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SSA-based stochastic subspace identification of structures from output-only vibration measurements

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Abstract. In this study an output-only system identification technique for civil structures under ambient vibrations is carried out, mainly focused on using the Stochastic Subspace Identification (SSI) based algorithms. A newly developed signal processing technique, called Singular Spectrum Analysis (SSA), capable to smooth a noisy signal, is adopted for preprocessing the measurement data. An SSA-based SSI algorithm with the aim of finding accurate and true modal parameters is developed through stabilization diagram which is constructed by plotting the identified system poles with increasing the size of data matrix. First, comparative study between different approaches, with and without using SSA to pre-process the data, on determining the model order and selecting the true system poles is examined in this study through numerical simulation. Finally, application of the proposed system identification task to the real large scale structure: Canton Tower, a benchmark problem for structural health monitoring of high-rise slender structures, using SSA-based SSI algorithm is carried out to extract the dynamic characteristics of the tower from output-only measurements.

Keywords: state-space model; operational modal analysis; stochastic subspace identification; singular spectrum analysis; covariance-driven SSI; data-driven SSI

1. Introduction

Identification of modal parameters using ambient excitation is more feasible for large engineering structures. The extraction of features from these measurements and the analysis of these features to determine the current state of health of the system using spaced measurement provide a tool for structural health monitoring and damage detection. In recent years, identification approaches using only measured response data were investigate widely. These approaches can be categorized as time-domain methods and frequency-domain methods. One of the frequency-domain methods, referred to as frequency domain decomposition (FDD) has been used for ambient analysis by using the singular value decomposition (SVD) of the output spectrum matrix (Brincker *et al*. 2001). As for the time-domain methods for ambient data analysis, there were several researches which include: eigensystem realization algorithm (ERA) (Juang and Pappa 1985), and stochastic subspace identification (Van Overschee *et al*. 1996, Peeters and De Roeck 2001).

For large scale civil structures such as bridges, the input excitation to the structural system is unknown, the output-only Stochastic Subspace Identification (SSI) is suitable for the identification

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and monitoring of these structures excited by ambient vibrations. There are several varieties of SSI technique such as Covariance-driven (SSI-COV), Data driven (SSI-DATA). SSI-DATA algorithms were fully enhanced by Van Overschee and De Moor (Van Overschee *et al*. 1996). The core of output-only identification through SSI-DATA is the orthogonal projection carried out by LQ decomposition (Van Overschee *et al*. 1996, Peeters and De Roeck 2001), followed by the SVD used to extract the system subspace. There are variants of the Data-driven algorithm which correspond to a different choice of weighting matrices before factorizing the projection matrix. The well-known SSI-DATA algorithms include CVA, N4SID, MOESP and IV-4SID (Larimore 1994, Van Overschee and De Moor 1994, Verhaegen and Dewilde 1992). The reference-based SSI algorithms were also developed in (Peeters and De Roeck 2001, Peeters and De Roeck 1999) and applied to the identification of a steel transmitter mast and a pre-stressed concrete bridge (the Z-24 bridge in Switzerland). Besides, application of the SSI-DATA algorithm to investigate the dynamic characteristics of a cable-stayed bridge had been studied in (Weng *et al.* 2008). Other application can be found in the identification of offshore structure, rotating machinery, aircraft structure and bridge structure (Abdelghani *et al.* 1997, Abdelghani *et al*. 1999, Bassville *et al*. 2007, Boonyapinyo and Janesupasaeree 2010).

As opposed to SSI-DATA, the SSI-COV algorithm avoids the computation of orthogonal projection, instead, it is replaced by converting raw time histories in an assemble of block covariances which is called Toeplitz matrix. The SSI-COV algorithm appears early as the Modified Instrumental Variable method, with applications in laboratory tests, such as the identification of a vertical steel clamped-free beam and modal analysis of a carrying bogie (Basseville *et al*. 1993).

Different family of SSI-COV exist, a very famous identification algorithm is a combined approach of the Natural Excitation Technique (NExT) (James *et al.* 1992) and the Eigensystem Realization Algorithm (ERA) (Juang and Pappa 1985) to find modal parameters from ambient response. It is called as the Natural Excitation Technique and Eigensystem Realization Algorithm (NExT-ERA) (Caicedo *et al*. 2004) which has been applied to the identification and damage detection of a 4-story 2-bay steel IASC-ASCE Benchmark structure. The same algorithm is also applied to the identification of a cable stayed bridge in (Zhang *et al.* 2005), where alternative form to construct the stabilization diagram (Peeters 2000) was proposed.

It was proved, for output-only measurements, the Stochastic Subspace Identification technique is a multivariate identification technique for extracting system dynamic characteristics and to be numerically stable, robust to noise perturbation and suitable for conducting non-stationarity of the ambient excitations although its stationary assumption is violated. In this paper, the covariance driven stochastic subspace identification (SSI-COV) was used to extract the dynamic characteristics of a high-rise tower from its ambient vibration measurement under operational conditions. Through numerical study a newly developed signal processing technique, called Singular Spectrum analysis (SSA), capable to smooth a noisy signal, is adopted for pre-processing the data before the SSI. Discussion on the dimension of Toeplitz matrix as well as the number of singular values and modal order on the construction of stability diagram of model frequencies was also made through sensitivity analysis.

2. Output-only covariance-driven stochastic subspace identification (SSI-COV)

Assuming a structure under consideration is being excited by unmeasurable stochastic input forces, the discrete time stochastic state-space model can be expressed as

$$\begin{aligned} \boldsymbol{x}_{k+1} &= \boldsymbol{A}x_k + \boldsymbol{w}_k \\ \boldsymbol{y}_k &= \boldsymbol{C}x_k + \boldsymbol{v}_k \end{aligned} \tag{1}$$

where $x_k \in \Re^{2n\times 1}$ is the state vector and $y_k \in \Re^{l\times 1}$ is the measurement vector, $\boldsymbol{w_k}$ and $\boldsymbol{v_k}$ represent the system noise and measurement noise respectively. The Covariance- Driven Stochastic Subspace Identification method (SSI-COV) stems from the need to solve the problem through identifying a stochastic state-space model (matrices $\boldsymbol{A}$ and $\boldsymbol{C}$) from output-only data. The first step is to gather the measurement vectors in a Hankel Data matrix

$$\boldsymbol{H} = \frac{1}{\sqrt{N}} \begin{bmatrix} \boldsymbol{y}_1 & \boldsymbol{y}_2 & \cdots & \boldsymbol{y}_N \\ \boldsymbol{y}_2 & \boldsymbol{y}_3 & \cdots & \boldsymbol{y}_{N+1} \\ \cdots & \cdots & \cdots & \cdots \\ \boldsymbol{y}_1 & \boldsymbol{y}_{i+1} & \cdots & \boldsymbol{y}_{i+N-1} \\ \hline \boldsymbol{y}_{i+1} & \boldsymbol{y}_{i+2} & \cdots & \boldsymbol{y}_{i+N} \\ \boldsymbol{y}_{i+2} & \boldsymbol{y}_{i+3} & \cdots & \boldsymbol{y}_{i+N+1} \\ \cdots & \cdots & \cdots & \cdots \\ \boldsymbol{y}_{2i} & \boldsymbol{y}_{2i+1} & \cdots & \boldsymbol{y}_{2i+N-1} \end{bmatrix} = \begin{bmatrix} \boldsymbol{Y}_p \\ \hline \boldsymbol{Y}_f \end{bmatrix} \tag{2}$$

where $\boldsymbol{Y}_p$ denotes the past measurements and $\boldsymbol{Y}_f$ denotes the future measurements. It can be easily found that the block Toeplitz matrix can be obtained by a multiplication between future and transpose of past measurements

$$T_{1/i} = \begin{bmatrix} \boldsymbol{R}_i & \boldsymbol{R}_{i-1} & \ldots & \boldsymbol{R}_1 \\ \boldsymbol{R}_{i+1} & \boldsymbol{R}_i & \ldots & \boldsymbol{R}_2 \\ \cdots & \cdots & \cdots & \cdots \\ \boldsymbol{R}_{2i-1} & \boldsymbol{R}_{2i-2} & \ldots & \boldsymbol{R}_i \end{bmatrix} = \boldsymbol{Y}_f(\boldsymbol{Y}_p)^T \tag{3}$$

where the block output covariance with time lag i is defined as $\boldsymbol{R}_i$

$$\boldsymbol{R}_i = E[\boldsymbol{y}_k \boldsymbol{y}_{k-i}^T] \tag{4}$$

The Toeplitz matrix can be factorized into the extended observability matrix $\mathbf{O}_i \in \Re^{li\times 2n}$ and the reversed extended stochastic controllability matrix $\Gamma_i \in \Re^{2n\times li}$, as shown below

$$\boldsymbol{T}_{1/i} = \boldsymbol{O}_i \boldsymbol{\Gamma}_i = \begin{bmatrix} \boldsymbol{C} \\ \boldsymbol{CA} \\ \cdots \\ \boldsymbol{CA}^{i-1} \end{bmatrix} \begin{bmatrix} \boldsymbol{A}^{i-1}\boldsymbol{G} & \ldots & \boldsymbol{AG} & \boldsymbol{G} \end{bmatrix} \tag{5}$$

Singular Value Decomposition (SVD) is the tool used to perform the above mentioned factorization

$$\boldsymbol{T}_{1/i} = \boldsymbol{USV}^T = (\boldsymbol{U}_1 \ \ \boldsymbol{U}_2)\begin{pmatrix} \boldsymbol{S}_1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \boldsymbol{V}_1^T \\ \boldsymbol{V}_2^T \end{pmatrix} = U_1 S_1 V_1^T \tag{6}$$

where $\boldsymbol{U} \in \Re^{li \times li}$ and $\boldsymbol{V} \in \Re^{li \times li}$ are orthonormal matrices, and $\boldsymbol{S}$ is a diagonal matrix containing positive singular values in descending order. Comparing Eqs. (5) and (6), the matrix $\boldsymbol{O}_i$ which contains the system matrices ($\boldsymbol{A}$ and $\boldsymbol{C}$) can be computed by splitting the SVD in two parts

$$\mathbf{O}_i = \mathbf{U}_1 \mathbf{S}_1^{1/2} \tag{7}$$

$$\Gamma_i = \mathbf{S}_1^{1/2} \mathbf{V}_1^T \tag{8}$$

From $\boldsymbol{O}_i$ matrix, the system matrices ($\boldsymbol{A}$ and $\boldsymbol{C}$) can be obtained easily. In MATLAB notation, the $\boldsymbol{C}$ matrix is just the first block of $\boldsymbol{O}_i$

$$\boldsymbol{C} = \boldsymbol{O}_i(1{:}l,:) \tag{9}$$

System matrix $\boldsymbol{A}$ can be computed by exploiting the shift structure of the extended observability matrix $\boldsymbol{O}_i$

$$\begin{bmatrix} \boldsymbol{CA} \\ \boldsymbol{CA}^2 \\ \cdots \\ \boldsymbol{CA}^i \end{bmatrix} = \begin{bmatrix} \boldsymbol{C} \\ \boldsymbol{CA} \\ \cdots \\ \boldsymbol{CA}^{i-1} \end{bmatrix} \boldsymbol{A} \tag{10}$$

Therefore:

$$\boldsymbol{A} = \boldsymbol{O}_i(1{:}l(i-1),:)^{\lambda} \boldsymbol{O}_i(l+1{:}li,:) \tag{11}$$

where $(\cdot)^{\lambda}$ denote pseudo-inverse. The system matrix $\boldsymbol{A}$ is extracted by taking advantage of the shift structure of matrix $\boldsymbol{O}_i$, and the pseudo-inverse of $\boldsymbol{O}_i$ is to determine the system matrix $\boldsymbol{A}$ in a least square sense. The modal frequencies and effective damping ratios can be computed by conducting eigenvalue decomposition of the system matrix $\boldsymbol{A}$, and the corresponding eigenvectors multiplied by the output matrix $\boldsymbol{C}$ are the observed mode shapes. Frequencies and damping ratios that were estimated from the SSI-COV can be used as one of the features for monitoring the structure. In operational modal identification one of the difficulties raises, i.e., the ambient excitation under normal operating conditions is not measured and often nonstationary.

As opposite to the covariance-driven stochastic algorithm, the data-driven method (SSI-DATA) avoids the calculation of covariance. Instead, the data reduction step is accomplished by projecting the row space of the future outputs into the row space of past outputs. Covariances and projections are closely related, in that they are both intended to eliminate uncorrelated noise contributions. Discussion on the performance of SSI-COV and SSI-DATA on signal processing will be made through numerical studies in the following sections.

3. Discussion of SSI-COV on developing the stability diagram

The first step of SSI-COV method is to form the covariance blocks with correlation of time lag "*i*" among the measurement data, and form the "Toeplitz matrix" which has the same elements along its diagonal. The Singular Value Decomposition (SVD) is then used as the principal tool to extract the system information from the Toeplitz matrix. It is important to note that if full sensors are used to establish the correlation matrix, the formed covariance block is a square matrix, but Toeplitz matrix may or may not have the same number of block rows and columns. This depends on different ways to construct the stabilization diagram. Besides, in real world applications, noise is always present at any measurement, and there is no prior information about the number of modes that can be extracted from the data. Therefore, a stabilization diagram is used to discriminate between noise or spurious poles and true system poles. Follow this discussion there are several ways to establish the stabilization diagram:

1st version: First the maximum dimension of the Toeplitz matrix is decided, then perform SVD, and let the system matrix order increases from a lower value till reaching a user defined maximum dimension of the Toeplitz matrix. The advantage of this version is that only one SVD has to be performed; less time is consumed in the construction of the stabilization diagram. The drawback is that, there is no clear criterion to ensure that the chosen maximum dimension of the Toeplitz matrix is sufficient or not to reveal the true system information. With increasing order of the system matrix $\boldsymbol{A}$, more noise information, which is separated from the measurement data by SVD, will be included in the system matrix $\boldsymbol{A}$ and consequently, more noise or spurious poles will appear on the diagram. For this purpose, modal transfer norm (Reynders *et al*. 2008) was introduced to clear out the large number of spurious poles at higher orders and thus clarifying the stabilization diagram. The concept behind this version to construct the stabilization diagram is that, even including more spurious poles in the system matrix $\boldsymbol{A}$, the true system modes (including frequencies, damping ratios, and mode shapes) extracted from $\boldsymbol{A}$ will remain stable.

2nd version: Determine the order of system matrix $\boldsymbol{A}$ by observing the variation of the singular values, and then, increase the size of the Toeplitz matrix both rows and columns by holding the order of system matrix unchanged. For convenience, this version will called the "square Toepliz matrix" or "original form" because it increments both columns and rows, and the Toeplitz matrix remains squared. The main drawback of this is, first, time consuming, and second, the system matrix order must be defined in advance. For field measurement data, generally there is no clear gap on the distribution of singular values as it appears in the numerical simulation. The advantage of this version is that, one do not have to try the maximum Toeplitz matrix size, since its required size may vary from case to case. Increase of the Toeplitz matrix dimension, meaning that more orthogonal components will be decomposed from the signal and therefore a better separation between signal and noise. It means that increment of the order of the extended observability matrix, more information is used to determine the modal parameters and therefore, more accurate estimate is expected.

3rd version: This is a modification of the second version. Since Toeplitz matrix has not necessary to be a square matrix, an alternative way to construct the stabilization diagram is to increase only block rows of the Toeplitz matrix by keeping the number of block columns and the order of system matrix $\boldsymbol{A}$ as constant (Zhang *et al.* 2005). For convenience, this version will called the "rectangular Toeplitz matrix" or "alternative form" becauses it only increases rows but not columns of the Toeplitz matrix. The advantage of this method is that, it conserves the data property of the stabilization diagram in a least square sense (it is much faster than the 2^{nd} version). The difficulty on the choice of number of block columns constitutes the main drawback of this method, since in this version block rows are always greater than columns, but the number of block columns will determine how many components will be decomposed from the covariances of the signal. If the number of columns of the Toeplitz matrix is lower than the required, it will lead to an unstable diagram on the estimation of the lower modes in the presence of noise which can lead to poor estimation of modal parameters. On the contrary, the use of square Toeplitz matrix do not have to worry about the noise effect and the determination of number of columns, since noise in the measurement data will only delay the outcome of a stable diagram.

Having reviewed the advantages and drawbacks to construct the stabilization diagram, the use of rectangular Toeplitz matrix may be recommended for subspace identification because of the prior knowledge about the noise content. But in the first step identification, the use of square Toeplitz matrix is recommended although it may imply more time consumption. As a short summary, SSI-COV technique is robust against noise and signal non-stationarity. The method strongly relies on SVD, which decompose the projection or covariance of the signal by taking advantage of the orthogonality between vectors of the obtained basis U and V^T in SVD. Either after projection (SSI-DATA) or after correlation (SSI-COV), there will always have some non-removed residuary noise or non-stationarity signal, particularly in the case of non-white ambient excitation. This situation may violate the assumptions of SSI algorithms, but its effects can be dropped out by increasing the size of the projection or the covariance matrix, because more orthogonal components the matrix be decomposed (by selecting only the subspace corresponding to those theoretically “non-zero” singular values), better the system-related information will be clearly separated from the noise.

4. Preprocessing of data for SSI-COV analysis: Singular spectrum analysis

SSA is a novel non-parametric technique used in the analysis of time series based on multivariate statistics. This method was firstly applied to extract tendencies and harmonic components in meteorological and geophysical time series (Elsner and Tsonis 1996, Alonso *et al*. 2005). Except the extraction of tendency, SSA can be applied to smooth a noisy signal, to extract seasonality component, or to detect the singularities. Basically, this method is capable of decomposing the original series into a summation of principal components, so that each component in this sum can be identified as a tendency, periodic components (stationary), nontationary signal or noise. The SSA procedure consists of four steps: (1) embedding, (2) singular value decomposition (SVD), (3) grouping, and (4) reconstruction.

In this study SSA will be used as a pre-processing tool to extract the principal components from its measurements. In the beginning, the frequency domain decomposition (FDD) is employed on the original measurements to construct the distribution of the singular values with respect to frequency which can be served as the reference for identifying the close-space model frequencies. Since both SSA and SSI-COV depends primary on SVD, and it is reasonable that the number of singular values be chosen in SSI-COV must be less than that chosen in SSA; in the best case, the number of components chosen in SSA coincide exactly with that of the system, i.e., noise was totally filtered. The main drawback of using SSA as a pre-processing tool before SSI-COV is that, the estimated damping ratio is not reliable because it is always much lower than the true damping ratio. The reason is that, SSA can only extract the sufficient information (principal components) to obtain an accurate estimate of the system natural frequencies, but some portions of the signal mixed with noise were filtered out which loosing in this way the possibility to obtain a good damping ratio estimation.

Based on the results from numerical study the summary of procedures to conduct SSA-SSI-COV is listed as follows:

(a) Assemble Hankel Data matrix with number of block rows as large as possible (this usually depends on the memory capacity of the computer and the time available to conduct the SVD), and the number of columns will determine the available data point to construct the covariance matrix (Toeplitz matrix). Usually the number of columns is much larger than the number of block rows, and this latter will determine the number of principal components of the signal to be decomposed.

(b) After performing SVD to the data Hankel matrix constructed in the first step of SSA, one can select a set of principal components from the plot of the distribution of singular values (for example: select 95% of singular spectrum) to reconstruct the signal.

(c) Reconstruct the signal, if necessary, by repeating it for different set of choice.

(d) Construct the covariance or Toeplitz matrix by calculating the correlation with the reconstructed signal.

(e) Perform SSI-COV by conducting SVD to the covariance matrix and plot the variation of singular values. (The size of the covariance matrix will be the largest number of block rows that will be reached in the stabilization diagram)

(f) Repeat step 4 and step 5 for different choice of SV from the result of SSA.

(g) Selecting the result which shows the change of slope is remarkable in the SV distribution from SSI-COV analysis and check if the required information is included through the plot of Fourier spectrum.

(h) Decide the system order which will be approximately the same at the end of the slope change in SV distribution, and construct the stabilization diagram.

5. Sensitivity study of the SSI-based algorithms

A comprehensive numerical simulation studies were carried out to understand the sensitivity of SSI-based algorithms by selecting different size of Hankel matrix and model order and pay special attention in their effects on the development of stabilization diagram of the identified system natural frequencies.

Example 1: Effect of noise on modal parameter estimation

Consider a multiple degree of freedom system with well-separated modal frequencies and its structural response was simulated using white noise as excitation. In this simulation study, if a spatially white noise was added on system responses (either acceleration or velocity measurements), there is no problem for SSI-based algorithms to identify accurately the modal frequencies. Because the first step of SSI algorithm, such as correlation or projection, can effectively cancel out the added white noise once at all. Therefore, even adding 200% of noise in RMS sense (noise to signal ratio), error on the identified frequencies is less than 4% for the 1^{st} mode and less than 1% for the remaining modes. If one adds noise which is correlated with the output measurement (such as in acceleration measurements, the external input acceleration can be treated as a measurement noise), by increasing the order of projection or covariance matrix, frequencies as well as the mode shapes

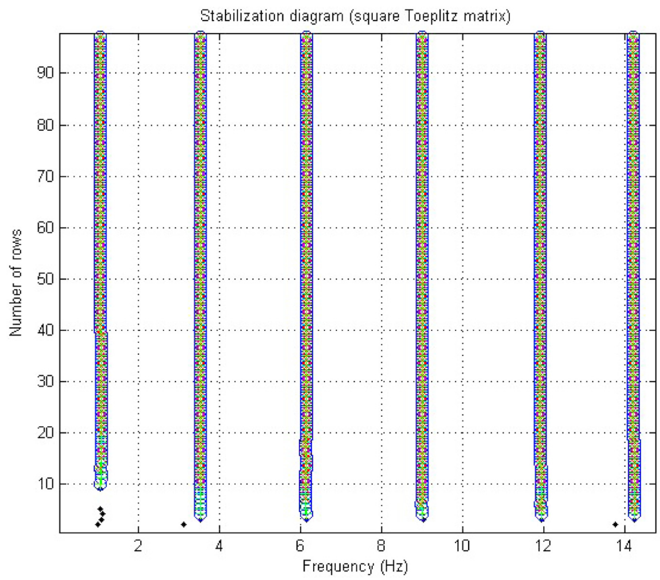
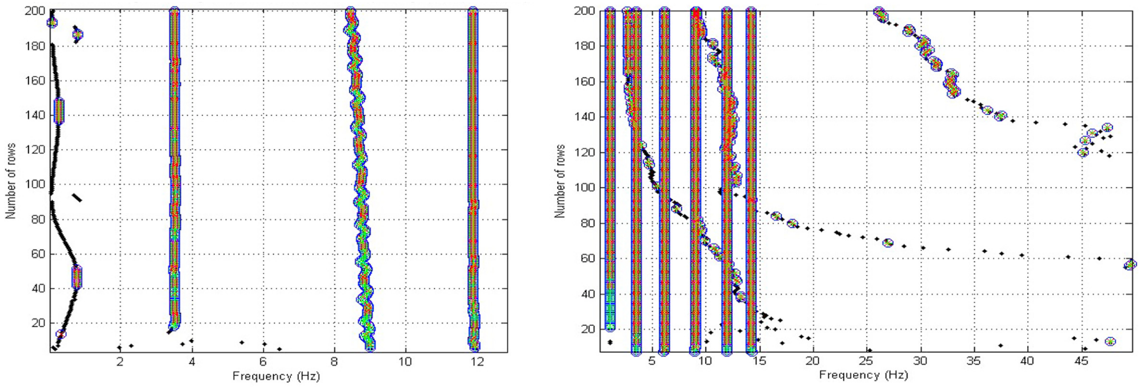


Fig. 1 Stabilization diagram of a 6-DOF system by using square Toeplitz matrix. Rows in Toeplitz matrix are increased from 2 to 100 and the system matrix order is 12

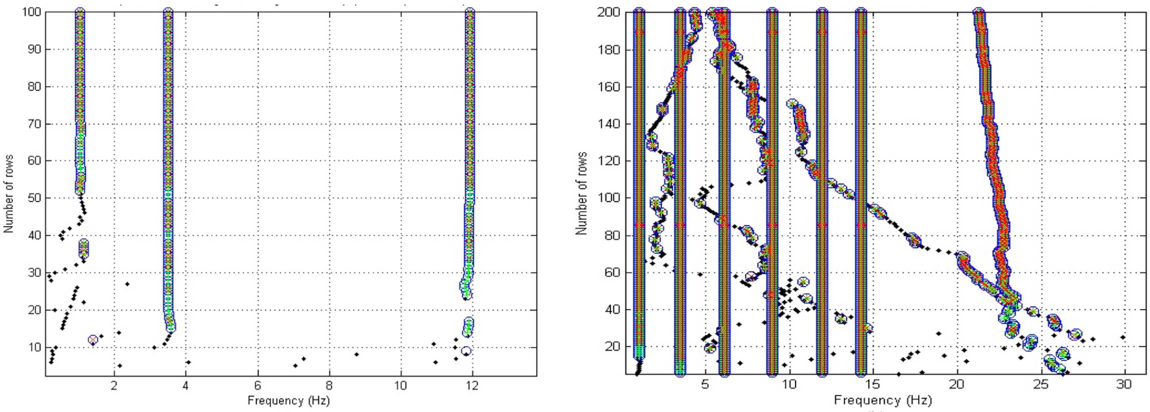
can also be accurately identified. Through observation, lower modes are more affected by the noise effects and may need larger row number in Toeplitz matrix so as to have a stable modal parameters. If the number of block columns used to fix the noise in SVD is not sufficient, lower modes will not stabilize at all no matter how many rows are used. The required minimum number of block columns depends on the noise content of the signal. Different from the modal frequencies, damping is very sensitive to the addition of any type of noise even it is very small; the error can be as big as to 100%.

Consider the shaking table test data of a 6-story steel frame as an example (Wong *et al*. 2009). The first six system natural frequencies are: 1.12 Hz, 3.63 Hz, 6.33 Hz, 9.21 Hz, 12.09 Hz and 14.33 Hz. Acceleration response data of white noise excitation (with maximum peak ground acceleration scaled to 50 gal) from each floor was collected for identification. In this case, although intuitively one may think that the six-story Moment Resistant Frame can be well modeled as a 6-DOF system since only 6 sensors are used, therefore the order of system matrix A can be determined as 12, however, in fact, a real world structure has infinite DOF, moreover, there is no guarantee that all modes will be well excited. It is important to note that, all SSI algorithms rely on SVD to extract the system matrices. Fig. 1 shows the stabilization diagram of this 6-DOF system by using square Toeplitz matrix. Rows in Toeplitz matrix are increased from 2 to 100 and the system matrix order is 12. The system natural frequencies can be identified clearly. If the system order is chosen to be less than 12, as one can see, the corresponding modal parameters to the selected 8 singular values is not stable at all even with 5 block columns and with 200 block rows, as shown in Fig. 2(a). Fig. 2(b) shows for the case in which more singular values are chosen, the system poles remain stable, and the numerical poles are transported in the diagram is spinning round in the plot. Thus, it is concluded that, if the alternative version of stabilization diagram is used and if one cannot choose the system order with confidence, one should choose more singular values because an underestimation of the system order will result in a not stable diagram even for actual system poles.



(a) (b)

Fig. 2 (a) For case of underestimation of the system order by using rectangular Toeplitz matrix (With 8 singular values and 5 block rows) and (b) for case of overestimation of the system order for rectangular Toeplitz matrix. (with 18 singular values and 3 block columns)



(a) (b)

Fig. 3 Stabilization diagram from using square Toeplitz matrix: (a) with 6 singular values (case of underestimation) and (b) with 22 singular values (case of overestimation)

Now the underestimation or overestimation for square Toeplitz matrix is investigated. Consider the same 6-DOF system, if only six singular values were chosen, the result of the stability diagram is shown in Fig. 3(a). The result does not mean that each identified pole represents the certain mode of the signal. The reason is that SVD just decomposes the signal in their principal components ordered by their projected norm in the subspace. Besides, due to the insufficient singular values be chosen for the system matrix, the diagram takes several rows to stabilize. Different from the case of using six SV, overestimate the system poles can happen if 22 singular values are used. The stability diagram for the overestimated case is shown in Fig. 3(b). It is concluded that if one cannot choose the system order with confidence, it is necessary to choose more singular values because an underestimation of the system order will result for actual system poles.

Example 2: Two close-spaced frequencies blended with time-varying signals

Basically, it is not appropriate to use linear SSI algorithm and apply stabilization diagram to

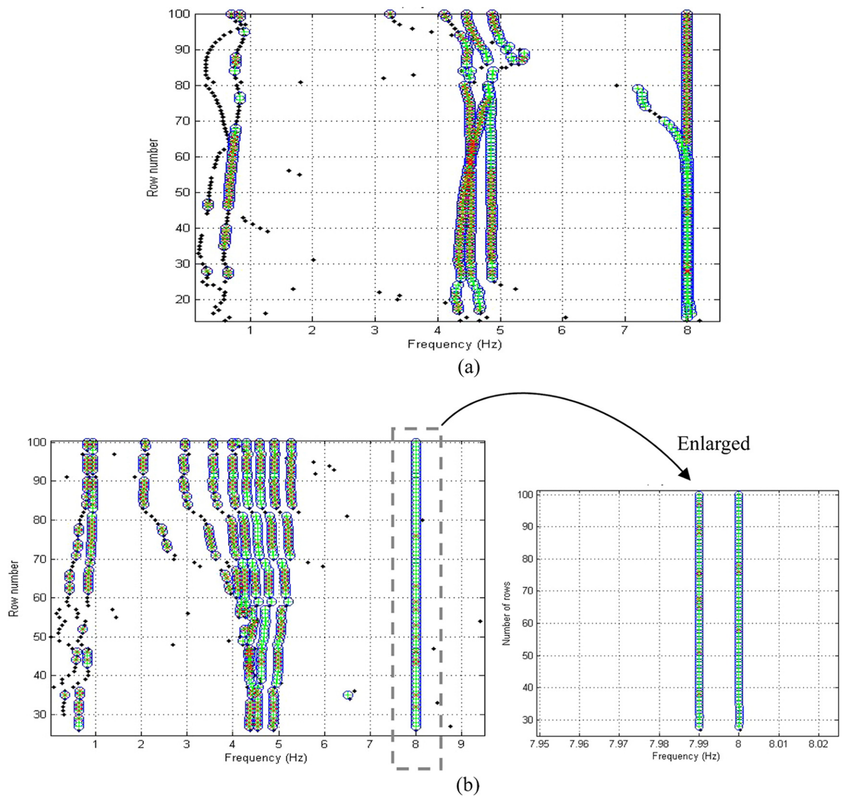


Fig. 4 The developed stability diagram from using SSI-COV algorithm with different system orders: (a) Using system order of 14 and (b) using system order of 26

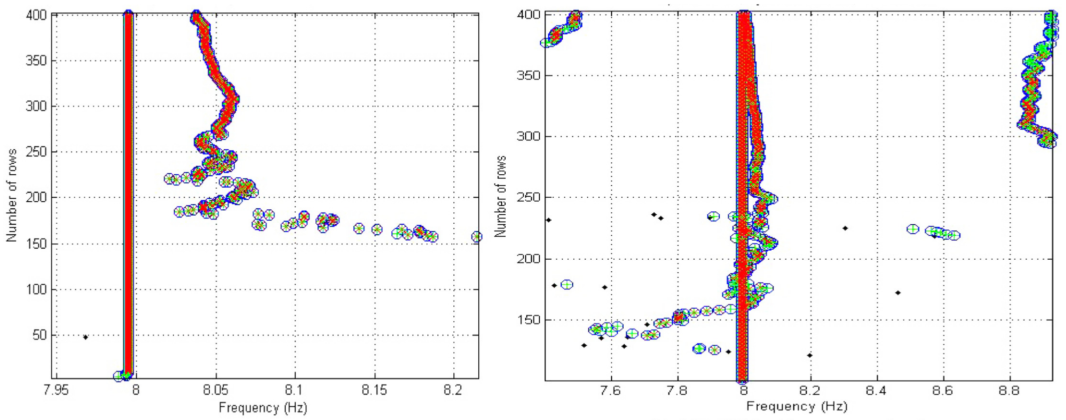
identify time-varying signals, but the stabilization diagram can be used to check the system with time-varying modal parameters. Consider a signal with the combination of three sinusoidal functions: two sine waves with close frequencies, f_1 = 7.99 Hz and f_2 = 8.00 Hz, and a time-varying term $sin(t^2)$

$$y = sin2\pi f_1 t + sin2\pi f_2 t + sint^2 \tag{12}$$

The SSI-COV algorithm was used with different order. Fig. 4 shows the stability diagram by using two different sets of system orders (with order of 14 and 26, respectively). The closely-spaced frequencies will appear as only one frequency if only a few model orders (order of system matrix $\boldsymbol{A}$) were chosen. As the model order increases to a sufficient level, these two close-spaced frequencies will be "split" which is a well known phenomena in stabilization diagram. Since there is a time-varying signal blended in with the same power, it contains, in other words, "a lot of frequencies" and consequently the closely-spaced frequencies cannot be revealed until the system order reaches a certain number. On the other hand, the time-varying component will appear in the stabilization diagram as a "combination" of a lot of equivalent frequencies which will not stabilize at all in the diagram but they follow certain pattern.

Example 3: Effect of Noise on system with close-spaced frequencies

Consider the case of a signal which constituted by two sine waves with frequency of 7.99 Hz and 8.00 Hz and also added 10% noise. Even with such closely-spaced system frequencies, in the noise

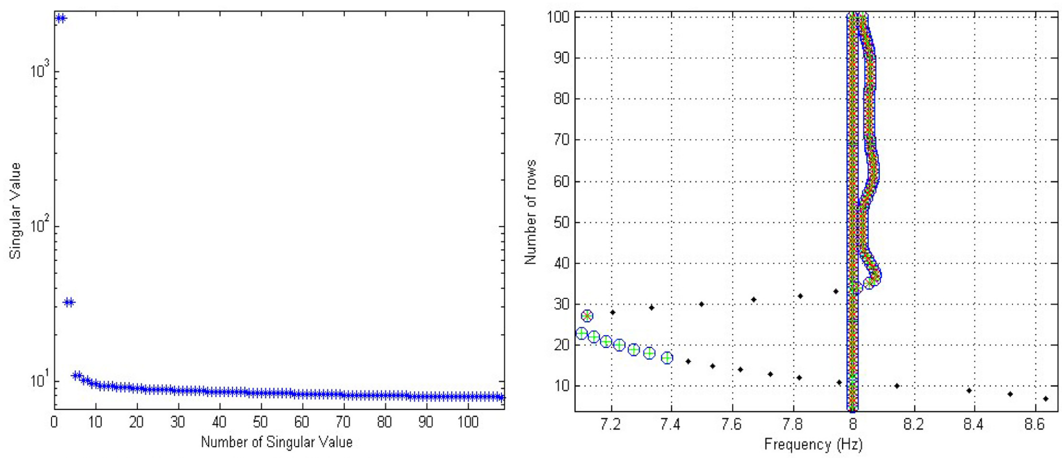


(a) SSI-COV with system order 4 (b) SSI-COV with system order 8

Fig. 5 Stabilization diagram generated from SSI-COV for signal contains 7.99 Hz and 8.00 Hz sine waves and added 10% noise; (a) with system order of 4 and (b) with system order of 8

free case, there is no problem for SSI-based algorithms to identify these two frequencies. However, once the noise is added, the SSI-based algorithm can only identify one clear frequency. Fig. 5(a) shows the result of using SSI-COV algorithm by selecting 4 singular values to identify the signal with two closed-space frequencies and with 10% noise contaminated in the measurement. Since SVD does not decompose the signal according to the frequency order, one may think that in the case of noise contamination, the system order may not be chosen exactly as 4 for signal with two frequencies. If the system order was chosen as 8, as shown in Fig. 5(b), and the quality in the stabilization diagram was improved, but it stabilized after approximately 260 rows in SSI-COV.

If a good subspace based pre-processing tool is used, such as SSA which was discussed in the previous section, noise can be filtered out before entering to the SSI algorithm. The use of SSA before SSI-COV is an option to filter out the noise and to improve the identification quality of the stabilization diagram. Fig. 6(a) shows the variation of singular values obtained in SSA with the size



(a) Singular Value Decomposition of 1000x3000 Hankel matrix, SSA. (b) SSA-SSI-COV, system order: 4, 4 SV.

Fig. 6 (a) Singular spectrum from SVD of Hankel matrix in SSA with dimension of 1000 by 3000 and (b) stabilization diagram constructed using SSA-SSI-COV with 4 Singular values of Hankel matrix is set to 1000 by 3000. Therefore, the data can be decomposed into 1000 components, and it is clearly observed that 4 singular values can be separated from the data effectively. Fig. 6(b) shows the stability diagram by using SSA-SSI-COV with system order 4. The identified frequencies and damping ratios at 100 rows are: 7.995 Hz, 8.0278 Hz, and 0.00028, 0.0167 respectively. Therefore, even with noise disturbance, with the pre-processing technique the two closely-spaced frequencies can be identified correctly.

6. Application: system identification of Canton Tower

The Canton Tower is located at Guangzhou, China. It is a super high-rise tube-in-tube structure with a height of 610 m, as shown in Fig. 7. This structure comprises a reinforced concrete inner tube and a steel outer tube with concrete-filled tube (CFT) columns. There are 37 floors connecting the inner tube and the outer tube. The outer tube consists of 24 CFT columns, uniformly spaced in an oval while inclined in the vertical direction. The inner tube is an oval shape but with constant dimension of 14 m by 17 m in plan, but its centroid differs from the centroid of the outer tube. The Hong Kong Polytechnic University is in charge of the implementation of the long-term SHM both during the construction as in the service stage. More details can be found in references (Ni *et al.* 2009, Ni *et al*. 2012). The data were recorded from 18:00 pm on 19 January 2010 to 18:00 pm on 20 January 2010, lasting 24 hours. The acceleration, wind direction, wind speed and ambient temperature

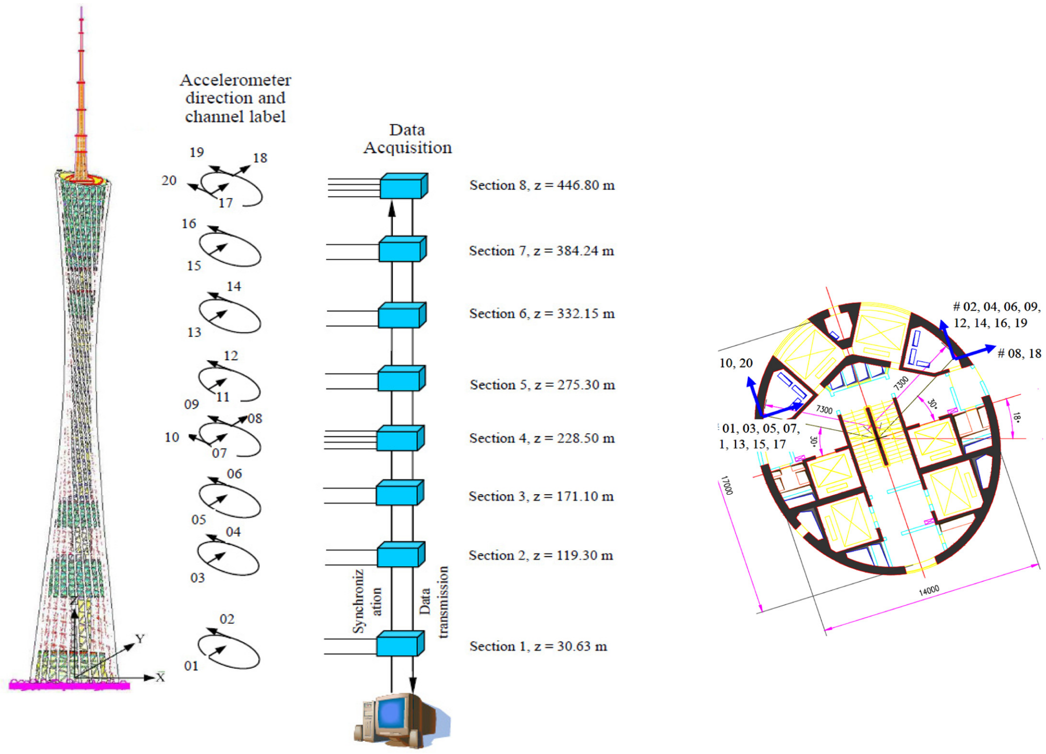


Fig. 7 A skematic diagram of Canton Tower which shows the locations of acceleometers at different levels. The locations of accelerometers in the floor plane is also shown

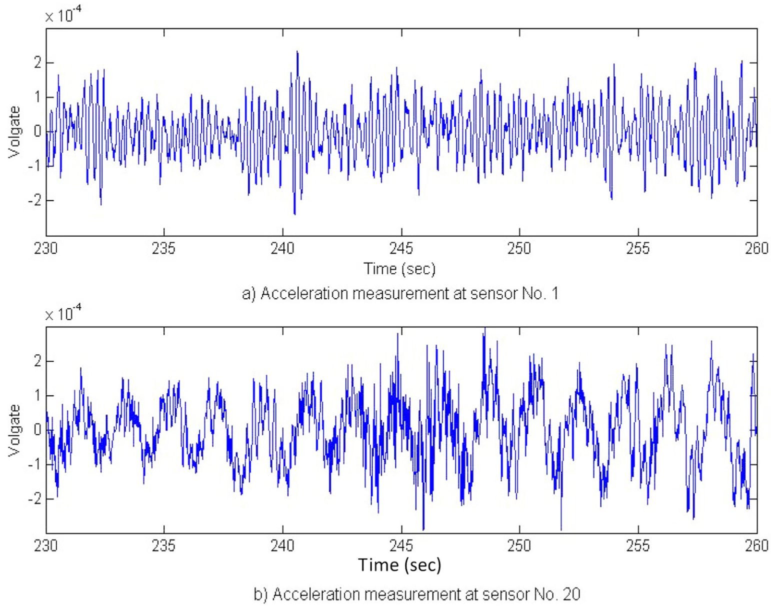


Fig. 8 A portion of the acceleration measurements for (a) the 1st sensor and (b) the 20th sensor

were measured during the period.

Twenty uni-axial accelerometers (Tokyo Sokushin AS-2000C) were employed for vibration measurements, the frequency range is DC-50 Hz (3 dB), amplitude range ±2 g, and the sensitivity 1.25 V/g. They were installed at eight levels as shown in Fig. 7, the 4th level and the 8th level were equipped with four uni-axial accelerometers, two for measurement of the horizontal acceleration along the long-axis of the inner structure and the other two for the short-axis. At the other six levels, each section was equipped with two uni-axial accelerometers, one along the long-axis of the inner structure and the other along the short-axis of the inner structure. Fig. 7 also shows a plan of the section and the measurement directions of acceleration. The sensors were fixed to the shear wall of the inner structure via a steel angle. The sampling frequency of the acceleration and wind data was 50 Hz. Fig. 8 shows the acceleration measurement at the first minutes of the 1st and 20th sensor which are located at a lower and a higher place of the tower respectively. Signals of the 20th sensor appear to contain long period signals (about 2 seconds) as compared to the measurement of 1st sensor.

Frequency Domain Decomposition (FDD) Before applying the SSI analysis, based on ambient vibration measurements, the FDD was used to identify the possible number of modes contained in the response measurements. The FDD spectrum was calculated using 131072 points for each set of data. Welch's periodogram (Caicedo *et al.* 2004) was used to estimate the power spectrum density function with a window length of 8192 points and an overlap of 4096 points, which leads to an estimate of power spectrum by averaging a total of 31 Fourier spectrums. Since the sampling rate is 50 Hz, the used window length of 8192 points leads to a frequency resolution of 0.006 Hz. The result of FDD spectrum is shown in Fig. 9. It is observed that in periodogram below 1.0 Hz there are several dominant frequencies. This power spectral density function will be used as a reference spectrum when conducting the SSI method.

SSI-COV (Covariance Driven) versus SSI-DATA (Data Driven)Algorithms To construct the stabilization diagram using SSI-COV, a total of 18000 data points were used in the correlation function, and the

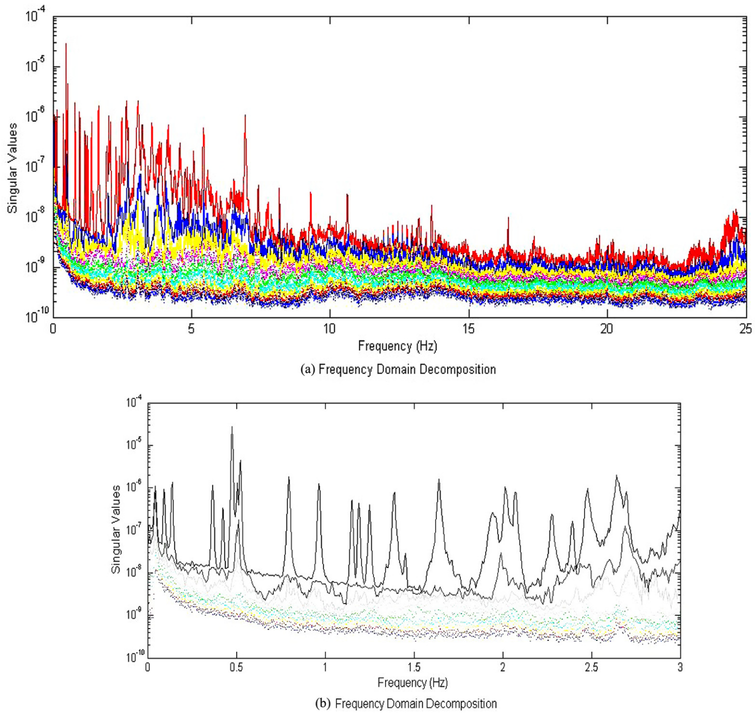


Fig. 9 (a) Plot of singular values with respect to frequencies using Frequency domain decomposition of the recorded accelerations from Canton Tower and (b) enlarged figure from FDD for frequency between 0.0~3.0 Hz

size of square Toeplitz matrix increases from 5 to 300 block rows, all 20 sensors were used in the computation. By observing the variation of singular values, one can note that it is difficult to identify a significant gap from the distribution of singular values. Therefore, it is hard to decide the

Table 1 Identified system natural frequencies of Canton Tower using four different approaches

FEM Mode Number					1	2	3	4	5	6	7
Theoretical		Freq	wind	wind	0.1100	0.1590	0.3470	0.3680	0.4000	0.4610	0.4850
SSI-COV	18000 points	Freq	0.0404	0.0409	0.0902	0.1392		0.3652	0.4243	0.4752	0.5060
	300 row	Damp	0.2885	1.0000	0.0693	0.0135		-0.0004	-0.0008	0.0018	0.0018
SSI-DATA	8000 points	Freq	0.0110	0.0339		0.1377		0.3655	0.4402	0.4774	
	280 row	Damp	1.0000	0.0727		0.0080		-0.0015	0.0048	-0.002	
SSA-SSI-COV	170 row	Freq	0.0373	0.0572		0.1388		0.3644	0.4238	0.4757	0.5063
95 SV, order 90		Damp	0.2412	0.3991		0.0122		0.0042	0.0039	0.0006	0.0031
SSA-SSI-COV	140 row	Freq	0.0345	0.0465	0.0958	0.1391		0.3648	0.4232	0.4756	0.5063
136SV, order 120		Damp	0.2368	0.1978	0.0700	0.0106		0.0024	0.0056	0.0006	0.0024

Table 1 Continued

Theoretical Mode Number				8	9	10	11	12	13	14	15	
Theoretical		Freq		0.7380	0.9020	0.9970	1.9970	1.1220	1.2440	1.5030	1.7260	
SSI-COV	180000 points	Freq	0.5223	0.7982		0.9649	1.1507	1.2031	1.2525	1.3891	1.6401	1.9463
	300 row	Damp	0.0000	0.0041		0.0013	0.0002	0.0058	0.0022	0.0036	0.0022	0.0053
SSI-DATA	80000 pints	Freq	0.5193	0.7977		0.9654	1.1794	1.2281		1.3848	1.6397	1.9428
	280 row	Damp	-0.0017	0.0009		-0.0004	0.0017	0.0010		0.0008	0.0003	0.0014
SSA-SSI-COV	170 row	Freq	0.5226	0.7986		0.9653	1.1512	1.2517		1.3899	1.6407	1.9446
95 SV, order 90		Damp	-0.0008	0.0022		0.0008	0.0007	0.0015		0.0020	0.0012	0.0031
SSA-SSI-COV	140 row	Freq	0.5226	0.7986		0.9652	1.1509	1.1932	1.2518	1.3899	1.6407	1.9445
136 SV, order 120		Damp	-0.0008	0.0020		0.0006	0.0006	0.0008	0.0013	0.0018	0.0010	0.0026

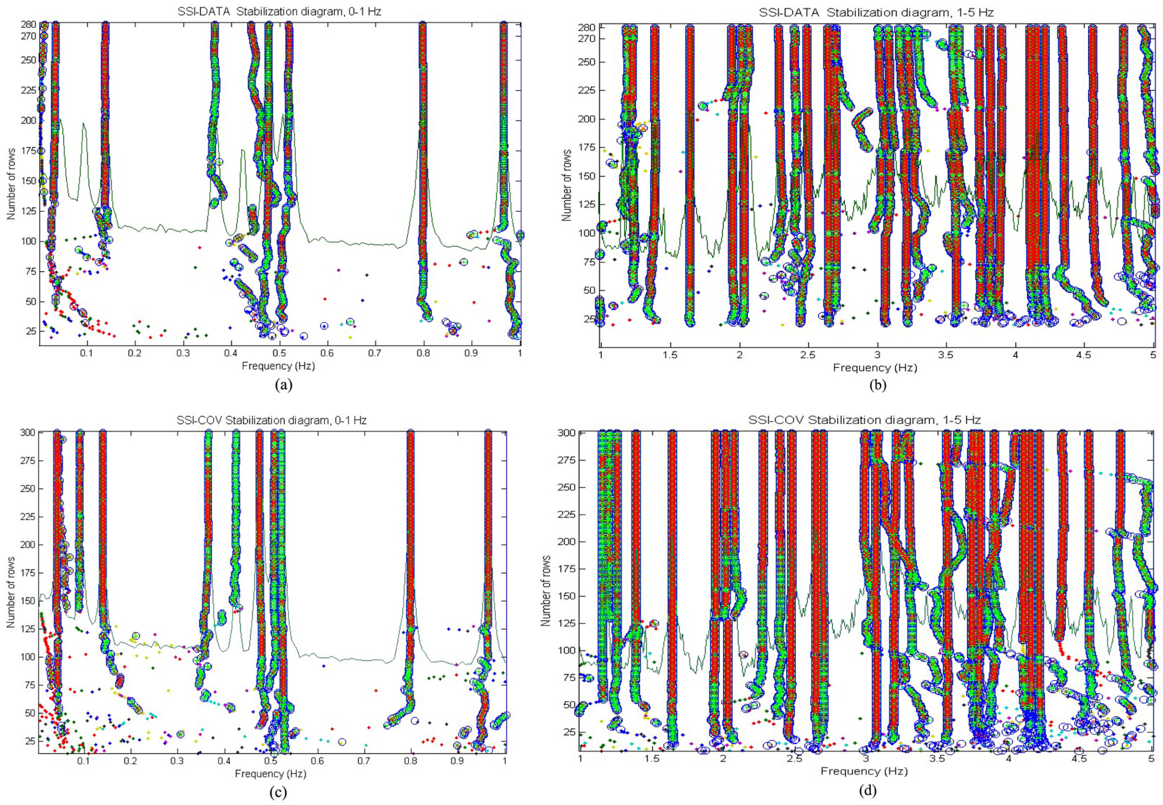


Fig. 10 Stability diagram constructed using SSI-DATA algorithm: (a) from frequency 0.0~1.0 Hz and, (b) from frequency 1.0~5.0 Hz, (c) from frequency 0.0~1.0 Hz and (d) from frequency 1.0~5.0 Hz

system order from this SV distribution. With an extra help, one can count the number of peaks that appear in the FDD spectrum and multiplied by two to get an estimate of the number of singular values. In our case, at the beginning, 90 singular values were chosen. The initial result is shown in Table 1 (1^{st} row). SSI-DATA is also used to identify the modal parameters of Canton Tower. The Hankel data matrix used in SSI-DATA is set to 8000 columns with rows increasing from 20 to 280 block rows (the actual maximum number of rows is $2 \times i \times \lambda = 2{\times}280 \times 20 = 11200$ rows). The system order is chosen as the same as SSI-COV. The results from SSI-DATA are also shown in this table for comparison (2^{nd} row). Fig. 10 shows that stability diagram from these two approaches. It is clear that, from the observation of stability diagram, to reach a stable result of system poles larger number of block rows may be required. But larger number of block rows may require more significant computation time.

Besides, unlike the SSI-COV method, application of SSI-DATA method to the tower data can not identify the first mode (0.0902 Hz) which the SSI-COV method can. But SSI-COV can not provide a good estimate of this first mode because the complex mode shape (from the observation of poles in complex plane) is a little sparse with "*R*" equal to 0.86 (*R* is the correlation coefficient between the imaginary and real part of the complex mode shape). The difficulty to identify the first mode may due to the fact that the dominant wind frequency may very close to the first fundamental mode of the tower; therefore, this frequency is most likely to be affected by the wind excitation frequency as well as the noise content. Generally, the lowest mode is the most affected by the noise. The other difficulty to identify the natural frequencies from the ambient vibration data of Canton Tower may due to the closely-spaced natural frequencies of the tower and also with the noise measurements. Although SSI-COV shows the ability to separate these close frequencies, there is still doubts due to their near zero damping ratio and complex model shape lying only in real or imaginary axis.

SSI-COV Algorithm with SSA as Data Pre-proposing Singular Spectrum Analysis, like the principal component analysis (PCA), was used to extract the major principal components of signals and filter out the undesired noise from measurements. In performing the SSA-SSI-COV analysis there are two parameters that have to be determined. One is the number of Singular Value (SV) to be chosen in conducting the SSA, and the other is the system order (n) to be determined in the SSI-COV analysis. In the beginning, 20 sensors measurements in the form of vector were placed at once

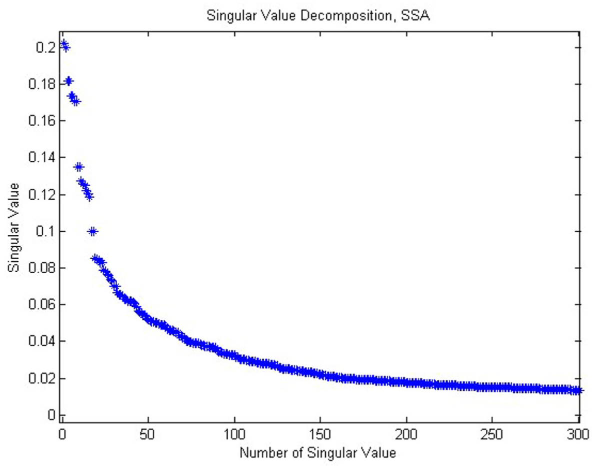


Fig. 11 Singular value decomposition of data Hankel matrix with 340 block rows and 15000 columns from SSA

in Hankel data matrix for SSA with the following dimensions: 340 × 20 block rows (totally 6800 rows) by 15000 columns. The variation of singular values from SSA is shown in Fig. 11. It is difficult to select a suitable number of singular values to be extracted from this figure because there is no significant gap on the distribution of singular values. Selection of percentage of singular values becomes an important issue to generate the stability diagram in the follow up SSI analysis.

From the working experience on the data of Canton Tower, if a specific number of singular value is chosen in conducting SDV from SSA as a prior, then distribution of SV from the SVD of the Toeplitz matrix in SSI-COV analysis can lead to a significant change of slope. Based on the recorded acceleration data from GNTVT, in the beginning, 312 singular values were chosen from the SVD of the data matrix of SSA and the jump of singular value in the SVD of Toeplitz matrix of SSI-COV analysis is almost imperceptible (as shown in Fig. 12(a)) and nothing is stable in plotting the stability diagram (as shown in Fig. 12(b)). When one try with smaller number of SV, e.g., 154

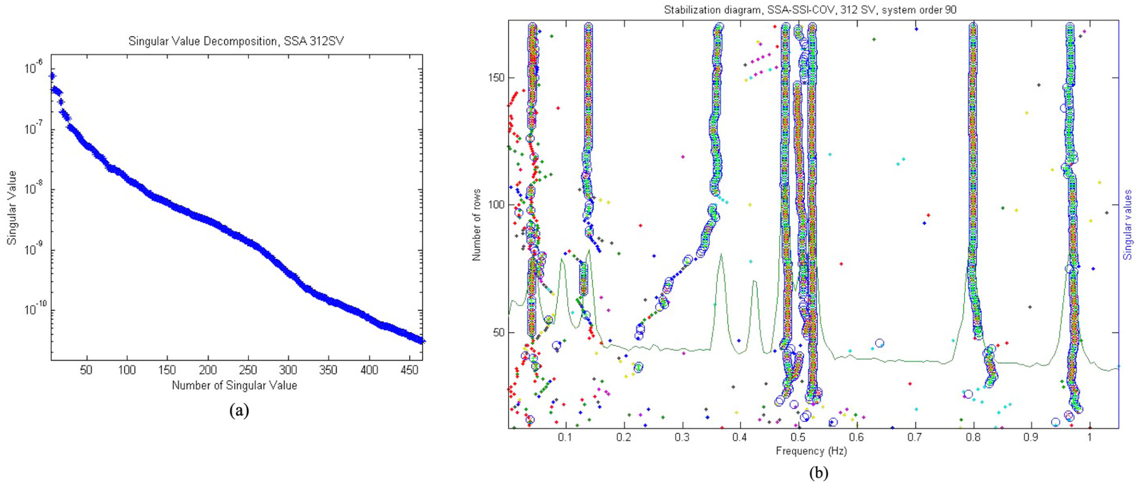
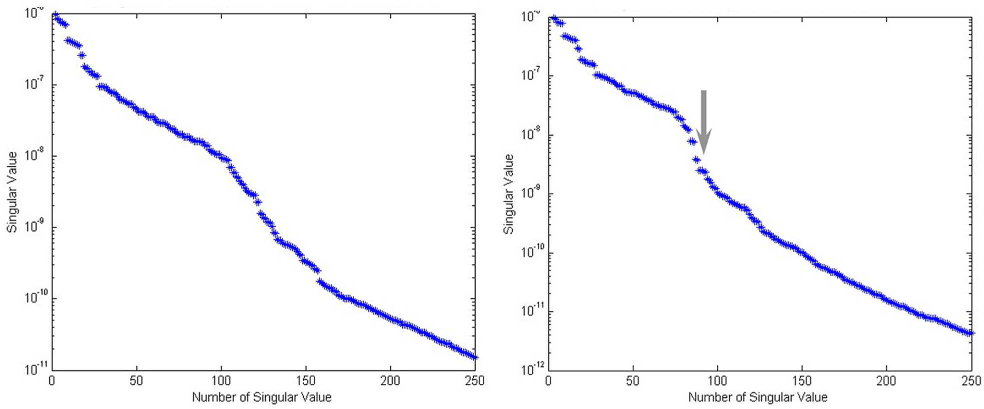


Fig. 12 (a) Singular value decomposition of Toeplitz matrix in SSI-COV with the reconstructed signal using SSA with 312 singular values and (b) stabilization diagram with 312 SV and system order 90



(a) (b)

Fig. 13 Plot the distribution of singular values from SSI-COV analysis: (a) using 136 SV in SSA to reconstructed data and (b) using 95 SV in SSA to reconstructed data

SV in SSA step, the change of slope in the SVD of Toeplitz matrix of SSI-COV analysis was a little more remarkable (as shown in Fig. 13(a)), this can be considered as approximately the first critical point), and the stabilization diagram was improved. If we continue to decrease the number of singular values in SSA, as shown in Fig. 13(b) in the case of using 95 SV in SSA, a critical point at order of $n = 90$ can be observed in performing the SVD of Toeplitz matrix of SSI-COV. This critical point gives the best identification on the determination of system order. Finally, two different sets of analysis were performed. First, 136 principal components (larger singular values) were extracted from SSA to reconstruct the response measurements, then the Toeplitz matrix was constructed based on the reconstructed data. The order of the system matrix $\boldsymbol{A}$ for this case is set to 120 and this value is fixed to construct the stabilization diagram (with increasing the size of the covariance matrix or Toeplitz matrix). The choice of 136 SV is more subjective because one has to try several times and plot SV in SSI-COV step to check if a change of slope appears.

In the case of Canton Tower, actually with 95 Singular Values the major peaks can be covered as compare to the result of FDD, but the peak corresponding to the first mode in FDD Spectrum is very fussy, therefore, one has to increase the number of components to be extracted in SSA so as to conserve the component corresponding to 1st mode. This means that the 1st mode is quite contaminated with noise (or with wind force frequency) and cannot be clearly identified. To find the frequency of the 1st mode, one have to increase the number of SV in SSA step, and also increase the system order in SSI-COV step. Fig. 14(a) shows the result using 95 singular values from SSA and with order of 90 in SSI, and Fig. 14(b) shows the result using 136 singular values from SSA and with order of 120 in SSI. The identified system natural frequencies are also shown in Table 1.

The major deficiency of using of 95 SV is the inability on the identification of the first mode.

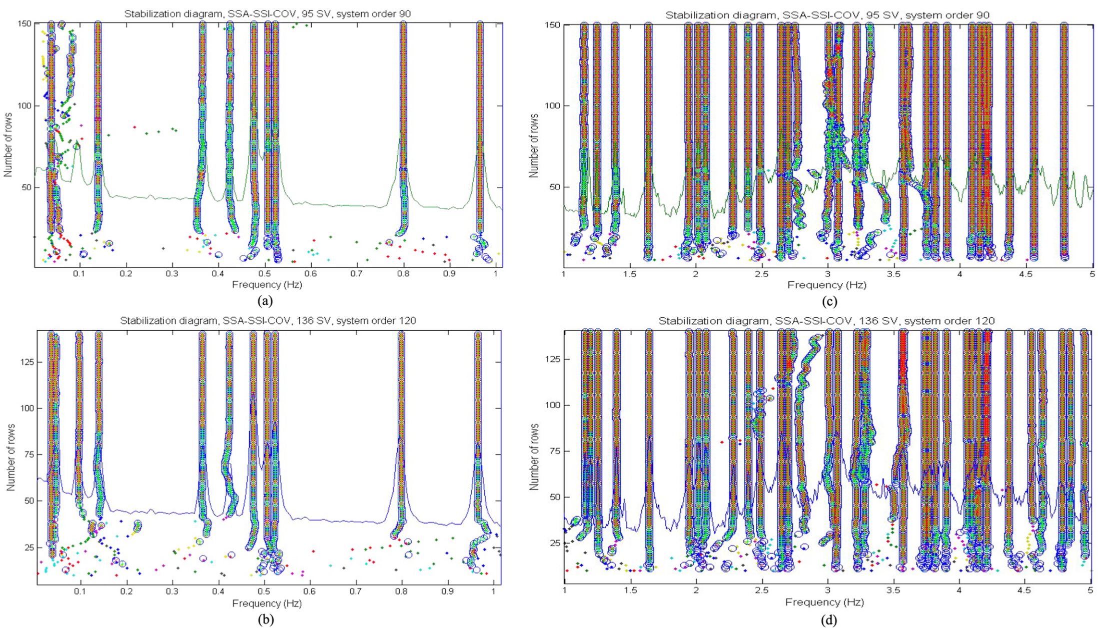


Fig. 14 Comparison on the stability diagram (between 0.0 Hz and 1.0 Hz) from SSA-SSI-COV result: (a) using 95 singular values from SSA and with order of 90 in SSI, (b) using 136 singular values from SSA and with order of 120 in SSI, (c) using 95 singular values from SSA and with order of 90 in SSI and (d) using 136 singular values from SSA and with order of 120 in SSI

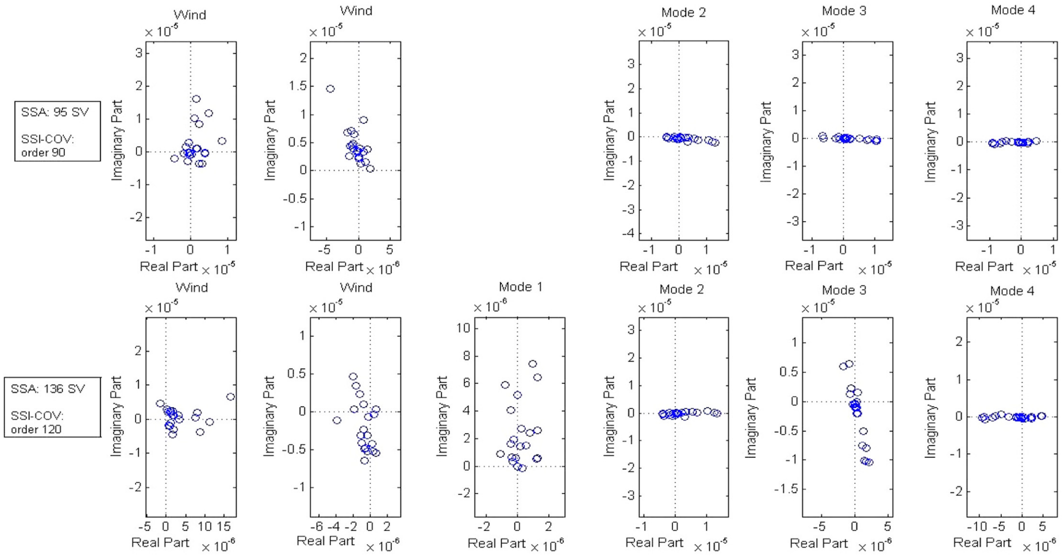


Fig. 15 Plot of the complex poles of the first 4 mode shapes identified by SSI-COV

This may due to the fact that this first mode is not well excited in addition to the interference of wind dominant frequency, or its corresponding signal component appears after using 95 SV and thus, was filtered out by SSA. Besides, the mode corresponding to approximately 1.2 Hz was filtered out by SSA. Table 1 also show the identified system natural frequency from using SSA-SSI-COV with 95 SV (90 order) and 136 SV. The modal parameter of all peaks shown in FDD spectrum was found by SSI-COV, the preliminary results of frequencies and damping ratios below 2 Hz are also shown in Table 1, The first two close frequencies: 0.0404 and 0.0409 Hz, which have damping ratio higher than the usual for civil structures, and bad correlation also exist in their mode shapes (as shown in Fig. 15 on the plot of poles in complex plane), hence, these two poles can be discarded. These first two identified poles may be the wind force excitation frequencies since the wind spectrum has its peak around 0.1 Hz.

7. Conclusions

The changes of features in a structural system may due to the change of environmental loading pattern, the nonlinear inelastic response of structure or structural damage when subjected to severe external loading. The detection of the change of features or damage in large structural system, such as buildings and bridges, can improve safety and reduce maintenance costs. Therefore, feature extraction and damage detection from vibration structures are the goals of SHM. In this study an output-only system identification technique for civil structures under ambient vibrations is carried out, mainly focused on the Subspace System Identification (SSI) based algorithms. To conduct the SSI-based algorithm a stabilization diagram is constructed by plotting the identified poles of the system meanwhile the model order increased. Through numerical simulation on the covariance-driven SSI, the following remarks are made:

(1) For SSI-COV it is better to assemble the Hankel data matrix with number of block rows as large

as possible, and the number of block columns will determine the available data point to construct the covariance matrix (Toeplitz matrix).
(2) If there is no sufficient block columns used to fix the noise in SVD, lower modes will not stabilize at all no matter how many rows are used. The required minimum number of block columns depends on the noise content of the signal.
(3) SSI-based algorithms can identify the equivalent linear system from nonlinear signals effectively, and the nonlinearity of signals do not interfere the stabilization diagram.
(4) SSI-COV technique is robust against the presence of measurement noise which is strongly depends on SVD. If the signal can be decomposed in more and more components through SVD, good separation between noise and system response can be achieved. Besides, larger number of block columns of the observability matrix is used, the larger the data point is provided to extract system matrices (***A***,***C***) in a least square sense.
(5) Comparative study between different approach, with and without using Singular Spectrum Analysis to pre-process the data, on determining the model order and selecting the true system poles is examined in this study. It is concluded that SSA can be used to reduce the noise effect from the concept of using principal components.

Identification task of the real large scale structure: Canton Tower, a benchmark problem for structural health monitoring of high-rise slender structures is carried out, from which the applicability of SSI-based algorithm is demonstrated. The use of SSA as a pre-processing tool for SSI-COV is a great help to improve the stabilization diagram and to extract rapidly the identifiable modes from measurements. In the case of using 95 SV from SSA, most of all modes are stabilizes starting from about the 25^{th} row or even earlier. On the contrary, by using SSI-COV without applying SSA for pre-processing the stabilization start approximately from the 100^{th} to 125^{th} row. These results are in significant savings of the computation time.

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