quad a_{12} = 0.0026, \quad a_{13} = 2.3080 \times 10^{-6}$$
$$a_{14} = 6.7568 \times 10^{-10}, \quad a_{23} = 0.0018, \quad a_{24} = 7.7001 \times 10^{-7}$$
$$a_{34} = 8.775 \times 10^{-4} \tag{23}$$

Then, the probability density matrix is obtained using Eq. (16)

$$Q = \begin{pmatrix} -0.0053 & 0.0053 & 0 & 0 & 0 \\ 1 & -1.0039 & 0.0039 & 0 & 0 \\ 0 & 1 & -1.0027 & 0.0027 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix} \tag{24}$$

Finally, the dynamic reliability of the system can be obtained using Eq. (20).

$$R(\tau) = -0.001\exp(-1.0091\tau) + 1.001\exp(-2.048 \times 10^{-5}\tau) \tag{25}$$

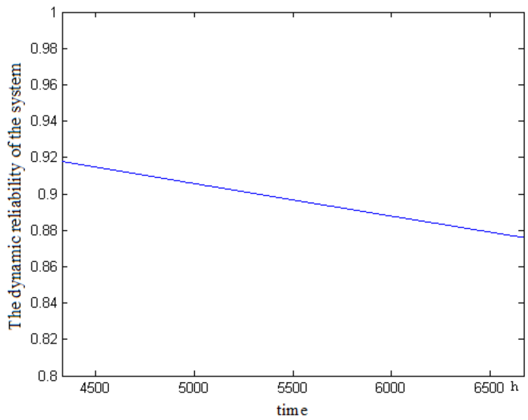

Fig. 2 The dynamic reliability of 3/4 (G) repairable system

The computed values of the system dynamic reliability from Eq. (25) are shown in Fig. 2. It is observed that the reliability of the system is descended to 0.9179 when the external random shocks are applied on the system longer than 6500 hours. In other words, the system has been serviced to 4333.3 hours. The fact that the system is repairable, from this moment the reliability of the system is decreased slowly.

When the external random shocks are applied on the system for 10000 hours or when the service period of the system reaches 6667 hours, the reliability of the system is decreased to 0.8760. In fact, at the end of its service life, the reliability of the system will also further decreased at a faster rate with increasing number of external random shocks and extended maintenance time.

5. Conclusions

The unit dynamic reliability index of repairable k/n (G) system under repeated random shocks has been established in the paper. The probability density matrix and the steady-state probabilities of the system can be retrieved from the transfer probability function. Finally, the dynamic reliability of the repairable k/n (G) system is obtained. It is shown that the proposed dynamic reliability prediction model of repairable k/n (G) system under repeated random shocks is feasible and conforms to the engineering practice.

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