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Updating finite element model using dynamic perturbation method and regularization algorithm

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Abstract. An effective approach for updating finite element model is presented which can provide reliable estimates for structural updating parameters from identified operational modal data. On the basis of the dynamic perturbation method, an exact relationship between the perturbation of structural parameters such as stiffness change and the modal properties of the tested structure is developed. An iterative solution procedure is then provided to solve for the structural updating parameters that characterise the modifications of structural parameters at element level, giving optimised solutions in the least squares sense without requiring an optimisation method. A regularization algorithm based on the Tikhonov solution incorporating the generalised cross-validation method is employed to reduce the influence of measurement errors in vibration modal data and then to produce stable and reasonable solutions for the structural updating parameters. The Canton Tower benchmark problem established by the Hong Kong Polytechnic University is employed to demonstrate the effectiveness and applicability of the proposed model updating technique. The results from the benchmark problem studies show that the proposed technique can successfully adjust the reduced finite element model of the structure using only limited number of frequencies identified from the recorded ambient vibration measurements.

Keywords: model updating; operational modal analysis; dynamic perturbation method; regularization algorithm; Canton Tower benchmark problem

1. Introduction

The finite element (FE) method is a powerful tool for structural design and analysis in civil engineering practice. The FE method can also be used for many other applications including structural condition assessment and health monitoring to provide a baseline for assessing current condition and predicting future performance (Doebling *et al*. 1996, Chen 2008). On the basis of the verified FE model, damage in a structure can be identified from the measured vibration modal data (Hu *et al*. 2001, Chen and Bicanic 2010). In general, the FE analytical model of an actual structure is constructed on the basis of highly idealised engineering design that may not fully represent all the physical aspects of the constructed structure. As a result, a significant discrepancy may exist between the dynamic characteristics predicted by the FE model and those identified from the actual tested structure. This problem arises not only from the modelling errors caused by simplified assumptions for complicated structures also from the parameter estimation errors due to the

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uncertainties in material and geometric properties. In order to minimise the discrepancy and to maximise the correlation between the FE analytical model and the actual tested structure, structural model updating methods are often utilised to adjust the analytical model by using the measured modal data of the actual structure.

Many investigations have been undertaken on FE model updating using vibration measurements over the past two decades (Mottershead and Friswell 1993, Friswell and Mottershead 1995). The model updating methods available can be broadly classified into two major groups, i.e., direct methods and iterative methods. The direct methods directly update the elements of stiffness and mass matrices and are a one-step procedure (Kabe 1985, Caesar and Peter 1987, Friswell *et al*. 1998, Yang and Chen 2009). This method allows the updated analytical model to reproduce measured vibration modal data, but there is no guarantee that it truly represents the physical properties of the actual structure concerned. On the other hand, the iterative parameter updating methods adopt the sensitivity of the parameters to update the FE analytical model (Link 1999). This method sets the errors between the analytical and measured data as an objective function, and attempts to minimize the chosen objective function by adjusting the pre-selected set of physical parameters of the FE model in question. Compared with the direct methods, the iterative methods such as the sensitivity-based parameter updating approach are more popular since it can be implemented in existing FE codes (Farhat and Hemez 1993, Ladeveze *et al*. 1994). Furthermore, there is a readily available physical explanation for each structural updating parameter, which is typical associated with the element stiffness and mass of the analytical model. For the sensitivity-based updating approach, its performance largely depends on the selections of an objective function and constraints, structural updating parameters and optimization techniques (Brownjohn and Xia 2000, Zhang *et al*. 2000, Wu and Li 2004, Jaishi *et al*. 2007). The objective function is often taken as the residuals between the modal properties of the analytical predictions and the measurement results such as frequencies. The selection of structural updating parameters requires considerable physical insight into the actual tested structure in order to correctly characterise the discrepancy between the analytical model and tested structure. Optimisation techniques such as the least-squares minimization method and genetic algorithm are often employed to solve the optimization problem in order to obtain optimum structural updating parameters. It should be pointed out that the sensitivity analysis and optimisation techniques used for model updating may not perform properly, when the number of the chosen structural parameters to be updated is large and a model refinement with relatively large modifications of structural parameters is required.

In this paper, an effective model updating method from measured vibration modal data is presented on the basis of the dynamic perturbation method and regularization algorithm. The exact relationship between the structural parameter modifications to be adjusted and the modal properties of the experimental structure is proposed by using the dynamic perturbation method. The structural updating parameters are properly chosen to reflect the modifications of stiffness and mass between the analytical model and tested structure at element level. An iterative solution procedure is provided to estimate the chosen structural updating parameters in the least squares sense, without requiring an optimisation method. The Tikhonov regularization algorithm incorporating the generalised cross-validation (GCV) method for determining the regularization parameter is then employed to produce reliable solutions for the structural updating parameters. The Canton Tower benchmark problem established by the Hong Kong Polytechnic University (HKPU) is employed to verify the proposed method for model updating (Ni *et al.* 2009, Chen *et al*. 2011). From the measured operational acceleration data provided in the benchmark problem, modal properties of the tower such as frequencies and mode shapes are identified by using typical operational modal analysis tools, e.g., the peak picking technique and the stochastic subspace identification technique. The identified modal properties are then adopted to update the reduced FE model established in the benchmark problem by using the proposed model updating method.

2. Dynamic perturbation method

For a dynamic structural system with global stiffness matrix $\boldsymbol{K}$ and mass matrix $\boldsymbol{M}$, the characteristic equation of an N Degrees of Freedom (DOFs) dynamic finite element model can be expressed as

$$\boldsymbol{K}\boldsymbol{\phi}_i = \omega_i^2 \boldsymbol{M}\boldsymbol{\phi}_i \tag{1}$$

where ω_i and $\boldsymbol{\phi}_i$ are the ith calculated frequency and the corresponding eigenvectors of the analytical model, respectively. In FE model updating, the FE model usually has uncertainties in modelling structural parameters such as stiffness and mass of the associated actual tested structure. The model uncertainties are then mainly related to the unknown perturbations of structural parameters such as difference in stiffness ($\Delta\boldsymbol{K}$) and mass ($\Delta\boldsymbol{M}$) between the analytical model and the tested structure. Therefore, the global stiffness matrix ($\tilde{\boldsymbol{K}}$) and mass matrix ($\tilde{\boldsymbol{M}}$) of the tested dynamic structure can be expressed as

$$\tilde{\boldsymbol{K}} = \boldsymbol{K} + \Delta\boldsymbol{K} \tag{2a}$$

$$\tilde{\boldsymbol{M}} = \boldsymbol{M} + \Delta\boldsymbol{M} \tag{2b}$$

Similarly, the characteristic equation for the tested structure is given by

$$\tilde{\boldsymbol{K}}\tilde{\boldsymbol{\phi}}_i = \tilde{\omega}_i^2 \tilde{\boldsymbol{M}}\tilde{\boldsymbol{\phi}}_i \tag{3}$$

where $\tilde{\omega}_i$ and $\tilde{\boldsymbol{\phi}}_i$ are the ith measured natural frequency and the corresponding mode shape for the tested dynamic system, respectively. Substituting Eq. (2) into the experimental characteristic equation in Eq. (3), yields

$$[(\Delta\boldsymbol{K} - \tilde{\omega}_i^2 \Delta\boldsymbol{M}) + (\boldsymbol{K} - \tilde{\omega}_i^2 \boldsymbol{M})]\tilde{\boldsymbol{\phi}}_i = 0 \tag{4}$$

Pre-multiplying Eq. (4) by $\tilde{\boldsymbol{\phi}}_i^T$ and using the analytical characteristic equation in Eq. (1), leads to

$$\boldsymbol{\phi}_i^T (\Delta\boldsymbol{K} - \tilde{\omega}_i^2 \Delta\boldsymbol{M})\tilde{\boldsymbol{\phi}}_i - (\tilde{\omega}_i^2 - \omega_i^2)\boldsymbol{\phi}_i^T \boldsymbol{M}\tilde{\boldsymbol{\phi}}_i = 0 \tag{5}$$

Similarly, pre-multiplying Eq. (4) by $\boldsymbol{\phi}_k^T$, when k is not equal to i, and using Eq. (1), gives

$$\boldsymbol{\phi}_k^T (\Delta\boldsymbol{K} - \tilde{\omega}_i^2 \Delta\boldsymbol{M})\tilde{\boldsymbol{\phi}}_i - (\tilde{\omega}_i^2 - \omega_k^2)\boldsymbol{\phi}_k^T \boldsymbol{M}\tilde{\boldsymbol{\phi}}_i = 0 \tag{6}$$

It should be noted that the eigenvectors of the analytical model $\boldsymbol{\phi}_i$ are linearly independent since its stiffness and mass matrices are symmetric. The mode shapes of the tested structure then can be expressed as a linear combination of the independent analytical eigenvectors, namely

$$\tilde{\boldsymbol{\phi}}_i = \sum_{k=1}^{N} C_{ik}\boldsymbol{\phi}_k \tag{7}$$

where C_{ik} are mode participation factors. Here, the analytical eigenvectors are assumed to be normalised as unity with respect to the mass of the analytical model.

Premultiplying Eq. (7) by $\boldsymbol{\phi}_k^T \boldsymbol{M}$, and using the mass normalisation of the analytical eigenvectors, yields

$$C_{ik} = \boldsymbol{\phi}_k^T \boldsymbol{M} \tilde{\boldsymbol{\phi}}_i \tag{8}$$

From Eq. (6), the mode participation factors C_{ik} in Eq. (8) are rewritten as

$$C_{ik} = \frac{\boldsymbol{\phi}_k^T(\Delta\boldsymbol{K} - \tilde{\omega}_i^2 \Delta\boldsymbol{M})\tilde{\boldsymbol{\phi}}_i}{(\tilde{\omega}_i^2 - \omega_k^2)} \tag{9}$$

In structural dynamic testing, modal information about natural frequency $\tilde{\omega}_i$ and the corresponding incomplete mode shape $\boldsymbol{\Psi}_i$ of the tested structure can be obtained. The measured mode then could be paired to the analytical eigenvector (restricted to the same dimensions as $\boldsymbol{\Psi}_i$), $\hat{\boldsymbol{\phi}}_k$, by using Modal Assurance Criterion (MAC) factors defined as

$$MAC(\hat{\boldsymbol{\phi}}_k, \tilde{\boldsymbol{\psi}}_i) = \frac{\left|\hat{\boldsymbol{\phi}}_k^T \; \tilde{\boldsymbol{\psi}}_i\right|^2}{\left|\hat{\boldsymbol{\phi}}_k^T \; \hat{\boldsymbol{\phi}}_i\right| \left|\tilde{\boldsymbol{\psi}}_i^T \; \tilde{\boldsymbol{\psi}}_i\right|} \tag{10}$$

Similar equation can be used for calculating MAC factors between the updated modes and the original analytical modes by replacing the measured mode in Eq. (10) with the updated one. From the definition, large MAC factors indicate a high degree of similarity between two mode shapes and small MAC factors represent little or even no correlation between two vectors. Here, only the frequencies identified from output-only modal identification method are utilised for model updating, and the information about identified mode shapes is only used for mode correlation checks.

3. Governing equations for model updating

In this study, system parameters, such as parameters for material and geometric properties, are employed to reflect the updating of structural parameters, e.g., stiffness matrix and/or mass matrix. It is assumed that system parameters characterise the structural parameters at element level. The perturbations of structural stiffness matrix and mass matrix are then defined as

$$\Delta\boldsymbol{K} = \sum_{j=1}^{Ne} \boldsymbol{K}_j \alpha_j \tag{11a}$$

$$\Delta\boldsymbol{M} = \sum_{j=1}^{Ne} \boldsymbol{M}_j \beta_j \tag{11b}$$

where Ne represents the total number of elements adopted in analytical model; α_j and β_j are stiffness and mass matrix parameters to be determined for model updating, respectively; $\boldsymbol{K}_j$ and $\boldsymbol{M}_j$ are the contributions of the jth element to the global stiffness matrix and mass matrix, respectively.

In order to ensure the uniqueness of a mode shape of the tested structural system, it is assumed that the mode shape for the tested structure is mass normalised in the form $\boldsymbol{\phi}_i^T\boldsymbol{M}\tilde{\boldsymbol{\phi}}_i = 1$, i.e., the mode participation factor in Eq. (8) $C_{ii} = 1$ (Chen 2005). Substituting Eq. (11) into Eq. (5), leads to

$$\sum_{j=1}^{Ne}\boldsymbol{\phi}_i^T\boldsymbol{K}_j\tilde{\boldsymbol{\phi}}_i\alpha_j - \tilde{\omega}_i^2\sum_{j=1}^{Ne}\boldsymbol{\phi}_i^T\boldsymbol{M}_j\tilde{\boldsymbol{\phi}}_i\beta_j = (\tilde{\omega}_i^2 - \omega_i^2) \tag{12}$$

Similarly, substituting Eq. (11) into Eq. (9), yields

$$C_{ik} = \frac{\sum_{j=1}^{Ne}\boldsymbol{\phi}_i^T\boldsymbol{K}_j\tilde{\boldsymbol{\phi}}_i\alpha_j - \tilde{\omega}_i^2\sum_{j=1}^{Ne}\boldsymbol{\phi}_k^T\boldsymbol{M}_j\tilde{\boldsymbol{\phi}}_i\beta_j}{(\tilde{\omega}_i^2 - \omega_i^2)} \tag{13}$$

It should be noted that the governing equations in Eqs. (12) and (13) represent the exact relationship between the change in structural parameters and the measured modal properties of the tested structure such as frequencies and mode shapes. Define the sensitivity coefficients associated with eigenmodes and structural parameters in a general form as

$$\tilde{a}_{iji} = \boldsymbol{\phi}_i^T\boldsymbol{K}_j\tilde{\boldsymbol{\phi}}_i, \quad \tilde{b}_{iji} = \boldsymbol{\phi}_i^T\boldsymbol{M}_j\tilde{\boldsymbol{\phi}}_i \tag{14}$$

$$a_{kjl} = \boldsymbol{\phi}_i^T\boldsymbol{K}_j\boldsymbol{\phi}_i, \quad b_{kjl} = \boldsymbol{\phi}_i^T\boldsymbol{M}_j\boldsymbol{\phi}_i \tag{15}$$

By using Eq. (7), the governing equation in Eqs. (12) and (13) are now rewritten as

$$\sum_{j=1}^{Ne}\tilde{a}_{iji}\alpha_j - \tilde{\omega}_i^2\sum_{j=1}^{Ne}\tilde{b}_{iji}\beta_j = (\tilde{\omega}_i^2 - \omega_i^2) \tag{16}$$

$$C_{ik} = \frac{\sum_{l=1}^{N}\sum_{j=1}^{Ne}(a_{kjl}\alpha_j - \tilde{\omega}_i^2 b_{kjl}\beta_j)C_{il}}{(\tilde{\omega}_i^2 - \omega_i^2)} \tag{17}$$

It should be noted that the system of linear equations in Eq. (16) is usually not determined, depending on the number of frequency measurements available. The Moore-Penrose pseudoinverse is then used to estimate the structural updating parameters in the least squares sense (Tikhonov and Arsenin 1977). In FE model updating, it is often assumed that the global mass matrix keeps unchanged before and after updating, that is $\Delta\boldsymbol{M} = 0$ or $\beta_j = 0$ in Eq. (11(b)), since the mass of the actual tested structure can usually be estimated at a relatively high level of accuracy.

Based on the basic equations described in Eqs. (16) and (17) with $\beta_j = 0$, an iterative solution procedure is now developed to solve for the stiffness updating parameters α_j. By utilising Eqs. (7) and (14), the nth approximations for the stiffness updating parameters $\alpha_j^{(n)}$ in Eq. (16) and the mode participation factors $C_{ik}^{(n)}$ in Eq. (17) are obtained from, respectively

$$\tilde{a}_{iji}^{(n-1)} = \boldsymbol{\phi}_i^T\boldsymbol{K}_j\tilde{\boldsymbol{\phi}}_i^{(n-1)} \text{ where } \tilde{\boldsymbol{\phi}}_i^{(n-1)} = \boldsymbol{\phi}_i + \sum_{k=1,k\neq i}^{N} C_{ik}^{(n-1)}\boldsymbol{\phi}_k \tag{18}$$

$$\sum_{j=1}^{Ne} \tilde{a}_{iji}^{(n-1)} \alpha_j^{(n)} = (\tilde{\omega}_i^2 - \omega_i^2) \tag{19}$$

$$C_{ik}^{(n)} = \frac{\sum_{l=1}^{k-1}\sum_{j=1}^{Ne} a_{kjl}\alpha_j^{(n)} C_{il}^{(n)} + \sum_{l=k+1}^{N}\sum_{j=1}^{Ne} a_{kjl}\alpha_j^{(n)} C_{il}^{(n-1)}}{(\tilde{\omega}_i^2 - \omega_i^2) - \sum_{j=1}^{Ne} a_{kjk}\alpha_j^{(n)}} \tag{20}$$

The iterative solution procedure is initiated by assuming that the initial mode participation factors are zero, i.e., $C_{ik}^{(0)} = 0$ where $k \neq i$. The initial sensitivity coefficient $\tilde{\alpha}_{iji}^{(0)}$ is calculated from Eq. (18) by using the known $C_{ik}^{(0)} = 0$. The first approximation for the stiffness updating parameters, $\alpha_j^{(1)}$, then can be obtained from the basic equations Eq. (19), which now become a set of linear equations because of the known $\tilde{\alpha}_{iji}^{(0)}$. A regularization method discussed in the following section can be applied to find a reliable solution in order to reduce the influence of the uncertainty in frequency measurements. After the estimate of stiffness updating parameters $\alpha_j^{(1)}$ is obtained, the next approximations for $C_{ik}^{(1)}$ then can be calculated from Eq. (20). Consequently, the set of basic equations is used recursively to compute further approximations for $\alpha_j^{(n)}$ and $C_{ik}^{(n)}$. The above recursive process is repeated until the convergence for stiffness updating parameters $\alpha_j^{(n)}$ is achieved, often after only a few iterations.

4. Solution by regularization method

Due to the inevitable noise in the natural frequency measurements of the tested structure, the solution of the structural updating parameters obtained from the Moore-Penrose pseudoinverse in Eq. (19) may not be stable. In order to reduce the influence of noise in measured frequencies on the performance of updating structural parameters, a regularization method is now employed to obtain robust solutions for the structural updating parameters. The linear system in Eq. (19) for solving the stiffness updating vector $\boldsymbol{\alpha}$, consisting of a total number of Ne unknowns, is now rewritten here as

$$\boldsymbol{A}_{(M\times Ne)}\boldsymbol{\alpha}_{(Ne\times 1)} = (\tilde{\boldsymbol{\omega}}^2 - \boldsymbol{\omega}^2)_{(M\times 1)} \tag{21}$$

where subscript M represents the total number of measured noisy frequencies of the tested structure and is generally less than the total number of unknowns, i.e., $M < Ne$. Let the singular value decomposition (SVD) of the sensitivity coefficient matrix $\boldsymbol{A}$ be

$$\boldsymbol{A}_{(M\times Ne)} = \boldsymbol{U}_{(M\times M)}\Sigma_{(M\times Ne)}\boldsymbol{V}^T_{(Ne\times Ne)} = \sum_{j=1}^{M}\sigma_j\boldsymbol{u}_j\boldsymbol{v}_j^T \tag{22}$$

where Σ is a diagonal matrix containing strictly non-negative and non-increasing singular values σ_j, and the orthonormal column vectors $\boldsymbol{u}_j$ of $\boldsymbol{U}$ and $\boldsymbol{v}_j$ of $\boldsymbol{V}$ are the left and right singular vectors, respectively. It can be shown that the ordinary least squares solution to Eq. (21) can be expressed as

$$\boldsymbol{\alpha} = \sum_{j=1}^{M}\frac{\boldsymbol{u}_j^T(\tilde{\boldsymbol{\omega}}^2 - \boldsymbol{\omega}^2)}{\sigma_j}\boldsymbol{v}_j \tag{23}$$

In order to obtain reliable solutions for the stiffness updating vector α, some sort of regularization of the problem is required to filter out the contributions of the noise contained in measured frequency vector $\tilde{\omega}$. One of the most commonly used regularization methods with a continuous regularization parameter is the Tikhonov regularization (Tikhonov and Arsenin 1977). The regularization replaces the original operation with a better-conditioned but related one and produces a regularized solution to the original problem. The Tikhonov regularized solution of a continuous regularization parameter θ is given in terms of the SVD in Eq. (23) as

$$\boldsymbol{\alpha}(\theta) = \sum_{j=1}^{M} f_j(\theta)\frac{\boldsymbol{u}_j^T(\tilde{\boldsymbol{\omega}}^2 - \boldsymbol{\omega}^2)}{\sigma_j}\boldsymbol{v}_j \tag{24}$$

in which $f_j(\theta)$ are the Tikhonov filter factors, depending on singular values σ_j and regularization parameter θ, defined as

$$f_j(\theta) = \frac{\sigma_j^2}{\sigma_j^2 + \theta^2} \approx \begin{cases} 1, & if\ \sigma_j >> \theta \\ \dfrac{\sigma_j^2}{\theta^2}, & if\ \sigma_j << \theta \end{cases} \tag{25}$$

A stable solution then can be obtained since the Tikhonov regularized solution coefficients $f_j(\theta)\left|\boldsymbol{u}_j^T(\tilde{\boldsymbol{\omega}}^2 - \boldsymbol{\omega}^2)\right|/\sigma_j$ gradually damp out as singular values decrease. The filter factors $f_j(\theta)$ increasingly filter out the contributions to the solution of $\boldsymbol{\alpha}(\theta)$ associated with the small singular values, whereas the contributions associated with the large singular values are almost unaffected. The Tikhonov regularization parameter θ depends on the extent of uncertainty in frequency measurements of the tested structure. In reality, the noise level for the measured frequencies is often unknown. The GCV method (Golub *et al.* 1979) is then employed to estimate the optimal value of the Tikhonov regularization parameter θ, since this method does not require a priori information about the noise level. The GCV estimate of Tikhonov regularization parameter θ is defined as

$$\boldsymbol{G}(\theta) = \frac{\frac{1}{M}\left\|[\boldsymbol{I} - \boldsymbol{A}\boldsymbol{A}^{\#}(\theta)](\tilde{\boldsymbol{\omega}}^2 - \boldsymbol{\omega}^2)\right\|_2^2}{\left\{\frac{1}{M}\,Trace[\boldsymbol{I} - \boldsymbol{A}\boldsymbol{A}^{\#}(\theta)]\right\}^2} \tag{26}$$

where $\boldsymbol{I}$ is the identity matrix, and $\boldsymbol{A}^{\#}(\theta)$ is the influence matrix, defined as

$$\boldsymbol{A}^{\#}(\theta) = \sum_{j=1}^{M}\frac{f_j(\theta)}{\sigma_j}\boldsymbol{v}_j\boldsymbol{u}_j^T \tag{27}$$

By using the SVD of the sensitivity coefficient matrix $\boldsymbol{A}$ in Eq. (22) and the regularised SVD solution $\boldsymbol{\alpha}(\theta)$ in Eq. (24), the residual norm in the numerator and the trace in the denominator of the GCV function in Eq. (26) can be computed from

$$\left\|[\boldsymbol{I} - \boldsymbol{A}\boldsymbol{A}^{\#}(\theta)](\tilde{\boldsymbol{\omega}}^2 - \boldsymbol{\omega}^2)\right\|_2^2 = \left\|(\tilde{\boldsymbol{\omega}}^2 - \boldsymbol{\omega}^2)\boldsymbol{A}\boldsymbol{\alpha}(\theta)\right\|_2^2 = \sum_{j=1}^{M}[(1 - f_j(\theta))\boldsymbol{u}_j^T(\tilde{\boldsymbol{\omega}}^2 - \boldsymbol{\omega}^2)]^2 \tag{28a}$$

$$Trace[\boldsymbol{I}-\boldsymbol{A}\boldsymbol{A}^{\#}(\theta)] = \sum_{j=1}^{M}(1-f_j(\theta)) \tag{28b}$$

The GCV function in Eq. (26) is then simplified as

$$\boldsymbol{G}(\theta) = \frac{M\sum_{j=1}^{M}[(1-f_j(\theta))\boldsymbol{u}_j^T(\tilde{\boldsymbol{\omega}}^2-\boldsymbol{\omega}^2)]^2}{\left[\sum_{j=1}^{M}(1-f_j(\theta))\right]^2} \tag{29}$$

Consequently, the Tikhonov regularization parameter θ can be obtained by minimising the GCV function given in Eq. (29). The regularised solution for the structural updating parameters $\boldsymbol{\alpha}(\theta)$ is then calculated from Eq. (24).

5. Canton Tower benchmark problem

5.1 Description of the benchmark problem

The Canton Tower, located in Guangzhou, China, is a supertall structure with a height of 610 m, consisting of a 454 m high main tower and a 156 m high antenna mast, as shown in Figs. 1(a) and

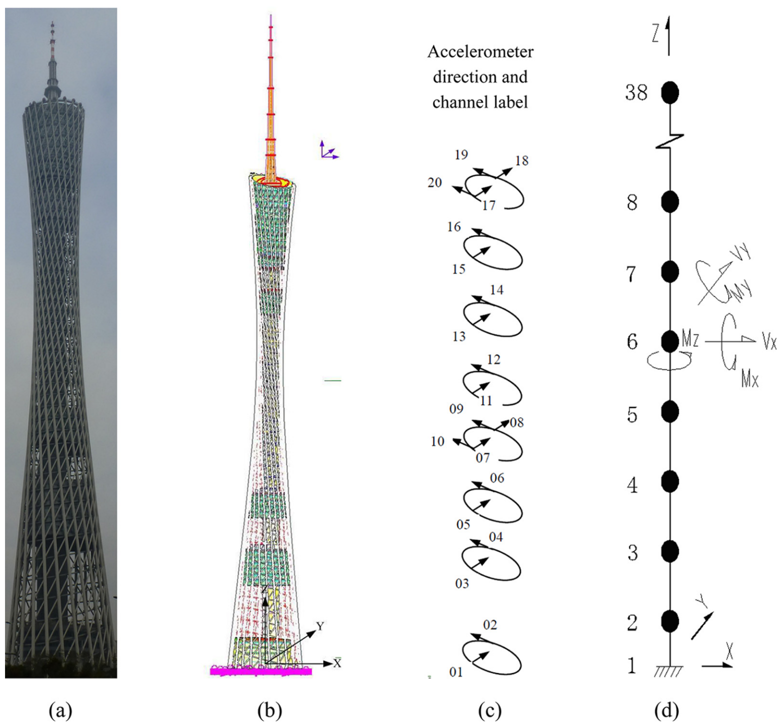

Fig. 1 The Canton Tower: (a) Actual structure, (b) full finite element model, (c) positions of installed accelerometers, and (d) reduced FE model (after Ni *et al*. 2012)

(b). The structure comprises a reinforced concrete inner tube and a steel outer tube with concrete-filled tube columns. During the construction of the tower, a sophisticated Structural Health Monitoring (SHM) system, consisting of more than 700 sensors including 20 accelerometers shown in Fig. 1(c), has been designed and implemented by the HKPU. In order to undertake SHM and associated studies, a reduced 3D beam model was established by the HKPU on the basis of the complex 3D full finite element model. In the reduced analytical model, the tower is modelled as a cantilever beam with 37 beam elements and 38 nodes, i.e., 27 elements for the main tower and 10 elements for the upper mast, as shown in Fig. 1(d) (Ni *et al*. 2012). The vertical displacement of the structure is ignored in the reduced analytical model, giving a total of 5 DOFs for each node, i.e., two horizontal translational DOFs and three rational DOFs. Therefore, each beam element has 10 DOFs and the reduced finite element model has a total of 185 DOFs with a fixed end at the base.

The implemented SHM system can provide the real-time monitoring of the structure at both in-construction and in-service stages. In order to obtain the operational modal properties of the tower, a total number of 20 uni-axial accelerometers were installed at eight different levels. Four uni-axial accelerometers were placed in the 4th and 8th floors and two uni-axial accelerometers were equipped in each of the remaining six floors, as shown in Fig. 1(c). The field ambient vibration measurement data then can be collected for many applications including operational modal analysis, model updating and structural condition assessment. The monitoring data recorded from 18:00pm on 19 January to 18:00pm on 20 January 2010 with a sampling frequency of 50 Hz by the SHM system were released by the HKPU for the Phase I of the Benchmark Problem for SHM of High-Rise Slender Structures.

5.2 Operational modal analysis

Data processing and modal identification for the tower are carried out by using the operational modal analysis software MACEC (Peeters and De Roeck 1999, De Roeck and Peeters 2011). In this study, only data recorded from 19:00pm to 20:00pm on 19 January 2010 are adopted for the operational modal identification. From the finite element analysis, the frequency range of interest lies between 0 and 2 Hz, containing at least the first fifteen frequencies within this range. As a result, re-sampling of the raw measurement data is necessary. For these measured data, a re-sampling and filtering from 50 to 5 Hz is carried out, leading to 18,000 data points with a frequency range from 0 to 2.0 Hz.

Two typical operational modal analysis techniques, i.e., the Peak Picking (PP) and Stochastic Subspace Identification (SSI) techniques, are employed in this study to extract modal properties such as frequencies, damping ratios and mode shapes from the recorded ambient vibration measurements. Peak picking is a frequency domain-based technique. This technique is often used for operational modal identification in civil engineering practice from ambient vibration measurements due to its simple implementation and fast processing speed. A practical implementation of the PP technique can be realized by the Averaged Normalized Power Spectral Densities (ANPSDs) in order to obtain a global picture of frequencies. The ANPSDs are calculated by converting the acceleration measurements to the frequency domain by a discrete Fourier transform and then by averaging the individual power spectral densities (Peeters and De Roeck 1999). As a result, the natural frequencies could be simply determined from the observation of the peaks on the graphs of the ANPSDs, as shown in Fig. 2. The PP technique however can only provide the operational deflection shapes and is unable to produce mode shapes.

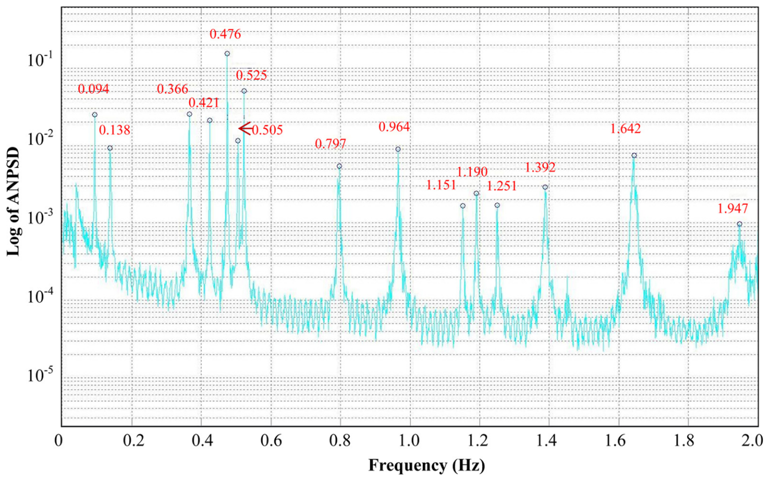

Fig. 2 Averaged normalized power spectral densities (ANPADs) of the measured acceleration data used for the Peak Picking (PP) technique

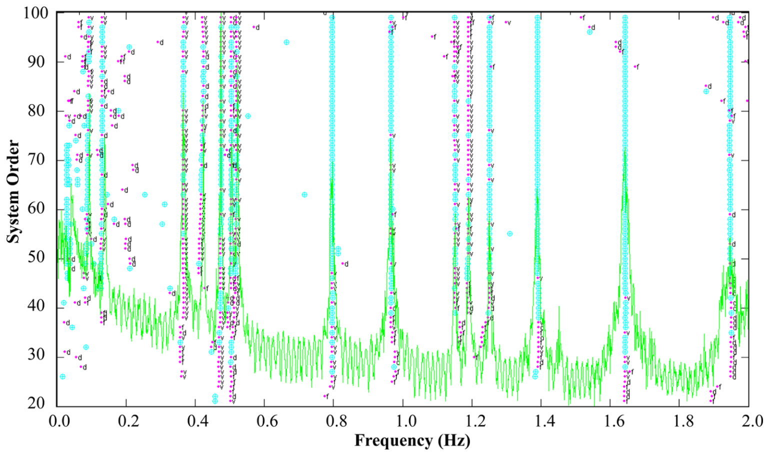

Fig. 3 The stabilization diagram of measured acceleration data used for the Stochastic Subspace Identification (SSI) technique

Stochastic subspace identification is a time domain method, which directly works with time dependent data, without requiring the conversions of the measured data into correlations or spectra. The SSI technique identifies the state space matrices on the basis of the measurements by using robust numerical techniques, such as singular value decomposition (Van Overschee and De Moor 1996). Once the mathematical description of the structure (the state space model) is determined, it is straightforward to extract natural frequencies from the stabilisation diagram, as shown in Fig. 3, and also to produce the associated damping ratios and mode shapes.

The modal properties identified by the PP technique and the SSI technique are then compared with the modal data obtained from the reduced FE model, as summarised in Table 1. The results show that the difference between the frequencies identified from the ambient vibration measurements and those from analytical model is relatively large, with the largest relative error in the fundamental natural frequency. However, the frequencies identified by the SSI technique are very close to those

Table 1 Comparison of modal data identified by operational modal analysis techniques from the monitored data with those calculated from the FE analytical model

Mode	FE model (Hz)	PP method (Hz)	SSI method (Hz)	Damping (%)	MAC value	Mode description
1	0.111	0.094	0.090	2.97	0.904	Short-axis bending
2	0.159	0.138	0.131	6.18	0.938	Long-axis bending
3	0.347	0.366	0.366	0.24	0.888	Short-axis bending
4	0.369	0.421	0.422	-1.50	0.888	Long-axis bending
5	0.400	0.476	0.474	0.07	0.869	Short-axis bending
6	0.462	0.505	0.504	0.38	0.104	Torsion
7	0.487	0.525	0.520	0.07	0.783	Long and short-axis bending
8	0.738	0.797	0.796	0.20	0.797	Short-axis bending
9	0.904	0.964	0.966	0.33	0.771	Long-axis bending
10	0.997	1.151	1.151	0.10	0.701	Short-axis bending
11	1.037	1.190	1.191	0.03	0.753	Long-axis bending
12	1.121	1.251	1.251	0.16	0.161	Torsion
13	1.245	1.392	1.390	0.35	0.793	Coupled bending and torsion
14	1.504	1.642	1.643	0.25	0.623	Coupled bending and torsion
15	1.726	1.947	1.946	0.59	0.609	Coupled bending and torsion

from the PP technique. The MAC diagonal values, calculated from the incomplete mode shapes identified by the SSI technique and the analytical eigenvectors restricting to the same DOFs, indicate good correlations between the identified and analytical modes, except two torsion modes, i.e., the 6^{th} and 12^{th} modes. The significant difference between the measured and analytical frequencies requires an updating of the finite element model. In this study, the modal data identified by the SSI technique are adopted for model updating.

5.3 Finite element model updating

Firstly, the first 12 measured frequencies including two torsion modes are utilised for model updating, without utilising the regularization algorithm. The updated modal properties such as frequencies and MAC diagonal values are summarised in Table 2. The results show that the updated frequencies are much closer to the frequencies identified from vibration measurements. The average of the absolute values of difference in frequencies reduces from 11.94% before updating to only 1.34% after updating. The obtained MAC diagonal values indicate that the updated mode shapes correlate well with the original modes of the FE model and also have good correlation with the modes identified from field measurements, except two torsion modes.

The results in Fig. 4 show the updated stiffness parameters of the reduced FE model from first 12 measured frequencies including two torsion modes. The obtained stiffness updating parameters range from -25.57% to 44.63%, with an average of the absolute values of 15.72%. This means that large modifications of structural stiffness are required in order to achieve significantly small difference between the updated and measured frequencies. The results for the required large modifications of the structural stiffness of the tower appear unreasonable in practice, since the reduced FE model was established on the basis of the sound understanding of the tower structure

Table 2 Updated modal properties of the FE analytical model using first 12 measured frequencies including two torsion modes

Mode	Tested (Hz)	Before updating			After updating			
		Frequency (Hz)	Difference (%)	MAC value	Frequency (Hz)	Difference (%)	MAC with tested	MAC with FE
1	0.090	0.111	23.81	0.904	0.089	-1.19%	0.805	0.976
2	0.131	0.159	21.19	0.938	0.130	-0.44%	0.839	0.978
3	0.366	0.347	-5.17	0.888	0.365	-0.37%	0.839	0.981
4	0.422	0.369	-12.50	0.888	0.422	0.13%	0.822	0.987
5	0.474	0.400	-15.59	0.869	0.458	-3.42%	0.818	0.976
6	0.504	0.462	-8.46	/	0.499	-1.14%	/	0.792
7	0.520	0.487	-6.31	0.783	0.516	-0.77%	0.807	0.976
8	0.796	0.738	-7.21	0.797	0.788	-0.95%	0.771	0.975
9	0.966	0.904	-6.44	0.771	0.974	0.80%	0.734	0.993
10	1.151	0.997	-13.34	0.701	1.124	-2.35%	0.766	0.998
11	1.191	1.037	-12.86	0.753	1.163	-2.33%	0.733	0.998
12	1.251	1.121	-10.38	/	1.224	-2.19%	/	0.886
		Average	11.94%*	0.829#		1.34%*	0.793#	0.960#

*: Average of the absolute values of deference in frequencies
#: Average of MAC values excluding two torsion modes

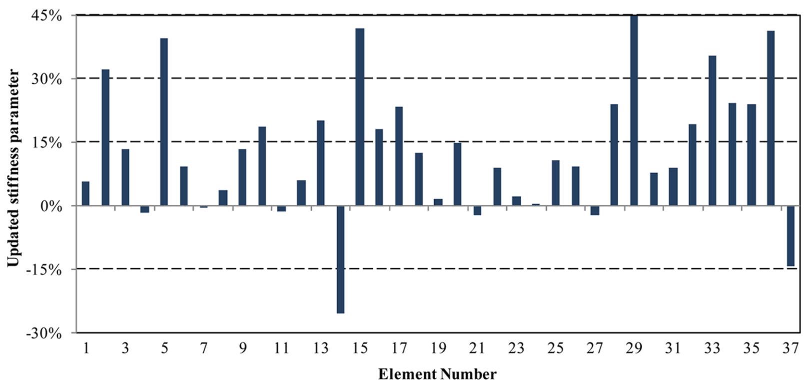

Fig. 4 Updated stiffness parameters of the FE analytical model using first 12 measured frequencies including two torsion modes

and should not be far away from the actual structure. On the other hand, the inevitable uncertainties in the operational modal properties identified from the field ambient vibration measurements may affect the accuracy of the model updating.

A regularization technique, i.e., the Tikhonov regularization incorporating the GCV method, is now introduced to reduce the influence of uncertainty in modal data measurements on model updating. Here, two cases with different number of measured frequencies adopted for model updating are considered, i.e., with first 10 and 13 measured frequencies without including two torsion modes. In the solution procedure, the optimum regularization parameters of the Tikhonov regularization are determined by minimising the GCV function, as shown in Fig. 5. In the case with 10 frequencies

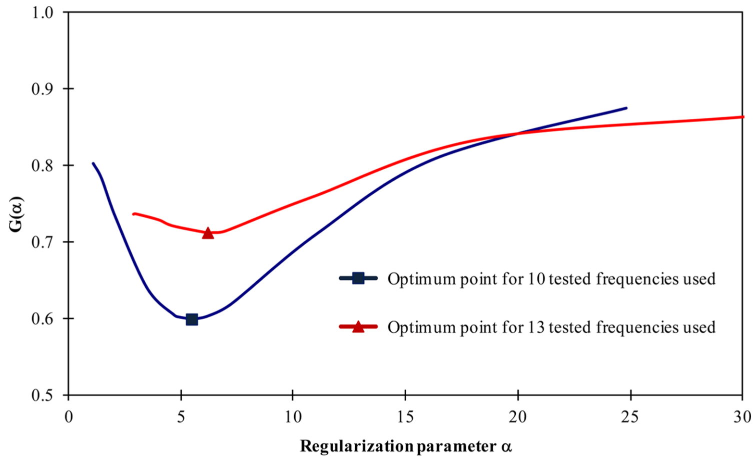

Fig. 5 The GCV as a function of Tikhonov regularization parameter θ for cases with 10 and 13 measured frequencies adopted for updating, giving optimum regularization parameters

Table 3 Updated modal properties of the FE analytical model using 10 and 13 measured frequencies without including two torsion modes

Mode	10 tested frequencies used				13 tested frequencies used			
	Frequency (Hz)	Difference (%)	MAC with tested	MAC with original	Frequency (Hz)	Difference (%)	MAC with tested	MAC with original
1	0.093	3.76%	0.773	0.937	0.094	5.01%	0.788	0.945
2	0.142	8.14%	0.824	0.936	0.143	8.82%	0.837	0.942
3	0.367	0.19%	0.881	0.996	0.365	-0.38%	0.875	0.995
4	0.402	-4.71%	0.917	0.998	0.402	-4.79%	0.912	0.996
5	0.428	-9.66%	0.844	0.998	0.427	-9.88%	0.841	0.996
6	/	/	/	/	/	/	/	/
7	0.506	-2.69%	0.892	0.995	0.501	-3.67%	0.880	0.995
8	0.781	-1.80%	0.832	0.997	0.782	-1.69%	0.839	0.997
9	0.940	-2.65%	0.832	0.996	0.939	-2.78%	0.842	0.995
10	1.004	-12.72%	0.683	0.999	1.008	-12.40%	0.698	0.997
11	1.055	-11.42%	0.810	0.998	1.057	-11.24%	0.821	0.996
12	/	/	/	/	/	/	/	/
13	/	/	/	/	1.254	-9.76%	0.799	1.000
14	/	/	/	/	1.504	-8.46%	0.623	1.000
15	/	/	/	/	1.747	-10.23%	0.607	0.987
	Average	5.77%*	0.829#	0.985#		6.85%*	0.797#	0.988#

*: Average of the absolute values of deference in frequencies used for updating
#: Average of MAC diagonal values of modes used for updating

used, the optimum regularization parameter is taken as θ=5.466 where the singular values of the SVD range from 0.445 to 24.819. Meanwhile, the optimum regularization parameter of θ=6.180 is chosen for the case with 13 frequencies used where the singular values of the SVD vary between 0.103 and 30.441. The results indicate that the optimum Tikhonov regularization parameter can be

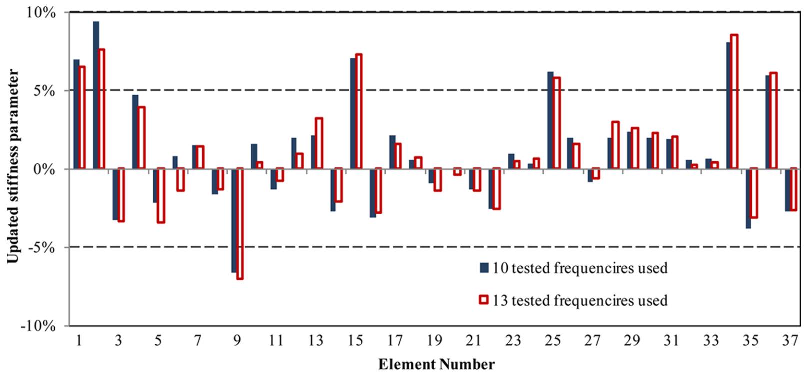

Fig. 6 Updated stiffness parameters of the FE analytical model using 10 and 13 measured frequencies without including two torsion modes

obviously determined by minimising the GCV function.

The updated modal properties of the reduced FE model by using the regularization algorithm are given in Table 3 for the two cases with different number of measured frequencies used for model updating. The average of the absolute values of the difference in frequencies used in updating reduces from 12.44% before updating to 5.77% after updating for the case with 10 tested frequencies used and from 11.90% to 6.85% for case with 13 tested frequencies used, respectively. The obtained high MAC diagonal values indicate that the updated mode shapes match well the original analytical eigenvectors and experimental mode shapes as well. Only small difference exists between the updated modal properties obtained by using 10 tested frequencies and those obtained by using 13 tested frequencies.

Fig. 6 shows the results for the regularised stiffness updating parameters of the analytical model for the two cases with 10 and 13 frequency measurements used for model updating. The values of stiffness updating parameters are now much smaller, comparing with the results obtained without using the regularization algorithm shown in Fig. 4. In the case with 10 measured frequencies used in model updating, the obtained stiffness updating parameters range from -6.63% to 9.42% with an average of the absolute values of 2.82%. For the case with 13 frequency measurements, similar results are obtained and the updated stiffness parameters vary between -7.00% and 8.52% with an average of the absolute values of 2.74%. The results from these two cases with different number of frequency measurements are very close to each other, in terms of the adjusted stiffness parameters and updated frequencies as well. These results appear more reasonable in practice since only relatively small stiffness modifications in the reduced FE model are required in order to obtain relatively small difference in the updated and identified frequencies.

6. Conclusions

An effective approach is proposed for updating finite element model of a complex actual structure with limited information about measured modal data available. The proposed dynamic perturbation method provides the exact relationship between the perturbation of structural parameters and the modal properties of the actual tested dynamic structure. Structural updating parameters are chosen to characterise the difference in structural parameters between the analytical model and tested structure at element level. On the basis of the developed dynamic perturbation method, an iterative solution procedure is proposed to be solved for the chosen structural updating parameters in the least squares sense. In order to obtain reliable solutions for the structural updating parameters, the regularization algorithm based on the Tikhonov solution incorporating the GCV method is employed to filter out the influence of modal data measurement uncertainties on the predictions of structural updating parameters. Finally, the Canton Tower benchmark problem is utilised to demonstrate the applicability and effectiveness of the proposed method for updating the reduced finite element model by using the operational modal properties identified from ambient vibration measurements.

Based on the case study involving the Canton Tower benchmark problem, the following conclusions are drawn. (1) The proposed approach is capable of successfully updating structural parameters such as stiffness at element level, requiring only limited information on operational modal data such as frequency measurements. The proposed approach needs only little computation effort to obtain the structural updating parameters, and provides optimised solutions for model updating in the least squares sense without requiring optimisation methods. (2) The proposed approach performs well for various scenarios considered and gives reasonable predictions of structural updating parameters even in the cases where relatively large modifications in structural parameters and/or modal properties exist between the analytical model and tested structure. (3) The predictions of stiffness updating parameters may be unreasonable when a regularization algorithm is not employed in model updating, since significant modifications of structural stiffness may be needed in order to minimise the difference between the updated and measured frequencies. (4) A regularization algorithm, such as the Tikhonov regularization method incorporating the GCV method, should be utilised to give reasonable solutions for the structural updating parameters by reducing the influence of measurement errors in modal data. The GCV method can effectively give an obvious optimum Tikhonov regularization parameter by minimising the GCV function. (5) Only relatively small modifications of structural stiffness may be sufficient for updating the reduced FE model of the Canton Tower benchmark problem, while the frequencies of the updated analytical model are close enough to the frequencies identified from the field monitored vibration data.

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