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Beam structural system moving forces active vibration control using a combined innovative control approach

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Abstract. This study proposes an innovative control approach to suppress the responses of a beam structural system under moving forces. The proposed control algorithm is a synthesis of the adaptive input estimation method (AIEM) and linear quadratic Gaussian (LQG) controller. Using the synthesis algorithm the moving forces can be estimated using AIEM while the LQG controller offers proper control forces to effectively suppress the beam structural system responses. Active control numerical simulations of the beam structural system are performed to evaluate the feasibility and effectiveness of the proposed control technique. The numerical simulation results show that the proposed method has more robust active control performance than the conventional LQG method.

Keywords: AIEM; LQG; active control

1. Introduction

To control the magnitude of bridge vibrations it is critically important to be able to accurately predict the bridge response to the action of crossing vehicles. The dynamic response of structures under moving loads is an important problem in bridge design and reliability evaluation. There are many research works (Yau 2009, Reis and Pala 2009, Chang *et al*. 2009, Yang *et al*. 2010, Huang *et al*. 2011, Simsek 2011) devoted to vehicle-induced vibration in suspension bridges. To control the magnitude of bridge vibrations it is critically important to be able to accurately predict the bridge response to the action of crossing vehicles and the resulting vehicle responses. Under such conditions the control concept applied to a beam structural system requires serious consideration. Control approaches have rapidly developed over the last two decades to provide structural safety and serviceability.

The control approaches for structures are clustered into three main categories, passive, semi-active and active control techniques. Passive techniques are normally performed using dynamic vibration absorbers or isolators. In terms of passive control technique, the unwanted vibration problem can be effectively solved using passive control techniques. In semi-active techniques viscous dampers are installed in the structure (Onoda *et al*. 1996). Active control technology has played an important role in suppressing the dynamic responses of continuous or discrete structural elements especially under unpredictable excitations. The control forces in active

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control techniques are generated by electric actuators imposed on the tendons or on a stiff cantilever fixed to the end of a beam (Frischgesel *et al*., Reckmann *et al*. 1998). A valuable literature survey on active control theory has been provided by Meirovitch (1997). The vibration control of structural systems under moving loads was investigated using various active control algorithms(Kwon *et al*. 1998, Wang *et al*. 1998 and 1999, Klasztorny 2001, Zribi *et al*. 2006, Nikkhoo *et al*. 2007, Yu *et al*. 2008, Şimşek *et al*. 2009). Song *et al*. (2006) reviewed piezoelectric application as a smart material in civil structure control.Rofooei and Nikkhoo (2009) recently studied the dynamic behavior of a smart Kirchhoff plate excited by a moving mass traveling at arbitrary trajectories.In their study a full state linear classical optimal algorithm was applied to control center point deflection of the plate using piezoelectric patches as actuators. They proved the efficiency of such a control system under various load cases. Stancioiu and Ouyang (2011) investigated the structural modification problem to feedback control design.

Zarfam *et al*. (2012) proposed a linear optimal control algorithm with displacement- velocity feedback as a solution to suppress the beam response on an elastic foundation. Min *et al*. (2012) intends to explore the dynamic interaction behaviors between actively controlled maglev vehicles and guide way girders by considering the nonlinear forms of electromagnetic force and exact current. In all of the above-mentioned methods the control force is obtained using the displacement and velocity feedback responses of the structures. In other words, the external load influence is not considered in the optimal controller design because the external load disturbances are immeasurable or inestimable in the control force calculation. The first objective in this study is to develop a novel method for the required control forces in a complete active control system.

In this study, we first investigate active control technique application to reduce a beam structural system subjected to moving forces. The active control technique is a synthesis algorithm of the AIEM and LQG controller. The AIEM uses the recursive form to process the measurement data. As opposed to the batch process, using the recursive form is an on-line process that has higher efficiency. The feasibility of the proposed method is verified using numerical active vibration control simulations for a beam structural system under moving forces. The AIEM first estimate on-line moving forces; meanwhile, an active LQG controller applies the same inverse control forces on a beam structural system. The control results show that the proposed method is more effective in suppressing vibration in a beam structural system than the conventional LQG method.

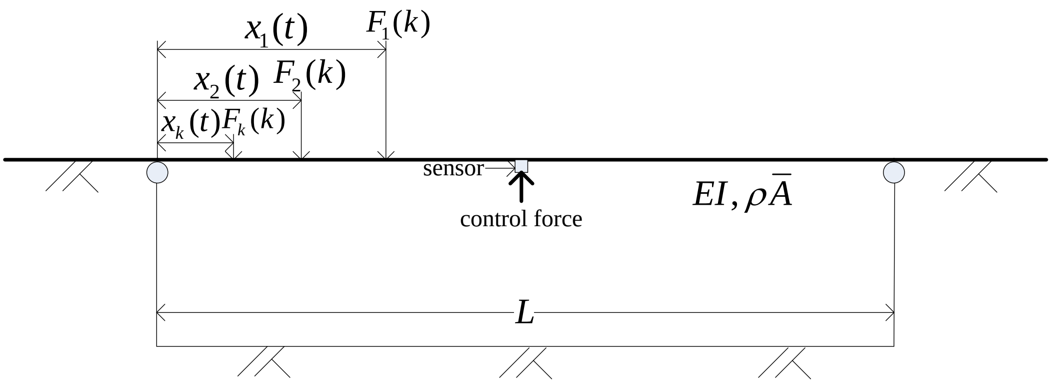

Fig. 1 The bridge structure model of the multi-vehicle input moving forces

2. Problem formulation

Numerical simulations of a bridge structure are investigated here to illustrate the practicability and accuracy of the proposed approach in estimating unknown input moving forces. As shown in Fig. 1 the bridge structure is modeled as a simple beam with a total span length L, constant flexural stiffness EI, constant mass per unit length ρ and viscous proportional damping C.

The active control force inputs on a beam structural system. The unknown input can be inversely estimated first using the adaptive input estimation method; meanwhile, the conventional LQG method is then used to find the active control force.The beam is assumed to be a Bernoulli-Euler beam in which the shear deformation and rotary inertia effects are not taken into account. Considering a group of vehicle forces moving from left to right at a constant speed, a beam structural system under moving forces and control forces,the equation of motion can be expressed as (Tommy *et al*. 2006)

$$\rho\bar{A}\frac{\partial^2 u(x,t)}{\partial t^2}+C\frac{\partial u(x,t)}{\partial t}+EI\frac{\partial^4 u(x,t)}{\partial x^4}=\sum_{k=1}^{N}F_k(t)\delta(x-x_k(t))+U(t) \tag{1}$$

where $\bar{A}$ is cross section of the beam, $u(x,t)$ is the beam displacement, $F_k(t)$ is the vehicle forces, $\delta(t)$ is Dirac delta function, $x_k(t)=v_k t$ is the position of the kth vehicle force, v_k the speed of the kth vehicle and $U(t)$ the control force vector,respectively. Based on modal superposition the solution for Eq. (1) can then be expressed as

$$u(x,t)=\sum_{n}^{\infty}\Phi_n(x)Y_n(t) \tag{2}$$

where $\Phi_n(x)$ is the nth mode shape function and $Y_n(t)$ is the nth modal amplitude of the beam. Substituting Eq. (2) into (1) and multiplying each term by $\Phi_r(x)$, integrating it over the beam length and then applying orthogonal conditions, the equation of motion in terms of the modal amplitude can be rewritten as

$$M_n\ddot{Y}_n(t)+C_n\dot{Y}_n(t)+K_nY_n(t)=F_n(t)+U_n(t) \tag{3}$$

where

$$M_n=\int_0^L \rho\bar{A}[\Phi_n(x)]^2\,dx \tag{4}$$

$$K_n=\int_0^L EI[\Phi_n''(x)]^2\,dx \tag{5}$$

$$F_n(t)=\int_0^L\sum_{k=1}^{N}F_k(t)\delta(x-x_k(t))[\Phi_n(x)]dx \tag{6}$$

$$U_n(t)=\int_0^L U(t)\left[\Phi_n(x)\right]dx \tag{7}$$

$M_n, K_n, F_n(t), U_n(t)$ are the modal mass, modal stiffness, modal force and modal control force of the nth mode, respectively, and $C_n = \alpha M_n + \beta K_n$ is modal proportional damping, where α, β are constants with proper units.

Input estimation is based on the state-space analysis method. We must construct a state-space model of the beam structural system before applying the input estimation method. In converting to the state-space model the state variables of the second order dynamic system with n degrees of freedom are represented by a $2n \times 1$ state vector, i.e., $X = \begin{bmatrix} Y(t) & \dot{Y}(t) \end{bmatrix}^T$. From Eq. (1), the continuous-time state equations and measurement equations of structure system can be written as:

$$\dot{X}(t) = AX(t) + BF_n(t) + EU_n(t) \tag{8}$$

$$Z(t) = HX(t) \tag{9}$$

where

$$A = \begin{bmatrix} 0_{n\times n} & I_{n\times n} \\ -M_n^{-1}K_n & -M_n^{-1}C_n \end{bmatrix},$$

$$B = \begin{bmatrix} 0_{n\times n} \\ M_n^{-1} \end{bmatrix},$$

$$E = \begin{bmatrix} 0_{n\times n} \\ M_n^{-1} \end{bmatrix}$$

$$H = \begin{bmatrix} I_{2n\times 2n} \end{bmatrix}$$

$$X(t) = \begin{bmatrix} X_1(t) & X_2(t) & \cdots & X_{2n-1}(t) & X_{2n}(t) \end{bmatrix}^T$$

A, B and E are constant matrix composed of mass, damping and stiffness of the beam structure system, $X(t)$ is the state vector, $Z(t)$ is the observation vector and H is the measurement matrix.

Noise interference exists in the practical circumstances. The noise interference was not considered in Eqs. (8) and (9). In order to approximate to the truth, the statistical characteristic noise interference was added in the state equations and measurement equations of structural system. This random noise interference was represented by Gaussian white noise. The statistical characteristic of a random variable was described in detail with the probability distribution and density function. However, the statistical characteristic of a random process can be represented with the mean and variance characteristic value of the random variable (Fun 1995). For the above reasons Eq. (8) is discretized over time intervals of length Δt and created the statistical mathematical system dynamic modal of the state-vector associated with process noise input (Bogler 1987). Eq. (8) then becomes

$$X(k+1) = \Phi X(k) + \Gamma[F(k) + w(k)] + \Lambda U(k) \tag{10}$$

where

$$X(k) = \begin{bmatrix} X_1(k) & X_2(k) & \cdots & X_{2n-1}(k) & X_{2n}(k) \end{bmatrix}^T$$

$$\Phi = \exp(A\Delta t)$$

$$\Gamma = \int_{k\Delta t}^{(k+1)\Delta t} \exp\left\{A\left[(k+1)\Delta t - \tau\right]\right\} B d\tau$$

$$F(k) = \left[F_1(k) \quad F_2(k) \quad \cdots \quad F_{n-1}(k) \quad F_n(k)\right]^T$$

$$U(k) = \left[U_1(k) \quad U_2(k) \quad \cdots \quad U_{n-1}(k) \quad U_n(k)\right]^T$$

$$w(k) = \left[w_1(k) \quad w_2(k) \quad \cdots \quad w_{n-1}(k) \quad w_n(k)\right]^T$$

$X(k)$ represents the state vector, Φ is the state transition matrix, Γ and Λ are the coefficient matrices of $F(k)$ and $U(k)$, respectively. $F_1(k)$ isthe deterministic moving dynamic input sequence. $U(k)$ is the control force vector. Δt is the sampling interval. The $w(k)$ process noise vector is assumed to be zero mean and white noise with variance $E\left\{w(k)w^T(k)\right\} = Q\delta_{kj}$, $Q = Q_W \times I_{2n\times 2n}$.

Here Q is the discrete time process noise covariance matrix and δ_{kj} is the Kronecker Delta.

In order to consider the measurement noise the measure Eq. (9) is expressed as

$$Z(k) = HX(k) + v(k) \tag{11}$$

where

$$Z(k) = \left[Z_1(k) \quad Z_2(k) \quad \ldots \quad Z_{2n}(k)\right]^T$$

$$v(k) = \left[v_1(k) \quad v_2(k) \quad \ldots \quad v_{2n}(k)\right]^T$$

$Z(k)$ is the observation vector, $v(k)$ represents the measurement noise vector. $v(k)$ isassumed to be zero mean and white noise with variance $E\left\{v(k)v^T(k)\right\} = R\delta_{kj}$, $R = R_V \times I_{2n\times 2n}$, R is the discrete time measurement noise covariance matrix and H is the measurement matrix.

3. AIEM combined LQG control technique design

For standard linear quadratic Gaussian problems the system under control is assumed to be described by the stochastic discrete-time state space equations as shown below (Lewis 1972)

$$X(k) = \Phi X(k-1) + \Lambda F(k-1) + \Gamma w(k-1) \tag{12}$$

where $w(k)$ is zero-mean white noises with variances Q. In general the input forces sequence $F(k)$ are neglected or assumed to be zeros in conventional LQG controller design. From Eq. (12) the conventional LQG control methodology for a system without input forces term was obvious.

That is to say, the system, Eq. (12) is not satisfactory for modeling most dynamic structures because there usually are external excitation forces. Therefore, we considered the case where the input forces were not zeros, i.e., Eqs. (8) and (9). However, the conventional LQG control methodology is not applicable to structures without neglecting the input disturbance forces because the entire input dynamic loads histories are not known a priori.

The conventional LQG controller has a specific level of interference suppression. It is weak in maintaining high performance in the suppression of external loads; which are complex and arbitrary style disturbances. In other words, in Eq. (8), if a time-varying load $G(k)$ exists, the optimal control method combining the Kalman filter and the LQG regulator will not be able to obtain the optimal control forces. To resolve this situation this study proposes combining the AIEM with the LQG control technique for active vibration control of the beam structural system. The AIEM can estimate the unknown dynamic inputs while the active LQG controller can apply the same inverse control forces on the structural system.

The AIEM is composed of a Kalman filter without the input term and the adaptive weighting recursive least square algorithm. The detailed formulation of this technique can be found in the research of Tuan *et al.* 1996. The Kalman filter optimal estimation equations are as follows:

The optimal estimate of the state is

$$\hat{X}(k+1/k) = \Phi \hat{X}(k/k-1) + K(k)\left[Z(k) - H\hat{X}(k/k-1)\right] \tag{13}$$

The bias innovation produced by the measurement noise and input disturbance is expressed by

$$\bar{Z}(k) = Z(k) - H\hat{X}(k/k-1) \tag{14}$$

The Kalman gain is

$$K(k) = \Phi(k+1/k)P(k/k-1)H^T(k)\left[H(k)P(k/k-1)H^T(k)+R\right]^{-1} \tag{15}$$

The covariance of residual is $S(k)$

$$S(k) = HP(k/k-1)H^T + R \tag{16}$$

The prediction error covariance matrix is

$$\begin{aligned} P(k+1/k) = {} & \Phi(k+1/k)P(k/k-1)\Phi^T(k+1/k) - \Phi(k+1/k)P(k/k-1)H^T(k) \\ & \times\left[H(k)P(k/k-1)H^T(k)+R\right]^{-1} H(k)P(k/k-1)\Phi^T(k+1/k) \\ & +\Gamma(k+1/k)Q\Gamma^T(k+1/k) \end{aligned} \tag{17}$$

The recursive least square estimator equations are as follows:
The sensitivity matrices are $B(k)$ and $M(k)$

$$B(k) = H[\Phi M(k-1) + I]\Gamma \tag{18}$$

$$M(k) = [I - K(k)H][\Phi M(k-1) + I] \tag{19}$$

The correction gain is expressed as

$$K_b(k) = \gamma^{-1}P_b(k-1)B^T(k)\left[B(k)\gamma^{-1}P_b(k-1)B^T(k) + S(k)\right]^{-1} \tag{20}$$

The error covariance of the input estimation process is

$$P_b(k) = \left[I - K_b(k)B(k)\right]\gamma^{-1}P_b(k-1) \tag{21}$$

The estimated earth motion acceleration is

$$\hat{F}(k) = \hat{F}(k-1) + K_b(k)\left[\bar{Z}(k) - B(k)\hat{F}(k-1)\right] \tag{22}$$

According to Eq. (14) the innovation with variation is caused by the measurement noise and the unknown input noise. If a large input variation exists $\bar{Z}(k)$ can be set as the abnormal or long-tail distribution. Huber used the least favorable probability density function (PDF), $f(v(k))$, to describe in a flexible manner the residual, which is either normal or abnormal. In this study, $\bar{Z}(k) = v(k)$. Here, the least favorable PDF $f(v(k))$ is defined as follows

$$f\left(\bar{Z}(k)\right) = \begin{cases} \dfrac{1}{\sqrt{2\pi\sigma}} e\left(-\bar{Z}(k)^2/2\sigma^2\right), & \left|\bar{Z}(k)\right| \le c \\ \dfrac{\alpha}{2} e^{-\alpha\left|\bar{Z}(k)\right|}, & \left|\bar{Z}(k)\right| > c \end{cases} \tag{23}$$

where c is the adjustable constant to adjust the robustness of the estimator. According to Eq. (23), if $\left|\bar{Z}(k)\right| \le c$, $f(v(k))$ is a normal distribution; if $\left|\bar{Z}(k)\right| > c$, $f(v(k))$ is a long-tail distribution. By adopting double-exponent density function to describe the possible abnormal samples and to lower the influence to the weight of estimation, the robustness can be obtained. Therefore, c can be regarded as a threshold, which is to functioning, $c = \sigma$, which is a reasonable threshold. σ is the standard deviation of the measurement error. In Eqs. (20) and (21), γ is a weighting factor using the adaptive weighting function in this study, which is formulated in (Tuan *et al*. 1998). That is

$$\gamma(k) = \begin{cases} 1, & \left|\bar{Z}(k)\right| \le \sigma, \\ \sigma/\left|\bar{Z}(k)\right|, & \left|\bar{Z}(k)\right| \le \sigma. \end{cases} \tag{24}$$

The weighting factor, $\gamma(k)$, as shown in Eq. (24), can be adjusted according to the measurement noise and input bias. In industrial applications the standard deviation σ is assumed as a constant value. The magnitude of the weighting factor is determined according to the modulus of bias innovation, $\left|\bar{Z}(k)\right|$. The unknown input prompt variation will cause a large bias innovation modulus. In the meantime a smaller weighting factor isobtained when thebias innovation modulus is larger. Therefore, the estimator accelerates the tracking speed and produces larger vibration in the estimation process.On the contrary, a smaller variation of unknown input causes a smaller bias innovation modulus. Meanwhile, the larger weighting factor isobtained according to the smaller bias innovation modulus.

In the optimal estimation portion of the LQG optimal control method, by substituting $\hat{F}(k)$ of Eq. (22) for $F(k)$ and substituting the control input in Eq. (13), the optimal state estimation equation can be rewritten as

$$\hat{X}(k+1/k) = \Phi\hat{X}(k/k-1) + K(k)\left[Z(k) - H\hat{X}(k/k-1)\right] + \Gamma\hat{F}(k) + \Lambda U(k) \tag{25}$$

The performance index is defined as

$$J_i(F) = E\left\{\frac{1}{2}\hat{X}^T(N)Q_0\hat{X}(N) + \frac{1}{2}\sum_{k=i}^{N-1}\left[\hat{X}^T(k)Q_1\hat{X}(k) + U^T(k)Q_2U(k)\right]\right\} \tag{26}$$

where $Q_1 \geq 0$, $Q_2 \geq 0$ and $Q_0 \geq 0$ are all symmetric weighting matrices. The optimal feedback control force vector can be obtained by using the separation theorem (Kwakernaak *et al*. 1972)

$$U(k) = -K_r(k)\hat{X}(k/k-1) \tag{27}$$

Here the regular gain $K_r(k)$ is given by

$$K_r(k) = \left[\Lambda^T P_2(k+1)\Lambda + Q_2\right]^{-1}\Lambda^T P_2(k+1)\Phi \tag{28}$$

where $P_2(k)$ is the discrete-time Ricatti equation solution. The Ricatti equation is shown below

$$P_2(k) = \Phi^T\left\{P_2(k+1) - P_2(k+1)\Lambda\left[\Lambda^T P_2(k+1)\Lambda + Q_2\right]^{-1}\Lambda^T P_2(k+1)\right\}\Phi + Q_1, \quad k \leq N \tag{29}$$

$$P_2(N) = Q_0 \tag{30}$$

According to Eq. (29), $P_2(k)$ can be obtained by inversely calculating from $k = N$ to $k = 1,$. The method combining AIEM and the LQG active controller is presented using the AIEM to estimate $\hat{F}(k)$ and combining Eq. (25) to obtain the optimal state estimate, $\bar{X}(k+1/k)$, which can be further substituted in Eq. (27). The combination of AIEM and the LQG controller is illustrated in Fig. 2.

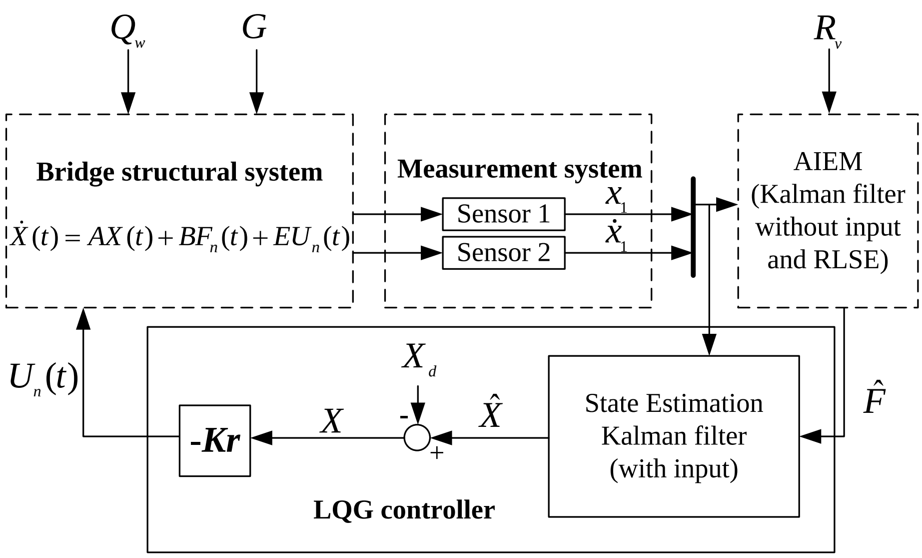

Fig. 2 Flowchart of the AIEM combined with the LQG

4. Results and discussion

To verify the practicability and accuracy of the proposed approach in estimating unknown moving input forces, the bridge structure is modeled as a simple beam with a total span length $L = 30m$, constant flexural stiffness $EI = 1.27914 \times 10^{11} Nm^2$, constant mass per unit length $\rho = 1.2 \times 10^4 kg/m$ and the modal proportional damping $C_n = \alpha M_n + \beta K_n$, where $\alpha = 0.01, \beta = 0.001$, the mode shape function $\Phi_n = \sin(\pi x_k / L)$. The initial conditions of the error covariance are given as $p(0/0) = diag[10^4]$ for the KF and $p_b(0) = 10^4$ for the adaptive weighting recursive least-squares estimator. The simulation conditions are taken as: sampling interval $\Delta t = 0.001$s, sensitivity matrix $M(0)$ is null, the weighting factor is adaptive weighting function.

4.1. Singular-vehicle input moving force estimation and control

The singular-vehicle input moving force is simulated as a vehicle with a static weight $F_k = 400KN$ acting on the bridge structure, at a constant velocity of $v_k = 10m/\sec$ over the bridge. From Eq. (6) the time-varying input moving force is simulated as follows

$$F_n(t) = \begin{cases} F_k \sin(\pi v_k t / L) & t_i \le t \le t_d \\ 0 & 0 \le t \le t_i, t \ge t_d \end{cases} \tag{31}$$

where t_i represents start time of the vehicle entering the bridge, it is delayed by 0.3 s to clearly identify the simulation results, $t_d = L/v$ represent the terminal time the vehicle leaves the bridge. The dynamic responses of the bridge are solved using a numerical method with system noise and measurement noise. The parameters used in the numerical model are given as follows: covariance matrix of process noise $Q = Q_w \times I_{2n \times 2n}$, $Q = 10^8$, covariance matrix of measurement noise $R = R_w \times I_{2n \times 2n}$, $R = \sigma^2 = 10^{-10}$, state weighting matrices $Q_0 = Q_1 = Q_s \times I_{1 \times 1}$, $Q_s = 1 \times 10^5$, control weighting matrices $Q_2 = Q_c \times I_{1 \times 1}$, $Q_c = 1$. Fig. 3 shows the time histories of the singular-vehicle input moving force estimation result, measured and estimated displacement at the middle span.

The result reveals very good estimating ability. The estimation values converge to exact values quickly. The estimation results have demonstrated the validity of the present inverse estimation algorithm on the singular-vehicle input moving force. Fig. 4 shows that a smaller weighting factor can be chosen in the recursive least square method when a larger unknown is input into the system. Note that the faster the forgetting effect is, the lower the smoothing effect will be, that is, it introduces oscillation. The adaptive weighting factor $\gamma(k)$ is employed to compromise between the tracking capability upgrade and the loss of estimation precision. Fig. 5 shows the overall time histories of the control forces required for the proposed method and LQG method. By applying the active dynamic reaction which contains noise to the presented control algorithm, the time histories of the beam structural system responses with and without control are shown in Fig. 6. The AIEM combined with the LQG controller has better effective and precision performance than the conventional LQG controller. The effectiveness of the proposed active control technique was further demonstrated by tuning the process noise covariance matrix, $Q = 10^6$, measurement noise covariance matrix, $R = \sigma^2 = 10^{-10}$. The case has been compared using process noise variance

$Q = 10^6$ as shown in Fig. 7. From Fig. 7 we can see that the process noise covariance matrix will influence the ability of tracking true values. It shows that if the process noise variance Q decreases it will influence the estimation resolution. A smaller process noise variance will affect the time-varying force inputs tracking capability. The time histories of the beam structural system responses with and without control are shown in Fig. 8. The AIEM combined with the LQG controller has better effective and precision performance than the conventional LQG controller.

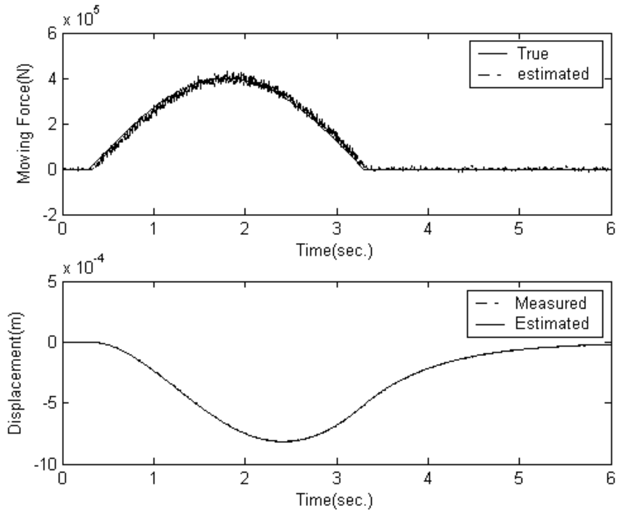

Fig. 3 Time histories of the estimated and true singular-vehicle input moving force and middle span displacement ($Q = 10^8$, $R = 10^{-12}$)

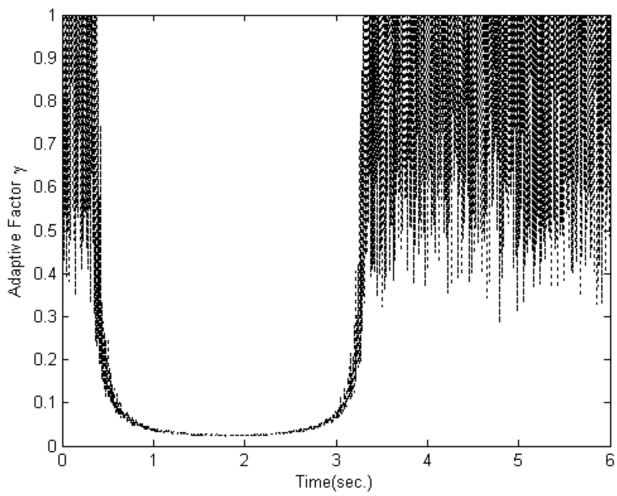

Fig. 4 Thevariance in the adaptive weighting factor. ($Q = 10^8$, $R = 10^{-12}$)

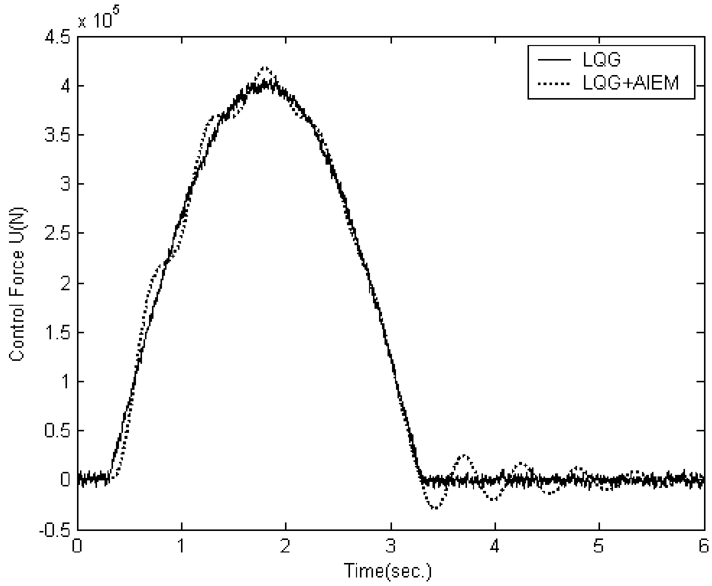

Fig. 5 Time histories of the control forces of a beam structural system under singular-vehicle input moving force ($Q=10^8$, $R=10^{-12}$)

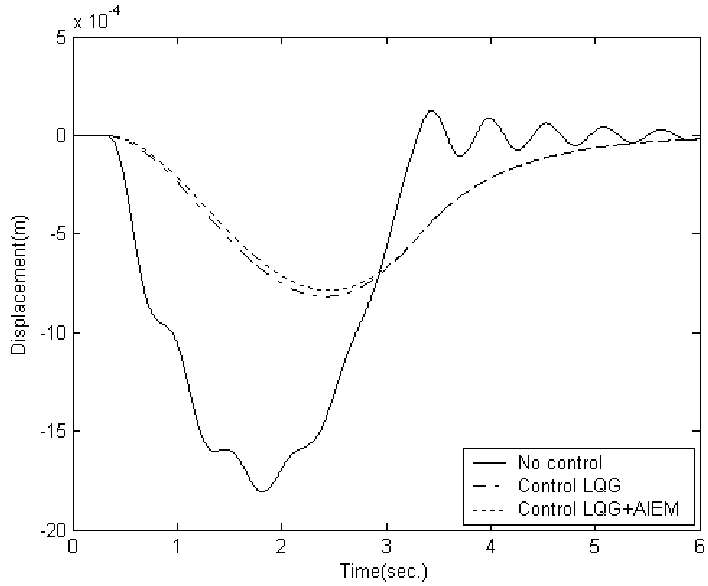

Fig. 6 Time histories of the displacements using the singular-vehicle input moving force($Q=10^8$, $R=10^{-12}$)

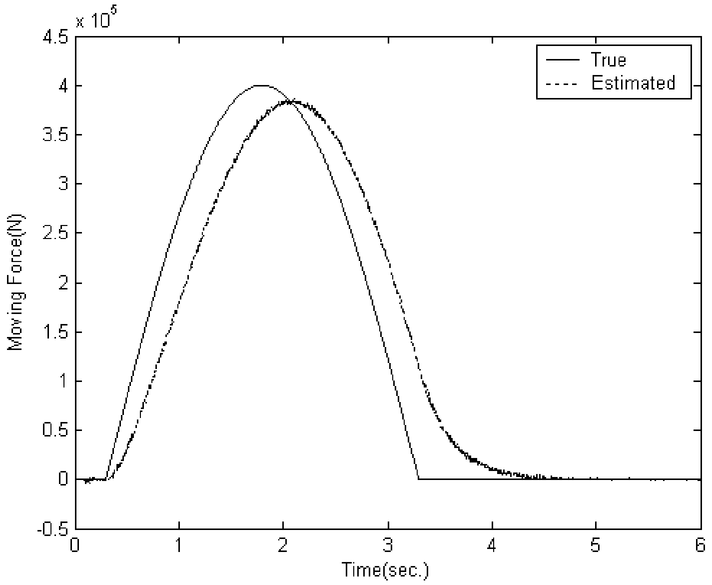

Fig. 7 Time histories of the estimated and exact singular-vehicle input moving force and middle span displacement ($Q = 10^6$, $R = 10^{-12}$)

4.2. Multi-vehicle input moving forces estimate

Three input moving forces are simulated using multiple vehicles with the static weight of the first vehicle input moving force $F_1 = 200(KN)$, the second vehicle $F_2 = 150(KN)$ and the third vehicle $F_3 = 100(KN)$ acting on the bridge structure. The velocity of the first vehicle is $v_1 = 8m/\sec$, the second vehicle $v_2 = 7m/\sec$ and the third vehicle $v_3 = 6m/\sec$ over the bridge, respectively.

The Kalman estimation parameters are $Q = 10^9$, $R = \sigma^2 = 10^{-12}$ .The bridge total span length $L = 20m$. The start time t_i of the first vehicle entering bridge was delayed 0.3 s to clearly identify the simulation results. The interval time of among vehicles entrancing bridge is 0.5 s. From Eq. (6), the time-varying input moving forces are simulated as follows

$$F_n(t) = \begin{cases} F_k \sin(\pi v_k t / L) & t_i \le t \le t_d \\ 0 & 0 \le t \le t_i, t \ge t_d \end{cases} \tag{32}$$

where $t_d = L / v_{k=1\sim3}$ represents termination time of the vehicles leaving bridge. The dynamic responses of the bridge were solved using a numerical method with system noise and measurement noise. The time histories of the multi-vehicle input moving forces estimation result were measured at middle span are shown in Fig. 9. Fig. 10 shows the overall time histories of the control forces required for the proposed method and LQG method. By applying the active dynamic reaction which contains noise to the presented control algorithm, the time histories of the beam structural system responses with and without control are shown in Fig. 11. According the simulation results the proposed control method, which combines the FIEM and the LQG controller, is suitable for dealing with the optimal control problem in the time-varying system model.

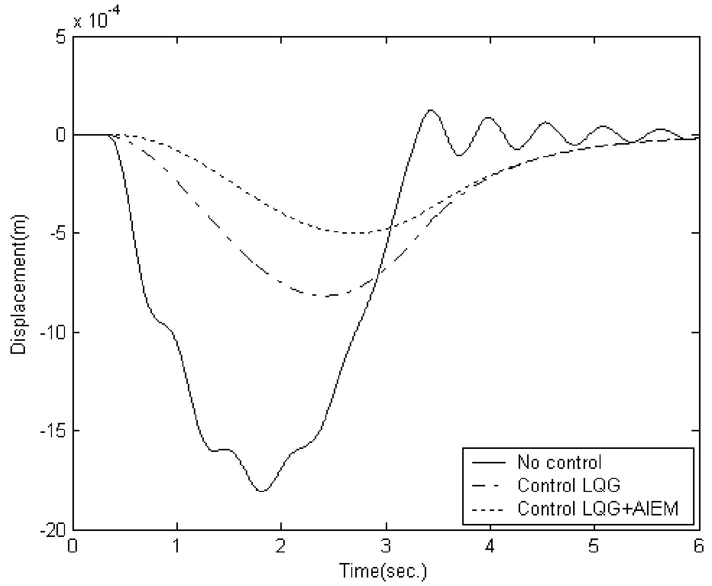

Fig. 8 Time histories of the displacements using the singular-vehicle input moving force($Q=10^6$, $R=10^{-12}$)

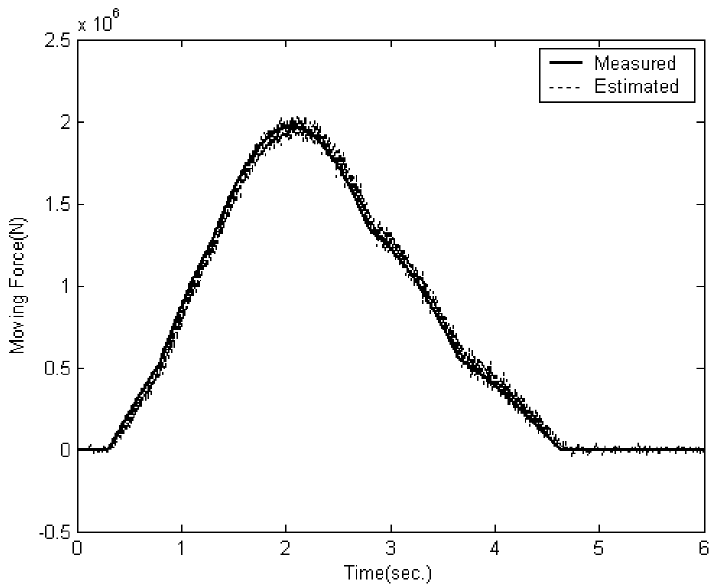

Fig. 9 Time histories of the estimated and exact mutli-vehicle input moving force and middle span displacement ($Q=10^9$, $R=10^{-12}$)

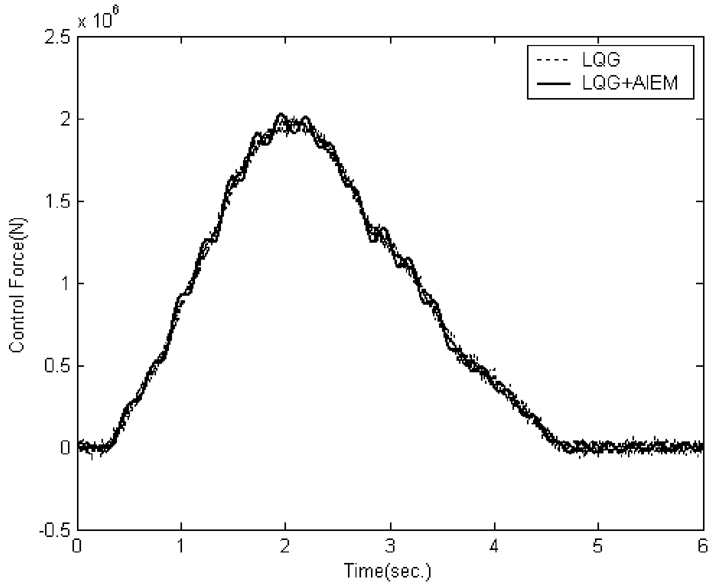

Fig. 10 Time histories of the control forces of a beam structural system under the multi-vehicle input moving force ($Q=10^{9}$, $R=10^{-12}$)

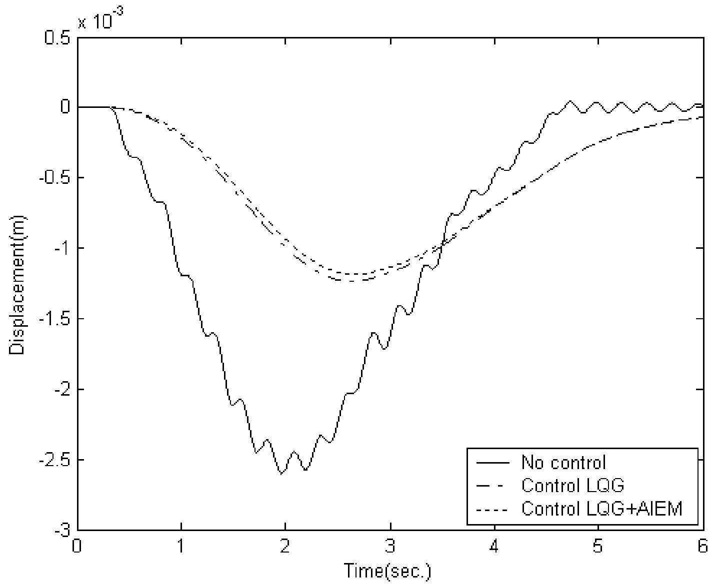

Fig. 11 Time histories of the displacements using the multi-vehicle input moving force($Q=10^{9}$, $R=10^{-12}$)

5. Conclusions

This study developed an active control technique for moving forces control in a beam structural system. Combing the excellent AIEM with LQG controller produced a feasible control approach that effectively reduces the beam structural system response3s.The control performance of the developed technique was evaluated through numerical simulations of the beam structural system under moving forces. The simulation results demonstrate that proposed technique is more effective in reducing the beam responses than the conventional LQG controller. Future work is being conducted to extend this application to a nonlinear structural system.

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