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Energy-efficiency enhancement and displacement-offset elimination for hybrid vibration control

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Abstract. New insights into our previously proposed hybrid-type method for vibration control are highlighted in terms of energy analysis, such as the assessment of energy efficiency and system stability. The hybrid method improves the bang-bang active method by combining it with an energy-recycling approach. Its simple configuration and low energy-consumption property are quite suitable especially for isolated structures whose energy sources are strictly limited. The harmful influence of the external voltage is assessed, as well as its beneficial performance. We show a new chattering prevention approach that both harvests electrical energy from piezoelectric actuators and eliminates the displacement-offset of the equilibrium point of structures. The amount of energy consumption of the hybrid system is assessed qualitatively and is compared with other control systems. Experiments and numerical simulations conducted on a 10-bay truss can provide a thorough energy-efficiency evaluation of the hybrid suppression system having our energy-harvesting system.

Keywords: hybrid-type vibration control; semi-active vibration suppression; piezoelectric; energy-transfer; switching control

1. Introduction

Flexible and lightweight structures are exposed to extensive vibrations that can cause mechanical failure and diminish their life span. For the purpose of damping full-scale stay cables, a shunt damping system was investigated (Kim *et al.* 2010). A new variable damping system was proposed to achieve much better isolation performance than the conventional semi-active control laws under single and fly-wheel force excitation environments (Oh and Choi 2011). Piezoelectric materials can exchange mechanical and electrical energies, and have been used for various purposes. A great amount of research has thus been devoted to combining piezoelectric materials and electrical equipment to suppress vibration in structures (Han *et al*. 1997, Sodano *et al.* 2004, Park *et al.* 2010, Lee *et al*. 2011) To control the vibration of a computer hard disk drive, a new shunt damping methodology was proposed and verified in experiment. (Lim and Choi 2007a,b).

Bang-bang active control is one of the simplest methods using a piezoelectric material. Its control input has one absolute value that changes in polarity. Its constant-voltage supplier is shared by multiple actuators, which simplifies the hardware configuration. In spite of the advantage of this

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simple configuration, the bang-bang method has the following two problems. The first problem is that the control performance is limited because the absolute value of the input voltage is constant. The second problem is that it risks the chattering phenomenon, which wastes valuable energy.

Another simple method is shunting a piezoelectric material with a circuit composed of an inductor and a resistor (Hagood and von Flotow 1991, Wu 1996). Such an inductive circuit has an electrical resonance that can work in a similar way as dynamic mass dampers. However, the performance of this passive method is degraded when the frequency of the electrical resonance is different from the frequency of the structure. That is why the passive method possesses limited robustness against model errors.

Several types of active-passive hybrid piezoelectric networks (APPN) have been proposed (Agnes 1995, Morgan and Wang 1998, Tang and Wang 2001) to reduce energy consumed in vibration control. This network integrates piezoelectric materials with an active voltage source and a passive shunting circuit composed of resistors and inductors, which are based on passive vibration suppression.

A number of semi-active vibration suppressors using an electric circuit with switches and piezoelectric materials have been proposed. The on-off switch is simply closed or opened to suppress the structural vibration. In one proposed method, a state-switching method is implemented with a switchable stiffness element (Clark 2000). Another method uses a state-switched vibration absorber to avoid potentially undesirable mechanical transients (Larson and Cunefare 2004). Recently, a promising semi-active (or semi-passive) method using an inductive circuit and a switch was proposed (Richard *et al*. 2000, Onoda *et al.* 2003). This method controls a circuit switch so that mechanical energy is converted to electrical energy, and the converted electrical energy is recycled to efficiently suppress vibration rather than immediately being dissipated. This energy-recycling mechanism is quite an excellent feature for achieving energy-efficient vibration suppression. It is called SSDI, LR-switching, RL-shunt and so on, depending on their switching strategies and circuits' components. For eliminating outer energy source, an analog autonomous self-powered switching system using analog electric devices to suppress vibration (Morari and Niederberger 2006, Liang and Liao 2010), which were difficult to retune parameters and to achieve sophisticated control. To overcome drawbacks of analog autonomous system, an innovative digital self-powered suppression unit was developed (Makihara *et al.* 2012). This digital autonomous system enables the suppressor to achieve complicated calculation for handling multi-mode vibrations (e.g., Kalman filter).

A hybrid vibration suppression method was proposed in order to improve the bang-bang active method by combining it with the aforementioned energy-recycling approach (Makihara 2005). The hybrid method recycles mechanical energy for vibration control, instead of providing the electrical energy entirely from external sources. Moreover, the energy-recycling approach enables the externally provided energy to be reused many times for vibration suppression. Qiu *et al*. (2009) presented an extensive latest overview of this field (i.e., switching control approach using piezoelectric materials for vibration suppression) that enables readers to quickly grasp the history and significance of semi-active and hybrid-type methods for vibration suppression.

2. Research objectives

The effectiveness of the hybrid approach with the chattering prevention method (Makihara 2005) was already confirmed in experiments and simulations; however, the following indispensable issues have not been addressed. Therefore, clarifying and discussing these unaddressed issues are major

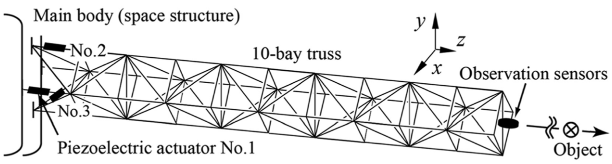

Fig. 1 Example of MDOF systems (extension mast stretching from a main body, e.g., astronomical satellite and space station): 10-bay truss with multiple actuators at fixed end, and sensors observing far-away astronomical objects

objectives of this paper.

Firstly, our previous paper only focused on beneficial aspects of additional external voltages to enhance the performance, but not on harmful ones of external voltages. We will cast light on the fact that higher piezoelectric voltage than necessary causes hazardous control-spillover (i.e., higher vibration-modes' excitation), as well as the voltage's beneficial effect for vibration suppression. It will be proven that a compromise value of the external voltage exists, considering its harmful and beneficial effects. We will discuss both aspects and show the detailed process to strike compromise for constructing ideal hybrid systems.

Secondly, our previous approach for chattering prevention had a serious problem; the trapped energy in actuators deviates the equilibrium point of structural displacement from its original point. The deviation of displacement exerts an adverse influence on many space-missions, such as space-observation and communication activities using antennas. The controversial displacement-offset will be removed by our new system.

Thirdly, the quantity of energy consumption has not been analyzed, despite it being referred to as the "low energy-consumption" method in our previous paper. The energy assessment is indispensable before actual usages in space, because any electrical energy or power is much more valuable in space than that on earth. In isolated space-missions, even a small amount of harvested energy should be collected as much as possible.

In this paper, to answer these aforementioned issues, in-depth discussion is presented through experiments and simulations on a 10-bay truss (See, Fig. 1). Astronomical satellites that observe distant targets (e.g., stars or galaxies) often extend long and flexible masts in space. Observation sensors, such as magnetometers or thermal plasma sensors, are mounted at the tip of these extension masts in order to avoid interference from the satellite's devices. These extension masts are usually composed of spindly members, and are called a truss structure. For these truss structures that have space observation missions, both displacement-offset and vibration can be significantly detrimental to tracking a target. In this study, a cantilevered 10-bay truss was used to simulate the long and flexible structure that stretches from the satellite's main body. Broadband frequency vibration analysis on our hybrid system has already made in our previous paper (Makihara 2005), so this paper does not focus on broadband vibrations to avoid overlap and redundancy of contents.

3. Hybrid vibration suppression

3.1 Equation for structure with piezoelectric actuators

Let us consider a truss having multiple piezoelectric actuators as shown in Fig. 1. Some abovementioned observation sensors are usually attached to the tip of the truss that extends from the main body (e.g., the space station or artificial satellites). The truss in Fig. 1 is rigidly fixed to the main body at the joint part. The number of the truss's nodes is 40 and that of its degree-of-freedom is 120, because the structure can vibrate in *x*-*y*-*z* coordinates due to no gravity in space. In the general missions of space-observation, free vibration due to satellites' attitude maneuver or unexpected impact is common, and forced vibration is quite rare. Accordingly, suppression of free vibration is focused on in this paper.

Each piezoelectric actuator is described with four variables (Jaffe *et al.* 1979): tensile load f, elongation u, voltage V, and electric charge Q as

$$f = k_p u - b_p V, \; Q = b_p u + C_p^S V \tag{1}$$

where k_p, b_p, and C_p^S are parameters of stiffness at constant voltage, piezoelectric coupling and capacitance at constant strain, respectively. We obtain the equation of motion of the truss, following the derivation of (Makihara 2007a). The motion equation for the truss having multiple actuators is written as

$$\boldsymbol{M}\ddot{\vec{u}} + \boldsymbol{D}\dot{\vec{u}} + \boldsymbol{K}\vec{u} = \boldsymbol{B}_V \vec{V} \tag{2}$$

where $\vec{u}$ is the displacement vector and $\vec{V}$ is the voltage vector of actuators. Here, $\boldsymbol{M}$, $\boldsymbol{D}$, $\boldsymbol{K}$, and $\boldsymbol{B}_V$ are the mass, damping, stiffness at constant voltage, and input matrices, respectively. The equation of motion in the modal coordinates is expressed as

$$\ddot{\vec{\eta}} + \Xi\dot{\vec{\eta}} + \Omega\vec{\eta} = \Phi^T \boldsymbol{B}_V \vec{V} \tag{3}$$

where $\vec{\eta}$ is the modal displacement vector and

$$\Phi \equiv [\vec{\phi}_1, \vec{\phi}_2, \ldots, \vec{\phi}_n], \; \Omega \equiv diagonal[\omega_i^2], \; \Xi \equiv diagonal[2\zeta\omega_i] \tag{4}$$

Here, the structure is assumed to have n degrees of freedom for the purpose of the generalization of discussion. The values for ω_i and $\vec{\phi}_i$ are obtained by solving the eigenvalue problem for the homogeneous part of Eq. (2), where $\Phi^T \boldsymbol{M} \Phi = \boldsymbol{I}$. Eq. (3) is transformed into

$$\dot{\vec{z}} = \boldsymbol{A}\vec{z} + \boldsymbol{B}\vec{V}, \; \vec{z} \equiv \left[\vec{\eta}^T, \dot{\vec{\eta}}^T\right]^T \tag{5}$$

where

$$\boldsymbol{A} \equiv \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\Omega & -\Xi \end{bmatrix}, \; \boldsymbol{B} = \begin{bmatrix} \boldsymbol{0} \\ \Phi^T \boldsymbol{B}_V \end{bmatrix} \tag{6}$$

If we apply an active control to the system, we can suppress the vibration of the system by supplying a control input voltage $\vec{V}$ to piezoelectric actuators. The LQ (Linear Quadratic) control theory (Junkins and Kim 1993) specifies that the control input voltage $\vec{V}$ that minimizes the performance index

$$J \equiv \int_0^{\infty} (\vec{z}^T \boldsymbol{Q} \vec{z} + \vec{V}^T \boldsymbol{R} \vec{V}) dt \tag{7}$$

is given as

$$\vec{V} = \vec{V}_T \equiv -\boldsymbol{F}\vec{z} \tag{8}$$

where $\boldsymbol{F}$ is the feedback matrix determined with the standard LQ scheme.

In the standard guideline of the LQ active control, the first and second terms on the right-hand side in Eq. (7) correspond to the state energy and the control energy, respectively, with appropriate $\boldsymbol{Q}$ and $\boldsymbol{R}$. The performance index $\boldsymbol{J}$ comprises the two competing factors. However, our hybrid method only makes reference to the polarity of $\vec{V}_T$ for its switching operation, and the meaning of $\boldsymbol{Q}$ and $\boldsymbol{R}$ in switching control systems is quite different from the standard guideline. This discussion and interpretation of these peculiar meaning regarding switching controls can be seen in (Makihara *et al*. 2007b)

3.2 Beneficial use of stored energy in chattering prevention

The basic concept of the hybrid method was elaborately explained in the previous paper (Makihara *et al.* 2005), so a concise outline is presented. The hybrid method is implemented with the circuit shown in Fig. 2. External voltage suppliers have a constant positive voltage of V_{ext} (> 0), and they are placed in parallel but in opposite directions. Since two diodes prevent electric current from flowing in one direction, they automatically stop the current when the current direction changes. The voltage across the piezoelectric actuator is thus controlled by the switch and the diodes. In other words, the aim of our hybrid system is to suppress vibration only by controlling the switch in the jth circuit, instead of modulating V_j directly. Our control logic for the hybrid control is that the jth switch is controlled so that V_j has the same polarity as V_{Tj} in Eq. (8). Thus, V_{Tj} is the target value of V_j for switching criteria of circuit switch.

In active control systems, we usually see a high-frequency change of the control input when vibration is sufficiently suppressed, especially in bang-bang vibration controls. This phenomenon is

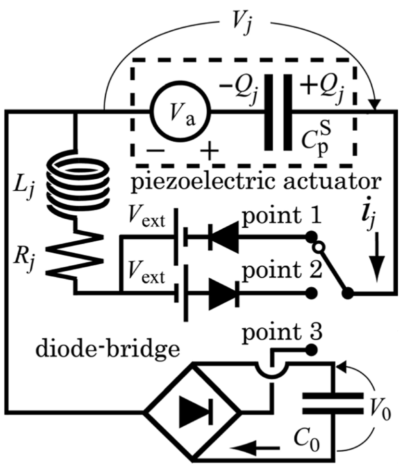

Fig. 2 Energy-storing electric circuit for hybrid vibration suppression

usually called chattering, and it leads to the waste of supplied energy and the excitation of high frequency dynamics. Switch control systems also experienced chattering when unnecessarily large energy was stored in the actuator even after vibration had been sufficiently suppressed (Onoda *et al.* 2003). Chattering may occur when the electrical energy in the piezoelectric actuator is decreased even without current flow (Makihara *et al.* 2005).

Also in our new hybrid system, by monitoring the stored energy, we determine the need to pause hybrid vibration suppression to prevent chattering. After the pause action, the stored energy is transferred to the extra capacitor through the diode-bridge (see Fig. 2). The hybrid method resumes its operation to suppress an increasing vibration when electrical energy reaches the value it had at the last switching. The piezoelectric voltage is monitored to detect the increment or decrement of the stored energy. Here, $\tilde{V}_j$ is defined as the jth actuator's voltage immediately after the current stops flowing, and is referred to as the reference voltage of the jth actuator. This study basically follows the original control scheme in our previous paper (Makihara *et al* 2005). The control logic with chattering prevention that is derived from energy-flow analysis can be written as

$$\begin{aligned} &\text{If } V_{Tj} < 0\text{: connect to point 1 until } V_{Tj} \geq 0 \text{ when } |V_j| - \varepsilon|\tilde{V}_j| \geq 0 \\ &\qquad\qquad\quad \text{connect to point 3 when } |V_j| - \varepsilon|\tilde{V}_j| < 0 \\ &\text{If } V_{Tj} \geq 0\text{: connect to point 2 until } V_{Tj} < 0 \text{ when } |V_j| - \varepsilon|\tilde{V}_j| \geq 0 \\ &\qquad\qquad\quad \text{connect to point 3 when } |V_j| - \varepsilon|\tilde{V}_j| < 0 \end{aligned} \tag{9}$$

The clear explanation of the chattering prevention using point 3 was written in detail in our paper, and is thus not repeated here to save the paper-space. ε is a marginal parameter and is set to be 0.95 by trial and error. This parameter prevents the controller from being activated too sensitively by the sensor noise or the temporal voltage relaxation just after voltage reversion. This chattering prevention harvests electrical energy from the actuator and stores it in the capacitor C_0 (see Fig. 2), whereas the previous approach merely traps electrical energy in the actuator. The diode-bridge is a full-wave bridge rectifier utilizing four diodes in a bridge configuration to convert alternating current into direct current.

3.3 Advantages of new chattering prevention approach

The new hybrid approach has three advantages over the previous one. The first advantage is that it removes the cause of the chattering problem. As revealed in the next section, quite a high value of piezoelectric voltage can have a negative effect on hybrid vibration suppression. Higher voltage than necessary brings negative influence, rather than positive one. This is because the sharp modulation of high voltage easily excites residual vibration modes. From this viewpoint, the new prevention method is much safer than the previous one for some structures whose higher modes can be easily excited, such as large and flexible space structures. The second advantage is that the new method can eliminate the displacement-offset of the equilibrium point of structures (See, Fig. 1). In contrast, the previous system deviated the equilibrium point of displacement, which had a detrimental effect on some applications, for example, observing or tracking distant stars and galaxies. The third advantage is that harvested energy can be used for other applications requiring electrical energy, which appeals to isolated structures with a limited energy source This energy-harvesting process improves the hybrid method in terms of energy efficiency. On the subject of isolated structures with limited energy sources, such as space satellites and sea platforms, minimizing energy consumption and recycling energy are requirements. Considering these advantages, the new hybrid approach excels the previous one, which will be seen in this study.

4. Numerical simulations of hybrid vibration suppression

4.1 Configuration for simulation

We conducted numerical simulations on a 10-bay truss having a piezoelectric actuator (Figs. 3 and 4). This simulated truss design was also used in the vibration suppression experiments discussed later. The axial member weighed 35.7 g, and the diagonal member weighed 46.3 g Each node weighed 67.9 g. Each axial member had a stiffness of $1.99{\times}10^6$ N and a length of 0.38 m. The diagonal member had a length of $\sqrt{2}$ times that of the axial member and the same stiffness as the axial member. The values in the mathematical model of the piezoelectric actuator expressed in Eq. (1) were $k_p = 5.8\times10^6$ N/m, $b_p = 3.0$ N/V, and $C_p^S = 1.2\times10^{-5}$ F, based on experimental setup in the later chapter. Regarding the truss with a single actuator, the frequency of the first vibration mode (the first bending mode in *x-z* coordinates) at the shunt circuit (i.e., constant voltage) was 10.7 Hz, and that at the open circuit (i.e., constant electric charge) was 11.6 Hz, respectively. This first bending mode in *x-z* plane was excited and was suppressed in simulations, because the

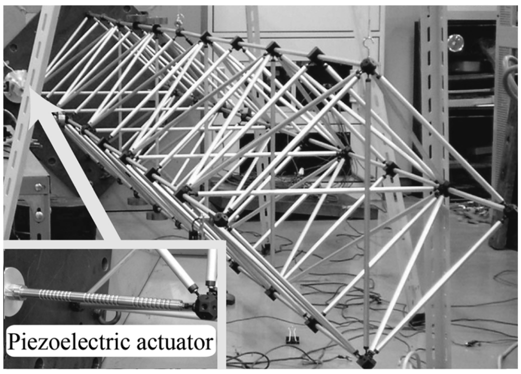

Fig. 3 Experimental setup for hybrid vibration suppression and magnified view of piezoelectric actuator

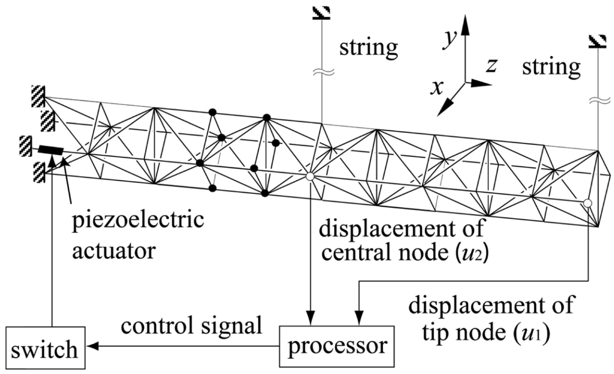

Fig. 4 Control flow of hybrid vibration suppression experiment

excitation was the same as experiments. The inductance in the circuit was $L = 2.7 \times 10^{-3}\mathrm{H}$, based on the experiments. Piezoelectric actuators have an equivalent resistance that decreases with the increase in the cycle number of input sinusoidal voltage (Makihara *et al.* 2006). The measurement of actual actuators based on the research conclusion determined the circuit resistance to be $R = 4.0\ \Omega$. The damping ratio ζ of each mode was 3.6×10^{-3}. These values were based on the experimental system parameters. The following numerical simulation had a time step of 1.0×1.0^{-7} s, which was sufficiently small compared with the highest mechanical vibration mode (5.0×10^{-4} s period) and the electrical vibration (1.0×1.0^{-7} s period). However, to avoid unrealistic investigation, the controller changed its switch every period of 3.0×10^{-4}, which was the time step of the experimental hardware.

4.2 Energy transition of hybrid vibration suppression

We first simulated hybrid vibration suppression for the truss to analyze the energy transition of the systems. The first bending mode in *x-z* plane was excited and was suppressed. The initial modal velocity of the first mode was 5.0×10^{-2} mkg$^{1/2}$/s, and the initial modal displacements and velocities of all the other modes were set to 0. The lowest mode was suppressed with the hybrid method. To obtain V_T from the LQ control scheme, the weighting matrix $\boldsymbol{Q}$ in Eq. (7) was set to

$$\boldsymbol{Q} = diagonal[1, 1/\omega_1^2] \tag{10}$$

The set of $\boldsymbol{Q}$ was explained in (Makihara *et al.* 2007b) for the space missions that place great importance on displacement. Since the truss had a single actuator, $\boldsymbol{R}$ was the scalar value. Here, $\boldsymbol{R}$ was optimized as 1.0×1.0^1, with which the performance index,

$$I_R = \int_{T_s}^{T_e} RMS[\vec{u}]dt \tag{11}$$

was minimal where T_s=0.0 s and T_s=2.0 s. The RMS function provided the root mean square of the displacement vector $\vec{u}$. The displacement vector was adopted as the performance index, because the displacement is the most important factor for static space-observation.

Fig. 5 shows time histories of vibration suppression for the hybrid method with an external voltage of 1.0 V and the semi-active method (i.e., V_{ext}=0.0 V). By switching between points 1 and 2, voltage alternately took positive and negative values. Since the hybrid method had a larger change in voltage due to the voltage supply, it had a large absolute value of voltage. Consequently, it suppressed the vibration more quickly than the semi-active method. The controller decided the control pause at $t = 0.28$ s according to the chattering prevention logic. The stored electrical energy equivalent to 4.6 V at $t = 0.28$ s was harvested from the actuator and stored in the capacitor. Due to the harvest, the piezoelectric voltage became zero afterwards. The harvested energy can be used for many applications, instead of being simply dissipated or trapped in the actuator. Thanks to the prevention logic, the chattering phenomenon never occurred even after vibration had been sufficiently suppressed.

An energy assessment was done to clarify the vibration suppression mechanism from the viewpoint of energy exchange. E_{sum} was defined as the sum of mechanical vibration energy (i.e., kinematic and potential energies) of the truss. E_{con} was the mechanical energy related to controlled modes, and E_{res} was the mechanical energy related to residual modes. From Eq. (3), these energies satisfied the following relation

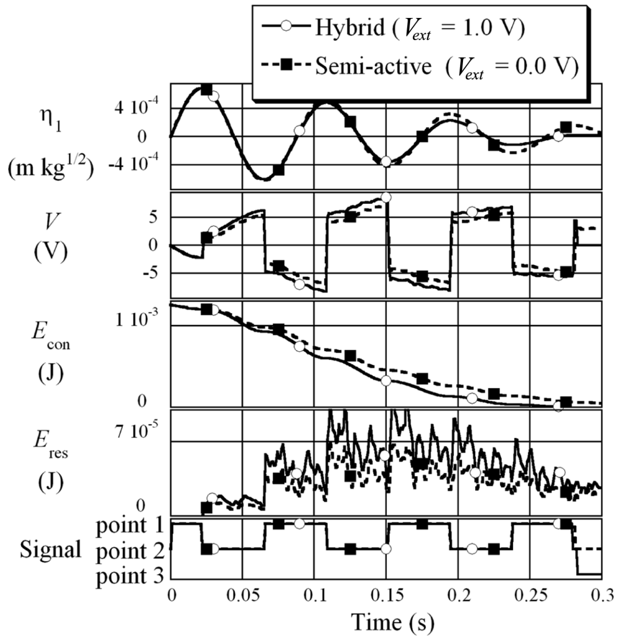

Fig. 5 Efficiency comparison of single-mode vibration suppression

$$E_{sum} = E_{con} + E_{res} \equiv \frac{1}{2}\dot{\vec{\eta}}^T \cdot \dot{\vec{\eta}} + \frac{1}{2}\vec{\eta}^T \Omega \vec{\eta} \tag{12}$$

Fig 5 shows time histories of the energies of controlled modes and residual modes. E_{con} monotonically decreased, and E_{res} increased in both methods. The hybrid method (V_{ext} = 1.0 V) excited residual modes more than the semi-active method (V_{ext} = 0.0 V), although it suppressed the controlled mode η_1 better. These different tendencies were ascribed to the external voltage. The external voltage increased the amplitude of the residual modes, while it decreased the amplitude of the controlled mode. There seems to be an optimal value of the external voltage to achieve effective hybrid vibration suppression. This optimal value for the external voltage will be discussed next.

4.3 Excitation of residual vibration modes

To evaluate the energy of the truss in hybrid vibration suppression, we calculated time-averaged values of mechanical energy

$$[E]_{con} = \frac{1}{T_e - T_s}\int_{T_s}^{T_e} E_{con} dt \tag{13}$$

$$[E]_{res} = \frac{1}{T_e - T_s}\int_{T_s}^{T_e} E_{res} dt \tag{14}$$

Here, $[E]_{con}$ represents the suppression effect of controlled modes, and $[E]_{res}$ represents the excitation effect of residual modes. Fig. 6 plots $[E]_{con}$, $[E]_{res}$, and $[E]_{sum}$ as a function of external voltage value with T_s=0.0 s and T_s=2.0 s, where $[E]_{sum}$ is the sum of $[E]_{con}$ and $[E]_{res}$. The structure

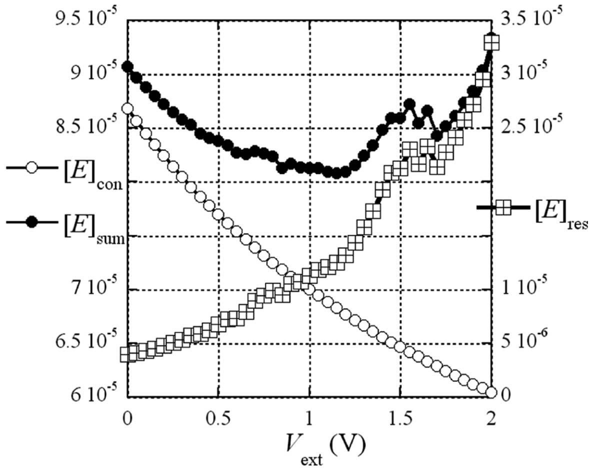

Fig. 6 Energy evaluation as a function of external voltage (controlled-mode energy $[E]_{\mathrm{con}}$, residual-mode energy $[E]_{\mathrm{res}}$, and sum of all energies $[E]_{\mathrm{sum}}$)

and the controller were the same as the previous simulation. The controlled modes' energy decreased with the increase in the external voltage value, and the residual modes' energy increased. The value of $[E]_{sum}$ was minimized when external voltage was around 1.0 V for this truss. The hybrid control system had an optimal value of external voltage as a result of the compromise between the controlled modes' suppression and residual modes' excitation.

4.4 Comparison of input energy for vibration suppression

In general, much attention is paid to energy consumption for vibration suppression. The amount of input energy in the hybrid method was assessed with control effort indices. We compared cost efforts for four systems (new hybrid, original hybrid, and LQ active, and bang-bang active systems). In the active LQ system, the piezoelectric actuator was directly controlled so that $\vec{V}$ in Eq. (5) should be equal to $\vec{V}_T$. The cost effort

$$E_{CA} \equiv \int_{T_s}^{T_e} \dot{\vec{u}}^T \cdot (\boldsymbol{B}_V \vec{V}) dt \tag{16}$$

was calculated for two active systems (LQ and bang-bang active systems), which was derived from Eq. (2). The subscript CA stands for the Control effort of Active systems. In the following comparison, we discuss the simple case where the system has a single actuator and a single circuit. However, to ensure the generality of the theoretical analysis, the cost effort was defined for a system having multiple actuators and circuits. In contrast to the active LQ system, for hybrid systems, the cost effort

$$E_{CH} \equiv \int_{T_s}^{T_e} \dot{\vec{Q}}^T \cdot \vec{V}_{ext}\, dt \tag{16}$$

was defined, where $\vec{Q}$ is the charge vector and $\vec{V}_{ext}$ is the external voltage vector. Here, E_{CH} represents the input energy that the external voltage supplies to the system via the circuit. The subscript CH stands for the Control effort of Hybrid systems. Eqs. (15) and (16) look different; however, both represent the amount of external work or energy consumption for the system, and can be compared qualitatively as the cost effort. In the new hybrid system, electrical energy is harvested from the actuator and stored in the extra capacitor, rather than being simply trapped in the actuator. E_{CH} includes this usable energy, as well as consumed energy in the vibration suppression operation. This means that E_{CH} is overestimated as an index of consumed energy only for vibration suppression. To evaluate the net amount of consumed energy, we subtract the harvested energy from E_{CH} and calculate the cost effort

$$E_{CH2} \equiv \int_{T_s}^{T_e} \dot{\vec{Q}}^T \cdot \vec{V}_{ext}\, dt - \left[\frac{1}{2} \vec{V}_0^T C_0 \vec{V}_0 \right]_{|t = T_e'} \tag{17}$$

where $\vec{V}_0$ is the voltage vector of extra capacitors and C_0 is the diagonal capacitance of extra capacitors. $\vec{V}_0$ is initially zero at $t = T_S$.

The hybrid system was controlled according to the switching logic with the chattering prevention in Eq. (9). The structure and the controller were the same as in the previous simulation. Based on the previous analysis (see Fig. 6), the hybrid system's external voltage was determined to be 1.0 V. The scalar value of $\boldsymbol{R}$ was determined as 1.0×10^1 so that I_R for the new hybrid system was minimized (1.3×10^{-5} ms), where $T_S = 0.0$ s and $T_e = 2.0$ s. The original hybrid system also used the same weighting matrices for fair comparison. In contrast to the hybrid systems, LQ active system performance can be modulated as desired because it directly depends on the amplitude of the feedback gain. Thus, we designed an LQ system whose I_R was the same as the minimum value for the hybrid systems (i.e., 1.3×10^{-5} ms). Accordingly, a controller with $\boldsymbol{R} = 2.5\times10^1$ was selected as a representative of LQ systems. Similarly to the LQ system, we designed a bang-bang active system whose I_R value was 1.3×10^{-5} ms, and in consequence, the bang-bang controller had the combination of $V_{ext} = 5.5$ V and $\boldsymbol{R} = 1.0\times10^3$.

Table 1 compares cost efforts in Eqs. (15) and (17), where $T_S = 0.0$ s and $T_e = 2.0$ s. We should remember that all systems had the same suppression performance (the same I_R value) for fair comparison. Among them (new hybrid, original hybrid, LQ active, and bang-bang active systems), the new hybrid system marks the smallest cost effort. This leads to the conclusion that the hybrid method is the most energy-efficient vibration suppression method. The bang-bang system required the largest cost effort, meaning that it is the least energy-efficient in suppressing vibration. Through these numerical simulations, the superiority of the hybrid approach was determined in light of energy efficiency. The amount of stored energy obtained from the energy-harvester depends both on the system configuration and on the magnitude of vibration. In this configuration, the amount may look small. However, keep in mind, in isolated space structures, even a small amount of the harvested energy is quite valuable and ought to be collected as much as possible, which is very different from the harvested energy's value on the earth.

Table 1. Cost effort for vibration suppression

Control method	New hybrid	Original hybrid	LQ active	Bang-bang
Performance Index	E_{CH2}	E_{CH}	E_{CA}	E_{CA}
V_{ext}	1.0 V	1.0 V	variable	5.5 V
Cost effort (J)	8.44×10^{-4}	9.77×10^{-4}	1.30×10^{-3}	5.84×10^{-3}

5. Experiment of hybrid vibration suppression

We carried out vibration suppression experiments on the 10-bay cantilevered truss shown in Fig. 3. Due to hanging 2.95-m strings compensating for the truss's weight, the motion of the structure was restricted to the *x-z* plane. Two displacement sensors were installed at the tip and the central nodes to measure *x*-directional displacements, u_1 and u_2, as shown in Fig. 4. Vibrations in the *x-z* plane were excited and suppressed in the following experiments. A commercial piezoelectric transducer (NEC/TOKIN ASB171C801NP0) was used. It had a length of 0.22 m and a mass of 93 g and was composed of 1300 piezoceramic layers.

The control flow was as follows (see Fig. 4). First, measured values of u_1 and u_2 were sent through an analog-digital converter to a processor. Next, the processor conducted modal estimation and calculated $\vec{V}_T$. It sent, if necessary, a signal through a digital-analog converter to the switch. On receiving the signal, the switch would change its connection point. The modal displacements were estimated by using the Kalman filter (Junkins and Kim 1993), and the state vector was composed of modal displacements and velocities, η_1, η_2, $\dot{\eta}_1$, and $\dot{\eta}_2$. The mathematical formulation of the Kalman filter for this truss structure was elaborately written in (Makihara *et al.* 2007)

We conducted a single-mode vibration suppression experiment using a piezoelectric actuator, both to see if the hybrid method suppressed vibration and to see if chattering prevention worked as intended. The first bending mode in the *x-z* plane (11.6 Hz) was excited by using a permanent magnet and a voice coil connected to the floor. After the excitation, the voice-coil circuit was opened so that the vibration of the truss structure would not be damped by the energy dissipating from the voice-coil circuit. Because we focused on free vibration that is common for space structures, the voice-coil circuit was cut off after the excitation.

Fig. 7 shows time histories for the hybrid system with the new chattering prevention. The external

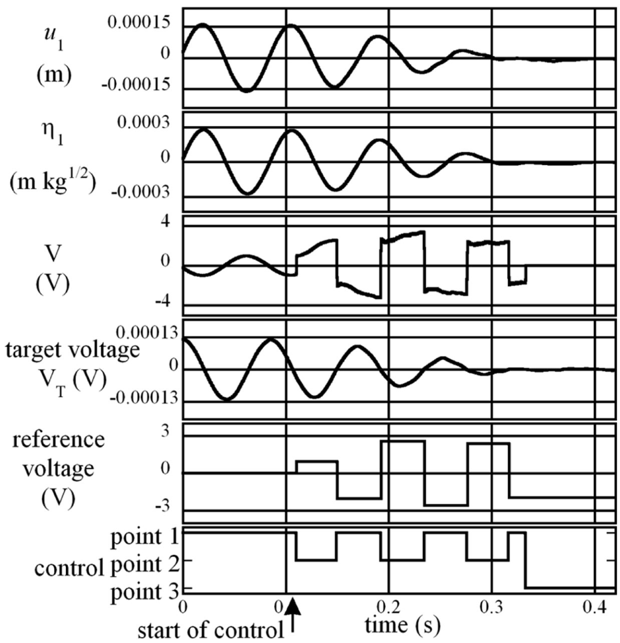

Fig. 7 Suppression experiment of new hybrid method using energy-storing circuit (with displace-offset prevention)

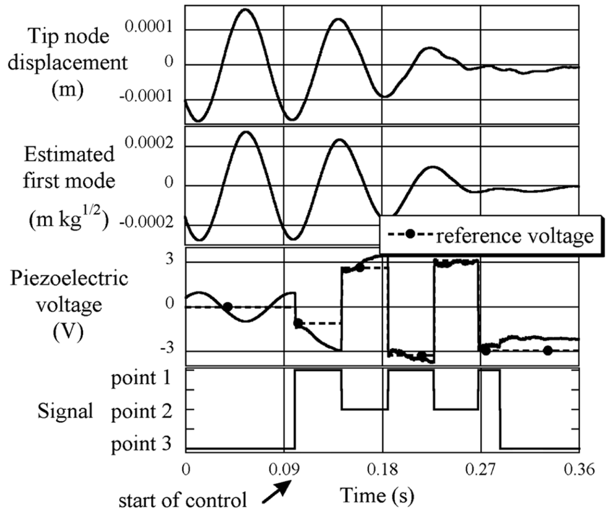

Fig. 8 Suppression experiment of original hybrid method (without displace-offset prevention) that is cited from (Makihara 2005)

voltage value was 0.4 V. Until $t = 0.11$ s, the truss had free vibration, and the circuit was disconnected so as not to dissipate the energy in the circuit. During $t = 0$ and $t = 0.11$ s, small damping of vibration without any control can be seen. After $t = 0.11$ s, the free vibration was suppressed by using the hybrid and the energy-recycling semi-active methods. To obtain V_T for suppressing the first vibration mode, we set $\boldsymbol{Q}$ and $\boldsymbol{R}$ was optimized in a similar way to the numerical simulations. The hybrid method suppressed vibration well. The controller paused the hybrid control at $t = 0.33$ s, and the circuit switch was connected to point 3. Consequently, piezoelectric voltage became zero very quickly. The chattering prevention worked as intended, and electrical energy was harvested at $t = 0.33$ s. It should be noted that u_1 was almost zero after $t = 0.33$ s, and the method eliminated any displacement-offset from the original equilibrium point. Fig. 8 is quoted from the previous paper (Makihara *et al.* 2005), and shows the experimental result using the original hybrid method. When Figs. 7 and 8 are compared, after vibration is enough suppressed, the obvious disappearance of displacement-offset can be seen only in Fig. 7. The offset elimination is the primary importance for many scientific and engineering space-missions, such as astronomical observation and star tracking. This improvement makes the hybrid vibration suppression all the more valuable. Simulation and experimental results (Figs. 5 and 7) are similar, indicating that the modeling of the system for simulations is valid for the actual design.

6. Conclusions

This paper presents new insights into hybrid vibration suppression combining a bang-bang active approach and an energy-recycling approach. We investigated essential issues of the hybrid method that had not yet been tackled.

Firstly, we assessed the negative effect of additional external voltages, as well as the positive effect. The major negative one is that higher piezoelectric voltage than necessary can cause harmful control-spillover. We revealed that the hybrid control system had an optimal value of external voltage as a result of the compromise between the beneficial effect (good control-performance) and the adverse effect (control spillover). The clearly demonstrated process to compromise both effects and to determine the value of external voltage can provide a more reasonable approach to deciding hybrid configurations, rather than a trial-and-error approach.

Secondly, a new chattering prevention method was shown to be effective not only in preventing chattering but also in obviating the displacement-offset of the equilibrium point of structures. The offset elimination is quite essential especially in space-observation missions of astronomical satellites. The method using the diode-bridge circuit is inherently the energy-harvesting approach, indicating that we can make use of the harvested energy efficiently for any kind of beneficial utilization in isolated space-missions.

Thirdly, we proved that energy consumption for the new hybrid method was lower than common active controls. This drawn conclusion leads to our hybrid method's high-adaptability to many machines and devices in energy-saving or energy-shortage environments.

Both numerical simulations and experiments on the 10-bay truss demonstrated various aspects of versatility of the hybrid method. The investigations in this paper comprise a comprehensive evaluation of the hybrid method with the energy-harvester, and pave the way for its practical application.

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