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Update the finite element model of Canton Tower based on direct matrix updating with incomplete modal data

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Abstract. In this paper, the structural health monitoring (SHM) benchmark problem of the Canton tower is studied. Based on the field monitoring data from the 20 accelerometers deployed on the tower, some modal frequencies and mode shapes at measured degrees of freedom of the tower are identified. Then, these identified incomplete modal data are used to update the reduced finite element (FE) model of the tower by a novel algorithm. The proposed algorithm avoids the problem of subjective selection of updated parameters and directly updates model stiffness matrix without model reduction or modal expansion approach. Only the eigenvalues and eigenvectors of the normal finite element models corresponding to the measured modes are needed in the computation procedures. The updated model not only possesses the measured modal frequencies and mode shapes but also preserves the modal frequencies and modes shapes in their normal values for the unobserved modes. Updating results including the natural frequencies and mode shapes are compared with the experimental ones to evaluate the proposed algorithm. Also, dynamic responses estimated from the updated FE model using remote senor locations are compared with the measurement ones to validate the convergence of the updated model.

Keywords: finite element model updating; direct updating method; incomplete measurement; benchmark problem; system identification; response estimation

1. Introduction

A sophisticated structural health monitoring system has been implemented by a research group of Professor Y.Q. Ni from the Hong Kong Polytechnic University (HKPU) for the 610 m high Canton tower (formerly named Guangzhou New TV Tower) (Ni *et al*. 2009, 2012, Chen *et al*. 2011, Niu *et al*. 2011, Ye 2011). An international SHM benchmark problem for high-rise structures has been developed by taking the instrumented Canton Tower as a host structure. Some tasks for this SHM benchmark problem have been proposed and Task I is to carry out identification with the output only field measurement data and model updating of the reduced finite element (FE) model of the Canton Tower with particular methods (Ni *et al*. 2012). A 3D full FE model is built based on the drawings of the tower and then a reduced FE model which can also be downloaded is generated

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from the full FE model. The main purpose is to find a reduced model with enough accuracy for future tasks.

FE model updating is the process of tuning the FE model so that the updated FE model gives a better reflection of the measured data. It is important for its application in structural analysis, structural control, damage detection and health monitoring. This topic has been well studied during the past several decades and many literatures including a well known technical book have been devoted to the subject (Mottershead and Friswell 1993, Friswell and Mottershead 1995, Link 1999).

The conventional FE model updating methods can be broadly classified into two categories: sensitivity-based parameterization methods and the direct updating methods (Friswell and Mottershead 1995, Link 1999, Weng 2010). In the sensitivity-based parameterization methods, parametric models of the structures are required. The methods allow a wide choice of physically meaningful parameters, preserve the matrix properties of symmetry, sparseness and positive-definiteness, and guarantee the structural connectivity. The aim is to update the physical parameters of the FE model, such as material properties, element stiffness or physical dimensions of the model; therefore the method have the advantage of identifying parameters that can directly affect the dynamic characteristics of the structure. Due to these merits, such FE updating methods have been becoming more popularly used (Yu and Lan 2012, Cheng *et al*. 2009, Jaishi and Ren 2005, Fritzen *et al*. 1998, Moller and Friberg 1998). However, selection of parameters to be updated is very critical to a successful model updating and requires engineering judgments (Friswell *et al*. 2001, Ahmadian *et al*. 1997, Zhang *et al*. 2000). For updating FE model of large-size structures, if the requested updating parameters are not completely selected, the updated model does not represent the real structure and cannot reproduce the required dynamic properties accurately. On the contrary, more selected updating parameters would not only needs expensive computation of eigenvalues and the associated sensitivities but also leads to the problem of ill-condition or spillover (Brownjohn 2001). Besides, it is necessary to select the most effective updating parameters that produce a genuine improvement in the modeling of the structures. In some cases model updating depends on the users' experiences so the model updating results are subjective (Friswell *et al*. 1998, Yuen 2010, Yuen 2012, Carvalho *et al*. 2007).

The direct model updating methods try to find the updated matrices (stiffness and/or mass) which produce the real responses of a structure as closely as possible. In these methods, the elements in the system matrices are treated as variables. Although this kind of methods do not generally maintain structural connectivity and the corrections suggested may not always physically meaningful, the direct updating methods have been used extensively for FE model refinement and damage detection (Mottershead and Friswell 1993, Friswell and Mottershead 1995, Link 1999), e.g., Yuen (2010, 2012) and Carvalho *et al*. (2007) proposed direct updating methods using incomplete measured modal data.

The reduced FE model of the Canton tower is simplified from its full 3D FE model with certain assumptions. It is difficult and subjective to select physically meaningful parameters for model updating of such a large size structure. Therefore, FE model of Canton tower is updated by an algorithm in the category of direct matrix updating in this paper. The proposed algorithm is based on the theorem of eigen-structure preserving updating matrix proposed by Carvalho *et. al.* (2007), but it does not require modal expansion approach. The algorithm minimizes the modal differences between the measurements and the updated FE model and preserves the modal properties of the unobserved modes.

The paper is structured as follows. In the next section, the formulation of the FE Model of the Canton tower, field vibration measurement and modal parameter identification, and the comparisons of modal parameters from measured data and those from the reduced-order FE model are described. The proposed direct matrix updating algorithm for updating FE model is presented in Section 3. The results of the model updating of the Canton tower are verified in Section 4. Finally, some conclusions are given in Section 5.

2. SHM benchmark problem of the Canton Tower

The Canton Tower located in the city of Guangzhou, China, is a super-tall structure of 610 m high. It consists of a 454 m high main tower and a 156 m high antenna mast, as shown by Fig. 1 (Ni *et al*. 2012). With completion of structural construction in May 2009, the Canton Tower was open for operation during the 2010 Asia Games. A sophisticated structural health monitoring system has been designed and implemented by HKPU for both in-construction and in-service monitoring (Ni *et al*. 2009, 2012, Chen *et al*. 2011). Monitoring data are exceedingly useful for detecting anomalies in loading and response and assessing the structural performances. Therefore, an SHM benchmark problem for high-rise structures has been developed by taking the instrumented Canton Tower as a host structure. This benchmark problem provides an international platform for efficient comparisons of various SHM-related methodologies and algorithms with the use of real-world monitoring data from such a large-scale structure, and to narrow the gap currently existing between the research and the practice of SHM.

Some tasks for this SHM benchmark problem have been proposed. Task I is to carry out

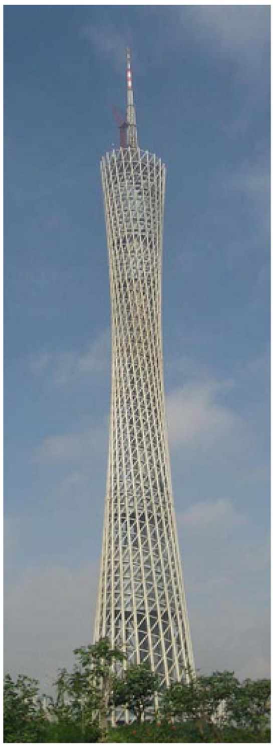
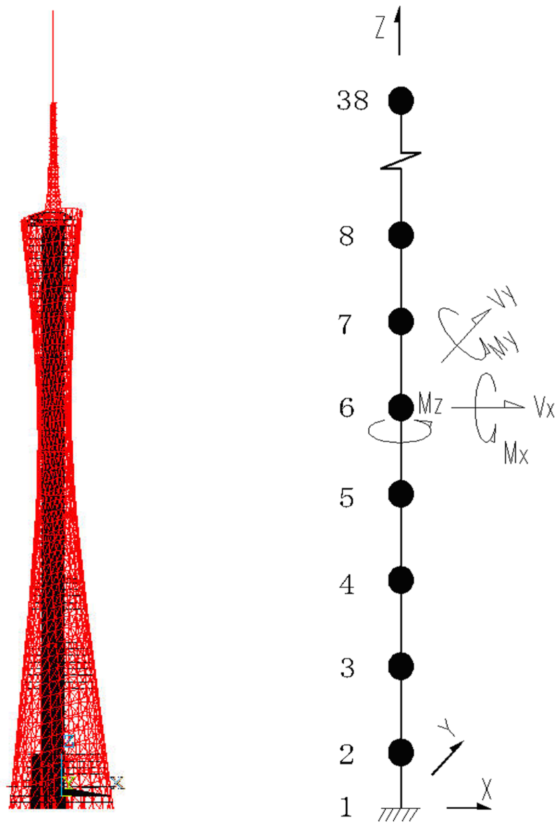


Fig. 1 The Canton Tower

identification with the output only field measurement data and finite element model updating of the Canton Tower. The main purpose is to find a reduced model with enough accuracy for future tasks (Ni *et al*. 2012).

2.1 Finite element model of the Canton Tower

A 3D full FE model shown in Fig. 2(a) was established by HKPU (Ni *et al*. 2009, 2012). The full FE model contains 122,476 elements, 84,370 nodes, and 505,164 degrees of freedom in total. However, such a full-order model is too complicated for SHM and related studies. Therefore, a reduced model was also established based on this full model by HKPU with the following assumptions: (1) the floor systems are assumed as rigid body, (2) each segment between two adjacent floors is modeled as a linear elastic beam element, and (3) the masses are lumped at the corresponding floors (Ni *et al*. 2012). Consequently, the whole structure is modeled as a cantilever beam with 37 beam elements and 38 nodes as shown by Fig. 2(b). Due to model simplification, the dynamic properties of the reduced model inevitably differ from those of the full model. By using sensitivity based model updating method, the reduced model was fine tuned so that its dynamic characteristics match those of the full model as closely as possible (Ni *et al*. 2012).



(a) (b)

Fig. 2 The FE model of the Canton Tower: (a) Full-order 3D FE Model and (b) reduce-order FE Model

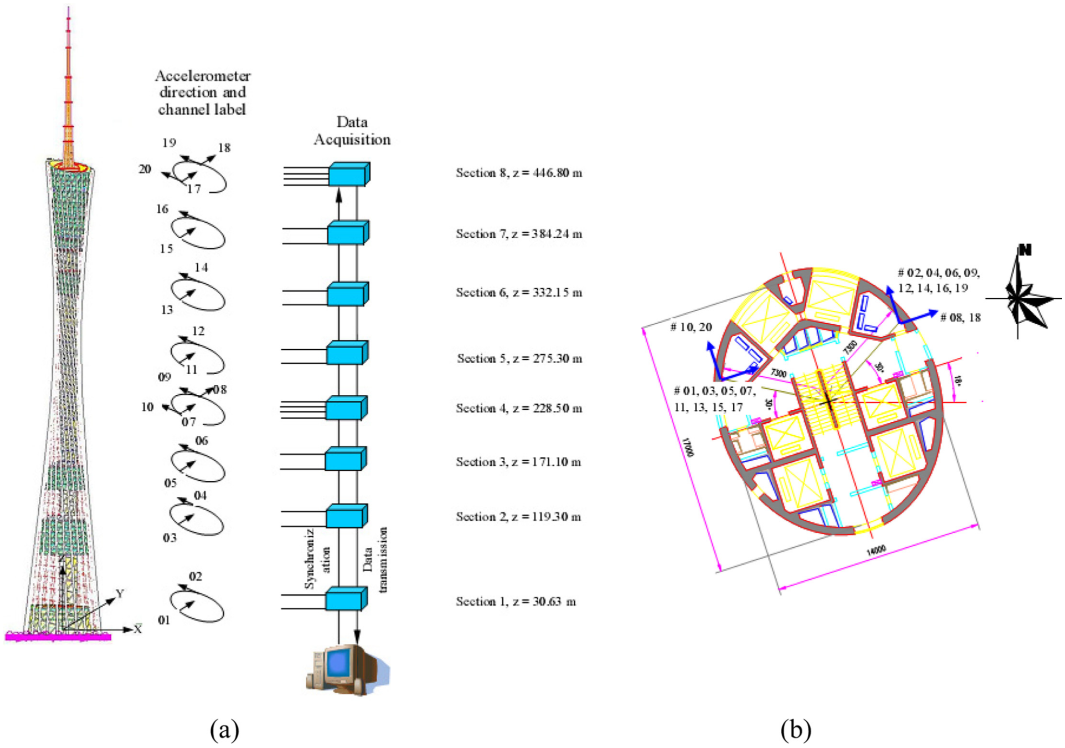


Fig. 3 Deployment of accelerometers on the Canton tower: (a) Position of accelerometers and (b) direction of measured accelerations and channel labels

2.2 Field vibration measurement and modal parameter identification

A sophisticated SHM system consisting of over 700 sensors of sixteen types has been designed and implemented by HKPU to monitor the tower in both construction and service stages (Ni *et al*. 2012). In order to obtain the tower's dynamic responses, 20 uni-axial accelerometers were deployed at 8 sections along the tower as shown in Fig. 3(a). Four uni-axial accelerometers were installed at section 4 and 8, two for measurement horizontal vibrations along the long-axis of the inner tube and the other two for the short-axis of the inner tube. At other six levels, each section is equipped with two uni-axial accelerometers, one for the long-axis of the inner tube and the other for the short-axis of the inner tube. From the plan position of accelerometers and direction of acceleration shown in Fig. 3(b), it is noted that the directions of uni-axial accelerometers has 18^{o} with the detections of the coordinate system for establishing the reduced model.

A total of 24-hour ambient vibration measurement data from the 20 uni-axial accelerometers are used for the modal parameter identification of the Canton tower. The measurement data are provided by the HKPU at the link "http://www.cse.polyu.edu.hk/benchmark/index.htm". Before the identification is performed, the measured data are first de-trended, which enables the removal of the DC-components that can badly influence modal identification results. Since the Canton tower is such a super-tall structures with the fist dozens of vibration frequencies below 1 Hz, pre-processing of measured data by re-sampling and filtering from 50 Hz to 2.5 Hz are performed to reduce the amount of data and filter out the influence of high frequency measurement noise.

Currently, the two popular methods for modal parameter identification are the Enhanced Frequency Domain Decomposition (EFDD) method in the frequency-domain (Brincker *et al*. 2001) and the Stochastic Subspace Identification (SSI) method in the time-domain (Overschee and Moor 1996). In this paper, EFDD is applied to identify the modal parameters of the Canton Tower based on the ambient vibration measurement data.

In the EFDD method, the relationship between the unknown input *x(t)* and the measured responses *y(t)* can be expressed as

$$\boldsymbol{G}_{yy}(\omega) = \boldsymbol{H}^{*}(\omega)\boldsymbol{G}_{xx}(\omega)\boldsymbol{H}^{T}(\omega) \tag{1}$$

in which, $\boldsymbol{G}_{xx}$ is the input power spectral density (PSD) matrix, $\boldsymbol{G}_{yy}$ is the output PSD matrix, $\boldsymbol{H}(\omega)$ is frequency response function (FRF), and superscripts "*" and "T" denote the complex conjugate and transpose, respectively. The FRF can be expressed as

$$\boldsymbol{H}(\omega) = \sum_{r=1}^{n}\left(\frac{\boldsymbol{R}_r}{i\omega-\lambda_r}+\frac{\boldsymbol{R}_r^{*}}{i\omega-\lambda_r^{*}}\right) \tag{2}$$

where n is the number of modes, λ_r is the pole and residue $\boldsymbol{R}_r = \boldsymbol{\phi}_r\gamma_r$, $\boldsymbol{\phi}_r$ is the r-th mode shape and γ_r is the contributed vector of the shape.

Assume the input is white noise with constant PSD matrix, i.e., $\boldsymbol{G}_{xx}(\omega) = \boldsymbol{C}$, the output PSD matrix $\boldsymbol{G}_{yy}$ can be obtained as

$$\boldsymbol{G}_{yy}(\omega) = \sum_{r=1}^{n}\left(\frac{\boldsymbol{A}_r}{i\omega-\lambda_r}+\frac{\boldsymbol{A}_r^{*}}{i\omega-\lambda_r^{*}}+\frac{\boldsymbol{A}_r}{-i\omega-\lambda_r}+\frac{\boldsymbol{A}_r^{*}}{-i\omega-\lambda_r^{*}}\right) \tag{3}$$

in which residue $\boldsymbol{A}_r$ is

$$\boldsymbol{A}_r = \sum_{k=1}^{n}\left(\frac{\boldsymbol{R}_k^{*}\boldsymbol{C}\boldsymbol{R}_r^{T}}{-\lambda_r-\lambda_k^{*}}+\frac{\boldsymbol{R}_k\boldsymbol{C}\boldsymbol{R}_r^{T}}{-\lambda_r-\lambda_k^{*}}\right) \tag{4}$$

In the case of light damping, the residue becomes proportional to the mode shape vector, i.e, $\boldsymbol{A}_r \approx d_r\boldsymbol{\phi}_r^{*}\boldsymbol{\phi}_r^{T}$ where d_r is a scalar constant. Therefore, near the modal frequency ω_r, Eq. (3) can be rewritten by ignoring the last two items as

$$\boldsymbol{G}_{yy}(\omega) \approx \sum_{r=1}^{n}\left(\frac{d_r\boldsymbol{\phi}_r^{*}\boldsymbol{\phi}_r^{T}}{i\omega-\lambda_r}+\frac{d_r\boldsymbol{\phi}_r^{*}\boldsymbol{\phi}_r^{T}}{-i\omega-\lambda_k^{*}}\right) \tag{5}$$

On the other hand, the estimate of the output PSD $\boldsymbol{G}_{yy}(\omega)$ known at discrete frequencies $\omega = \omega_i$ can be decomposed by taking the Singular Value Decomposition (SVD) of the matrix as

$$\boldsymbol{G}_{yy}(\omega) = \boldsymbol{U}(\omega)\boldsymbol{S}(\omega)\boldsymbol{U}^{T}(\omega) \tag{6}$$

where $\boldsymbol{S}(\omega)$ is a diagonal matrix holding the singular values and $\boldsymbol{U}(\omega)$ is a unitary matrix containing the singular vectors. It is clear that Eqs. (5) and (6) have the same form. Thus, the natural frequencies can be evaluated from the peaks of the 1st singular values plotted versus frequency while the corresponding 1st singular vectors at these frequencies are the associated mode shapes.

Table 1 Comparisons of modal properties between FE model and identification

Mode and Type	Frequency (Hz)					MAC	
	Experiment	FE Model	Error (%)	Updated FE Model	Error (%)	FE Model	Updated FE Model
1 (BY)	0.094	0.110	17.0	0.094	0.02	0.89	0.97
2 (BX)	0.138	0.159	15.2	0.138	0.08	0.94	0.98
3 (BX)	0.366	0.346	5.5	0.364	0.33	0.87	0.98
4 (BY)	0.424	0.370	12.9	0.396	6.90	0.89	0.99
5 (BY)	0.475	0.397	16.4	0.459	2.95	0.86	0.98
6 (T)	0.505	0.461	8.9	0.513	1.58	-	-
7 (BX)	0.522	0.485	7.1	0.522	0.63	0.78	0.96
8 (BX)	0.795	0.738	7.1	0.793	0.84	0.76	0.91
9 (BY)	0.964	0.903	6.3	0.963	1.17	0.75	0.89
10 (BX)	1.150	0.997	13.3	1.112	3.3	0.70	0.90
11 (BX)	1.191	1.037	12.9	1.131	5.0	0.82	0.88
12 (T)	1.250	1.122	10.3	1.248	0.16	-	-
Average			11.1		1.9	0.87	0.94

Mode Type: *BX*, *BY*: bending mode in *X* and *Y* directions, respectively; *T*: torsional mode

2.3 Comparisons of modal parameters

Based on the reduced FE model of the Canton tower, the natural vibration frequencies and mode shapes of the tower can be calculated. The first 13 natural frequencies are listed in the 3rd column in Table 1. From the vibration mode shapes, it is noted that the 6th and 12th modes are the torsion modes of the tower.

Identification of modal parameters with the measurement data of the Canton Tower is conducted by the EFDD method. Utilizing the 24 hours ambient vibration data measured by the 20 uni-axial accelerometers and the procedures in section 2.2, the natural frequencies and modes shapes are identified. Identification results of the first 13 frequencies are listed in the 2nd column in Table 1 for comparison. It is noted that natural frequencies estimated from FE model have discrepancies with the corresponding identification values. The maximum difference is near 17%, which is quite large.

For the comparisons of mode shapes, the modal assurance criteria (MAC) values between the identification mode shapes and the FE analysis are calculated by

$$MAC(i) = \frac{(\phi_i^T \hat{\phi}_i)^2}{(\phi_i^T \phi_i)(\hat{\phi}_i^T \hat{\phi}_i)} \tag{7}$$

in which ϕ_i and $\hat{\phi}_i$ denote the numerical and experimental mode shape vector at the measured DOFs, respectively. Since only section 4 and 8 are deployed with four uni-axial accelerometers for measuring rotational vibration, the torsion modes of the tower can not be fully identified. By comparing the estimated mode shapes from the reduced FE model and the corresponding mode shapes identified from the measurement data, it is found the average of MAC value is 0.87.

From the above comparisons of model parameters, it is noted that the reduced FE model needs to be updated further to reduce its discrepancies of model parameters with the corresponding experimental identification values.

3. A direct matrix updating algorithm for updating FE model

Since the reduced FE model of the Canton tower is simplified from its full 3D FE model, it's difficult and subjective to select physically meaningful parameters for updating the reduced FE model. Therefore, an algorithm in the category of direct matrix updating is proposed for updating the reduced FE model of the Canton tower using incomplete measured modal data.

Assume that the identified experimental natural frequencies $\hat{\lambda}_1, \hat{\lambda}_2, \ldots, \hat{\lambda}_m$ and the responding incomplete measured mode shape matrix $\hat{\boldsymbol{\Phi}} = (\hat{\boldsymbol{\phi}}_1, \hat{\boldsymbol{\phi}}_2, \ldots, \hat{\boldsymbol{\phi}}_m)$ are used for model updating, in which $\hat{\boldsymbol{\Phi}}$ has a dimension of $p \times m$ with p measured DOFs. $\boldsymbol{M}$ and $\boldsymbol{K}$ are the mass and stiffness matrices of the reduced FE model with n degrees of freedom (DOFs), respectively. The matrices of eigenvalues and eigenvectors of the FE model are denoted by $\boldsymbol{\Lambda}$ and $\boldsymbol{\Phi}$, respectively. These matrices can be partitioned as

$$\boldsymbol{\Lambda} = \begin{pmatrix} \boldsymbol{\Lambda}_1 & \\ & \boldsymbol{\Lambda}_2 \end{pmatrix} \; ; \; \boldsymbol{\Phi} = (\boldsymbol{\Phi}_1 \quad \boldsymbol{\Phi}_2) \tag{8}$$

where ; $\boldsymbol{\Lambda}_1 = diag(\lambda_1, \lambda_2, \ldots, \lambda_m)$; $\boldsymbol{\Lambda}_2 = diag(\lambda_{m+1}, \lambda_{m+2}, \ldots, \lambda_n)$; $\boldsymbol{\Phi}_1 = (\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, \ldots, \boldsymbol{\phi}_m)$; $\boldsymbol{\Phi}_2 = (\boldsymbol{\phi}_{m+1}, \boldsymbol{\phi}_{m+2}, \ldots, \boldsymbol{\phi}_n)$

Then, these eigenvalues and eigenvectors satisfy the eigenvalue equations as

$$\boldsymbol{K\Phi}_1 = \boldsymbol{\Lambda}_1 \boldsymbol{M\Phi}_1 \; ; \; \boldsymbol{K\Phi}_2 = \boldsymbol{\Lambda}_2 \boldsymbol{M\Phi}_2 \tag{9a}$$

and the orthogonality relations as

$$\boldsymbol{\Phi}_1^T \boldsymbol{M\Phi}_2 = 0 \; ; \; \boldsymbol{\Phi}_1^T \boldsymbol{K\Phi}_2 = 0 \tag{9b}$$

The mass matrix $\boldsymbol{M}$ is not updated, only the stiffness matrix $\boldsymbol{K}$ is updated by the following updating formula [23]

$$\boldsymbol{K}^* = \boldsymbol{K} + \boldsymbol{M\Phi}_1 \boldsymbol{\Psi} \boldsymbol{\Phi}_1^T \boldsymbol{M} \tag{10}$$

in which $\boldsymbol{\Psi}$ is an unknown parametric matrix with dimension $m \times m$ with m being the number of identified mode shapes. Usually, only limited numbers of modes can be fully identified from ambient vibration data, therefore the size of the unknown parameters in matrix $\boldsymbol{\Psi}$ is not large.

The motivation for direct updating the stiffness matrix by the formula in Eq. (10) is that such a updated model satisfies the following eigenvalue equation utilizing the relations in Eqs. (9(a) and (b))

$$\boldsymbol{K}^* \boldsymbol{\Phi}_2 = \boldsymbol{K\Phi}_2 + \boldsymbol{M\Phi}_1 \boldsymbol{\Psi} \boldsymbol{\Phi}_1^T \boldsymbol{M\Phi}_2 = \boldsymbol{K\Phi}_2 = \boldsymbol{\Lambda}_2 \boldsymbol{M\Phi}_2 \tag{11}$$

i.e., the eigenvalues and eigenvectors corresponding to the unmeasured ones remain unchanged by such an updating approach (Carvalho *et al*. 2007).

For the estimation of unknown parametric matrix $\boldsymbol{\Psi}$, Carvalho *et al*. (2007) proposed an algorithm to first estimate the unknown part of the incomplete measured mode shapes $\hat{\boldsymbol{\Phi}}$ with mode shape expansion and then calculate the matrix $\boldsymbol{\Psi}$. In this paper, no model expansion technique is required. The unknown parameters in the matrix $\boldsymbol{\Psi}$ are determined by minimizing the following objective function defined as

$$J = \boldsymbol{R}^T \boldsymbol{W} \boldsymbol{R} \tag{12}$$

where $\boldsymbol{W}$ is a diagonal weighting matrix, $\boldsymbol{R}$ is the residual vector defined as

$$\boldsymbol{R} = [\boldsymbol{R}_f \quad \boldsymbol{R}_\phi]^T \tag{13}$$

and

$$\boldsymbol{R}_f(i) = \frac{|\lambda_i^* - \hat{\lambda}_i|}{\hat{\lambda}_i} \ ; \ \boldsymbol{R}_\phi(i) = 1 - MAC(i) = 1 - \frac{(\boldsymbol{\phi}_i^{*T}\hat{\boldsymbol{\phi}}_i)^2}{(\boldsymbol{\phi}_i^{*T}\boldsymbol{\phi}_i^*)(\hat{\boldsymbol{\phi}}_i^T\hat{\boldsymbol{\phi}}_i)} \ ; (i=1,\ 2, \ldots,\ m) \tag{14}$$

λ_i^* and $\boldsymbol{\phi}_i^*$ are the *i-th* eigenvalue and eigenvector of the updated FE model.

Thus, the objective function consists of the residuals of frequencies and mode shapes between the experimental modal identification values and those of the updated model where relative important between the residuals is represented by the weighting matrix $\boldsymbol{W}$. Therefore, unknown parameters in the matrix $\boldsymbol{\Psi}$ are determined by the minimizing the modal properties differences between the measurements and the updated FE model.

To minimize the objective function defined in Eq. (12), the function *fmincon* with sequential quadratic programming algorithm within the Optimization Toolbox of MATLAB is used. This is one of the most widely used algorithms for nonlinear optimization problem.

The direct matrix updating algorithm for updating FE model can solve the problem of incomplete modal measurement in practice as the mode shapes are not required to be measured at all degrees of freedom. Moreover, it does not need any model reduction or modal expansion technique. It minimizes the modal differences between the measurements and the FE model and preserves the modal properties of the unobserved modes.

4. Updating the FE model of the Canton Tower

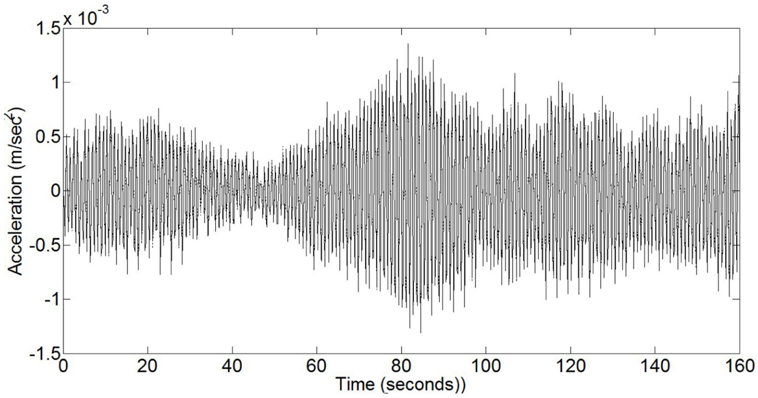
The reduced FE model of the Canton tower is a cantilever beam with 37 beam elements and 38 nodes as shown by Fig. 2(b). Each node has 5 DOFs, i.e., two horizontal translational DOFs and three rotational DOFs. As a result, each element has 10 DOFs and the entire model has 185 DOFs in total. Then, based on the proposed direct matrix updating algorithm, the reduced FE model is updated using the experimental identified natural frequencies and incomplete measured mode shapes, but the rotational modes are excluded for FE model updating.

In the model updating algorithm for the FE model of Canton tower, n=185, m=10 and p=16, so the number of unknown parameters in matrix $\boldsymbol{\Psi}$ is: $(m+1)\times m/2=55$. It is well known that natural frequencies can be identified much more accurately than mode shapes, therefore weighting matrix $\boldsymbol{W}$ is a diagonal matrix and the weight coefficients are selected as 10 for all frequency differences, 2.0 for the first six bending mode shapes, 0.5 for the other four higher bending mode shapes.

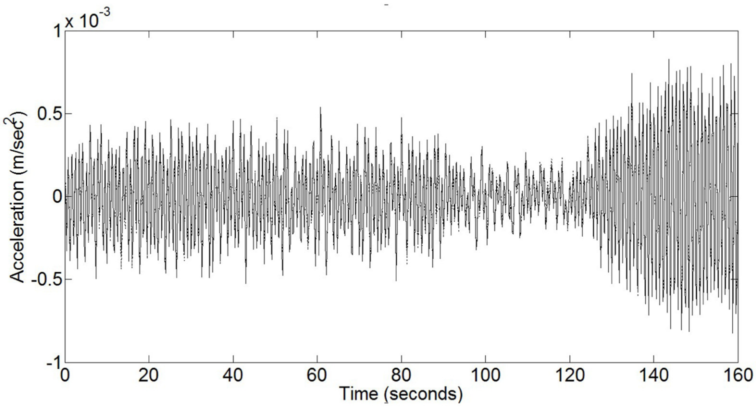
The model updating for the FE model is computational efficient with limited CPU time consuming as it is direct matrix updating algorithm. It only needs a small number of eigenvalues and eigenvectors associated with the experimental observed modal data but does not require the computation of the complete set of the eigenvalues and eigenvectors of the FE model. This is important especially for updating large structures such as the Canton tower.

The first 13 natural frequencies from the updated FE model of the tower are shown in the 5th column in Table 1. From the comparisons of errors in frequencies shown in 4th and 6th columns, respectively, it is noted that that the average error of frequencies is reduced significantly from 11.1% before model updating to 1.9% after model updating. Also, the mode shapes from the updated FE model are also compared with observed experimental results by MAC values as shown in the 8th column in Table 1. It is also noted that the MAC values also have been improved after updating with the average value increases from 0.87 to 0.94. Therefore, modal parameters estimated from the updated model are more compatible with the measurement modal data.

To further check the accuracy of the updated FE model of the tower by the proposed algorithm, the convergence of the updated FE model is verified through the co-validation between different sensor measurements. Kammer (1997) proposed a method for estimating structural response using remote sensor locations. Accelerometers were used as sensor throughout the analysis. Two approaches were considered. The first requested as many sensors as there are modes responding in the data. The second approach required as many sensors as the number of input locations. Since experimental measurement data were recorded from ambient vibration, in which the number input force locations are much larger than the number of sensors, the first approach is applied herein. Assume that $\boldsymbol{y}$ represents the measured acceleration response. The all 20 measurement responses are separated into sensor and desired location partitions, represented by $\boldsymbol{y}_s$ and $\boldsymbol{y}_d$, respectively. Here, $\boldsymbol{y}_d$



(a)



(b)

Fig. 4 Comparisons of acceleration responses at desired location: (a) Comparison of acceleration responses at Sensor No. 9 and (b) comparison of acceleration responses at Sensor No. 12

Table 2 Comparisons of estimation error

Desired locations	$\|y_d\|$ (10^{-4})	$\|\hat{y}_d\|$ (10^{-4}) (FE Model)	E_d (FE Model)	$\|\hat{y}_d\|$ (10^{-4}) (Updated FE Model)	E_d (Updated FE Model)
No.9	3.34	2.74	17.8%	0.31	9.15%
No.12	2.76	3.02	9.3%	0.14	5.0%

is randomly selected. Based on mode superposition, acceleration responses at sensor location can be estimated by

$$\boldsymbol{y}_s = \chi_s \ddot{\boldsymbol{q}} \tag{15}$$

in which χ_s is the matrix of mode shape that respond in the output at the sensor locations and $\ddot{\boldsymbol{q}}$ is the vector of corresponding modal accelerations.

Assume that the first 10 modes are excited by ambient excitation. The number of measurement sensors is greater that the number of modes assumed in the response data, so the acceleration responses at desired location can be estimated by a least-squares approach as

$$\hat{\boldsymbol{y}}_d = \chi_d [\chi_s^T \chi_s]^{-1} \boldsymbol{y}_s \tag{16}$$

where $\hat{\boldsymbol{y}}_d$ represents the estimated value of the acceleration responses at desired locations. The convergence of the updated FE model is evaluated by comparing the difference between $\hat{\boldsymbol{y}}_d$ and $\boldsymbol{y}_d$, denoted by E_d as

$$\boldsymbol{E}_d = \frac{\|\hat{\boldsymbol{y}}_d - \boldsymbol{y}_d\|}{\|\boldsymbol{y}_d\|} \tag{17}$$

where $\|\cdot\|$ is defined as the root mean square value of a vector.

In Fig. 4, the estimated value of the acceleration responses from the updated model at section 5 in Fig. 2 along the long axis of the inner tube (measurement sensor No. 9) are plotted as dotted curve for a time segment of 160 seconds (data recorded from 01:20:40 am on Jan. 19, 2010) and compared with the corresponding measured data denoted by solid curve. Analogously, comparisons of the acceleration responses at section 6 in Fig. 2 along the long axis of the inner tube (measurement sensor No. 12) are also shown in Fig. 4(b) for a time segment of 160 seconds (data recorded from 01:24:40 am on Jan. 19, 2010). Table 2 shows the comparisons of estimation error for the acceleration responses from the FE models before and after model updating for a period of 2000 seconds. From the above comparisons, convergence of the updated FE model of the Canton tower by the proposed algorithm is verified.

5. Conclusions

In this paper, the SHM benchmark problem of the Canton tower is studied. With the output only field measurement data from the 20 accelerometers deployed on the tower, some modal frequencies and mode shapes at measured degrees of freedom of the tower are identified by the EFDD algorithm. The identified natural frequencies of the first 13 modes are found to differ from those of the reduced FE model with 11.05% error in average, which indicates the need for updating the reduced FE model. An algorithm based on direct matrix updating is proposed for updating the reduced FE model using incomplete identified modal data. The algorithm can avoid the problem of subjective selection of updated parameters and directly updates model stiffness matrix without model reduction or modal expansion approach. It is computational efficient as it only needs a small number of eigenvalues and eigenvectors associated with the experimental observed modal data but does not require the computation of the complete set of the eigenvalues and eigenvectors of the FE model. The updated model not only possesses the measured modal frequencies and mode shapes but also preserves the modal frequencies and modes shapes in their normal values for the unobserved modes.

The updated FE model of the Canton tower has been evaluated by the reduction of the discrepancy in modal properties between the updated FE model and the measured counterparts when compared with the original FE model. Also, convergence of the updated FE model has been validated through the co-validation between different sensor measurements.

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References

Ahmadian, H., Gladwell, G.M.L. and Ismail, F. (1997), "Parameter selection strategies in finite element model updating", *J. Vib. Acoust*., **119**(1), 37-45.

Brincker, R., Zhang L.M. and Andersen P. (2001), "Modal identification of output-only systems using frequency domain decomposition", *Smart Mater. Struct*., **10**(3), 441-445.

Brownjohn, J.M.W., Xia, P.Q., Hao, H. and Xia, Y., (2001), "Civil structure condition assessment by FE model updating: methodology and case studies", *Finit. Elem .Anal. Des*., **37**(10), 761-775.

Carvalho, J., Biswa N., Datta, B.N., Gupta, A. and Lagadapati, M. (2007), "A direct method for model updating with incomplete measured data and without spurious modes", *Mech. Syst. Signal Pr*., **21**(7), 2715-2731.

Chen, W.H., Lu, Z.R., Lin, W., Chen, S.H., Ni, Y.Q., Y., Xia, Y. and Laio, W.Y. (2011), "Theoretical and experimental modal analysis of the Guangzhou New TV Tower", *Eng. Struct*., **33**(12), 3628-3646.

Cheng, L., Xie, H.C. and Spencer, B.F. Jr. (2009), "Optimized finite element model updating method for damage detection using limited sensor information", *Smart Struct. Syst*., **5**(6), 681-697.

Friswell, M.I. (2001), "Finite-element model updating using experimental test data: Parametrization and regularization", Philos. T. R. Soc. A., Mathematical, **359**(1778), 169-186.

Friswell, M.I. and Mottershead, J.E. (1995), *Finite element model updating in structural dynamics*, Kluwer Academic Publishers.

Friswell, M.I., Inman, D.J. and Pilkey D.F. (1998), "The direct updating damping and stiffness matrices", *AIAA J*., **36**(3), 4591-493.

Fritzen, C.P., Jannewein, D. and Kiefer, T. (1998), "Damage detection based on model updating methods", *Mech. Syst. Signal Pr*., **12**, 163-186.

Jaishi, B. and Ren,W.X. (2005), "Structural finite element model updating using ambient vibration test results", *J. Struct. Eng. - ASCE*, **131**(4), 617-628.

Link, M. (1999), "Updating of analytical models-review of numerical procedures and application aspects", *Proceedings of the Structural Dynamics forum SD2000*, Los Alamos.
Kammer, D.C. (1997), "Estimation of structural response using remote sensor locations", *J. Guid.Control Dynam*., **20**(3), 501-508.
Moller, P.W. and Friberg, O. (1998), "Updating large finite element models in structural dynamics", *AIAA J*., **36**(10), 1861-1868.
Mottershead, J.E. and Friswell M.I. (1993), "Model updating in structural dynamics: a survey", *J. Sound Vib*., **167**(2), 347-375.
Ni, Y.Q., Xia, Y., Liao, W.Y. and Ko, J.M. (2009), "Technology innovation in developing the structural health monitoring system for Guangzhou New TV Tower", *Struct.Control Health Monit*., **16**(1), 73-98.
Ni, Y.Q., Xia, Y., Lin, W., Chen, W.H. and Ko, J.M. (2012), "SHM benchmark for high-rise structures: a reduced-order finite element model and field measurement data", *Smart Struct. Syst*., in this issue.
Niu, Y., Kraemer, P. and Fritzen, C.P. (2011), "Operational modal analysis for the Guangzhou New TV Tower", *Proceedings of the Society for Experimental Mechanics Series*, **7**, 211-220.
Overschee, V.P. and De Moor, B. (1996), *Subspace Identification for Linear Systems: Theory-implementation-applications*, Kluwer Academic Publishers, Dordrecht, The Netherlands.
Weng, S. (2010), *A New Substructuring Method for Model Updating of Large-scale Structures*, Ph.D. Thesis, Department of Civil and Structural Engineering, The Hong Kong Polytechnic University, Hong Kong.
Yu, E. and Lan, C. (2012), "Seismic damage detection of a reinforced concrete structure by finite element model updating", *Smart Struct. Syst*., **9**(3), 253-271.
Ye, J.H. (2011), *Finite Element Model Simplification and Updating of Guangzhou New TV Tower*, Master Thesis, Sun Yat-Sen University, China.
Yuen, K.V. (2010), "Efficient model correction method with model measurement", *J. Eng. Mech. - ASCE*, **136**(1), 91-99.
Zhang, Q.W., Chang, C.C. and Chang, T.Y.P. (2000), "Finite element model updating for structures with parametric constraints", *Earthq. Eng. Struct. D*., **29**(7), 927-944.