Smart Structures and Systems, *Vol. 14, No. 3 (2014) 397-418*
DOI: http://dx.doi.org/10.12989/sss.2014.14.3.397

A new method to detect cracks in plate-like structures with though-thickness cracks

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(Received November 30, 2012, Revised February 11, 2014, Accepted February 15, 2014)

Abstract. In this paper, a simple two-step method for structural vibration-based health monitoring for beam-like structures have been extended to plate-like structures with though-thickness cracks. Crack locations and severities of plate-like structures are detected using a hybrid approach. The interval wavelet transform is employed to extract crack singularity locations from mode shape and support vector regression (SVR) is applied to predict crack serviettes form crack severity detection database (the relationship of natural frequencies and crack serviettes) using several natural frequencies as inputs. Of particular interest is the natural frequencies estimation for cracked plate-like structures using Rayleigh quotient. Only the natural frequencies and mode shapes of intact structures are needed to calculate the natural frequencies of cracked plate-like structures using a simple formula. The crack severity detection database can be easily obtained with this formula. The hybrid method is investigated using numerical simulation and its validity of the usage of interval wavelet transform and SVR are addressed.

Keywords: plate-like structures; Rayleigh quotient; interval wavelet transform; natural frequency estimation; crack detection; support vector regression

1. Introduction

It is well known that the modal parameters (natural frequencies and mode shapes) of cracked elastic structures differ from their healthy conditions. Some comprehensive literature survey of research activities regarding the vibration-based structural health monitor of various structures with cracks is reviewed in the works by Dimarogonas (1996), Doebling *et al*. (1998), Montalvão *et al*. (2006), Fan and Qiao (2011). In these paper, many vibration-based cracked/damaged methods for beam-like and plate-like structures are summarised, such as natural frequency based methods (Adams *et al*. 1978, Nandwana and Maiti 1997, Lele and Maiti 2002, Patil and Maiti 2003, Maiti

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http://www.techno-press.org/?journal=sss&subpage=8 ISSN: 1738-1584 (Print), 1738-1991 (Online)

and Patil 2004, Patil and Maiti 2005, Chasalevris and Papadopoulos 2006, Sekhar 2008, Li and He 2011), displacement mode shape based method(Ren and De 2000a, 2000b, Rajasekaran and Varghese 2005, Han *et al*. 2005, Ren and Sun 2008, Koo *et al*. 2008, Gökdağ and Kopmaz 2009, 2010, Gökdağ 2011), curve mode shape based method(Pandey *et al*. 1991, Abdel and De 1999, Sampaio *et al*. 1999, Wahab 2001, Qiao *et al*. 2007, Chandrashekhar and Ganguli 2009, Tomaszewska 2010, Jiang and Ma 2012), etc.

Because natural frequency can be easily and cheaply obtained from measured vibration responses, the natural frequency based methods have drawn special attention. Generally, there are two procedures to accomplish the crack detection in structures. The first procedure is the so-called forward problem analysis, which considers the construction of a cracked stiffness matrix exclusively for the crack section and the computation of crack detention database for dynamic parameters. The cracks require a large number of refined elements in the local areas and thus complicate the computational process. Therefore, wavelet finite element method (Chen *et al.* 2004, Han and Ren 2005, 2006, Xiang *et al*. 2007a, Chen *et al*. 2010, 2012, He and Ren 2012) with reduced number of elements was proposed to detect cracks in structures (Chen, *et al*. 2005, Li *et al*. 2005, Chen *et al*. 2006, Xiang *et al*. 2007b, Chen *et al*. 2009, Xaing *et al*. 2011) more efficiently. However, both the finite element method and wavelet finite element method are complexity to construct the crack detection database. The second procedure is the inverse problem analysis, which measures natural frequencies and search for crack location and depth from the depth estimation database using optimization approaches, such as genetic algorithm (Kim *et al*. 2007, Yun *et al*. 2009), neural networks algorithm (Kim *et al*. 2008), support vector regression (Worden and Lane 2001, Isa and Rajkumar 2009), stochastic subspace identification and statistical pattern recognition method (Ren and Lin 2011, Lin and Ren 2011). However, so far only the single-crack detection methods are well established. The reason is that the natural frequency alone cannot provide enough information to construct a robust detection method to identify multiple cracks in structures. To minimize the differences (residuals) between analytical and experimental data, some studies were done. Jaishi and Ren presented finite element model updating technique based on modal flexibility residual (Jaishi and Ren 2006) and response surface method. However, the 'zero-setting' procedure described by Adams (Adamas *et al.* 1978) might be the simplest method.

Singularity detection from mode shape is attractive mainly because it is possible to distinguish the intact and cracked structures and further identify crack locations when the priori knowledge of the cracked zones is not available. Certain mode shapes associated with cracked beam structures contain local singularity information. However, we cannot directly observe singular location from the mode shape. Wavelet transform is a useful tool to extract singularity information from many different kinds of data. Therefore, it has been employed to detect crack locations in the literature (Ren and De 2000a, b, Rajasekaran and Varghese 2005, Han *et al*. 2005, Ren and Sun 2008, Koo *et al*. 2008, Gökdağ and Kopmaz 2009, 2010, Gökdağ 2011). However, the usage of wavelet transform to extract singularity positions will not lead to reliably crack severities detection results.

To overcome the difficult of the above motioned methods, Xiang proposed a simple method to detect cracks in beam-like structures (Xiang *et al*. 2012a). The method presented a hybrid of curve mode shape and several natural frequencies of cracked beam. An arbitrary curve mode shape of slender beam was decomposed using wavelet transform to detect crack locations. Rayleigh quotient was applied to estimate the natural frequencies of beam-like structures with various boundary conditions. Only one formula was necessary to estimate the possibly natural frequencies to construct the crack severities detection database. Finally, the particle swarm optimization (PSO) was employed to predict the crack severities from crack severities detection database.

The purpose of the present work is to extend the simple method to plate-like structures and further establish a simply and relatively robust method to detect the locations and severities of multiple through thickness cracks in plate-like structures using interval wavelet transform and SVR. With this approach, certain peak locations in one of the wavelet details of the mode shape are considered as the crack locations. To simply contract the crack severities detection database, the natural frequencies estimation formula is derived for cracked plate-like structures using Rayleigh quotient. The resulting inverse problem of natural frequency based method is then solved by SVR.

This paper is organized as follows. In section 2, the natural frequency estimation formula is derived using fracture mechanics theory and Rayleigh quotient. In section 3, a detection technique for the cracked plate-like structures, called the hybrid mode shape and natural frequencies approach, is proposed. The interval wavelet transform and SVR are briefly reviewed and applied to detect crack locations and severities, respectively. In section 4, the natural frequencies of a cracked simply supported plate are estimated and the results are compared with the results obtained with finite element method. The crack detention approach is also verified using numerical simulation. Finally, conclusions are summarized in section 5.

2. Natural frequencies estimation formula

Fig. 1 shows a plate containing n though-thickness cracks oriented at different angles $\phi_i\,(i=1,2,\cdots,n)$. The coordinate (x_{ci}, y_{ci}), $(i=1,2,\cdots,n)$ denotes the crack location i in plates. l_x and l_y are the length and width of the plate, respectively, $2b_i\,(i=1,2,\cdots,n)$ are the crack lengths, M_x and M_y are the bending moments perpendicular to x-axis and y-axis, respectively.

The m^{th} natural angular frequency ω_m (for the intact plate and the corresponding natural frequency $f_m=\omega_m/2\pi$) and $\overline{\omega}_m$ (for the plate with n though-thickness cracks and the corresponding natural frequency $\bar{f}_m=\overline{\omega}_m/2\pi$) can be approximately obtained using Rayleigh quotient as

$$\omega_m^2=\frac{U_m}{T_m} \tag{1}$$

and

$$\overline{\omega}_m^2=\frac{\overline{U}_m}{\overline{T}_m} \tag{2}$$

where U_m and $\overline{U}_m$ are the strain energy of the intact and cracked plates, respectively, T_m and $\overline{T}_m$ are respectively the kinetic energy of the intact and cracked plates. According to the assumption that the volume is a tiny loss of materials between intact and cracked structures, we have

$$T_m\cong\overline{T}_m \tag{3}$$

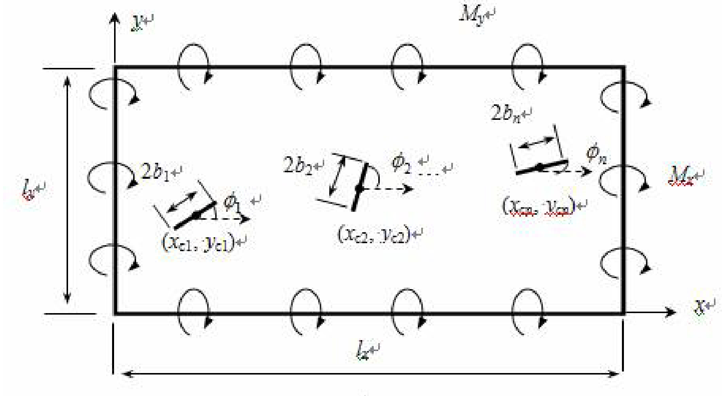

Fig. 1 A plate containing *n* though-thickness cracks oriented at different angles $\phi_i\,(i=1,2,\cdots,n)$

Therefore, consideration of Eq. (1) to Eq. (3), we have

$$\frac{\bar{f}_m}{f_m}=\frac{\overline{\omega}_m}{\omega_m}=\sqrt{(\frac{\overline{U}_m}{\overline{T}_m})/(\frac{U_m}{T_m})}=\sqrt{\frac{\overline{U}_m}{U_m}}=\sqrt{1-\frac{\Delta U_m}{U_m}} \tag{4}$$

where $\Delta U_m = U_m - \overline{U}_m$ denote the difference between the strain energies of the intact and cracked plates, which is equal to the strain energy stored in the cracks in its m^{th} mode (Kannappan and Shankar 2007).

Using binomial expansion, we have

$$\frac{\bar{f}_m}{f_m}=1-\frac{1}{2}\frac{\Delta U_m}{U_m}-\frac{1}{8}(\frac{\Delta U_m}{U_m})^2+\cdots \tag{5}$$

Neglecting higher order terms, we get

$$\bar{f}_m \cong f_m(1-\frac{1}{2}\frac{\Delta U_m}{U_m}) \tag{6}$$

In order to calculate $\bar{f}_m$, we have to determine U_m, f_m and ΔU_m, respectively.

The strain energy of the intact plate can be calculated based on Kirchhoff plate theory as

$$U_m=\frac{D}{2}\int_0^{l_y}\int_0^{l_x}\left[(\frac{\partial^2\psi_m}{\partial x^2})^2+(\frac{\partial^2\psi_m}{\partial y^2})^2+2\mu(\frac{\partial^2\psi_m}{\partial x^2})(\frac{\partial^2\psi_m}{\partial y^2})+2(1-\mu)(\frac{\partial^2\psi_m}{\partial x\partial y})^2\right]\mathrm{d}x\mathrm{d}y \tag{7}$$

where ψ_m is the m^{th} mode shape of the intact plate, μ is the Possion's ratio and *D* is the flexural rigidity of the plate given by

$$D=\frac{Eh^3}{12(1-\mu^2)} \tag{8}$$

where h is the thickness of the plate and E is the Young's modulus.

According to linear elastic fracture mechanics theory, the decrease in the strain energy of n cracks under m^{th} modal vibration is equal to the energy stored in the cracks, which can be expressed in stress intensity factor (SIF) K_I^i at the crack tip, shown by Kobayashi (Tada *et al.* 2000) as

$$\Delta U_m = \sum_{i=1}^{n} \frac{1}{E} \int_{A_i} (K_I^i)^2 \mathrm{d}A_i \tag{9}$$

where A_i is the area of i^{th} crack.

The SIF K_I^i $(i = 1,2,\cdots,n)$ for the i^{th} through-thickness crack in the plate have been provided by Boduroglu and Erdogan (1983) expressed as

$$K_I^i = \frac{6M_{0m}^i}{h^2} f(\gamma_i)\sqrt{b_i} \tag{10}$$

where M_{0m}^i is the moment per unit width acting along the normal of the i^{th} crack, the relative crack length $\gamma_i = 2b_i / l_y$. It is note that $f(\gamma_i)$ is the finite width correction factor (a function of the relative crack length γ_i), which is different for the thick and the thin plates.

For the thick plate, $f(\gamma_i)$ is not just a function of relative crack length, but also a function of width to thickness ratio $\delta = l_y / h$, and some cases can be calculated by (Boduroglu and Erdogan 1983)

$$f(\gamma_i)_{\delta=4} = 21.5\gamma_i^5 - 34.6\gamma_i^4 + 20.15\gamma_i^3 - 3.39\gamma_i^2 - 0.53\gamma_i + 1 \tag{11}$$

$$f(\gamma_i)_{\delta=8} = 37.97\gamma_i^5 - 52.28\gamma_i^4 + 24.63\gamma_i^3 - 0.16\gamma_i^2 - 1.63\gamma_i + 1 \tag{12}$$

$$f(\gamma_i)_{\delta=12} = 2.12\gamma_i^5 + 8.22\gamma_i^4 - 16.82\gamma_i^3 + 11.17\gamma_i^2 - 2.94\gamma_i + 1.01 \tag{13}$$

$$f(\gamma_i)_{\delta=16} = 26.65\gamma_i^5 + 8.09\gamma_i^4 - 34.84\gamma_i^3 + 19.03\gamma_i^2 - 3.91\gamma_i + 1.01 \tag{14}$$

$$f(\gamma_i)_{\delta=20} = -79.64\gamma_i^5 + 144.38\gamma_i^4 - 98.15\gamma_i^3 + 31.75\gamma_i^2 - 4.88\gamma_i + 1 \tag{15}$$

For the thin plate, $f(\gamma_i)$ can be calculated by (Wilson and Thompson 1971)

$$f(\gamma_i) = \left[\frac{1+\mu}{3+\mu}\right]\sqrt{\pi} f_1(\gamma_i) \tag{16}$$

where

$$f_1(\gamma_i) = 12\gamma_i^5 - 14.7\gamma_i^4 + 5.6\gamma_i^3 + 0.02\gamma_i^2 + 0.002\gamma_i + 1 \tag{17}$$

As shown in Fig. 2, suppose ζ_i is a running variable representing the instantaneous crack length and substituting Eq. (10) into Eq. (9), we have

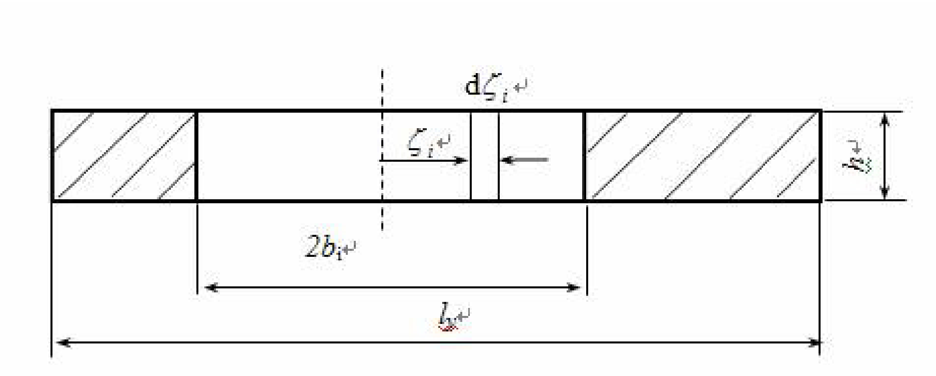

Fig. 2 Cross section at the location of the i^{th} crack $\phi_i\,(i=1,2,\cdots,n)$

$$\Delta U_m = \sum_{i=1}^{n} \frac{36(M_{0m}^{i})^2}{Eh^4} \int_{A_i} [f(\frac{2\zeta_i}{l_y})]^2 \zeta_i \,\mathrm{d}A_i \tag{18}$$

Substituting $\mathrm{d}A_i = h\mathrm{d}\zeta_i$ into Eq. (18), we have

$$\begin{aligned}\Delta U_m &= \sum_{i=1}^{n} \frac{36(M_{0m}^{i})^2}{Eh^3} \int_{-b_i}^{b_i} [f(\frac{2\zeta_i}{l_y})]^2 \zeta_i \,\mathrm{d}\zeta_i \\ &= \sum_{i=1}^{n} \frac{72(M_{0m}^{i})^2}{Eh^3} \int_{0}^{b_i} [f(\frac{2\zeta_i}{l_y})]^2 \zeta_i \,\mathrm{d}\zeta_i\end{aligned} \tag{19}$$

As the length of cracks is denoted by $2b_i\,(i=1,2,\cdots,n)$, the crack locations are identified by x and y coordinates of its mid point, i.e., (x_{ci}, y_{ci}), ($i=1,2,\cdots,n$).

M_{0m}^{i} can be expressed in terms of the moments M_x and M_y along x and y axes, respectively and twisting moment M_{xy} as

$$M_{0m}^{i} = [M_x \sin^2\phi_i + M_y \cos^2\phi_i + 2M_{xy} \sin\phi_i \cos\phi_i]x = x_{ci} \text{ and } y = y_{ci} \tag{20}$$

where the curvatures and the twist are

$$\begin{cases} M_x = -D\left(\dfrac{\partial^2 \psi_m}{\partial x^2} + \mu \dfrac{\partial^2 \psi_m}{\partial y^2}\right) \\ M_y = -D\left(\dfrac{\partial^2 \psi_m}{\partial y^2} + \mu \dfrac{\partial^2 \psi_m}{\partial x^2}\right) \\ M_{xy} = -D(1-\mu)\dfrac{\partial^2 \psi_m}{\partial x \partial y} \end{cases} \tag{21}$$

Substituting Eq. (21) into Eq. (20), we get

$$
\begin{aligned}
M_{0m}^{i} = -D[& \left(\frac{\partial^2 \psi_m}{\partial x^2} + \mu \frac{\partial^2 \psi_m}{\partial y^2} \right) \sin^2 \phi_i + \left(\frac{\partial^2 \psi_m}{\partial y^2} + \mu \frac{\partial^2 \psi_m}{\partial x^2} \right) \cos^2 \phi_i \\
& + 2(1-\mu) \frac{\partial^2 \psi_m}{\partial x \partial y} \sin \phi_i \cos \phi_i]_{x = x_{ci} \text{ and } y = y_{ci}}
\end{aligned} \tag{22}
$$

From Eqs. (6), (7), (19) and (22), we can see clearly that on of the main advantages of the natural frequency estimation formula developed herein is that, only the m^{th} natural frequency f_m and mode shape y_m of the intact plate are required to solve the natural frequency of the cracked plate.

The natural frequency f_m and the mode shape ψ_m of the intact plate can be obtained using the close-from solution with the boundary condition of all edges simply supported (Timoshenko 1974), exact solutions with every kind of boundary conditions (Wu 2007) or finite element method for complex plate-like structures with every kind of boundary conditions (Zienkiewicz *et al*. 2005).

For a rectangle plate with the boundary condition of all edges simply supported, the close-from solutions of both f_m and ψ_m are given by Timoshenko (Timoshenko 1974) as

$$
f_m = \frac{\pi}{2} \left(\frac{n_1^2}{l_x^2} + \frac{n_2^2}{l_y^2} \right) \sqrt{\frac{D}{\rho h}} \tag{23}
$$

$$
\psi_m = \sin \frac{n_1 \pi x}{l_x} \sin \frac{n_2 \pi y}{l_y} \tag{24}
$$

where n_1 and n_2 are the coefficients. When $n_1 = n_2 = 1$, we obtain the first natural frequency (the fundament frequency) f_1. The other frequency f_m ($m \geq 2$) is determined by the plate length l_x, plate width l_y, the coefficients n_1 and n_2.

Therefore, according to Eq. (6), we can use symbolic or numerical integrals to calculate the natural frequency $\bar{f}_m$ of the plate with n though-thickness cracks.

3. Crack detection approach

3.1 Interval wavelet transform

In this section, we give a brief description and the reason to the usage of the interval wavelet transform. When the wavelet transform is applied to decompose finite data sets or signals, boundary distortion phenomenon will inevitably occur (Cohen *et al*. 1993). This phenomenon would influence the singularity detection results. To avoid boundary distortion phenomenon, several methods based on signal extension on the boundaries are presented, such as zero-padding, symmetrization and smooth padding, etc. (Strang and Nguyen 1996). Interval wavelet transform is a natural way to overcome boundary distortion phenomenon (Cohen *et al*. 1993). Interval wavelets are the construction of special boundary wavelets together with the usual wavelets for the interior within the interval to generate a multi-resolution analysis (MRA) on the interval (Cohen *et al*. 1993).

The good performance of interval wavelet decomposition of mode shape is verified for beam-like and plate-like structures by Xiang (Xiang *et al*. 2011). Therefore, in this paper, we use CDV3 (3 denote the vanish moment of Daubechies wavelet) wavelets as a tool to decompose the first mode shape and the source files of WAVELAB (Donoho *et al*. 2012) to code program so as to decompose the first mode shape and further detect crack locations in though-thickness plate-like structures. More details about interval wavelet transform can be seen in the paper written by Cohen, Daubechies and Vial (Cohen *et al*. 1993). Fig. 3 gives the boundary scaling functions and wavelets of CDV3.

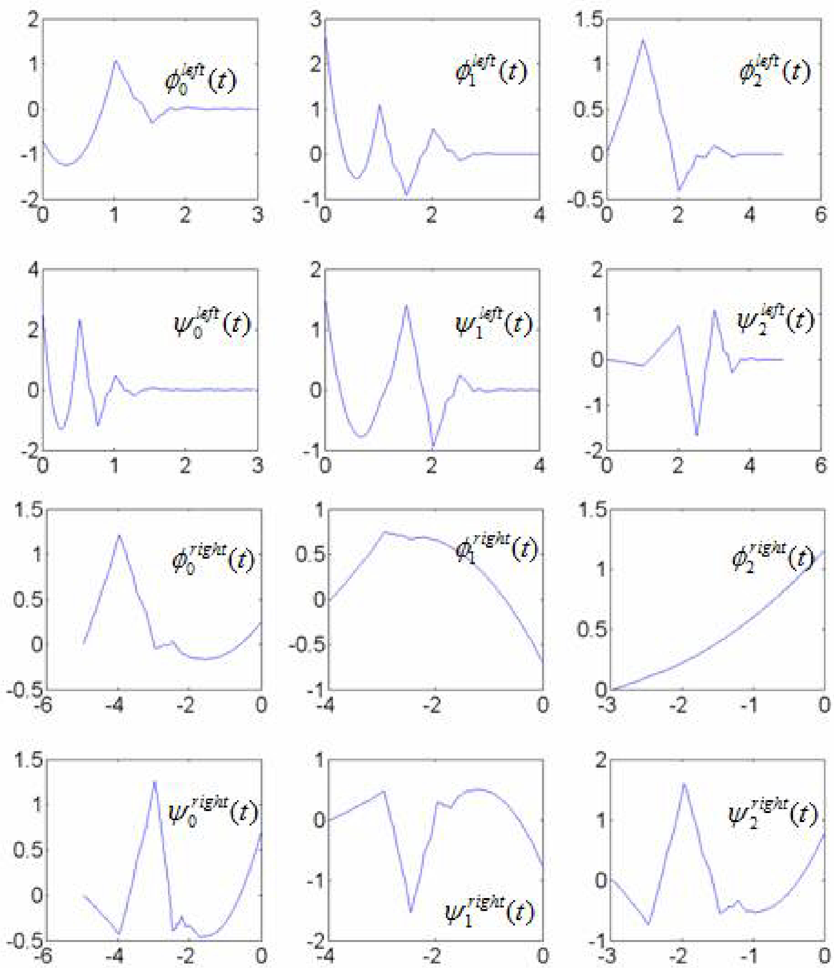

Fig. 3 Boundary scaling functions and wavelets of CDV3

3.2 Support vector regression

In machine learning, support vector machine (SVM) is supervised learning models with associated learning algorithms that analyze data and recognize patterns, used for classification (Support vector classification, SVC) and regression (Support vector regression, SVR) analysis (Alex and Bernhard 2004). SVR has been used for predictive data analysis with many applications in various areas of study. For optimization problems, as the SVR leads to a convex optimization problem and thus the optimal solution must be global (Alex and Bernhard 2004). The basic idea of the SVR is to map the input data into the feature space via a nonlinear map.

Consider the problem of approximating a set of data D

$$D = \{(\mathbf{x}_i, y_i)\}_{i=1}^{l}, \quad \mathbf{x}_i \in \mathbf{R}^n, \quad y_i \in \mathbf{R} \tag{25}$$

with a linear function

$$f(\mathbf{x}) = <\mathbf{w}, \mathbf{x}> + b \quad \text{with} \quad b \in R \tag{26}$$

where $\mathbf{x}$ is the inputs data, $\mathbf{w}$ is the weight vector, $<\mathbf{w}, \mathbf{x}>$ represents the dot product of the $\mathbf{x}$ and $\mathbf{w}$. To ensure that the norm can be minimized, we write this problem as a convex optimization problem, i.e.

minimize $\frac{1}{2}\|\mathbf{w}\|^2$

$$\text{subject to} \quad \begin{cases} y_i - <\mathbf{w}, \mathbf{x}_i> - b \le \varepsilon \\ <\mathbf{w}, \mathbf{x}_i> + b - y_i \le \varepsilon \end{cases} \tag{27}$$

where ε denotes the tolerance error. To deal with infeasible constraints of the optimization problem, ξ_i and ξ_i^* are introduced, and the formulation can be recast as

minimize $\frac{1}{2}\|\mathbf{w}\|^2 + C\sum_{i=1}^{l}(\xi_i + \xi_i^*)$

$$\text{subject to} \quad \begin{cases} y_i - <\mathbf{w}, \mathbf{x}_i> - b \le \varepsilon + \xi_i \\ <\mathbf{w}, \mathbf{x}_i> + b - y_i \le \varepsilon + \xi_i^* \\ \xi_i, \quad \xi_i^* \ge 0 \end{cases} \tag{28}$$

where constant $C > 0$ determines the trade-off between the flatness of f and the amount up to which deviations larger than ε are tolerated.

Applying the Lagrange multipliers, we have

$$\text{minimize} \quad \begin{cases} -\frac{1}{2}\sum_{i=1}^{l}(\alpha_i - \alpha_i^*)(\alpha_j - \alpha_j^*) < x_i, x_j > \\ -\varepsilon\sum_{i=1}^{l}(\alpha_i + \alpha_i^*) + \sum_{i=1}^{l} y_i(\alpha_i - \alpha_i^*) \end{cases}$$

$$\text{subject to} \quad \sum_{i=1}^{l}(\alpha_i - \alpha_i^*) = 0\,, \quad \alpha_i \quad \alpha_i^* \in [0, C] \tag{29}$$

where α_i, α_i^* are Lagrange multipliers, and the support vector expansion can be expressed as follows

$$f(\mathbf{x}) = \sum_{i=1}^{l}(\alpha_i - \alpha_i^*) < \mathbf{x}_i, \mathbf{x} > +b \tag{30}$$

For non-linear cases, the dot product is replaced by a kernel function $K(\mathbf{x}_i, \mathbf{x})$, and the expansion for the non-linear cases is

$$f(\mathbf{x}) = \sum_{i=1}^{l}(\alpha_i - \alpha_i^*)K < \mathbf{x}_i, \mathbf{x} > +b \tag{31}$$

It notes that the loss functions can also be introduced to the SVR algorithm. Generally, there are four loss functions, i.e., Quadratic, Laplace, Huber and ε- insensitive loss functions. Kernel functions are also an important issue to obtain a high precision regression model of the training data. Linear, polynomial, radial basis, sigmoid, Spline, B-spline and Fourier functions are often selected as SVR kernel functions.

Many investigations show that on one hand, the ε-insensitive guarantees a sparse set of support vectors and leads to a fast calculation, on the other hand, to train a data set without priori knowledge, the radial basis functions (RBF) is the best one (Alex and Bernhard 2004). Therefore, they are selected in the present work.

3.3 A two-step approach to detect crack locations and severities

The basic idea of the two-step approach proposed in the literature and crack locations and severities were detected by two-steps (Xiang *et al*. 2012a, b). In order to improve the accuracy of locations detection in the first step, the wavelet transform (Xiang *et al*. 2012b) or interval wavelet transform (Xiang *et al*. 2012a) was employed to analyze an arbitrary modal shape. In the wavelet transform, the spatially distributed modal shape was decomposed into one approximation signal and three detailed signals. The wavelet shrinkage signals was proposed and applied to a specific scale, which can clearly identify the damage positions of a plate by showing peaks at the corresponding locations (Xiang *et al*. 2012b). To further decrease the boundary distortion phenomenon, interval wavelet transform was employed and the boundary distortion would be enormously suppressed (Xiang *et al*. 2012a). Once the crack locations have been identified, the next step is to detect the damage severities by natural frequency based method using finite element method (Adams *et al*. 1978, Nandwana and Maiti 1997, Lele and Maiti 2002, Patil and Maiti 2003, Maiti and Patil 2004, Patil and Maiti 2005, Chasalevris and Papadopoulos 2006, Sekhar 2008, Li and He 2011).

In the present investigation, the diagram of the two-step approach shown in Fig. 4, which is the improvement from Fig. 3 in the literature (Xiang *et al*. 2012b)

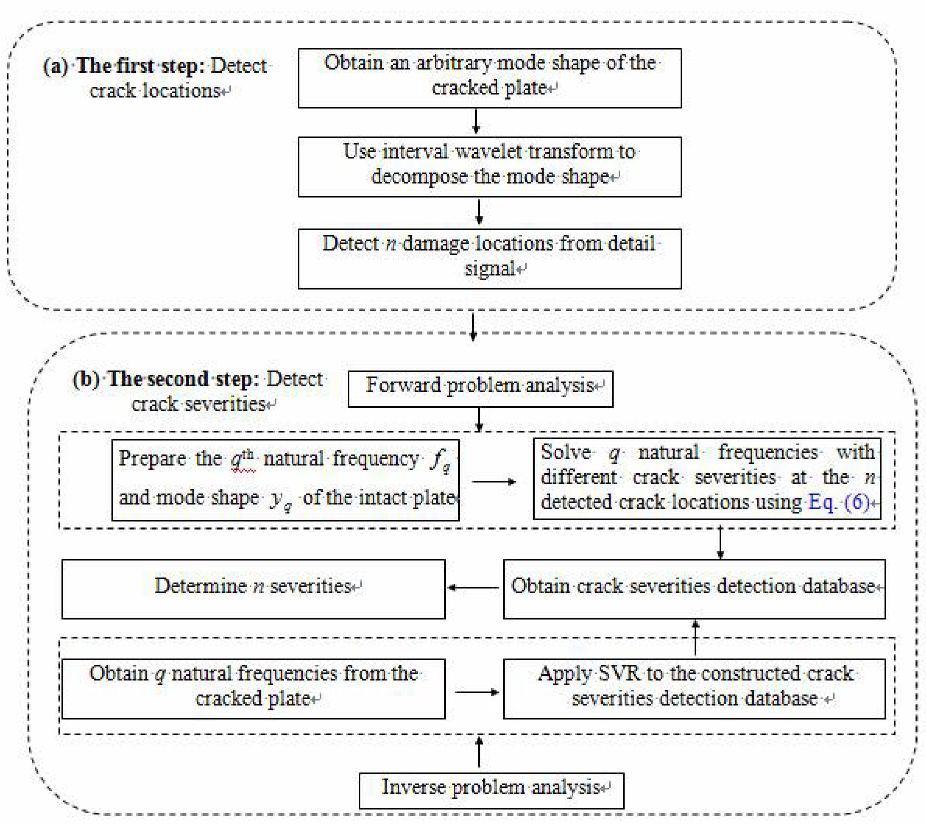

Fig. 4 The block diagram of the two-step damage detection method

3.3.1 Detect crack locations

Obtain an arbitrary mode shape of the cracked plate by experimental modal analysis (EMA). In the present investigation, because we use numerical simulation to testify the performance of the present approach, the finite element method is employed to calculate the mode shapes. Then we use interval wavelet transform to decompose the mode shape and the peaks in the detail signal indicate the crack locations $(x_{ci}, y_{ci}), i = 1,2,\cdots,n$ and the corresponding angles ϕ_i. Obviously, if the peaks or sudden changes are not available, it means that no cracks are available in plates. In addition, the relative crack locations can be represented by $\alpha_{ci} = x_{ci}/l_x$ and $\beta_{ci} = y_{ci}/l_y$.

3.3.2 Detect crack severities

To detect crack severities, there are two problems to be proceeded, one is the forward problem analysis and the other is the inverse problem analysis. In forward problem analysis, to construct crack depth detection database (the relationship between natural frequencies and crack severities) for q natural frequencies using Eq. (6), the q^{th} natural frequency f_q and mode shape y_q of the intact plate should be previously obtained using close-from solution (Timoshenko 1974), exact solutions (Wu 2007) or finite element method (Zienkiewicz *et al.* 2005). Therefore, we continuously compute natural frequencies versus different relative crack lengths $\gamma_i\,(i=1,2,\cdots,n)$ for the known crack locations (x_{ci}, y_{ci}), $i=1,2,\cdots,n$ and the corresponding angles ϕ_i, as follows

$$\bar{f}_m = F_m(\gamma_1, \gamma_2, \cdots, \gamma_n) \qquad (m=1,2,\cdots,q \geq n) \tag{32}$$

where F_m denote the relationship between relative crack lengths $\gamma_1, \gamma_2, \cdots, \gamma_n$ and the corresponding natural frequencies $\bar{f}_m\,(m=1,2,\cdots,q)$. In order to evaluate *n* crack depths in the plate-like structure with though-thickness cracks, the least *q* should be equals to *n*. Eq. (32) is also called the crack severities detection database.

In inverse problem analysis, based on Eq. (32), we have

$$(\gamma_1, \gamma_2, \cdots, \gamma_n) = F^{-1}\,(\bar{f}_1, \bar{f}_2, \cdots, \bar{f}_q) \qquad (q \geq n) \tag{33}$$

From Eq. (33), we can see clearly that the inverse problem to determine the relative crack lengths γ_i of *n* cracks is essentially a optimization problem. In this study, we adopt the SVM toolkit programmed by Professor Gunn of the University of Southampton (Gunn 1998).

To detect *n* cracks in the plate-like structure, *q* natural frequencies are employed as training samples and *n* relative crack lengths γ_i act as test samples to obtain a trained SVR model. According to the SVR algorithm, the training and test samples are

$$\left\{\mathbf{X}_i, H_i^s\right\}_{i=1}^{l}, \mathbf{X}_i = \{\bar{f}_1, \bar{f}_2, \cdots, \bar{f}_q\} \text{ for } H_i^s = \gamma_1, \gamma_2, \cdots, \gamma_n \tag{34}$$

where $\mathbf{X}_i$ and H_i^s are respectively the training and test samples, and *l* is the number of samples. For each training and prediction, *n* loops are needed to obtain relative crack lengths γ_i. It is desirable to use normalized, non-dimensional parameters to speed up the computational process (Alex and Bernhard 2004). Therefore, prior to the training of the SVR model, all data are separately normalized to be bounded by [-1, 1]. Take a dataset $\mathbf{g} = \{g_1, g_2, \cdots, g_n\}$ for example, the normalized dataset $\tilde{\mathbf{g}}$ is calculated by the formula $\tilde{\mathbf{g}} = (\mathbf{g} - \bar{g}) / \max\left|\mathbf{g} - \bar{g}\right|$, where the mean value $\bar{g} = \dfrac{1}{n}\sum_{i=1}^{n} g_i$. The corresponding renormalized formula is $\mathbf{g} = \tilde{\mathbf{g}} \times \max\left|\mathbf{g} + \bar{g}\right|$.

In practical applications, the large difference between the measured frequencies and the computed ones may make solutions irrelevant. For this reason, the model updating technique (Xiang *et al*. 2012a) is adopted for model updating and hence reducing the difference.

4. Numerical simulation

In section 4.1, several examples are given to validate the approximate natural frequencies formula. The suggested crack detection approach is also introduced in section 4.2.

4.1 The validity of approximate natural frequencies formula

4.1.1 Single crack

In this study, we consider a rectangle plate simply supported on all edges with one though-thickness crack. The dimensions of the plate are: length l_x = 105 mm, width l_y = 100 mm and thickness h = 5 mm. Material properties are: Young's modulus $E = 1.923 \times 10^{11}$ N/m^2 , Poisson's ratio $\mu = 0.33$ and material density ρ =7810 kg/m^3.

The relative crack locations $\alpha_{c1} = x_{c1} / l_x$ and $\beta_{c1} = y_{c1} / l_y$, the crack angles ϕ_1 and the relative crack lengths γ_1 of eight crack cases are listed in Table 1.

Table 1 gives the first sixth natural frequencies using Eq. (6) and those by Shell63 element in commercial FEM software ANSYS (Moaveni 2003). Additional analyses are performed to examine the performance of the present natural frequency approximation formula. For this purpose, we use more than 50000 triangular shell63 elements to calculate the first six natural frequencies. In Table 2, the maximum errors for the first six natural frequencies of the eight cases are 0.146%, 0.293%, 0.063%, 0.121%, 0.079% and 0.036%, respectively. It indicates that the performance of the natural frequency estimation formula is good.

4.1.2 Multiple cracks

Consider a same plate as shown in *Example 1* with three though-thickness cracks. The relative crack locations $\alpha_{ci} = x_{ci} / l_x$ and $\beta_{ci} = y_{ci} / l_y$, the crack angles ϕ_i and the relative crack lengths γ_i of five crack cases are listed in Table 3, where i=1,2,3.

In each crack case, a different combination of relative crack locations, angle orientations and relative crack lengths is consider to check the general applicability of the present natural frequency estimation formula Eq. (6) to the plate with multiple cracks. As shown in Table 4, we use more than 60000 triangular shell63 elements to calculate the natural frequencies as a benchmark. For the five cases, the maximum relative errors for mode 1 to mode 6 are 0.105%, 0.316%, 0.428%, 0.262%, 0.079% and 0.038%, respectively. The small errors between the present method and FEM indicate the present approximate equation is likely to be reliable.

Table 1 Crack cases for a simply supported plate with one crack

Crack case	Relative crack location		Orientation ϕ_1	relative crack lengths γ_1
	$\alpha_{c1} = x_{c1} / l_x$	$\beta_{c1} = y_{c1} / l_y$		
1	0.25	0.25	90°	0.05
2	0.25	0.25	90°	0.10
3	0.35	0.25	30°	0.05
4	0.35	0.25	30°	0.10
5	0.40	0.25	30°	0.05
6	0.40	0.25	30°	0.10
7	0.25	0.40	0°	0.15
8	0.25	0.40	0°	0.20

Table 2 Comparison of finite element method and the present solution for a simply supported plate with one crack

Case		1	2	3	4	5	6	7	8	Uncracked
Mode1 (Hz)	Shell63	1721.2	1720.4	1720.1	1716.3	1720.1	1716.3	1710.7	1702.7	1721.5
	Present	1721.1	1720.1	1720.8	1718.8	1720.6	1717.8	1708.9	1698.9	
	Error(%)	0.006	0.017	0.041	0.146	0.029	0.087	0.105	0.223	
Mode2 (Hz)	Shell63	3308.5	3302.4	3310	3308.5	3310.4	3310	3289.8	3275.1	3310.5
	Present	3307.8	3299.8	3308.5	3302.3	3308.9	3304.1	3285.5	3265.5	
	Error(%)	0.021	0.079	0.045	0.187	0.045	0.178	0.131	0.293	
Mode3 (Hz)	Shell63	5295.8	5293.0	5289.5	5269.8	5288.6	5266	5277.7	5261.8	5296.8
	Present	5295.7	5292.1	5289.3	5266.5	5288.2	5262.3	5276.5	5260.2	
	Error(%)	0.002	0.017	0.004	0.063	0.008	0.07	0.023	0.03	
Mode4 (Hz)	Shell63	5956.3	5948.9	5958.1	5955.7	5956.8	5951.1	5946.6	5938	5959
	Present	5955.8	5946.3	5958.8	5958.3	5958.8	5958.3	5944.1	5932.2	
	Error(%)	0.008	0.044	0.012	0.044	0.034	0.121	0.042	0.098	
Mode5 (Hz)	Shell63	6881.0	6868.9	6879.2	6862.4	6882.2	6873.3	6850.8	6822.1	6885.9
	Present	6880.3	6863.5	6879.2	6858.8	6882.3	6871.6	6847.5	6816.8	
	Error(%)	0.010	0.079	0.000	0.052	0.001	0.025	0.048	0.078	
Mode6 (Hz)	Shell63	9528.0	9514.1	9533.4	9532.1	9529.8	9518.3	9514.6	9497.4	9534.3
	Present	9528.4	9510.5	9534.0	9533.2	9530.2	9517.9	9515.7	9500.8	
	Error(%)	0.004	0.038	0.006	0.012	0.004	0.004	0.012	0.036	

By summarizing the above comparisons, the rectangle plates with one and three though-thickness cracks indicate that reasonably good calculation accuracy can be achieved for the crack problems by the proposed approximate natural frequency estimation formula.

The main advantage of the present natural frequency approximation formula is the simplicity. Only the natural frequencies and mode shapes of uncracked plate is needed to estimate the natural frequencies of plate with multiple though-thickness cracks. For the simply supported plate, the close-form solution of the natural frequencies and mode shapes can be used. However, for the more complicated plate structures, the natural frequency estimation equation can also be used to estimate the approximate natural frequencies of the plates with though-thickness cracks. The key problem is that the natural frequencies and mode shapes of the uncracked complicate plate structures should be provided firstly, which can be solved easily using finite element method.

Table 3 Crack cases for a simply supported plate with three cracks

Crack case	i	Relative crack locations		Orientations ϕ_i	relative crack lengths γ_i
		$\alpha_{ci} = x_{ci} / l_x$	$\beta_{ci} = y_{ci} / l_y$		
1	1	0.25	0.25	90°	0.05
	2	0.45	0.45	90°	0.10
	3	0.75	0.75	0°	0.15
2	1	0.25	0.25	0°	0.15
	2	0.45	0.45	0°	0.05
	3	0.75	0.75	90°	0.20
3	1	0.15	0.25	30°	0.05
	2	0.35	0.35	90°	0.15
	3	0.55	0.65	0°	0.10
4	1	0.25	0.45	45°	0.05
	2	0.45	0.25	30°	0.20
	3	0.75	0.35	0°	0.15
5	1	0.25	0.35	30°	0.05
	2	0.55	0.15	45°	0.15
	3	0.65	0.65	60°	0.10

4.2 Crack detection for simply supported plate with two though-thickness cracks

As shown in Fig. 1, suppose the plate dimensions and the material properties are: length l_x =1000 mm, width l_y =1000 mm and thickness h = 20 mm. Material properties are: Young's modulus E = 20.6×10^{11} N/m^2, Poisson's ratio $\mu = 0.3$ and material density $\rho = 7860$ Kg/m^3.

4.2.1 Detection crack locations

Suppose the two crack location coordinates are: ($x_{c1} = 0.1$, $y_{c1} = 0.1$) and ($x_{c2} = 0.6$, $y_{c2} = 0.7$), the crack angles are: $\phi_1 = 90^\circ$ and $\phi_2 = 0^\circ$, the relative crack lengths are: $\gamma_1 = 0.1$, $\gamma_2 = 0.2$, the finite element meshes (127×127) using Shell63 is shown in Fig. 5.

The second mode shape is employed to show how to determine the crack locations. It worth to point out that an arbitrary mode shape can be used to obtain the same results (Xiang *et al.* 2011, Xiang and Liang 2012b). For the analysis, CDV3 is used to decompose only one level. Fig. 6 shows the decomposition results of the second mode shape. The approximation signal A as shown in Fig. 6(a) is essentially a approximation signal where the singularity components are removed.

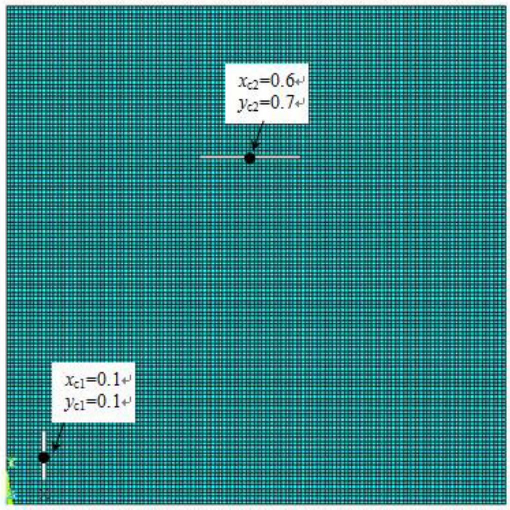

Fig. 5 The finite element model of thin plate with two though-thickness cracks

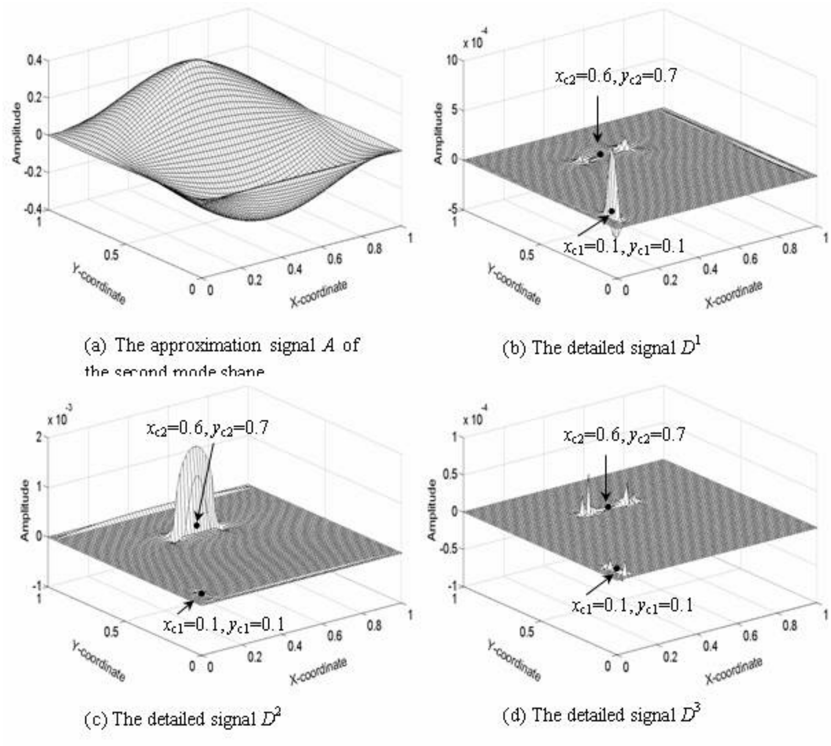

Fig. 6 The first mode shape and its wavelet decomposition using Db3 interval wavelets at level 1

The three detailed signal D^1, D^2 and D^3 are shown in Fig. 6 (b)-(d), respectively. As shown in the figures, D^1, D^2 and D^3 are all sensitive to damage singularity. Moreover, similar to the damage detection method using interval wavelet transform (Xiang *et al.* 2011), the severe boundary distortions that usually occur in wavelet coefficient computation near the signal edges (Xiang and Liang 2012b) are not shown in the CDV3 (interval wavelet transform) decomposition. The above results suggest that all of the three detailed signal D^1, D^2 and D^3 can be used to detect crack locations for plate with though-thickness cracks. The detected two crack angles and crack location coordinates are $\phi_1^* = 90^\circ$, $\phi_2^* = 0^\circ$, $x_{c1}^* = 0.1$, $y_{c1}^* = 0.1$, $x_{c2}^* = 0.6$, $y_{c2}^* = 0.7$, respectively, as shown in D^1, D^2 and D^3. However, we can not determine the crack severities when the small measurement errors are introduced (Xiang *et al.* 2011, Xiang and Liang 2012b). Therefore, the two-step hybrid method provides for a more reliable crack detection strategy.

Table 4 Comparison of finite element method and the present solution for a simply supported plate with three cracks

Case		1	2	3	4	5
Mode1 (Hz)	Shell63	1710.2	1707.7	1705.5	1691.2	1712.6
	Present	1708.8	1705.9	1703.9	1691	1712.2
	Error(%)	0.082	0.105	0.094	0.012	0.023
Mode2 (Hz)	Shell63	3293.3	3262.3	3285.7	3286.3	3291.2
	Present	3292	3252	3283.6	3278.4	3292.0
	Error(%)	0.039	0.316	0.064	0.24	0.024
Mode3 (Hz)	Shell63	5242.5	5230.6	5255.8	5143.5	5256.8
	Present	5235.4	5216.8	5250.3	5121.5	5240.4
	Error(%)	0.135	0.264	0.105	0.428	0.312
Mode4 (Hz)	Shell63	5914	5897.9	5941.9	5905.0	5949.7
	Present	5908.3	5896.2	5945.3	5920.0	5934.1
	Error(%)	0.096	0.029	0.057	0.254	0.262
Mode5 (Hz)	Shell63	6791.3	6737.1	6855.9	6809.5	6844.3
	Present	6768	6724.5	6854.3	6795.7	6872.1
	Error(%)	0.343	0.187	0.023	0.203	0.406
Mode6 (Hz)	Shell63	9477.7	9398.4	9474.8	9376.5	9480.0
	Present	9470.6	9381.1	9496.2	9351.7	9446.3
	Error(%)	0.075	0.184	0.226	0.264	0.355

4.2.2 Detection crack severities

We adopt the SVM toolkit programmed by Professor Gunn of the University of Southampton (Gunn 1998) to detect crack severities.

To detect two though-thickness cracks in the plate, the first three natural frequencies f_1, f_2 and f_3 calculated by Eq. (6) are employed as training samples and two relative crack lengths γ_1 and γ_2 act as test samples to obtain a trained SVR model.

According to the SVR algorithm, the training and test samples are

$$\{\mathbf{X}_i, H_i^s\}_{i=1}^{l}, \mathbf{X}_i = \{f_1, f_2, f_3\} \text{ for } H_i^s = \gamma_1, \gamma_2 \tag{35}$$

where $\mathbf{X}_i$ and H_i^s are respectively the training and test samples, and l is the number of samples. For each training and prediction, two loops are needed to obtain two relative crack lengths γ_1 and γ_2. ε- insensitive (ε=0.000001, suggested by Gunn(1998)) and radial basis functions are employed as loss and kernel functions respectively. Suppose the selected γ_1 are in the range of 0.01 to 0.2 with step 0.01, whereas γ_2 are in the range of 0.01 to 0.4 with the same step. The data for training samples constitute a 800×3 matrix and test samples lead to a 800×1 vector. When we the trained SVR to predict the two relative crack lengths γ_1^* and γ_2^*, the input parameters should be the measured first three natural frequencies. In the present, the first three natural frequencies are calculated by about 60000 Shell63 elements.

Table 5 lists the crack severities prediction results with combination (the width of RBF p_1 = 10 and C = 50) for the seven different cases. The absolute errors are defined by δ_1=| γ_1^* - γ_1 |×100%, δ_2=| γ_2^* - γ_2 |×100%, respectively. The maximum absolute errors δ_1 and δ_2 for seven cases are respectively 2.8% and 2.9%. The numerical simulation validate the SVR model and also indicated that the SVR performance.

Table 5 Crack severities detection results (p1 = 10 and C = 50)

Case	γ_1	γ_2	f_1^*	f_2^*	f_3^*	γ_1^*	δ_1	γ_2^*	δ_2
1	0.05	0.05	97.234	242.71	243.26	0.053	0.3	0.041	0.9
2	0.05	0.1	96.981	241.12	243.25	0.07	2	0.076	2.4
3	0.1	0.1	96.974	241.08	243.2	0.072	2.8	0.076	2.4
4	0.1	0.15	96.558	238.55	243.17	0.095	0.5	0.132	1.8
5	0.1	0.20	95.99	235.28	243.03	0.12	2	0.198	0.2
6	0.15	0.20	95.975	235.17	242.95	0.122	2.8	0.199	0.1
7	0.15	0.3	94.36	227	242.43	0.128	2.2	0.329	2.9

Note: δ_1=| γ_1^* - γ_1 |×100%, δ_2=| γ_2^* - γ_2 |×100%

How to choose the trade off parameters p_1 (the width of RBF) and C to obtain the good support vectors is a difficult task in theory analysis society and application field (Gunn 1998). To apply the SVR for the detection of cracks in structures, it notes that the simulation investigation can be preceded at first to obtain the good parameters p_1 and C. Because the similarity of simulation and experimental/practical of the same structures, these parameters can also be employed to deal with the experimental/practical data.

5. Conclusions

In this paper, the crack detection approach of though-thickness plate-like structures has been investigated and the basic idea of this approach is deriving from a simple crack detection method for beam-like structures. The procedure involves two steps. The first step is the application of the first mode shape to detect the possibly existence and locations of cracks in plate-like structures. The second step is the employment of SVR to predict crack severities from the crack severity detection database. In particular, the interval wavelet transform is applied to decompose the first mode shape to eliminate boundary distortion phenomena. The natural frequencies estimation formula is derived for cracked plate-like structures using Rayleigh quotient and the relationship of natural frequencies and crack severities can be easily constructed by a simple formula. It is observed that the numerical results from both the natural frequencies estimation formula matched very well with those obtained by the finite element method. The hybrid crack detection approach is also verified using numerical simulations.

The further work is to evaluate the natural frequencies of cracked thick plate/shell structures or other complicated structures. The possible way is to employ numerical analysis method, such as finite element analysis method and boundary element method, etc., to calculate natural frequency and strain energy of the intact structures. Then according to linear elastic fracture mechanics theory, we can determine the energy stored in the cracks. Finally, the natural frequencies of cracked structures will be calculated by Eq. (6). Moreover, the crack detection approach is similar to the present method.

Acknowledgments

The authors are grateful to the support of the Alexander von Humboldt Foundation (ID: 1140023), the National Science Foundation of China (Nos. 51175097, 51075314), the Zhejiang Provincial Natural Science Foundation for Excellent Young Scientists (No. LR13E050002) and the Project-sponsored by SRF for ROCS, SEM. This paper is also supported by the Zhejiang Technologies R&D Program of China (No. 2014C31103).

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