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Probabilistic optimal safety valuation based on stochastic finite element analysis of steel cable-stayed bridges

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Abstract. This study was intended to efficiently perform the probabilistic optimal safety assessment of steel cable-stayed bridges (SCS bridges) using stochastic finite element analysis (SFEA) and expected life-cycle cost (LCC) concept. To that end, advanced probabilistic finite element algorithm (APFEA) which enables to execute the static and dynamic SFEA considering aleatory uncertainties contained in random variable was developed. APFEA is the useful analytical means enabling to conduct the reliability assessment (RA) in a systematic way by considering the result of SFEA based on linearity and nonlinearity of before or after introducing initial tensile force. The appropriateness of APFEA was verified in such a way of comparing the result of SFEA and that of Monte Carlo Simulation (MCS). The probabilistic method was set taking into account of analytical parameters. The dynamic response characteristic by probabilistic method was evaluated using ASFEA, and RA was carried out using analysis results, thereby quantitatively calculating the probabilistic safety. The optimal design was determined based on the expected LCC according to the results of SFEA and RA of alternative designs. Moreover, given the potential epistemic uncertainty contained in safety index, failure probability and minimum LCC, the sensitivity analysis was conducted and as a result, a critical distribution phase was illustrated using a cumulative-percentile.

Keywords: steel cable-stayed bridges; advanced probabilistic finite element algorithm; stochastic finite element analysis; reliability assessment; probabilistic safety; optimal design

1. Introduction

Reliability assessment (RA), unlike existing deterministic method or safety factor-introducing method, enables to carry out the structural safety assessment in a logical manner, considering the inherent uncertainty of random variable included in the structure (Choi *et al.* 2006, Ang and Tang 2007). However, as it requires highly complex analysis of nonlinear state function, it's known to have had a great deal of constrains (Cornell 1969, Hasofer and Lind 1974, Rackwitz and Fiessler 1978). As an alternative to deal with such a problem, RA using Monte Carlo Simulation (MCS) as illustrated in Fig. 1 has commonly been accepted. RA by MCS allows calculating the accurate simulation result, but it is too time-consuming because of the need for repeating the numerical analysis, which is not appropriate to such a complex structure as steel cable-stayed bridges (SCS bridges) and so it's been used as the verification means of approximate analysis (Rubinstein 1981, Hwang and Nowak 1991). Thus, it is necessary to develop the analytical tool which enables to efficiently carry out the

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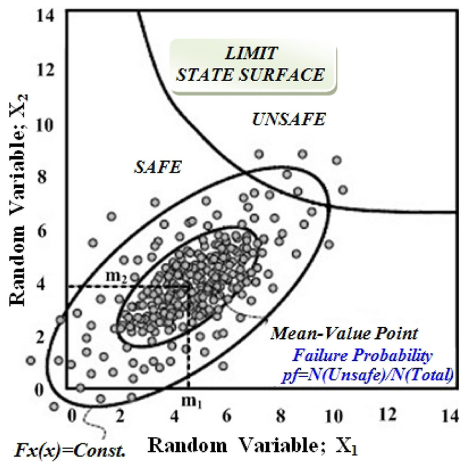


Fig. 1 Reliability assessment using Monte Carlo Simulation method

RA based on probabilistic method, considering the uncertainty of random variable. Meanwhile, probabilistic approach was the assessment method which can identify the mean and standard deviation of structural response resulting from the uncertainty of random variable through the simple numerical analysis (Kleiber and Hien 1991, Choi and Noh 1996, Chakraborty and Dey 1998, Xiaozhen Li and Yan Zhu 2010, Lucia Faravelli *et al.* 2011, Noh 2011). Thus, it is known as the method which will be used very effectively in executing the RA of the major social infrastructure facilities (Chang and Chang 1994, Lei and Qiu 1998, Achintya and Sankaran 2000). However as it has been applied to such a complex structure as SCS bridges, further study in various ways needs to be carried out (Cho *et al*. 2009).

It was reasonable in terms of economic efficiency to determine the optimal design based on the minimum life-cycle cost (LCC), and it has been investigated by many studies (Lee *et al*. 2006, Han and Ang 2008, Han and Park 2009, Li *et al.* 2009). The minimum LCC should be evaluated considering the uncertainties included in cost items (Ang 2006). In this study, linear and nonlinear initial shape analyses (ISAL and ISANL) of SCS bridges were executed based on a trial and error method. And then advanced probabilistic finite element algorithm (APFEA) which can perform the static and dynamic stochastic finite element analysis (SFEA) was developed, considering the final equilibrium before or after introducing initial tensile force and by linear and nonlinear static analysis as initial state. Summarizing the definition of APFEA:

· APFEA is the advanced assessment means which can evaluate the mean and standard deviation responses of SCS bridges, which is dependent on aleatory uncertainty of random variable using only a single SFEA.

· Coefficient of variation (COV) which is required for RA can efficiently be evaluated by formalizing the perturbation model to make it adaptable to existing reliability theories.

· APFEA can be used in estimating the safety index and failure probability of SCS bridges, making use of Equivalent Normal Distribution Transformation (ENDT) method, after setting the limit state function.

The validity of APFEA was proved in such a way of comparing numerical analysis result of a beam-cable structure using MCS program and coefficient of correlation obtained as a result of a regression analysis. The probabilistic method was set in an attempt to evaluate the effect of initial shape, nonlinearity and earthquake characteristics as analytical variables. The mean and standard

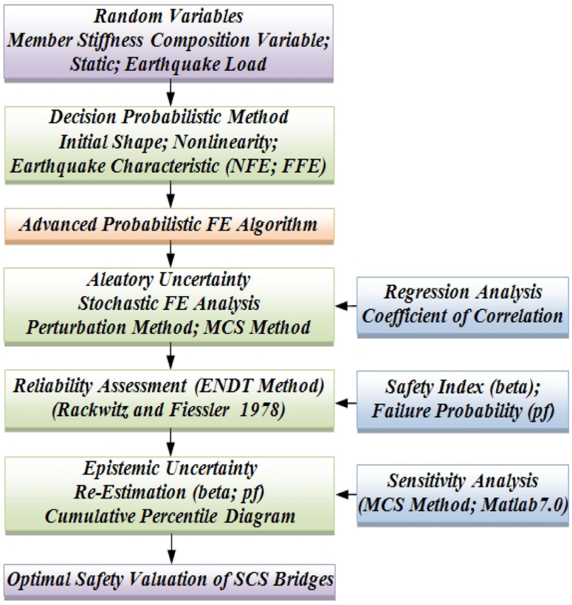


Fig. 2 Procedures of optimal safety valuation for steel cable-stayed bridges

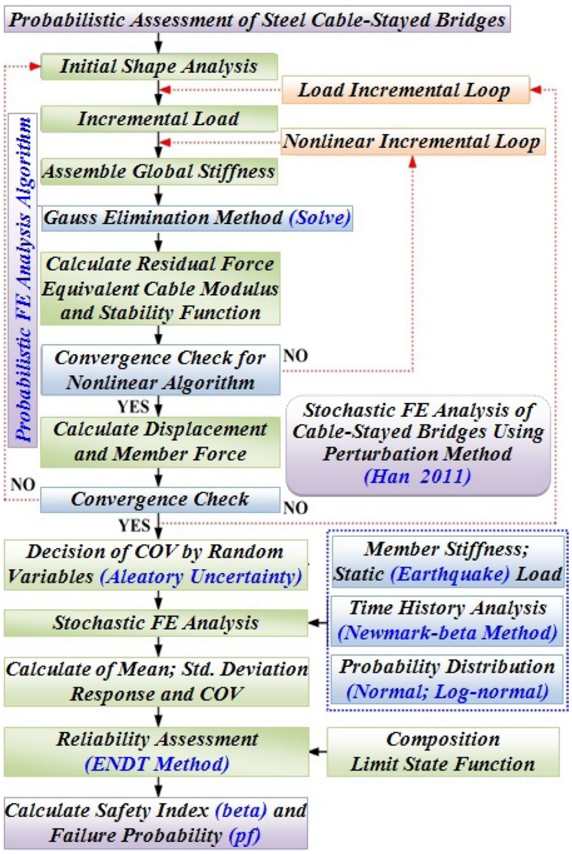


Fig. 3 Flowchart of advanced probabilistic finite element algorithm

deviation response of critical members and the aspect of COV by probabilistic method were evaluated, which was followed by RA to review the safety index and failure probability according to the aleatory uncertainty. For the optimal design of SCS bridges, the minimum LCC was calculated using the results of SFEA and RA of various design alternatives by the variation of member sections based on the standard design. The effect of epistemic uncertainty contained in the result of RA and minimum LCC was reevaluated through sensitivity analysis and the cumulative-percentile was illustrated accordingly. Thus, the consequences of this study are expected to provide the fundamental data with regard to the enhanced assessment method of SCS bridges as shown in Figs. 2 and 3 illustrates the flowchart of APFEA developed in this study.

3. Stochastic finite element analysis of steel cable-stayed bridges

3.1 Perturbation method and Advanced first-order second moment method

In this study, APFEA was developed based on the theory of perturbation method and following formalization process using computer language, Compaq Visual Fortran. When performing the SFEA, the earthquake load and member stiffness composition variables of SCS bridges were considered as random variables which includes aleatory uncertainty. Each random variable was assumed to be mutually non-correlative. Generally, probabilistic dynamic equilibrium equation of multi degree of freedom structure including random variable vector (α_i) is defined as Eq. (1) below (Kleiber and Hien 1991).

$$[M(\alpha_i)]\{\ddot{X}(t,\alpha_i)\}+[C(\alpha_i)]\{\dot{X}(t,\alpha_i)\}+[K(\alpha_i)]\{X(t,\alpha_i)\}=\{F(t,\alpha_i)\} \tag{1}$$

The zeroth-order dynamic equation calculated from Eq. (1) to the mean of random variable vector is defined as Eq. (2).

$$[M]^{(0)}\{\ddot{X}\}^{(0)}+[C]^{(0)}\{\dot{X}\}^{(0)}+[K]^{(0)}\{X\}^{(0)}=\{F\}^{(0)} \tag{2}$$

where, $[M]^{(0)}$=mass matrix; $[C]^{(0)}$=damping matrix; $[K]^{(0)}$=stiffness matrix; $\{F\}^{(0)}$=earthquake load; $\{\ddot{X}\}^{(0)}$; $\{\dot{X}\}^{(0)}$; $\{X\}^{(0)}$=acceleration, velocity and displacement responses

The first-order perturbation as described in Eq. (3) is calculated using a way of partial differential. After calculating Eq. (3) and performing the partial differential one more, the second-order perturbation as described in Eq. (4) can be obtained.

$$[M]^{(1)}\{\ddot{X}\}^{(1)}+[C]^{(0)}\{\dot{X}\}^{(1)}+[K]^{(0)}\{X\}^{(1)}=\{F\}^{(1)}-[M]^{(1)}\{\ddot{X}\}^{(0)}-[C]^{(1)}\{\dot{X}\}^{(0)}-[K]^{(1)}\{X\}^{(0)} \tag{3}$$

$$\begin{aligned}&[M]^{(0)}\{\ddot{X}\}^{(2)}+[C]^{(0)}\{\dot{X}\}^{(2)}+[K]^{(0)}\{X\}^{(2)}=\{F\}^{(2)}-[M]^{(2)}\{\ddot{X}\}^{(0)}-[C]^{(2)}\{\dot{X}\}^{(0)}-[K]^{(2)}\{X\}^{(0)}\\&-2([M]^{(1)}\{\ddot{X}\}^{(1)}+[C]^{(1)}\{\dot{X}\}^{(1)}+[K]^{(1)}\{X\}^{(1)})\end{aligned} \tag{4}$$

where, $(g)^{(0)}=(g)\big|_{\alpha=0}$; $(g)^{(1)}=\dfrac{\partial}{\partial\alpha_i}(g)\big|_{\alpha=0}$; $(g)^{(2)}=\dfrac{\partial^2}{\partial\alpha_i\partial\alpha_j}(g)\big|_{\alpha=0}$

g=dynamic equilibrium equation

The first-order perturbation as described in Eq. (3) to the random variable (member stiffness composition variables and earthquake load) can be defined as Eq. (5) The second-order perturbation as described in Eq. (4) can be calculated as Eq. (6) which is outlined as Eq. (7).

$$[M]^{(0)}\left\{\frac{\partial \ddot{X}}{\partial K}\right\}+[C]^{(0)}\left\{\frac{\partial \dot{X}}{\partial K}\right\}+[K]^{(0)}\left\{\frac{\partial X}{\partial K}\right\}=-\{X\}^{(0)}\ ;\ [M]^{(0)}\left\{\frac{\partial \ddot{X}}{\partial F}\right\}+[C]^{(0)}\left\{\frac{\partial \dot{X}}{\partial F}\right\}+[K]^{(0)}\left\{\frac{\partial X}{\partial F}\right\}=\{1\} \tag{5}$$

$$[M]^{(0)}\left\{\frac{\partial^2 \ddot{X}}{\partial K\partial F}\right\}+[C]^{(0)}\left\{\frac{\partial^2 \dot{X}}{\partial K\partial F}\right\}+[K]^{(0)}\left\{\frac{\partial^2 X}{\partial K\partial F}\right\}=2\left\{\frac{\partial X}{\partial F}\right\} \tag{6}$$

$$[M]^{(0)}\{\ddot{X}\}^{(2)}+[C]^{(0)}\{\dot{X}\}^{(2)}+[K]^{(0)}\{X\}^{(2)}=-2\{X\}^{(1)} \tag{7}$$

where, $\{\cdot\}^{(2)}=\left\{\dfrac{\partial^2(\cdot)}{\partial K\partial F}\right\}$

Thus, the mean of the displacement and member force of SCS bridges can be calculated using Eq. (2) and the standard deviation according to aleatory uncertainty of random variable can be evaluated using Eqs. (5)-(7). Advanced First Order Second Moment (AFOSM) is a method that can resolve the lack of invariance resulting from linear transformation of basic random variables to new variables in normal distribution and follows linear approximation at a probable failure point in the minimum distance from the origin of the transformed coordinate space (Hasofer and Lind 1974). AFOSM method can also resolve the correlation by transforming correlated random variables into non-correlated random variables (Fig. 4).

In this study, the RA of SCS bridges depending on the effect of aleatory uncertainty was carried out based on the result of SFEA obtained using APFEA, making use of ENDT method proposed by Rackwitz-Fiessler. ENDT method is the RA technique which enables to obtain accurate results even

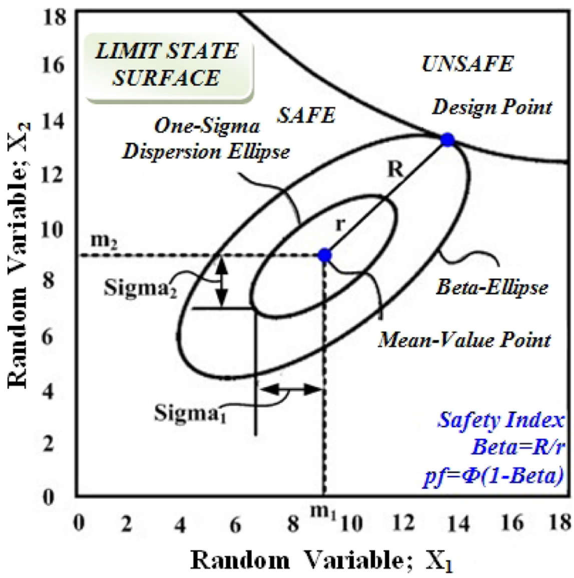


Fig. 4 Advanced first-order second moment method

in the case that the limit state function of basic variable having non-normal correlation distribution is nonlinear function (Rackwitz and Fiessler 1978). The basic concept of equivalent normal distribution function is to calculate the mean and standard deviation by transforming the non-normal variable to equivalent normal distribution so as to make the non-normal distribution value and probability density function value equal to standard normal probability distribution function and standard normal probability density function at the initial value of the assumption point.

3.2 Dynamic verification of advanced probabilistic finite element algorithm in a beam-cable structure

A free vibration analysis was carried out in an attempt to review the dynamic characteristic of the beam-cable structure as illustrated in Fig. 5 (Korea Institute of Construction Technology 2000). Table 1 shows the highlight of free vibration analysis result which indicates insignificant gap which was attributable to the error from numerical modeling. The SFEA for dynamic verification of APFEA was performed by applying the mean and standard deviation of the member stiffness composition variable of the beam-cable structure and the sinusoidal wave base motion (applying horizontal and vertical direction) that does not have a phase difference on two points. At that time, a damping ratio was discarded and each random variable including aleatory uncertainty was assumed to have had non-correlation, normal (member stiffness composition variable) and log-normal distribution (sinusoidal wave base motion) (Korea Institute of Construction Technology 2000).

Table 2 shows the highlight of COV of random variable and probability distribution from literature

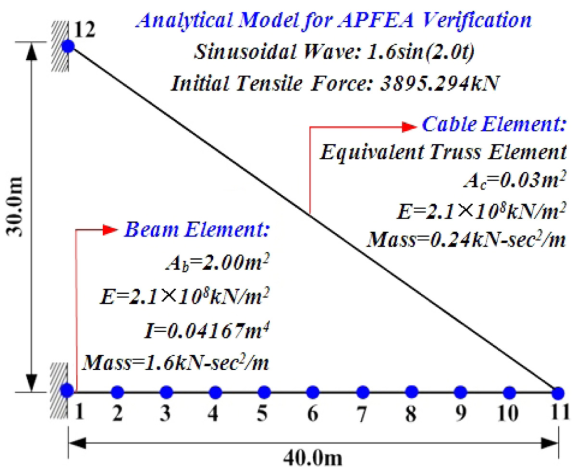


Fig. 5 Analytical verification model for advanced probabilistic finite element algorithm

Table 1 Natural frequency of a beam-cable structure (Hz)

Modes	(Korea Institute of Construction Technology 2000)	In this study
1	1.0033	1.0033
2	3.0296	3.0296
3	5.5017	5.5117
4	8.9434	8.9556
5	13.2360	13.2800

Table 2 Statistical parameters of random variables for verification of advanced probabilistic finite element algorithm

Random variables	Coefficient of variation (COV*)	Distribution type	References
Cross Section (A_b)	0.050 (Beam)	Normal	(Cheng and Xiao 2005)
Cross Section (A_c)	0.015 (Cable)	Normal	(Cho and Kim 2008)
Elasticity Modulus (E)	0.060	Normal	(Tabsh and Nowak 1991)
Moment of Inertia (I)	0.020	Normal	(Nowak and Collins 2000)
Sinusoidal Wave	0.200	Log-normal	Assume

*COV: Coefficient of Variation (as Standard Deviation Divided by Mean Value)

survey. For cable supported structures such as SCS bridges, determining the rational cable tensile force is more important. However, this process was accompanied with many difficulties. The initial tensile force of cable element (equivalent truss element) was determined as a result of initial shape analysis (Wang *et al*. 1993).

In a bid to verify APFEA developed in this study, the mean and standard deviation of vertical displacement, axial force and bending moment at 7th node, horizontal displacement and rotation at 11th node and cable tensile force were calculated. At the same time, numerical analysis using the existing MCS program (Han 2011) was carried out for mutual comparison and evaluation. To achieve the highest accuracy of the analytical results using MCS program, the frequency of the random sample collection should be infinite. However, as this is not realistically possible, the collection frequency has to be selected within an appropriate range. To conduct MCS, the frequency of the random sample collection was set at 10000 using an error-estimating equation with 5.0% significance level, as proposed by Shooman (Shooman 1968). Figs. 6 and 7 illustrate the goodness-of-fit of vertical displacement and cable tensile force using a regression analysis. The result from APFEA and MCS appeared to be generally in agreement, and given the assessment result was near 1.0 from the aspect of correlation coefficient, APFEA is considered to be appropriate. Since APFEA becomes more favorable as much as the frequency of simulation, compared to the numerical analysis using MCS, it's confirmed to be the advanced assessment tool which is very useful in

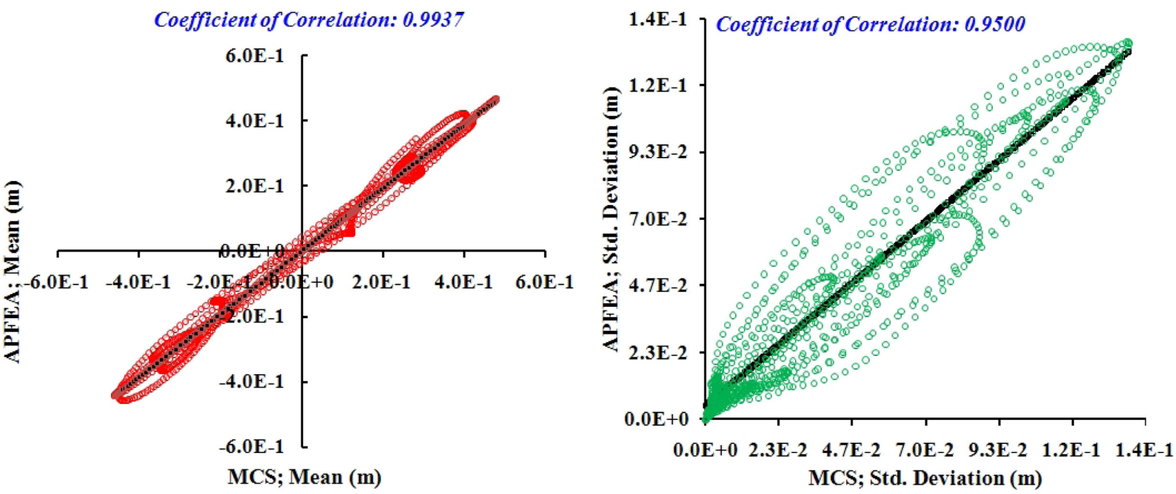


Fig. 6 Regression analysis results of vertical displacement

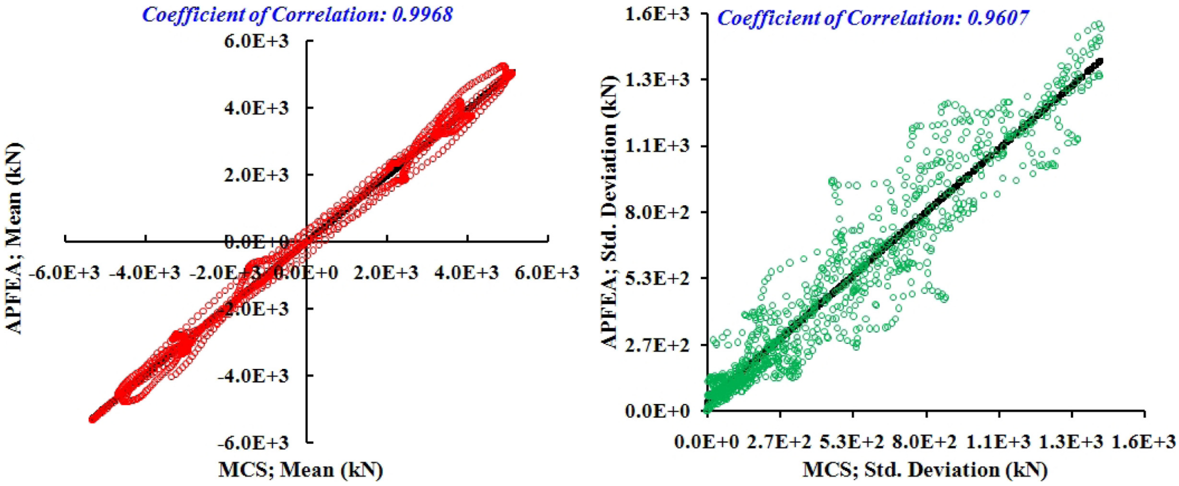


Fig. 7 Regression analysis results of cable tensile force

executing the SFEA of such a complex structure as SCS bridges. COV of maximum vertical displacement (7th node) by APFEA was 28.13% and COV of horizontal displacement and rotation (11th node) was 28.82% and 29.13%, respectively. COV of axial force and bending moment at 7th node was 35.42% and 28.92%, respectively, while COV of cable tensile force was estimated at 29.40%.

3.3 Stochastic finite element analysis under earthquake loads

The earthquakes occurred recently all over the world including Indonesia (2004), Sichuan (2008), Haiti, Chile (2010) and East Japan (2011) caused a huge casualty and economic damage. Nowadays, seismic design of the major social infrastructure facilities near the fault and the evaluation of seismic safety have been comprehensively underway. The characteristic of Near-Fault Earthquake (NFE) is different from Far-Fault Earthquake (FFE), which resulted in increased damage to the structure designed with traditional seismic design specification (Jennings 1997). Currently, evaluation of characteristic of NFE has been underway worldwide. However, the study on evaluation of dynamic response characteristic of SCS bridges, considering NFE effect, still remains at the beginning stage and thus, it's necessary to conduct the study in a systematic way in a bid to evaluate the seismic capacity assessment as well as to supplement the criterion (Han *et al*. 2010). Particularly, given the probabilistic method which can provide the rational information on dynamic response characteristic of SCS bridges considering earthquake characteristic, it would possibly serve the alternative in fact in the process of probabilistic safety assessment. The objective bridge is 11.3 m-width and 483 m-length (main span length: 344.0 m; side span length: 140.0 m) and is a continuous 3-span bridge. Fig. 8 shows the SFEA model (symmetrical model) of SCS bridges, with the locations of the critical members. The SFEA model was composed of with a total of 71 nodes, 34 equivalent truss elements and 68 frame elements. Fig. 9 illustrates the cross section of girder and the profile of pylon, while Tables 3 and 4 indicate the structure type and the cross section and material properties of the objective bridge (Han 2011).

A SFEA of SCS bridges was performed on assumption that the aleatory uncertainty of earthquake

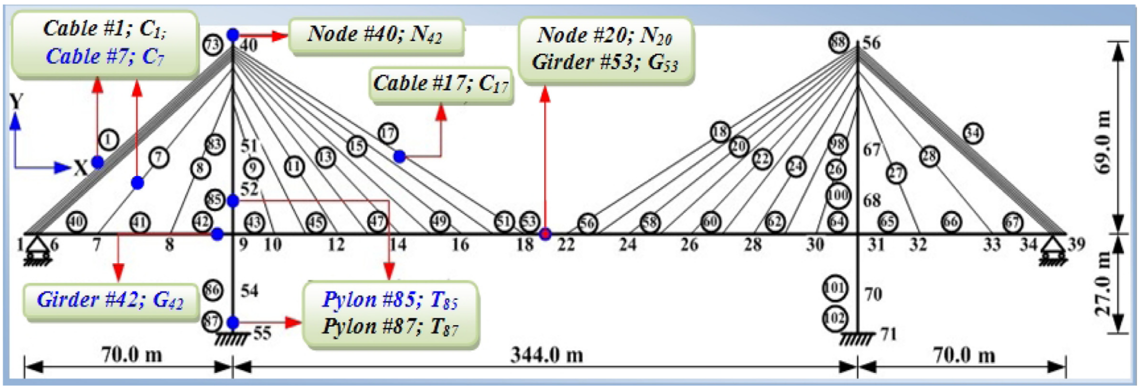


Fig. 8 Analytical model of a symmetrical steel cable-stayed bridge system

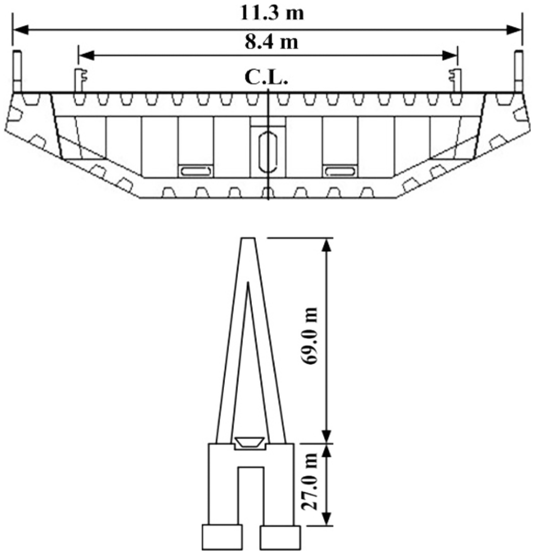


Fig. 9 Cross section of girder and profile of pylon

Table 3 Structural types of steel cable-stayed bridges

Span Length (m)	70.0(Side Span)-344.0(Main Span)-70.0(Side Span)
Girder Width (m)	11.30
Pylon Hight (m)	69.00
Cable Type	Semi-Fan Type
Class of Cable	Lock Coiled Cable
Girder Type	Streamlined Steel Box
Pylon Type	A-Type Steel Box
Supporting Point Condition of Pylon	Continuous 3-Span

load and member stiffness was a non-correlation random variable, respectively. Mass and damping were considered as fixed variable and the statistical parameters of random variables were indicated in Table 5 from the literature survey. The earthquake load was calculated by normalizing (0.07 (coefficient of earthquake area)×1.4 (importance factor)=0.098 g) the acceleration time history from

Table 4 Cross section and material properties of steel cable-stayed bridges

Cross section; material properties		Cross section (m^2)	Elasticity modulus (kN/m^2)	Unit weight (kN/m^3)	Moment of inertia (m^4)
Girder element	Side	0.9475	2.1×10^8	78.5	0.9475
	Center	0.4373			0.5544
Pylon element	Top	0.646	2.1×10^8	78.5	1.227
	Middle	0.525			0.482
	Bottom	0.619			0.534
	Pier	35.60	2.0×10^7	25.0	125.4
Cable element	1, 2, 3, 4, 5, 6, 17	0.00998	1.6×10^8	78.5	-
	7, 11, 12	0.00598			
	8, 9, 10	0.00426			
	13, 14, 15, 16	0.00762			

Table 5 Statistical parameters of random variables in dynamic stochastic finite element analysis

Random variables		Coefficient of variation (COV)	Distribution type	References
Cross section	δ_{cs}	0.050(Girder and Pylon)	Normal	(Cheng and Xiao 2005)
		0.015(Cable)		(Cho and Kim 2008)
Elasticity modulus	δ_{em}	0.060	Normal	(Tabsh and Nowak 1991)
Moment of inertia	δ_{mi}	0.020	Normal	(Nowak and Collins 2000)
Earthquake load (NFE; FFE)	δ_{el}	0.250	Log-normal	(Han and Park 2009)

Table 6 Characteristic of selected acceleration time history

Sites	Station No.	D_{rup}^*(km)	PGA (gal)		PGV (cm/s)		PGV/PGA	
			NS	*UD***	*NS*	*UD*	*NS*	*UD*
FFE	ILA035	104.77	52	11	9.9	2.1	0.190	0.191
NFE	TCU052	1.84	419	241	118.4	110.5	0.283	0.459

*D_{rup}: Distance from Rupture; ***UD*: Up-Down Component

different direction components (TCU052: NFE; ILA035: FFE) more obvious the earthquake characteristic. The earthquake load was applied simultaneously to longitudinal and vertical direction to elastic supporting point, considering the characteristic of NFE which has relatively larger vertical component (Korea's Ministry of Construction & Transportation (KMOCT) 2005, Han *et al*. 2010). Table 6 outlines the location of measuring point, distance from the fault plane and earthquake characteristic along with PGA by different directions (Wang *et al*. 1999). In such a process, damping ratio was 2.0% and newmark-beta was applied in conducting the numerical analysis (Chopra 1995).

A SFEA was carried out over the three schemes as follows:

Scheme 1 (initial shape (linear) analysis + static linear analysis + dynamic SFEA (FFE));

Scheme 2 (initial shape (nonlinear) analysis + static nonlinear analysis + dynamic SFEA (FFE));

Scheme 3 (initial shape (nonlinear) analysis + static nonlinear analysis + dynamic SFEA (NFE))

Fig. 10 shows the initial tensile force of the cable according to ISAL & ISNAL of the objective bridge (symmetrical system) using APFEA (Wang *et al*. 1993). Fig. 11 illustrates the vertical displacement of the girder obtained as a result of the initial shape analysis of SCS bridges by

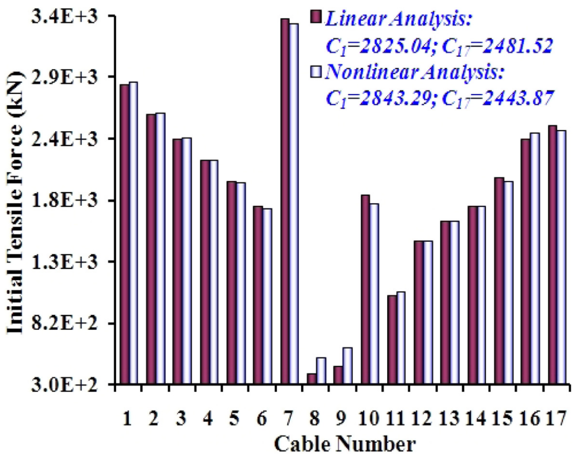


Fig. 10 Results of initial tensile force by advanced probabilistic finite element algorithm

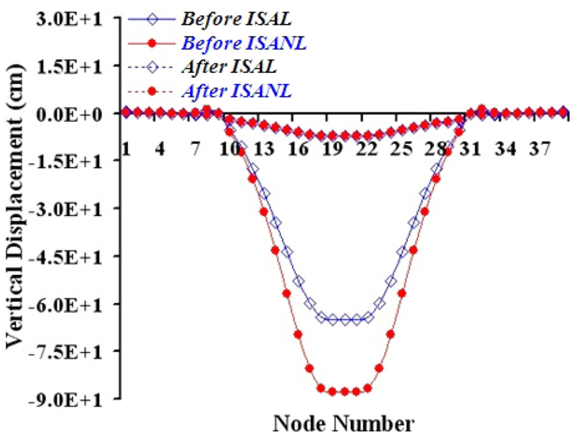
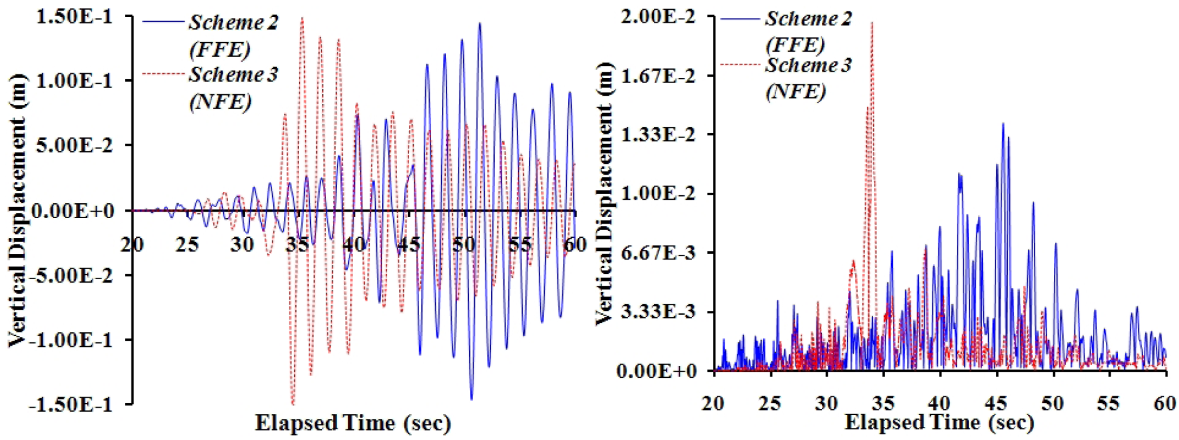


Fig. 11 Comparisons of initial shape by advanced probabilistic finite element algorithm

categorizing them into linear and nonlinear analysis. It indicates the significant difference from the response phases before and after the initial shape analysis.

Displacement of the standard design by probabilistic method was evaluated using APFEA, respectively, to 20^{th} node (central span girder; vertical displacement) and 40^{th} node (the highest member of pylon; horizontal displacement). The mean response of maximum (minimum) displacement of the girder was 15.47(-14.60), 14.50(-14.66) and 14.87(-15.68) cm. The maximum standard deviation response was 1.645, 1.400 and 1.995, respectively. The mean response of maximum (minimum) displacement of the pylon was 2.744(-2.632), 2.646(-2.478) and 2.590(-2.758) cm, and the maximum standard deviation response was 0.315, 0.280 and 0.385 cm, respectively. COV to the absolute maximum displacement response of the girder and pylon was found to be 10.633, 9.551, 12.723% and 11.480, 10.582, 13.959%, respectively. As indicated above, displacement response of Scheme 1 was found to be greater than Scheme 2 and such a phase appeared to be dependent on consideration of non-linearity in evaluating the static analysis. When it comes to Scheme 3 which considered NFE, it indicated the most sensitive response to displacement response of SCS bridges



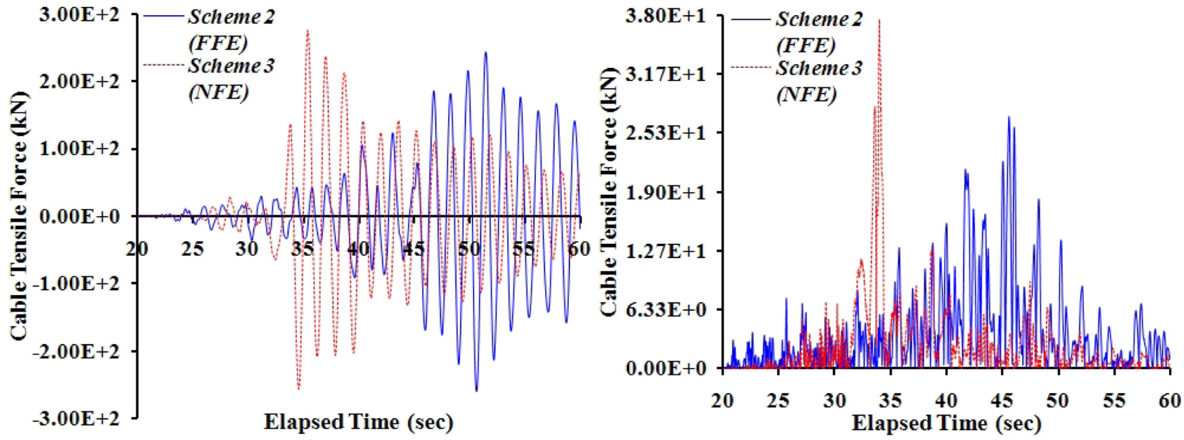
(a) Mean value (b) Std. deviation value

Fig. 12 Mean and std. deviation of vertical displacement values by advanced probabilistic finite element algorithm (Node #20)

which is the long-period structures and the response was accordingly greater than Scheme 1 or 2. Fig. 12 illustrates the result after comparing the mean and standard deviation response to vertical displacement (20^{th} node) of the girder in Scheme 2 & 3.

In reviewing the member response of the standard design by probabilistic method, axial force and bending moment of critical members were evaluated using APFEA. In case of the girder, mean response, standard deviation response and COV of the 42th member which is the connecting member of the girder and pylon and the 53^{rd} member which is the central member of main span. In case of pylon, 85^{th} member which is the connecting member of the girder and pylon and the 87^{th} member which is the lowest member were evaluated. The absolute maximum mean and standard deviation response to axial force and bending moment of the girder in Scheme 3 appeared to be greater than Scheme 1 or 2 from the aspect of COV. COV to axial force and bending moment of the 85th member of the pylon appeared to be similar with that of the girder but the analysis of the 87^{th} member of pylon showed contrastive result due to the effect of member stiffness. When it comes to Scheme 3 which considered NEF, the mean response to absolute maximum axial force (bending moment) of 42^{nd} and 53^{rd} member in Scheme 3 was 1345.60, 857.53 kN (1921.50, 5036.37 kN-m) and the maximum standard deviation response was 173.78, 117.25 kN (423.89, 491.02 kN-m). When the absolute maximum axial force (bending moment) was occurred, COV in Scheme 1~3 was 11.96, 10.68, 12.91(21.86, 16.52, 22.06)% and 11.81, 10.22, 13.67(8.17, 7.19, 9.75)%, respectively. The absolute maximum axial force (bending moment) of the 85^{th} and 87^{th} member was estimated at 1906.66, 1910.47 kN (2099.02, 1880 kN-m) and the maximum standard deviation response was 272.86, 390.41 kN (263.38, 432.32 kN-m). COV in Scheme 1~3 was 12.53, 11.09, 14.31(8.14, 8.93, 12.55)% and 30.50, 25.85, 20.44(29.59, 31.49, 22.99)%. The response of cable tensile force of the standard design by probabilistic method was evaluated using APFEA for the cable at the end of side span (1^{st} & 17^{th} cable) and 7^{th} cable on side span.

Fig. 13 illustrates the result after comparing the mean and standard deviation response in Scheme 2 & 3 with the 1^{st} cable. The absolute maximum mean and standard deviation response in Scheme 1 was evaluated relatively higher than those in Scheme 2 & 3. Such a phase appeared to be



(a) Mean value (b) Std. deviation value

Fig. 13 Mean and std. deviation of cable tensile force values by advanced probabilistic finite element algorithm (Cable #1)

Table 7 Response ratios of member force; displacement; cable tensile force

Critical nodes; members	Probabilistic method; member force by APFEA		Scheme 2/ Scheme 1	Scheme 3/ Scheme 1	Scheme 3/ Scheme 2
G_{42}	Bending Moment	Mean	1.491	1.269	0.851
		Std. Deviation	1.127	1.281	1.136
P_{85}		Mean	1.032	0.997	0.966
		Std. Deviation	1.132	1.537	1.358
G_{42}	Axial Force; Cable Tensile Force	Mean	0.918	1.040	1.133
		Std. Deviation	0.820	1.123	1.369
P_{85}		Mean	0.919	0.966	1.051
		Std. Deviation	0.813	1.103	1.357
C_7		Mean	0.947	0.987	1.042
		Std. Deviation	0.858	1.114	1.299
N_{20}	Displacement	Mean	0.948	1.014	1.070
		Std. Deviation	0.851	1.213	1.425

influenced by cable sag in the process of determining the initial tensile force, like the displacement response. When it comes to Scheme 3, mean response absolute maximum cable tensile force of the 1st (7th and 17th) cable was estimated at 275.58(111.87 and 272.52) KN while the maximum standard deviation response was 37.56(18.55; 30.17) kN. When the absolute maximum cable tensile force is occurred, COV by probabilistic method 11.67, 10.44, 13.63(14.68, 13.29, 16.58 and 9.28, 8.33, 11.07)%, indicating COV in Scheme 3 was evaluated relatively higher than others. To effectively evaluate the response aspect depending on probabilistic method, Table 7 shows the highlight as a result of comparing the mean and standard deviation response rate of the critical members and Fig. 14 illustrates the response of COV of critical members (Scheme 3/ Scheme 2).

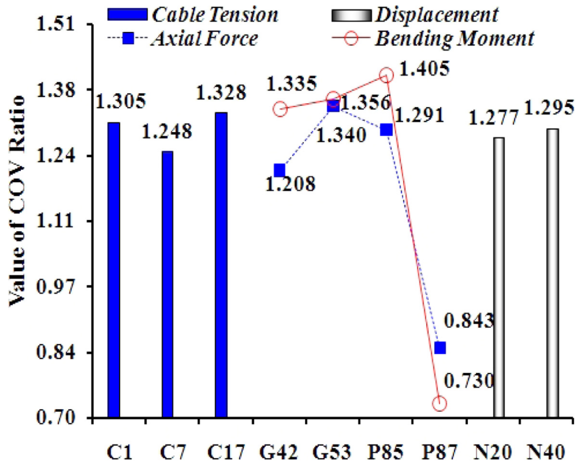


Fig. 14 COV ratios of critical member force, displacement and cable tensile force

4. Optimal design by minimization of life-cycle cost with reliability assessment results

4.1 Reliability assessment of steel cable-stayed bridges

RA was carried out using ENDT method based on Table 8. When it comes to ultimate resistant stress (δ_{urs}), COV and bias factor of the girder and pylon (cable) was assumed as 15.0%, 1.12 (12.0%, 1.12) and random variable was normal distribution with non-correlation. Ultimate resistant stress of the cable was 1030.0 Mpa and the girder and pylon were 190.0 Mpa (Korea's Ministry of Construction & Transportation (KMOCT) 2005). As the limit state function, member force response and cable tensile force by member stiffness, dead load, live load and earthquake load were defined as the function of random variable with aleatory uncertainty and defined as Eq. (8) (Korea's Ministry of Construction & Transportation (KMOCT) 2005). Aleatory uncertainty of member force response and cable tensile force resulting from dead & live load was estimated using the result of static SFEA (Han 2011).

$$g(\cdot)_{f_{g,p}} = f_{y_{g,p}} - \frac{P(\cdot)_{g,p}}{A(\cdot)_{g,p}} - \frac{M(\cdot)_{g,p}}{I(\cdot)_{g,p}} c_y;\ \ g(\cdot)_{fc} = f_{y_c} - \frac{T(\cdot)_c}{A(\cdot)_c} \tag{8}$$

where, $f_{g,p}$, f_c=stress in girders, pylons and cables, respectively;

$P(\cdot)_{g,p}$, $M(\cdot)_{g,p}$=axial force and bending moment in girders and pylons, respectively;

$T(\cdot)_c$=cable tensile forces; $A(\cdot)_{g,p}$, $A(\cdot)_c$=cross section of girders, pylons and cables, respectively;

$I(\cdot)_{g,p}$=moment of inertia of girders and pylons, respectively;

c_y=maximum distance from center of section;

$f_{y_{g,p}}$ and f_{y_c} are the ultimate resistant strengths of the girder, pylon and cable, respectively

The result of RA which is the quantitative index for probabilistic safety enables to identify the varying aspect depending on probabilistic method and thus, it will provide important information needed for structural design and seismic capacity review. According to Scheme 3, failure probability of the critical members was estimated at 2.095E-06, 8.410E-04, 7.497E-08 (1^{st}, 7^{th} and 17^{th} cable),

Table 8 Absolute maximum mean and std. deviation values by probabilistic method (unit: kN; kN-m)

Critical members	Probabilistic method; member force by APFEA		Static SFEA (Han 2011)		Dynamic SFEA		
			Scheme 1	Scheme 2; 3	Scheme 1	Scheme 2	Scheme 3
G_{42}	Bending Moment	Mean	25397.99	24430.97	1514.38	2257.64	1921.50
		Std. Deviation	2578.17	2588.26	330.98	373.00	423.89
		COV (%)	10.151	10.594	21.86	16.52	22.06
		Bias Factor*	1.08	1.08	1.08	1.08	1.08
P_{85}		Mean	10665.36	11224.90	2105.18	2172.66	2099.02
		Std. Deviation	1347.37	1359.83	171.39	193.97	263.38
		COV (%)	12.633	12.114	8.14	8.93	12.55
		Bias Factor	1.08	1.08	1.08	1.08	1.08
G_{42}	Axial Force; Cable Tensile Force	Mean	19496.39	19468.39	1293.67	1188.15	1345.60
		Std. Deviation	622.73	622.38	154.74	126.94	173.78
		COV (%)	3.194	3.197	11.96	10.68	12.91
		Bias Factor	1.05	1.05	1.08	1.08	1.08
P_{85}		Mean	35403.56	35532.80	1973.58	1813.84	1906.66
		Std. Deviation	1025.30	1025.66	247.27	201.13	272.86
		COV (%)	2.896	2.887	12.53	11.09	14.31
		Bias Factor	1.05	1.05	1.08	1.08	1.08
C_7		Mean	3861.87	3831.08	113.39	107.41	111.87
		Std. Deviation	272.90	269.601	16.65	14.28	18.55
		COV (%)	7.067	7.037	14.68	13.29	16.58
		Bias Factor	1.05	1.05	1.08	1.08	1.08

*Bias Factor: Ratio of Actual Values to Nominal Values

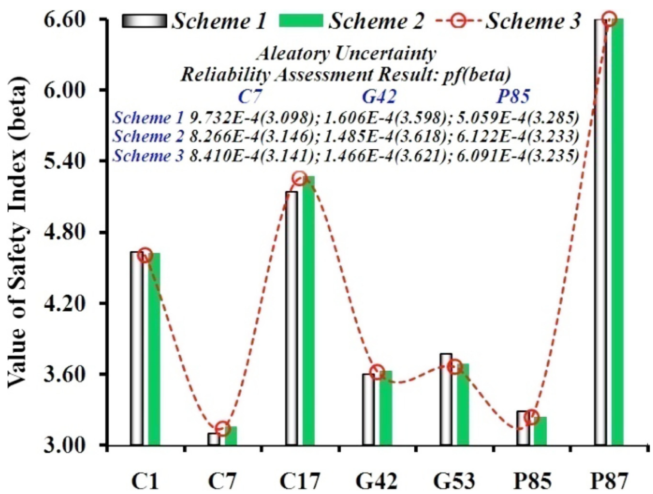


Fig. 15 Comparison of safety indices by probabilistic method

1.466E-04, 1.260E-04(42th and 53th girder) and 6.091E-04, 2.082E-11(85th and 87th pylon). Fig. 15 shows the comparison of safety indices of each member by probabilistic method. In order to determine optimal design method depending on scheme 3 which was considered NFE, SFEA and RA were also performed for the designs with varying member sections.

Table 9 System safety index and failure probability by probabilistic method

Alternative designs	Girder #42 (pf1)	Pylon #85 (pf2)	Cable #7 (pf3)	System Failure Probability (pfs)	Corresponding Safety Index (beta)
80-Percentile	2.113E-03	8.616E-03	2.804E-02	3.877E-02	1.7652
90-Percentile	5.144E-04	2.156E-03	4.849E-03	7.519E-03	2.4314
95-Percentile	2.694E-04	1.128E-03	2.007E-03	3.404E-03	2.7061
Standard Design	1.466E-04	6.091E-04	8.410E-04	1.597E-03	2.9485
105-Percentile	8.274E-05	3.395E-04	3.590E-04	7.812E-04	3.1628
110-Percentile	4.831E-05	1.951E-04	1.569E-04	4.003E-04	3.3526
120-Percentile	1.809E-05	7.025E-05	3.250E-05	1.208E-04	3.6709

Table 9 summarizes the system failure probability and corresponding safety indices. The failure probability of the bridge system was calculated the union of the maximum failure probabilities of the critical members (42^{th} girder, 85^{th} pylon and 7^{th} cable) because the modeling of SCS bridges is composed of series model. The failure probabilities of critical member were assumed statistically independent (Ang and Tang 2007).

4.2 Formulations of minimum life-cycle cost

Currently, the concept of the expected LCC has been applied to the design of bridges, as well as to the maintenance of individual bridges and bridge networks. The minimum LCC design of a structural system may be formulated as follow Eq. (9).

$$E(LCC) = \overline{COST}_{ICT} + \overline{COST}_{MCT} + \overline{COST}_{DCT} \tag{9}$$

where, E(LCC) = the expected life-cycle cost;

$\overline{COST}_{ICT} = \overline{COST}_{dct} + \overline{COST}_{cct} + \overline{COST}_{ltct}$ =the initial cost of a structure or system;

$\overline{COST}_{dct}$ =design costs; $\overline{COST}_{cct}$ =construction costs; $\overline{COST}_{ltct}$ =load testing costs;

$\overline{COST}_{MCT} = \overline{COST}_{cict} + \overline{COST}_{dict} + \overline{COST}_{rrct}$ =the maintenance cost over the lifetime of the structure;

$\overline{COST}_{cict}$ =current inspection costs; $\overline{COST}_{dict}$ =detail inspection costs;

$\overline{COST}_{rrct}$ =repair and reinforcement costs;

$\overline{COST}_{DCT}$ =the damage cost over the lifetime of the structure

In estimating LCC, there is seldom sufficient data or information to evaluate the maintenance cost. Therefore, estimating the maintenance cost may require reliance on experiences and judgments gained from similar structures. The maintenance cost of SCS bridges was assessed from information of economic reports and maintenance ordinances for bridge safety in Korea. In order to determine the optimal design based on minimum expected LCC, nine alternative designs were considered, including the standard one based on current code, by increasing and decreasing the member sections relative to the standard design. The initial cost for the standard design was based on information from construction reports. All of the initial costs for the standard design and the various alternative design sections were shown in Fig. 16.

The damage cost can be obtained from the failure probability and cost associated with damage (Koskito and Ellingwood, 1997). The damage cost may be composed of several cost items, as

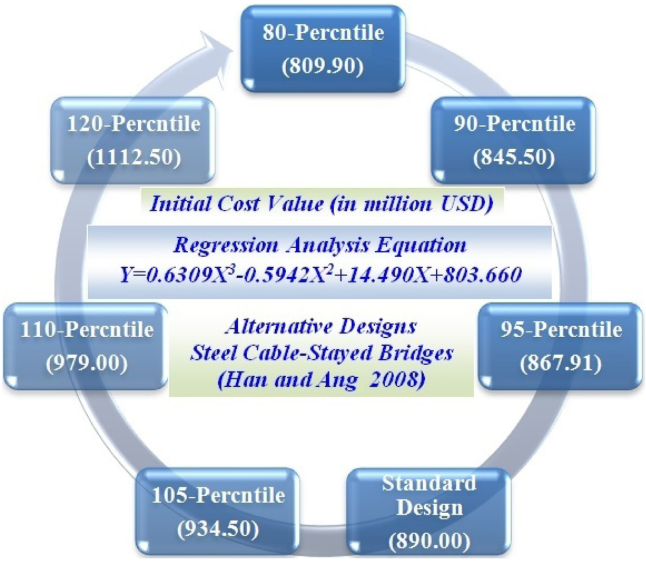


Fig. 16 Initial costs of various design alternatives

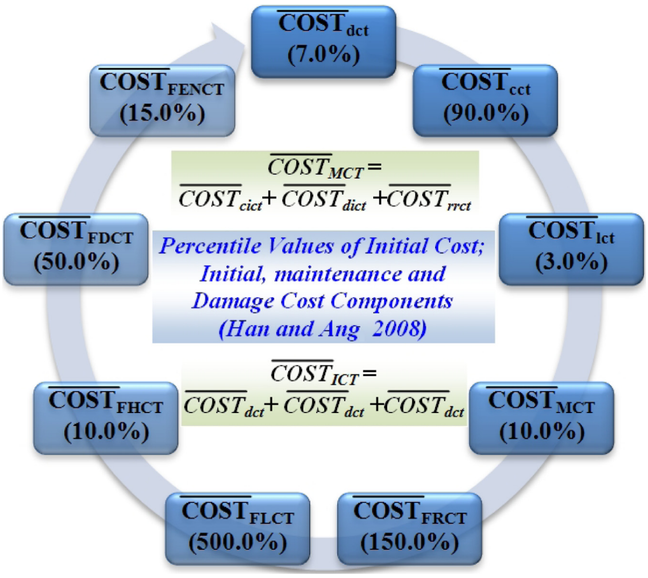


Fig. 17 Expected LCC items of standard design

shown in Eq. (10) (Frangopol and Lin 1997). In estimating the expected LCC of SCS bridges, the initial cost, the maintenance and damage cost items can be summarized by the ratio of initial cost as shown in Fig. 17. In this study, for maintenance cost of the standard design, 10% of initial cost was applied and for maintenance cost of each design alternative, it's assumed to be in inverse proportion to variation of initial cost. The expected LCC can be expressed considering present value factor (PVF) as indicated in Eq. (11) (Han and Park 2009).

$$\overline{COST}_{DCT} = \overline{COST}_{FRCT} + \overline{COST}_{FLCT} + \overline{COST}_{FHCT} + \overline{COST}_{FDCT} + \overline{COST}_{FENCT} \tag{10}$$

where, $\overline{COST}_{FRCT}$=bridge replacement costs; $\overline{COST}_{FLCT}$=loss of lives and equipment costs; $\overline{COST}_{FHCT}$=historical and cultural costs; $\overline{COST}_{FDCT}$=functional disruption costs; $\overline{COST}_{FENCT}$=social and environmental costs

$$E(LCC) = \overline{COST}_{ICT}$$

$$+ \; PVF(\overline{COST}_{MCT} + \overline{COST}_{FRCT} + \overline{COST}_{FLCT} + \overline{COST}_{FHCT} + \overline{COST}_{FDCT} + \overline{COST}_{FENCT}) \tag{11}$$

where, $PVF = \dfrac{[1-\exp(-\ln(1+Q_{COST})L_{STRUCTURE})]}{[\ln(1+Q_{COST})L_{STRUCTURE}]}$

$L_{STRUCTURE}$ =lifetime of SCS bridges; Q_{COST}=annual discount rate

4.3 Determination optimal safety assessment with minimum life-cycle cost

The results were depicted between the safety index and the expected LCC, while considering only the aleatory uncertainties, as shown in Fig. 18. The minimum LCC (970.75 in million USD) and optimal safety index (2.706) were estimated at turn over point when variation of damage cost begins to slow and variation of initial cost begins to increase. The safety index (failure probability) of optimal design calculated above needs to be re-evaluated considering the effect of epistemic uncertainty contained each random variable (Han and Ang 2008). With regard to epistemic uncertainty, it's assumed as indicated in Table 10 from the aspect of COV as recommended by the experts because of insufficient data on basic characteristic and evaluation. Sensitivity analysis was carried out using MCS, which was carried out using Matlab 7.0 and to make sure the accuracy of the sensitivity analysis, it's determined to perform the simulation 10000 times. Generally, structural capacity tends

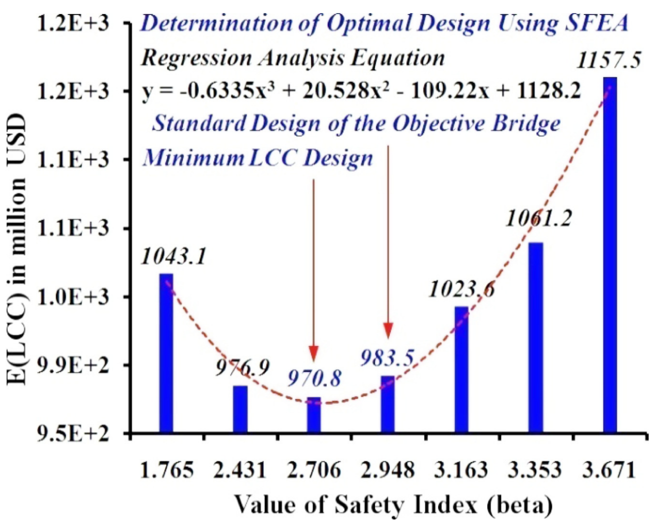


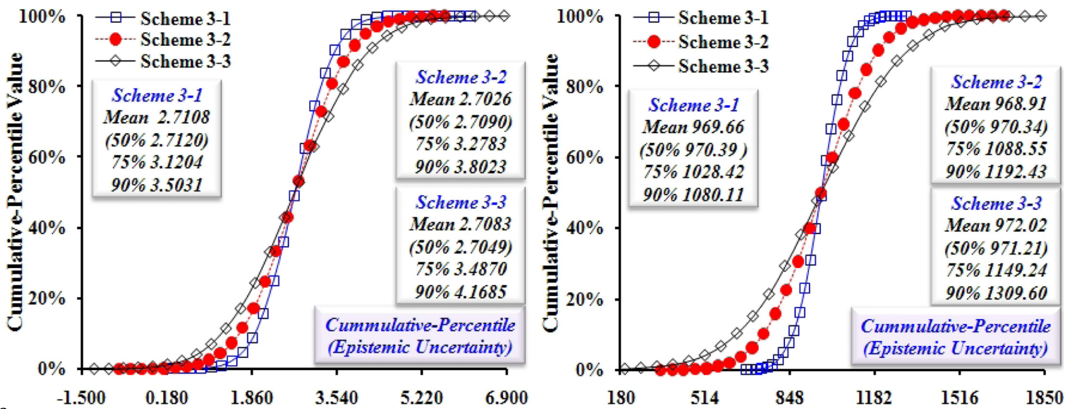
Fig. 18 Expected life-cycle cost versus safety index considering aleatory uncertainties

Table 10 Epistemic uncertainties in random variables for sensitivity analysis

Random variables		Coefficient of variation			Distribution types
		Scheme 3-1	Scheme 3-2	Scheme 3-3	
Ultimate strength	Δ_{urs}	0.05	0.10	0.15	Normal Type
Cross section	Δ_{cs}	0.04	0.05	0.06	Normal Type
Moment of inertia	Δ_{mi}	0.01	0.02	0.03	Normal Type
Dead load	Δ_{dl}	0.05	0.10	0.15	Normal Type
Live load	Δ_{ll}	0.15	0.20	0.25	Log-normal Type
Earthquake load	Δ_{el}	0.15	0.20	0.25	Log-normal Type

Table 11 Epistemic uncertainties in cost items for sensitivity analysis

Cost items	Coefficient of variation		
	Scheme 3-1	Scheme 3-2	Scheme 3-3
$\overline{COST}_{ICT}$	$N(1.0, 0.10)$	$N(1.0, 0.20)$	$N(1.0, 0.30)$
$\overline{COST}_{MCT}$	$N(1.0, 0.10)$	$N(1.0, 0.20)$	$N(1.0, 0.30)$
$\overline{COST}_{FRCT}$	10%	20%	30%
$\overline{COST}_{FLCT}$	30%	40%	50%
$\overline{COST}_{FHCT}$	30%	40%	50%
$\overline{COST}_{FDCT}$	30%	40%	50%
$\overline{COST}_{FENCT}$	70%	80%	90%
$\overline{COST}_{DCT}$		$7.250\,\overline{COST}_{ICT}$	
$Var(\overline{COST}_{DCT})$	$2.307\,\overline{COST}_{ICT}^{\,2}$	$4.146\,\overline{COST}_{ICT}^{\,2}$	$6.536\,\overline{COST}_{ICT}^{\,2}$
$COV(\overline{COST}_{DCT})$	$N(1.0, 0.210)$	$N(1.0, 0.281)$	$N(1.0, 0.353)$
Scheme 3-1	$Var(LCC)=(0.10\,\overline{COST}_{ICT})^2+(0.10\,\overline{COST}_{MCT})^2+(0.210\,\overline{COST}_{DCT})^2$		
Scheme 3-2	$Var(LCC)=(0.20\,\overline{COST}_{ICT})^2+(0.20\,\overline{COST}_{MCT})^2+(0.281\,\overline{COST}_{DCT})^2$		
Scheme 3-3	$Var(LCC)=(0.30\,\overline{COST}_{ICT})^2+(0.30\,\overline{COST}_{MCT})^2+(0.353\,\overline{COST}_{DCT})^2$		



to be conservatively evaluated in design process. That is, safety level and risk were evaluated by the standard which is higher than the actual. Thus, evaluation in this study was performed for the mean

(50-percentile), 75-percentile and 90-percentile of the RA result. In addition, the effect of epistemic uncertainty shall be considered in each cost item of optimal design. Table 11 summarizes the epistemic uncertainties (normal distribution) included in each cost item of optimal design method for implementing sensitivity analysis. Based on the information presented in Fig. 17 and Table 11, the mean and variance of the damage cost were defined by scheme. The COVs of the damage cost would be 0.210, 0.281 and 0.353 respectively. Fig. 19 illustrates the cumulative-percentile of safety indecies and minimum LCC which considers epistemic uncertainties using MCS.

5. Conclusions

Advanced probabilistic FE algorithm (APFEA) which enables to perform the static and dynamic SFEA was developed in this study. Based on the result assessment from probabilistic method, the basic information for probabilistic safety-optimal design evaluation of SCS bridges using the results of SFEA, RA and minimum LCC considering aleatory & epistemic uncertainty was proposed. The findings and conclusions of this study can be summarized as follows: APFEA enables to efficiently carry out the SFEA depending on aleatory uncertainty of random variable and is expected to serve the enhanced analytical tool which will systematically execute the RA of such complex structure as SCS bridges. Given the aspect that the result of SFEA was evaluated in various ways depending on consideration of initial shape and nonlinearity, further assessment process is a must for rational structural design of SCS bridges. Taking into account of the fact that the COV varies significantly depending on earthquake characteristic, seismic design and evaluation criterion shall be further supplemented. The failure probability by probabilistic method which was evaluated considering aleatory uncertainty and corresponding safety index were expected to offer the virtually useful assessment specification which is needed for quantitative safety of SCS bridges. When it comes to Scheme 3 which considered NFE, safety index and failure probability which serve the standard of structural capacity were evaluated as 3.120~3.487 and 3.926E-3~4.364E-3 (75-percentile) and 3.503~4.169 and 4.394E-3~5.197E-3 (90-percentile). In addition, the minimum LCC of optimal design method which is the barometer of cost efficiency was 1028.42~1149.24 (75-percentile) and 1080.11~1309.60 (90-percentile) respectively (in million USD) as a result of reviewing the effect of epistemic uncertainty. The assessment result with regard to epistemic uncertainty is expected to provide the fundamental data which is necessary for logical structural capacity assessment as well as to deal with the risk for the designer, client and decision-maker at design and construction stage. However, it's noteworthy that the result of sensitivity analysis may vary depending on phase of COV of epistemic uncertainty. Further static & dynamic SFEA shall be carried out, taking into account of aleatory & epistemic uncertainty included in ground condition or wind load as random variable and more in-depth study on optimal design of SCS bridges needs to continue, making use of RA result according to SFEA.

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