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Variability in bridge frequency induced by a parked vehicle

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Abstract. The natural frequency of a bridge is an important parameter in many engineering applications such as bridge seismic design and modal-based bridge health monitoring. The natural frequency of a bridge vibrating alone may differ from that vibrating along with a vehicle. Although such vehicle-induced variability in bridge frequency is revealed in several experimental and numerical simulation studies, few attempts have been made on the theoretical descriptions. In this study, both theoretically and experimentally, the variability in the bridge frequency induced by a parked vehicle is verified, and is therefore suggested to be considered in bridge-related engineering, especially for those cases with near vehicle-bridge resonance conditions or with large vehicle-to-bridge mass ratios. Moreover, the variability ranges could be estimated by an analytical formula presented herein.

Keywords: bridge engineering; bridge frequency; vehicle-bridge interaction (VBI); vibration-based health monitoring

1. Introduction

The natural frequency of a bridge is an important parameter in the field of bridge engineering. For bridge dynamics, the natural frequency primarily dominates the dynamic responses of a bridge under dynamic loadings, especially under seismic loadings. Also, for modal-based bridge health monitoring, the natural frequency serves as a preliminary damage indicator (Doebling *et al.* 1998, Carden and Fanning 2004, Fan and Qiao 2011).

The natural frequency of a bridge vibrating alone may differ from that vibrating along with a vehicle, due to the fact that the presence of the travelling or parked vehicles on the bridge introduces additional mass and interaction effects to the bridge. Such vehicle-induced frequency variability is of equal importance for the bridge dynamics and health monitoring. Under seismic loadings, the varied bridge frequency may cause the bridge responses deviate from the design responses wherein usually no existing vehicle is taken into consideration (Kim and Kawatani 2006). In modal-based bridge health monitoring, the bridge frequencies are altered due to changes in stiffness, mass, or supporting conditions caused by damages, and those alters are supposed to be identified from the bridge dynamic responses. However, the damage-induced frequency variability may be masked by the vehicle-induced frequency variability to a certain extent that the former is difficult to be identified, even is identified with false alarms. Therefore, the vehicle-induced variability in the bridge frequency should be made clear so that it can be removed from the

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measured data to avoid any misjudgment in bridge health conditions (Kim and Kawatani 2008). The variability in the bridge frequency is revealed in several experimental studies (Zhang *et al*. 2002, Kim *et al*. 2003, Macdonald and Daniell 2005, Sohn 2007, Zhang and Xiang 2011, Kim and Lynch 2012) and numerical simulation studies (De Roeck *et al.* 2002, Macdonald and Daniell 2005, Zhang and Xiang 2011). However, few attempts have been made on their theoretical descriptions. Once the theoretical descriptions are established, the variability in bridge frequency can be evaluated explicitly and the dominant factors and their sensitivity can be realized.

This study is thus to explore the vehicle-induced variability in the bridge frequency, both theoretically and experimentally, and to investigate the sensitivity of the factors that dominate the frequency variability. As an initial attempt, a parked vehicle is considered. In the theoretical study, an analytical formula for estimating the varied bridge frequencies induced by a parked vehicle is derived, in consideration of a simplified two-degree-of-freedom vehicle-bridge interaction model. In the experimental study, a field experiment on a highway bridge and a laboratory experiment on a scaled bridge are conducted for verifying both the vehicle-induced variability in bridge frequency and the accuracy and applicability of the derived analytical formula. In addition, a sensitivity analysis is performed utilizing the analytical formula to illustrate how the key factors dominate the vehicle-induced variability in the bridge frequency. Finally, several concluding remarks are drawn regarding the potential engineering applications.

2. Theoretical derivation

Fig. 1 shows a simplified vehicle-bridge interaction model for a bridge along with a vehicle parked at its midspan. In the model, the vehicle is modeled as a sprung mass m_v of single degree of freedom (DOF) supported by a spring of stiffness constant k_v, and the bridge as a Euler-Bernoulli beam of length L, constant rigidity EI, and constant mass density m^* per unit length. Since only frequency characteristics are of our concern, the damping effects for both vehicle and bridge are neglected without losing generality and the road surface roughness is not taken into consideration for its non-contribution to the frequency characteristics when a vehicle is parked on the bridge. Moreover, no other location besides the midspan of the bridge is considered to park a vehicle, because midspan is the location most affected by the presence of the vehicle as the variability of the system frequency is of concern; the above reason is especially dominant in conducting verification experiments efficiently within a limited amount of budget.

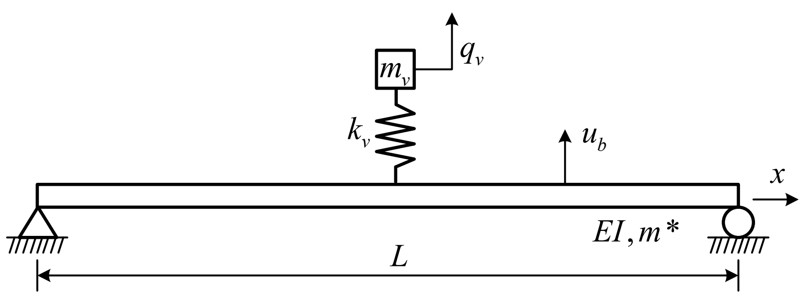


Fig. 1 Vehicle-bridge interaction model

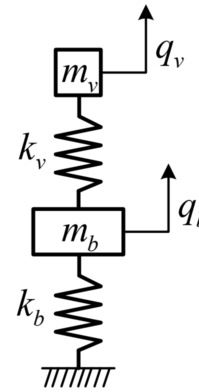


Fig. 2 Two-DOF model

The interaction model can be further represented by a two-DOF system as shown in Fig. 2, where the vehicle is represented by the same sprung mass as in Fig. 1 and the bridge by another sprung mass with the generalized mass m_b and generalized stiffness k_b given by the following formulas (Clough and Penzien 1993)

$$m_b = \int_0^L m^* \phi(x)^2 dx\,,\quad k_b = \int_0^L EI\phi''(x)^2 dx \tag{1,2}$$

where $\phi(x)$ is the shape function of the bridge, and a prime is the derivative with respect to the longitudinal coordinate x. Considering only the first mode of most significance, the shape function for a simply supported beam is $\sin(\pi x/L)$. m_b and k_b in Eqs. (1) and (2)are thus obtainable as

$$m_b = \frac{m^* L}{2},\quad k_b = \frac{\pi^4 EI}{2L^3} \tag{3,4}$$

The equations of motion for vehicle and bridge can be expressed as

$$m_v \ddot{q}_v + k_v (q_v - q_b) = 0 \tag{5}$$

$$m_b \ddot{q}_b - k_v (q_v - q_b) + k_b q_b = 0 \tag{6}$$

where q_v and q_b are displacements of vehicle and bridge, respectively. The above equations can then be assembled in a matrix form as

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \ddot{q}_v \\ \ddot{q}_b \end{Bmatrix} + \begin{bmatrix} \omega_{v,0}^2 & -\omega_{v,0}^2 \\ -\mu\omega_{v,0}^2 & \omega_{b,0}^2 + \mu\omega_{v,0}^2 \end{bmatrix} \begin{Bmatrix} q_v \\ q_b \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{7}$$

where $\omega_{v,0}$ and $\omega_{b,0}$ respectively denote the natural frequencies of the vehicle and bridge when they vibrate independently, and μ denotes the mass ratio of the vehicle to the bridge, as defined as follows.

$$\omega_{v,0} \equiv \sqrt{\frac{k_v}{m_v}}, \quad \omega_{b,0} \equiv \sqrt{\frac{k_b}{m_b}}, \quad \mu \equiv \frac{m_v}{m_b} \tag{8,9,10}$$

When the vehicle is parked at the midspan of the bridge, it may interact with the bridge when vibrating. In other words, those two systems act dependently as a so-called vehicle-bridge interaction system. For the interaction system, its natural frequency ω can be obtained by solving the corresponding eigen-problem

$$|\,[K] - \omega^2\,[M]\,| = 0 \tag{11}$$

where $[M]$ and $[K]$ are normalized mass and stiffness matrices, respectively, and are expressed as follows

$$[M] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad [K] = \begin{bmatrix} \omega_{v,0}^2 & -\omega_{v,0}^2 \\ -\mu\omega_{v,0}^2 & \omega_{b,0}^2 + \mu\omega_{v,0}^2 \end{bmatrix} \tag{12}$$

The solutions in square form to Eq. (11) are

$$\omega^2 = \omega_{b,0}^2 \times \frac{1}{2}\left[1 + \beta^2 + \mu\beta^2 \pm \sqrt{\left(1 + \beta^2 + \mu\beta^2\right)^2 - 4\beta^2}\,\right] \tag{13}$$

where β denotes the frequency ratio of the vehicle to bridge, i.e.

$$\beta \equiv \frac{\omega_{v,0}}{\omega_{b,0}} \tag{14}$$

It is then easy to obtain ω simply by taking the positive squared root of Eq. (13). For general uses, the above frequencies, ω, $\omega_{v,0}$ and $\omega_{b,0}$, in angular terms (rad/s) can be converted into cyclic terms, f, $f_{v,0}$ and $f_{b,0}$, respectively, by dividing them by 2π. It is clearly shown in Eq. (13) that the eigen-frequency of the vehicle-bridge interaction system is a product of $\omega_{b,0}$ and the factor in the bracket, rather than $\omega_{v,0}$ or $\omega_{b,0}$, indicating that the system frequency observed at the bridge may shift from the bridge frequency to the interaction-system frequency with a certain level as the presence of the parked vehicle.

The solution contains two expressions: one with the positive sign and the other with the negative sign in the bracket. Physically speaking, the expression that gives larger value should correspond to either the original vehicle or bridge frequency which has larger natural frequency when vibrating alone, and the expression that gives smaller value should correspond to that which has smaller natural frequency. To be more specific, for the case where the original bridge frequency is larger than the original vehicle frequency, the expression with positive sign should correspond to the bridge frequency and the expression with negative sign to the vehicle frequency. The reverse is also true. Generally, the bridge-corresponded natural frequency should dominate the responses of the vehicle-bridge interaction system since the bridge is larger in mass than the vehicle. For the sake of brevity and also for the topic of our major concern, the natural frequency of the interaction system will refer to the bridge-corresponded natural frequency without further indication.

The above theoretical formula can be used to calculate the natural frequencies of the vehicle-bridge interaction system once the mass ratio μ and frequency ratio β are given. Its accuracy is examined in the following experimental studies.

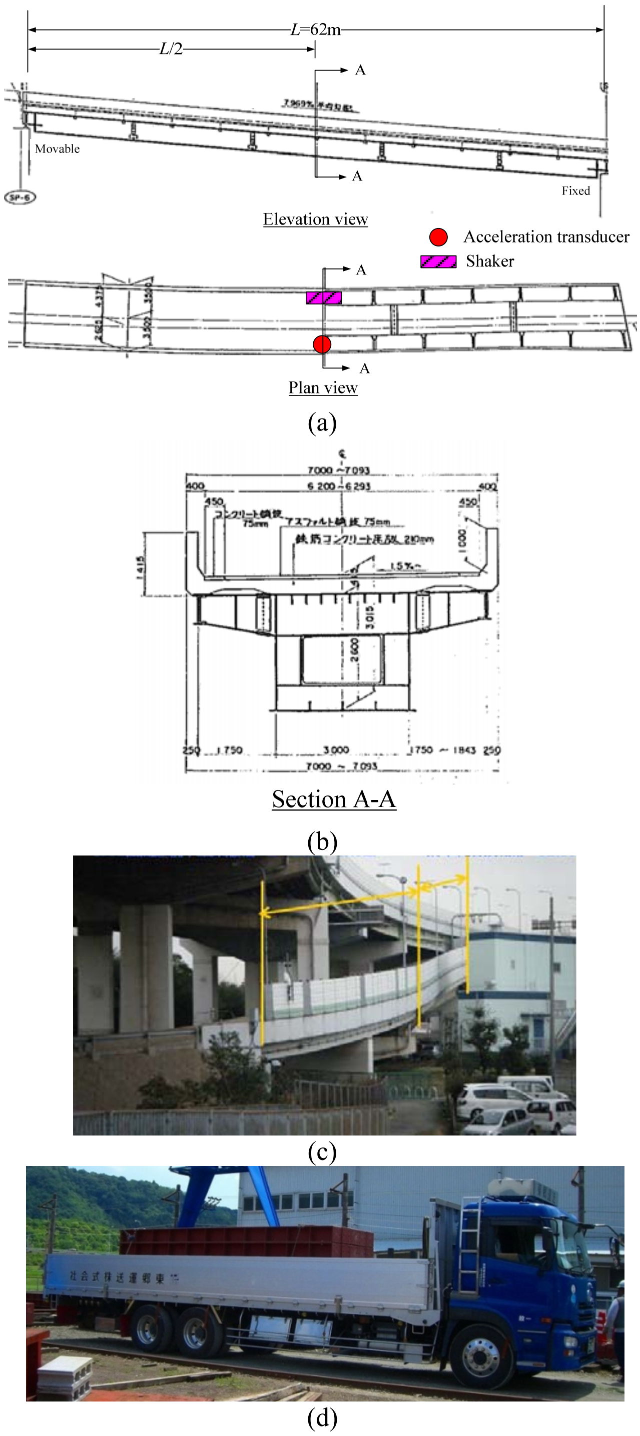


Fig. 3 Field experiment bridge and vehicle: (a) elevation and plane views of the test span, with instrumentation layout, (b) cross-section view of the test span, (c) photo of the test span and (d) photo of the experiment vehicle. (Partly adopted from original design drawings)

Experiment bridge

The experiment bridge was one span of an Entrance Viaduct, Hanshin Expressway No.4, located at the eastern edge of Osaka Bay, in Sakai City, Osaka Prefecture. It was a simply-supported steel box girder bridge of 62 m in span length, as shown in Fig. 3. According to the original design, the total mass of the bridge m^*L was about 620Mg.

Experiment vehicle

The experiment vehicle (see Fig. 3(d)) was a cargo truck of model LKG-CD5ZA, produced by UD Trucks Corp. The total weight, including the truck body and additional cargo loading, was about 250 KN (25.5 Mg in mass). To realize the dynamic properties of the truck, an independent free vibration test was performed, by letting the truck drop from a brick. From the test, the natural frequency of the truck is obtained as 2.86 Hz in vehicle body bounce motion, measured from the vehicle body right above the rear axle.

Instrumentation

The acceleration responses of both bridge and vehicle were measured with one-dimensional transducers of model ARF-10A and ARS-10A, and transmitted to and recorded by a data acquisition system of model DC-204R, both produced by Tokyo Sokki Kenkyujo Co., Ltd.. The nominal capacity of transducers is 10 m/s^2.

The shaker was 4905 N in weight (500 kg in mass) and was held by a set of jigs of 6180 N (630 kg in mass). The forced frequency was applied from 1.54 Hz to 1.60 Hz for bridge system alone (1st stage) and from 1.50 to 1.60 Hz for vehicle-bridge interaction system (2nd stage), both with an increment of 0.01 Hz. For each case, the actuating amplitude remained 18 mm, and the duration of actuating was kept more than 30 seconds to ensure the systems reach their steady states.

Layout

Fig. 3(a) shows the instrumentation layout of the field experiment. The field experiment was performed in two stages. For the 1st stage, the acceleration of the bridge system without the vehicle was measured. The shaker was installed at the midspan of the bridge and actuates vertically, and a vertical acceleration transducer was also mounted at the midspan. For the 2nd stage, the acceleration of bridge with the cargo truck parked at its midspan was measured. Besides the above mentioned shaker and acceleration transducer remaining at the same location, another vertical acceleration transducer was mounted at the vehicle body right above the rear axle of the experiment vehicle. Ideally the shaker should be mounted at the center of the longitudinal lane to avoid any possible torsion vibration induced by the shaker’s asymmetric input, but herein the shaker was mounted at the side of the lane due to the practical constraint: the center was parked by the experiment vehicle, therefore allowing no space for the shaker. Even so, the possible torsion vibrations were supposed to couple little with the bending vibrations.

Experiment results

Fig. 4 shows an interval of the steady acceleration response recorded on the bridge under the shaker excitation with a forced frequency of 1.56 Hz at the 1st stage, along with its corresponding Fourier spectrum. It can be identified from the spectrum that the dominant frequency of this response is 1.56 Hz and the corresponding amplitude is 0.0774 m/s^2.

By the same way, the time histories, including the acceleration responses of the experiment bridge recorded at the 1st stage and those of the experiment bridge and vehicle recorded at the 2nd stage for every forced frequency increment, are performed with Fourier transform to obtain their corresponding Fourier spectra. From those spectra, the dominant frequencies and corresponding amplitudes can be identified, as summarized in Table 1. It is observed that the identified bridge and vehicle frequencies are entirely consistent with the forced frequencies actuated by the shaker. Such a consistency offers a preliminary verification of the reliability of the recorded responses.

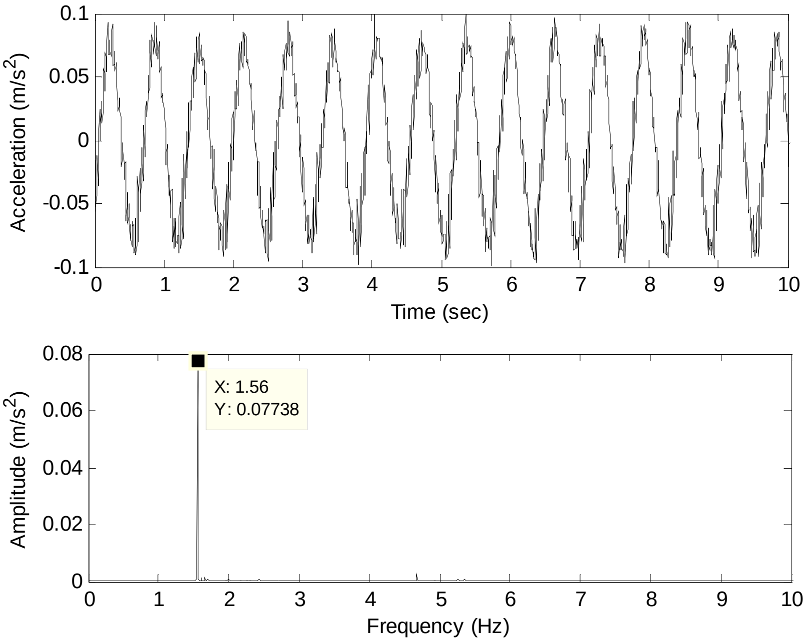
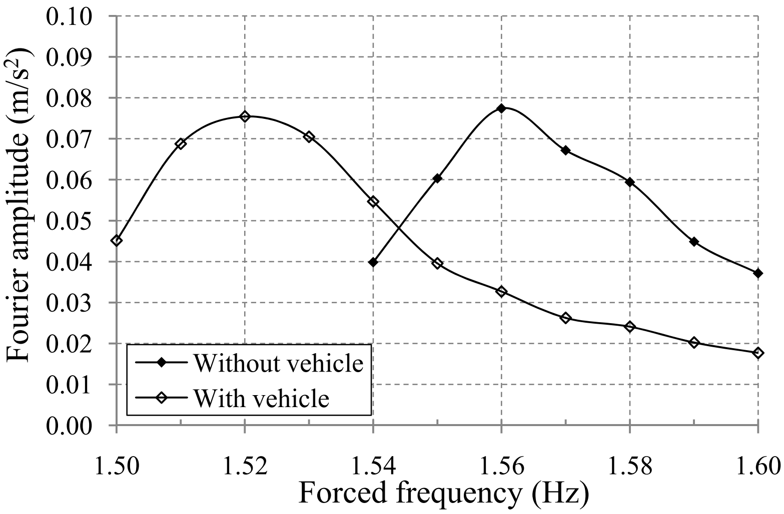


Fig. 4 An interval of the steady acceleration response recorded of the bridge (upper) and its corresponding Fourier spectrum (lower)

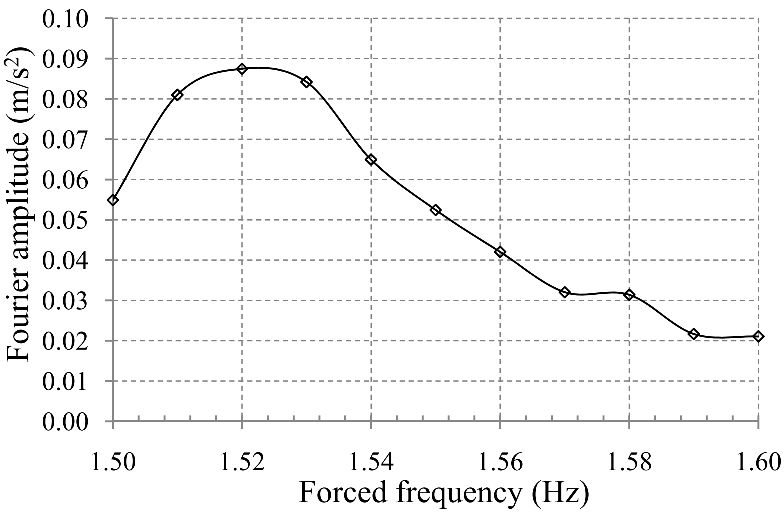
Table 1 Identified frequencies and amplitudes for bridge and vehicle responses

Bridge responses						Vehicle responses		
Stage 1			Stage 2			Stage 2		
f_f	f_i	A	f_f	f_i	A	f_f	f_i	A
1.54	1.54	0.0398	1.50	1.50	0.0451	1.50	1.50	0.0549
1.55	1.55	0.0603	1.51	1.51	0.0688	1.51	1.51	0.0810
1.56	1.56	0.0774	1.52	1.52	0.0754	1.52	1.52	0.0875
1.57	1.57	0.0671	1.53	1.53	0.0705	1.53	1.53	0.0842
1.58	1.58	0.0594	1.54	1.54	0.0547	1.54	1.54	0.0650
1.59	1.59	0.0449	1.55	1.55	0.0396	1.55	1.55	0.0525
1.60	1.60	0.0371	1.56	1.56	0.0327	1.56	1.56	0.0420
			1.57	1.57	0.0263	1.57	1.57	0.0320
			1.58	1.58	0.0241	1.58	1.58	0.0314
			1.59	1.59	0.0202	1.59	1.59	0.0217
			1.60	1.60	0.0177	1.60	1.60	0.0211

Note: f_f and f_i denote the forced frequency and identified frequency (Hz), respectively, and A denotes the corresponding amplitude (m/s^2).



(a)



(b)

Fig. 5 Fourier amplitude vs. forced frequency for (a) the bridge and (b) the vehicle

Then, the amplitudes corresponding to the forced frequencies are plotted for both bridge and vehicle, as shown in Fig. 5. The natural frequency of the systems is identified as the frequency corresponding to the peak of respective curve. By this way, the natural frequency for the bridge alone (1st stage) is identified as 1.56 Hz, while that for the vehicle-bridge interaction system (2nd stage) is 1.52 Hz. The latter frequency can be identified from either bridge or vehicle response, and the values identified from those two responses are identical, mutually verifying their reliability. More importantly, as the presence of the parked vehicle, a decrease in the bridge frequency, say of about 2.6% herein, is obviously illustrated.

Comparison with analytical results

The field-experiment results can be adopted to examine the validity of the analytical expression of the natural frequency in Eq. (13). With the identified natural frequencies of the bridge and vehicle, the value of the frequency ratio β is calculated from Eq. (14) as 2.86 / 1.56 = 1.833. Another parameter, the mass ratio μ, is calculated from Eq. (10) as 25.5 / (620 / 2) = 0.082. Substituting the above two parameters into Eq. (13), the analytical natural frequency f of the interaction system is obtained as 1.48 Hz, which matches rather well with that identified from the field experiment (1.52 Hz), merely with a deviation of 2.6%.

The slight deviation may be acceptable since the simplification is made in the analytical derivation: the vehicle was idealized as a SDOF system, with which the distribution pattern of the vehicle mass was not considered. Despite the simplification, the analytical formula in Eq. (13) is still qualified to preliminarily estimate the variability of the bridge frequency. Note that the mass of the shaker was neglected because it was too small compared to that of the bridge and therefore affect little to the accuracy of the analytical results.

3.2 Laboratory experiment on a scaled bridge

For the same purposes of verifying the variability of the bridge frequency experimentally, a laboratory experiment was performed on a scaled steel bridge. The experiment bridge and vehicle adopted herein were designed to offer similar frequency characteristics of real ones, rather than scaled down from a real bridge and vehicle following the scaling rule.

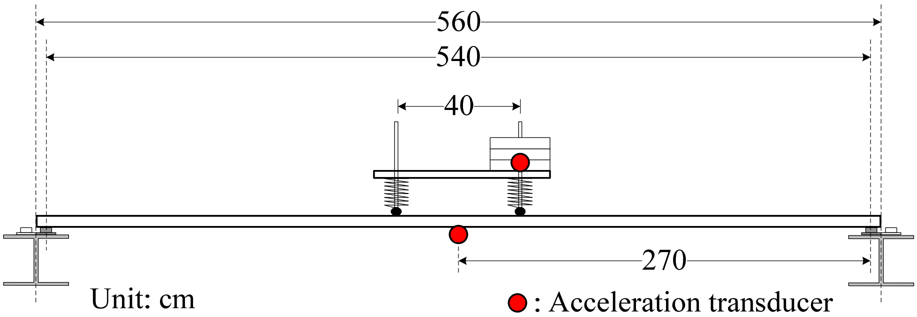
Similar to the field experiment, the bridge frequencies were measured at two stages: the 1st stage was for the bridge alone, and the 2nd stage was for the bridge along with a scaled vehicle parked at its midspan. The natural frequency of either the bridge system or the vehicle-bridge interaction system was measured by free vibration tests. During the experiments, the interaction system was firstly subjected to an initial displacement and then released suddenly, and the acceleration responses of the system were recorded during its free vibration. By performing a time domain analysis to the free-vibration responses, the natural frequencies of both the bridge system alone and vehicle-bridge interaction system can be identified. The details of the laboratory experiment and the experiment results are given as follows.

Experiment bridge

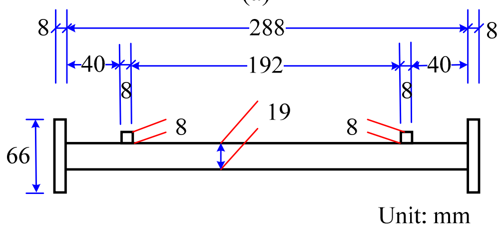
The experiment bridge was a simply-supported steel beam with a span length of 5.4 m (see Fig. 6). The density of steel is known as 7.8×10^{-3} kg/cm^3, and the designed cross-sectional area was 66.56 cm^2. Accordingly, the total mass of the scaled bridge m^*L was about 280kg.

Experiment vehicle

The experiment vehicle was a two-axle vehicle, assembled by a steel plate as vehicle body, four springs as suspension system, and four plastic wheels (see Fig. 6). The stiffness of the vehicle suspension system was able to be adjusted by replacing different springs, and the mass of the experiment vehicle was able to be adjusted by attaching steel blocks on it. The natural frequency of the experiment vehicle, as a function of vehicle stiffness and mass, varied accordingly. Two vehicle models, designated as V1 and V2 respectively, were adopted in this study. Both vehicles weighed 212 N (21.6 kg in mass), and the natural frequencies of V1 and V2 were measured as 3.023 Hz and 3.646 Hz, respectively.



(a)



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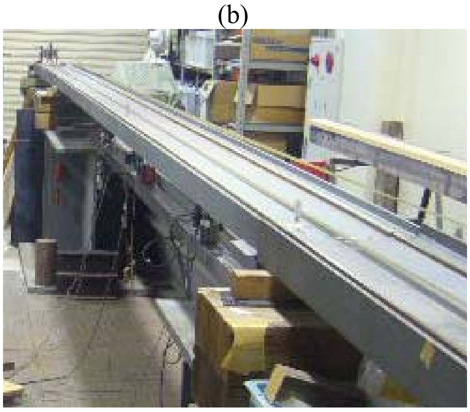

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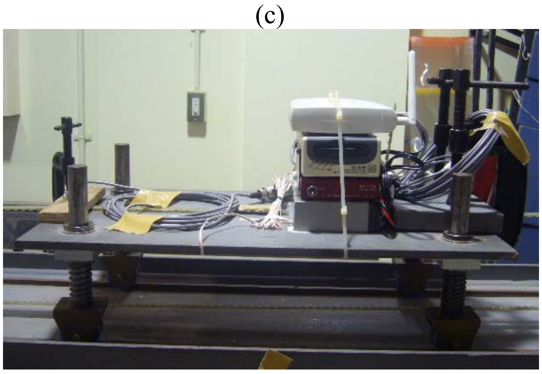


(d)

Fig. 6 Experiment scaled bridge and vhielce: (a) vertical longitudinal section of the bridge, with instrumentation layout, (b) vertical transverse section of the bridge, (c) photo of the bridge, and (d) photo of the vehicle

Layout

Similar to the field experiment, the laboratory experiment was also performed in two stages. For the 1st stage, no vehicle was placed on the bridge. A vertical acceleration transducer was mounted at the midspan of the experiment bridge. For the 2nd stage, the experiment vehicle, either V1 or V2 model, was placed at the midspan of the experiment bridge. The vertical acceleration transducer remained at the midspan, and another transducer was mounted above the rear axle of the experiment vehicle. To be brevity, the vehicle-bridge interaction system involving the experiment vehicle V1 was referred to as the V1-VBI system and that involving V2 as V2-VBI system.

Experiment results

One typical free-vibration response of the scaled bridge for the 1st stage is shown in Fig. 7, and those of the scaled bridge vibrating with the experiment vehicle V1 and V2 for the 2nd stage are shown in Figs. 8 (a) and 8(b), respectively. Those free-vibration responses clearly show that the experiment systems vibrate approximately in a SDOF manner with their respective fundamental mode. Preliminarily compared with the free-vibration response for the 1st stage (Fig. 7), the oscillation numbers of the responses for the 2nd stage (Fig. 8) are reduced in a fixed time span, implying that the oscillation frequencies are reduced in the 2nd stage. To be more precise, the natural frequencies of the experiment systems are identified by the following method.

Herein, the natural frequency of the experiment system is identified in time domain by taking the reciprocal of the natural period of the system, which can be theoretically defined as the time span between adjacent peaks or troughs of a free-vibration response. In practice, the natural period thus obtained may not be constant even within one single free-vibration response in some cases. In those cases, the natural period can be obtained in an average manner that the time span of several peaks/troughs is divided by the number of peaks/troughs. Opting for such a time domain analysis is to avoid poor resolutions in frequency domain analysis (e.g., fast Fourier transform) due to short analysis intervals available from free-vibration responses of the interaction systems which decay quickly.

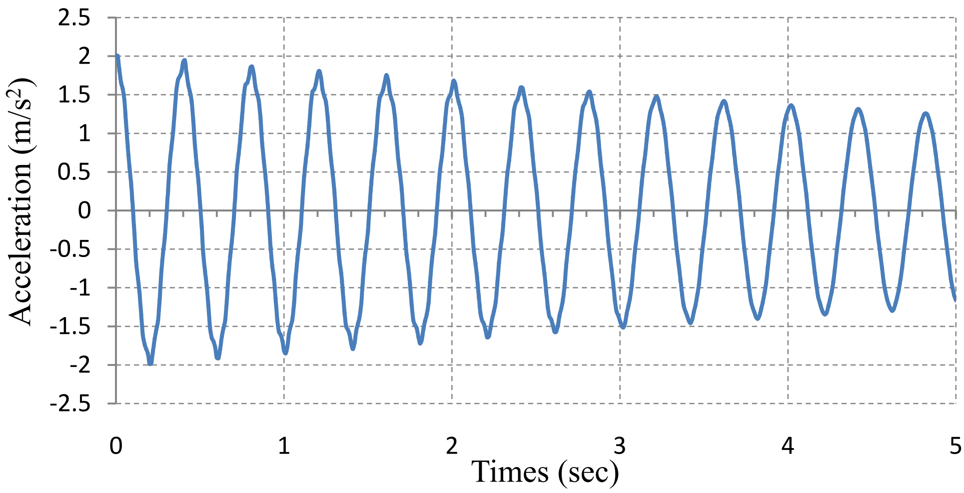
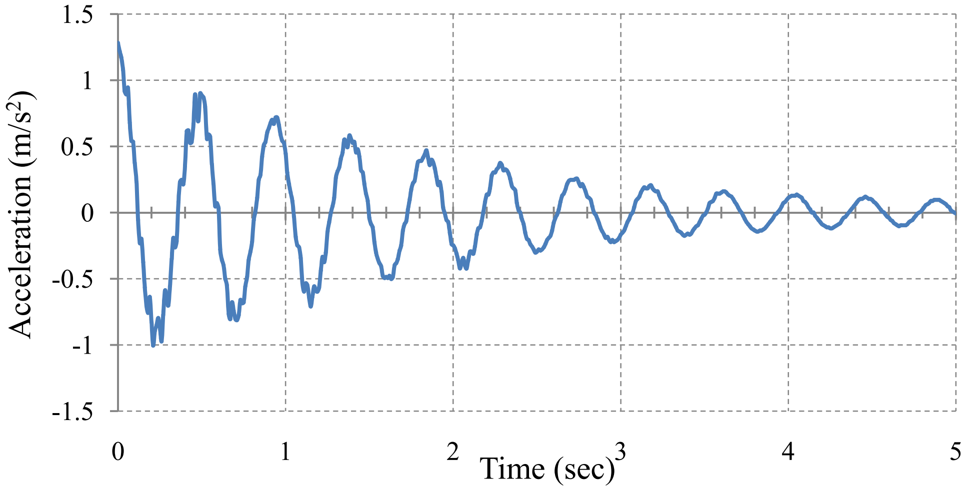
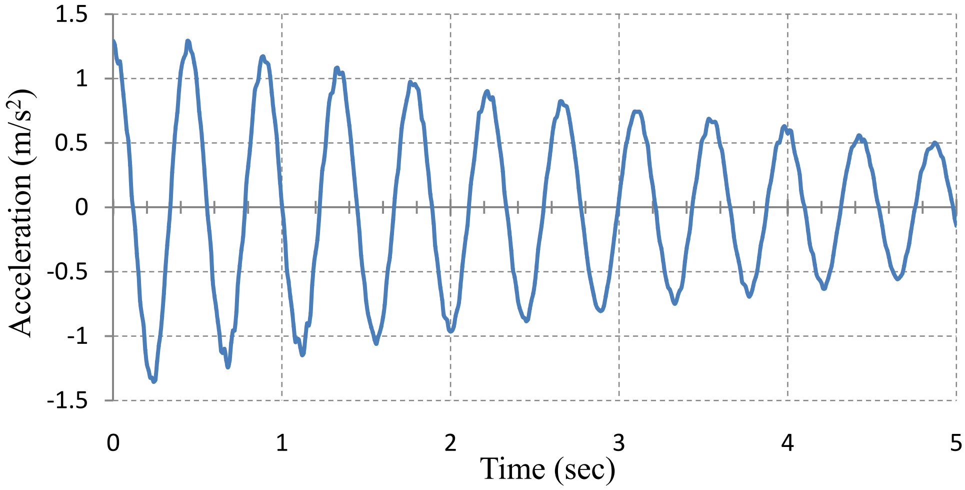


Fig. 7 Free-vibration response of the scaled bridge for the 1st stage



(a)



(b)

Fig. 8 Free-vibration response of the scaled bridge for the 2nd stage: (a) V1-VBI system, and (b) V2-VBI system

Taking the free-vibration response recorded of the bridge in the 1st stage for example (see Fig. 7), the natural period is identified as 0.40 sec; correspondingly, its natural frequency is calculated as 2.500 Hz. By the same way, the natural frequencies of V1-VBI and V2-VBI interaction systems can be identified for the 2nd stage, as listed in Table 2.For each case, 5 rounds of test are performed to ensure sample precisions. The precision of the identified frequency for the 1st stage is higher than that for the 2nd stage, indicated by the fact that the identified frequencies of 5 runs are identical for the 1st stage but those are slightly different for the 2nd stage, probably because the structural system in the 1st stage (a simple bridge) is much simpler and thus has less stochastic factors than that in the 2nd stage (a VBI system). From the bridge responses, the average natural frequency of V1-VBI interaction system is identified as 2.240 Hz, and that of V2-VBI system is

2.275 Hz. Besides from the bridge responses, the natural frequency of the above interaction systems can also be identified from the vehicle responses. The values identified from bridge and vehicle responses are approximately identical, mutually verifying their reliability.

Table 2 Identified system frequencies in the laboratory experiment

	Stage 1	Stage 2			
	Bridge system	V1-VBI interaction system		V2-VBI interaction system	
Round	Bridge	Bridge	V1	Bridge	V2
1	2.500	2.230	2.254	2.271	2.286
2	2.500	2.247	2.258	2.278	2.270
3	2.500	2.222	2.194	2.275	2.275
4	2.500	2.251	2.198	2.279	2.282
5	2.500	2.251	2.210	2.271	2.292
Average	2.500	2.240	2.223	2.275	2.281

Unit: Hz

Note: Values in "Bridge" columns are frequency values identified from the bridge responses, while those in "V1" and "V2" columns are identified from the V1 and V2 responses, respectively

Table 3 Comparison of analytical and experimental natural frequencies for the laboratory experiment

Model	f_a (Hz)	f_e (Hz)	*dev*. (%)
V1-VBI	2.176	2.240	2.9
V2-VBI	2.238	2.275	1.6

Note: f_a denotes the analytical frequency estimated by Eq. (9), f_e the natural frequency of the interaction system identified from the laboratory experiment, and *dev*. the deviation of f_a from f_e

What is more important is that the variability of the bridge frequency is readable from Table 2: as the vehicle V1 is placed at the midspan of the bridge, the natural frequency of the bridge is shifted from 2.500 Hz to 2.240 Hz with a decrease of 10.4%; similarly, as the vehicle V2 is placed at the midspan of the bridge, the natural frequency is shifted from 2.5 Hz to 2.275 Hz, with a decrease of 9%.

Comparison with analytical results

The above experiment results are then taken to examine the accuracy of the analytical expression of the natural frequency in Eq. (13). The analytical and experimental results for V1- and V2-VBI systems are summarized in Table 3. In considering the V1-VBI system first, the value of the frequency ratio β is calculated from Eq. (14) as 3.023 / 2.500 = 1.209 and the mass ratio μ from Eq. (10) as 21.6 / (280 / 2) = 0.154.Substituting the above two parameters into Eq. (13), the analytical frequency f of theV1-VBI system is obtained as 2.176 Hz, which matches quite well with the natural frequency obtained from the laboratory experiment, with a deviation of 2.9%. Next, in considering the V2-VBI system, the value of the parameter β is calculated as 3.646 / 2.5 = 1.458 and μ remains 0.154. Then the substitution of those two parameters into Eq. (13) yields the analytical frequency f of the V2-VBI system as 2.238 Hz. Such an analytical value also matches rather well with the value obtained from the laboratory experiment, with a deviation of as little as 1.6 %.

The little deviation between the analytical and experimental values of the natural frequency of the scaled vehicle-bridge interaction system verifies again the accuracy of the analytical formula in Eq. (13). It is indicated that this equation is qualified to preliminarily estimate the variability of the bridge frequency even though it was derived according to a simplified vehicle and bridge model.

4. Sensitivity analysis

It was verified that the natural frequency of the bridge with a parked vehicle may differ from that of the bridge alone, and that the natural frequency of the said vehicle-bridge interaction system can be preliminarily estimated by Eq. (13). One may then be interested in the sensitivity of the factors that dominate the variability in the natural frequencies of the interaction system.

From Eq. (13), it is observed that the natural frequency of the interaction system is a function of two parameters: the frequency ratio β and mass ratio μ of the vehicle to the bridge. The sensitivity of those two parameters is explored as follows.

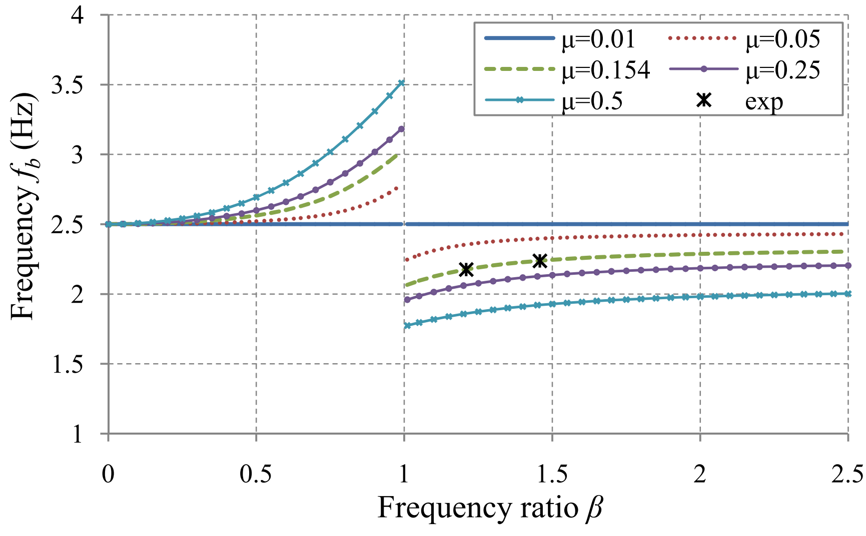


Fig. 9 Bridge-corresponded natural frequency f_b of the interaction system v.s. frequency ratio β

4.1 Frequency ratio

Calculated with Eq. (13), the bridge-corresponded natural frequency f_b (= $\omega_b/2\pi$) of the vehicle-bridge interaction system is plotted with respect to the frequency ratio β in Fig. 9. In this figure, the natural frequency $f_{b,0}$ (= $\omega_{b,0}/2\pi$) of the bridge alone is set equal to that of the scaled bridge in laboratory experiment, i.e., 2.5 Hz. The mass ratio μ is selected as 0.01, 0.05, 0.154, 0.25, and 0.5, where the case with μ = 0.01 represents a special case that the vehicle mass is relatively small compared to the bridge mass, the case with μ = 0.154 is analogous to the above laboratory experiment, and the other cases are assigned with specific intervals. In addition, the experimental values are plotted with (*) in the same figure as well, showing good agreement with the analytical values.

Several general trends can be observed from Fig. 9. Firstly, as the frequency ratio β approaches 0, i.e., as the bridge frequency is relatively larger than the vehicle frequency, the natural frequency f_b of the interaction system approximately remains the natural frequency $f_{b,0}$ of the bridge alone. It is indicated that a bridge with relatively high frequency may not affect the natural frequency of the interaction system. Secondly, in the interval of $0 < \beta < 1$, f_b is larger than $f_{b,0}$; as β increases from 0 to 1, f_b monotonously increases from $\omega_{b,0}$ with a larger and larger slope, reaching its maximum at the point $\beta = 1^-$. On the other hand, in the interval of $\beta > 1$, f_b is smaller than $f_{b,0}$; as β decreases from a value larger than 1, f_b monotonously decreases with a larger and larger slope, reaching its minimum at the point β = 1^+. A discontinuity exists at the point β = 1. Thirdly, such trends are more obvious with larger mass ratio μ. In the special case with relatively small vehicle mass, say μ= 0.01 herein, f_b is approximately equal to $f_{b,0}$ regardless of the β value, indicating that a vehicle with relatively small mass may hardly affect the natural frequency of the interaction system. With the same β value, the larger the μ value, the more the f_b deviates from $f_{b,0}$. As far as the variability in the natural frequency of the interaction system is concerned, the largest deviation of f_b from $f_{b,0}$ occurs at near resonance conditions.

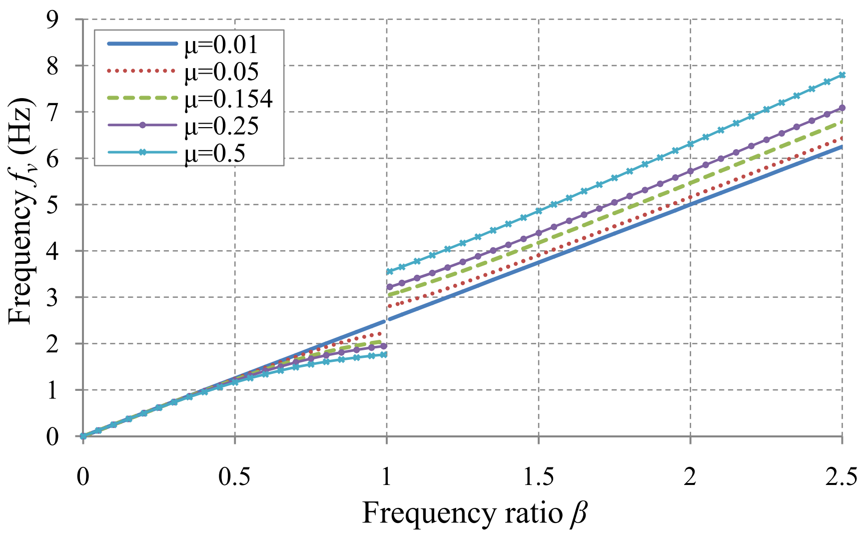


Fig. 10 Vehicle-corresponded natural frequency f_v of the interaction system v.s. frequency ratio β

The vehicle-corresponded natural frequency f_v of the interaction system is plotted with respect to the frequency ratio β as Fig. 10, with the same parameter values $f_{b,0}$ = 2.5 Hz and μ= 0.01, 0.05, 0.154, 0.25, and 0.5. For the case with relatively small μ value, say μ = 0.01, f_v approximately remains linearly proportional to β as was defined, indicating that the vehicle system with small mass hardly affect the natural frequency of the interaction system. Such a case can be regarded as the reference case for other cases with larger μ values, where deviations may occur. In the interval of $0 < \beta < 1$, f_v deviates downwards from the reference line; as β increases, the deviation of f_v also increases. In the interval of $\beta > 1$, f_v deviates upwards from that the reference line; as β increases, the deviation of f_v almost remains constant. Those observations can be useful for the approach of using a test vehicle to indirectly extract the bridge frequencies (Yang *et al*. 2004, Lin and Yang 2005).

From the engineering point of view, the application of the variability in bridge frequency can be preliminarily illustrated as follows. According to the statistical expression given by Tilly (1986), the natural frequency $f_{b,T}$ (in Hz) of a concrete highway bridge is related to the span length L (in meter) as

$$f_{b,T} = 82 \times L^{-0.9} \quad (\text{Hz}) \tag{15}$$

For highway bridges of span lengths within 30 to 50 m, whose estimated natural frequencies range from 2.43 Hz to 3.84 Hz, they may have higher probability of near resonance with roadway vehicles, whose natural frequencies in body bounce motion mostly fall within this range. It follows that they may have higher probability of large frequency variability induced by the roadway vehicles. For bridges of span lengths over 50 m, whose estimated natural frequencies are smaller than 2.43 Hz, they may have lower probability of large vehicle-induced frequency variability.

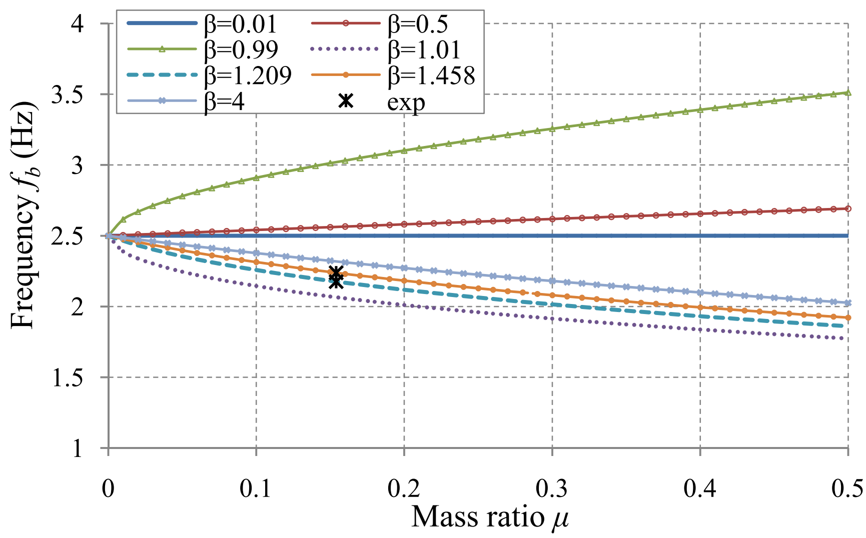


Fig. 11 Bridge-corresponded natural frequency f_b of the interaction system v.s. mass ratio μ

4.2 Mass ratio

The bridge-corresponded natural frequency f_b of the vehicle-bridge interaction system is plotted with respect to the mass ratio μ in Fig. 11. In this figure, the natural frequency $f_{b,0}$ of the bridge alone remains 2.5 Hz. The frequency ratio β is selected as 0.01, 0.5, 0.99, 1.01, 1.209, 1.458, and 4, where the case with β= 0.01 represents the special case that the frequency of the vehicle alone is extremely smaller than that of the bridge alone, the cases with β= 0.99 and 1.01 represent ones near resonance, and the cases with β= 1.209 and 1.458 are analogous to the laboratory experiments. Plotting the experimental values in the same figure as well, good agreement with analytical values is shown.

For the special case with β= 0.01, f_b is approximately identical to $f_{b,0}$, which can be regarded as the reference case for other cases with larger β values. The effect of μ on f_b can be obviously illustrated in Fig. 11: f_b deviates more from $f_{b,0}$ as μ goes larger for the same β value. When μ approaches 0, f_b approximately remains $f_{b,0}$,indicating again that a vehicle with relatively small mass hardly affect the natural frequency of the interaction system. As far as the variability in the natural frequency of the interaction system is concerned, larger deviations of f_b from $f_{b,0}$ occur at the conditions with larger μ values, say the conditions that a heavy vehicle is parked on a light bridge. In addition, such effects are more significant in near resonance conditions, say β = 0.99 and 1.01 herein, as was discussed earlier.

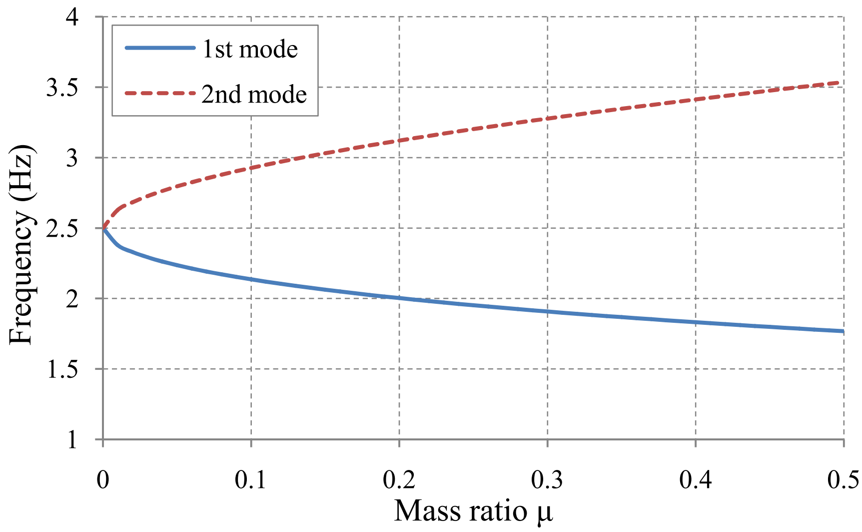


Fig. 12 Natural frequencies of the interaction system v.s. mass ratio μ in resonance case

The resonance case should be paid a special attention. It is the case that the natural frequency f_v of the vehicle alone is identical to the natural frequency f_b of the bridge alone, namely β = 1. The natural frequencies of the vehicle-bridge interaction system in resonance are plotted with respective to mass ratio μ in Fig. 12. It is observed that the natural frequencies of the two modes deviate symmetrically from the original one of vehicle or bridge alone. In comparison with non-resonance conditions, the deviation amounts in the resonance condition are larger for the same μ value. Similar to other non-resonance cases, the natural frequencies of the resonant interaction system deviates more as μ goes larger.

From the engineering point of view, bridges of shorter span may face larger vehicle-induced frequency variability, since they are generally lighter than those of longer span and a larger vehicle-to-bridge mass ratio can be expected. The reverse of the above statement is also true.

5. Conclusions

As the presence of a parked vehicle on a bridge, the bridge frequency may vary from the vehicle-bridge interaction system to the bridge system alone. Such variability is verified both theoretically and experimentally in this study. In the theoretical study, an analytical formula is presented for preliminarily estimating the varied natural frequencies of the vehicle-bridge interaction systems, based on a simplified two-degree-of-freedom system. In the experimental study, a field experiment was performed on a highway bridge and a laboratory experiment is performed on a scaled bridge. The experiment results are summarized as follows.

In the field experiment, a decrease of 2.6% in bridge frequency is observed when a truck with mass ratio of 0.082 and frequency ratio of 2.231 to the experiment bridge is parked at the midspan of the bridge. In the laboratory experiments, a decrease of 10.4 % is observed when a model vehicle with mass ratio of 0.154 and frequency ratio of 1.209 to the scaled bridge is parked at the midspan of the bridge, and a decrease of 9% is observed when another model vehicle with mass ratio of 0.154 and frequency ratio of 1.458 is parked on the same bridge. The above experiment results match rather well with the analytical results estimated with the present formula, with deviations of no more than 3%. Therefore, two major points are illustrated in the experimental study: one is to verify the variability in bridge frequency induced by a parked vehicle, and the other is to verify the accuracy and applicability of the present analytical formula.

With the analytical formula, the sensitivity of the major factors that dominate the variability in bridge frequency is studied. For the vehicle-to-bridge frequency ratio, the natural frequency of the interaction system deviates from that of the original bridge alone the most at near resonance conditions, namely those with close natural frequencies of the vehicle and bridge alone. As for the vehicle-to-bridge mass ratio, the larger the ratio, the larger the deviation from the natural frequency of the bridge system alone is observed. In contrast, a vehicle with relatively small vehicle-to-bridge frequency or mass ratio may hardly affect the natural frequency of the interaction system.

Such variability in bridge frequency is suggested to be considered in bridge-related engineering, say bridge seismic design or vibration-based health monitoring, wherever the variability range could be estimated by the present analytical formula or its derivatives. For health monitoring applications, for example, the estimated variability range of the bridge frequency induced by the presence of the vehicle can be removed so that the damage-induced frequency variability can be made clearer. Special attentions should be paid to those cases with near vehicle-bridge resonance conditions or with large vehicle-to-bridge mass ratios.

The conclusions drawn herein are based on a limited number of experiments of vehicles and bridges with simple boundary conditions. How to extend the analytical formula to estimate the varied natural frequency of interaction systems comprised of vehicles and bridges of various types and how to incorporate it into the existing frameworks of engineering applications are the next steps for this study. In addition, a more comprehensive numerical model can be utilized to verify the results derived herein with a simple model.

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