Beam-rotating machinery system active vibration control using a fuzzy input estimation method and LQG control technique combination

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Abstract. This study proposes an active control method to suppress beam-rotating machinery system vibrations. The present control method is a combination of the fuzzy input estimation method (FIEM) and linear quadratic Gaussian problem (LQG) algorithms. The FIEM can estimate the unknown input and optimal states by measuring the dynamic displacement, the optimal estimated states into the feedback control; thereby obtaining the optimal control force for a random linear system. Active vibration control of a beam-rotating machinery system is performed to verify the feasibility and effectiveness of the proposed algorithm. The simulation results demonstrate that the proposed method can suppress vibrations in a beam-machine system more efficiently than the conventional LQG method.

Keywords: active control; beam-rotating machinery; fuzzy input estimation method (FIEM); linear quadratic Gaussian problem (LQG); feedback control

1. Introduction

Rotating machinery, such as turbines, compressors and pumps are commonly used in the most important military and industrial applications. Ensuring the availability and reliability of rotating machinery is a very important task. Vibration amplitudes are a key index in monitoring rotating machinery. Vibrations caused by mass unbalance are a common problem in rotating machinery. Rotor unbalance occurs when the principal inertia axis of the rotor does not coincide with its geometrical axis and leads to synchronous vibrations and significant undesirable forces transmitted to the mechanical elements and supports. Under this condition, catastrophic failures caused by crack propagation can occur because the fatigue process is intensified. Therefore, significant research effort has been devoted to the development and improvement of mechanisms capable of attenuating undesirable vibrations in rotating machinery (Mahfoud *et al.* 2009, Simoes *et al.* 2007, Blanco *et al.* 2010).

The control approaches for rotating machines are clustered into three main categories, passive, semi-active and active control techniques. Passive techniques are normally performed by devices known as absorbers or isolators. In terms of passive control technique, the unwanted vibration problem can be effectively solved using passive control techniques. It is well established that the

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vibration of rotating machinery can be reduced by introducing passive devices into the system (Cunningham 1978, Nikolajsen and Holmes 1979). In semi-active techniques the vibration is attenuated through an indirect manner by changing the machine structural parameters, such as damping and stiffness (Kori *et al.* 2008, Ying *et al.* 2009, Zheng *et al.* 2009). Active control techniques promise vibration suppression over a broadband of frequencies in which the suppression is performed by incorporating active actuators to the machine to act directly against the vibratory loads (Cavalini *et al.* 2011).

Various active control technique methods have been developed. Optimal control methods, such as linear quadratic regulator (LQR) and LQG, are popular with many structural engineers. The traditional linear control technique has difficulty maintaining robust control performance under the random dynamic input condition. The optimal control theory-basis LQG has higher control performance when the system is in the uncertain disturbance condition. Therefore, it is popular with many engineers because the controlled framework is easier to engineer. Yang (1975) applied the optimal control theorem to control civil engineering structure vibrations under stochastic dynamic loads such as earthquakes and wind loads. He used the instantaneous optimal control method, which minimizes the quadratic performance index at every time instant, to overcome the deficiency of neglecting the earthquake input (Yang et al. 1987). That the input disturbances needed to be measurable was his chief defect. Moreover, the control performances were improved only slightly. Chen et al. (2007) proposed a mixed robust/optimal control approach is proposed to treat the active vibration control (or active vibration suppression) problems of flexible structural systems. Aldemir (2009) presented a simple evaluation of the disturbance weighting parameter of well-known minimax disturbance attenuation problems derived for earthquake-excited structures. Bakioglu et al. (2001) proposed a new numerical algorithm for the sub-optimal solution of the optimal closed-openloop control based on the prediction of near-future earthquake excitation using the Taylor series method and the Kalman filtering technique. Aldemir et al. (2001) presented the analytical solution of the modified linear quadratic regulator (MLQR) problem including a parameter α known as system stability order in the presence of unknown seismic excitation. Akhiev et al. (2002) used the multipoint performance index for the vibration suppression of earthquake-excited structures. Although the conventional LQG controller has a specific level of interference suppression, it is weak in maintaining high suppression performance of external complex, arbitrary disturbance loads. Therefore, better control methods require an interference compensator for robust control performance. The influence of external loads is not considered in optimal controller designs because external load disturbances are immeasurable or inestimable in control force calculation. The LQG controller includes a low pass filter that is not affected by high frequency load disturbance inputs. The controller will have poor performance because the low frequency load disturbance inputs are neglected in the control system (Lan et al. 2004).

This study first proposes the input estimated method combined with the LQG technique for active vibration control of a beam-rotating machinery system. The input estimation method can estimate dynamic loads online. An active LQG controller can apply the same inverse control forces on a structural system. The presented control method is a synthesized algorithm that can suppress the vibrations more effectively due to the actions of the proper control forces. The input estimation method has been successfully used to inversely solve 1-D and 2-D heat conduction problems (Tuan *et al.* 1997, 1998) and identify input forces for structural systems (Deng *et al.* 2006, Lee *et al.* 2008, Chen *et al.* 2008). The input estimation method uses the recursive form to process the measurement data. As opposed to the batch process, using the recursive form is an on-line process

that has higher efficiency. By directly synthesizing the robust input estimated method with the LQG controller, this work first presents an efficient robust active vibration control technique for a beam-rotating machinery system. By comparing these results with the conventional LQG technique, the efficiency, adaptability and robustness of the proposed control technique are demonstrated.

2. Mathematical model

This study presented the active vibration control of a beam-rotating machinery system in the random disturbance condition. A skeletal diagram and simplified mathematical model of the beam-rotating machinery system are shown in Fig. 1 (Mario 1986). The beam structure is modeled as a simple beam with a total span, L, the flexural stiffness constant, EI, and the viscous proportional damping coefficient, C. An eccentric rotating machine was mounted at the middle of the beam span. The machine is excited by a vibratory force generated by an unbalanced member, that is the eccentric distance, e_o . A simulation sensor was installed under the middle of the beam to measure the displacement. A control force was applied on the middle of the beam. The unknown vibratory force can be estimated from the beam displacement measured by the simulation sensor. Inversely estimating the vibratory input and active vibration control problems of a beam-rotating machinery system under uncertain disturbance is investigated in this study. Inversely estimating the unknown input and optimal estimated state are substituted for the control law state variable in the optimal control force calculation.



Fig. 1 A skeleton diagram (a), mathematics model (b) and free body diagram (c) of the beam-rotating machine system

The mass of the motor and eccentric rotor, m, the mass of the eccentric rotor, m' are shown in Fig. 1. From Figs. 1(b) and (c), the vertical displacement of the eccentric rotor mass, y_1 can be shown as

$$y_1 = y + e_0 \sin \overline{\omega} t \tag{1}$$

According to Fig. 1(b), the force balance conditions in the vertical direction include the inertia force in both the non-rotated and unbalanced mass. Therefore, the equations of motion for the system are shown as follows

$$(m - m')\ddot{y} + m'\ddot{y}_1 + c\dot{y} + ky = 0$$
⁽²⁾

Substituting Eq. (1) in Eq. (2) gives

$$(m-m')\ddot{y}+m'(\ddot{y}-e_0\overline{\omega}^2\sin\overline{\omega}t)+c\dot{y}+ky=0$$
(3)

Eq. (3) can be rewritten as

$$m\ddot{y} + c\dot{y} + ky = m'e_0\overline{\omega}^2 sin\overline{\omega}t \tag{4}$$

Dividing by *m* gives

$$\ddot{y} + 2\zeta \overline{\omega} \dot{y} + \overline{\omega}^2 y = \frac{m'}{m} e_0 \overline{\omega}^2 sin\overline{\omega} t$$
⁽⁵⁾

where $\overline{\omega} = \sqrt{k/m}$, $\zeta = \frac{c}{2m\overline{\omega}}$ and the active vibration force input, $F(t) = m'e_0\overline{\omega}^2 \sin\overline{\omega}t$. The active vibration force inputs on a beam-rotating machinery system. The unknown input can

be inversely estimated first using the fuzzy input estimation method. The conventional LQG method is then used to find the active control force. Assuming a beam-rotating machinery system under active vibration and control forces, the equations of motion can be written as below

$$\ddot{y} + 2\zeta\omega\dot{y} + \overline{\omega}^2 y = \frac{m'}{m}e_0\overline{\omega}^2\sin\overline{\omega}t + \frac{D}{m}U(t)$$
(6)

where *D* denotes the control force distribution matrix and U(t) the control force vector. The input estimation algorithm is a calculation method using the state space. Therefore, the state equation and the measurement equation must be constructed before applying this method. To satisfy this situation, the equality, $X = [y(t) \dot{y}(t)]^T$ is used to transfer the movement equation into the state space form. The continuous-time state equation and measurement equation of the structural system is presented as follows (Tuan *et al.* 1996)

$$X(t) = AX(t) + BF(t) + GU(t)$$
(7)

$$Z(t) = HX(t) \tag{8}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -\omega^2 & -2\zeta \overline{\omega} \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1/m \end{bmatrix}, G = \begin{bmatrix} 0 \\ D/m \end{bmatrix}, H = \begin{bmatrix} 1 & 0 \end{bmatrix}, X(t) = \begin{bmatrix} X_1 & X_2 \end{bmatrix}^T$$

A and B are both constant matrices composed of the *n*th natural frequency and the inertia moment

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of the structural system. X(t) is the modal state vector. F(t) is the input dynamic load. Z(t) is the observation vector and H is the measurement matrix.

Using the sampling time, Δt , to sample the continuous-time state Eq. (7), and assuming that the system model error, w(t), is Gaussian white noise with zero mean, the discrete-time state equation can be obtained as follows (Tuan *et al.* 1996).

$$X(k+1) = \Phi X(k) + \Gamma[F(k) + w(k)] + \Lambda U(k)$$
(9)

where

$$X(k) = [X_1(k) \ X_2(k)]^T$$
$$\Phi = exp(A\Delta t)$$
$$\Gamma = \int_{k\Delta t}^{(k+1)\Delta t} exp\{A[(k+1)\Delta t - \tau]\}Bd\tau$$
$$\Lambda = \int_{k\Delta t}^{(k+1)\Delta t} exp\{A[(k+1)\Delta t - \tau]\}Gd\tau$$

X(k) is the discrete state vector. Φ is the state transition matrix. Γ and Λ are the coefficient matrices of F(k) and U(k), respectively. F(k) is the deterministic dynamic input sequence. U(k) is the control force vector. Δt is the sampling interval, and w(k) is the processing error vector, which is assumed as Gaussian white noise with variance $E\{w(k)w^T(k)\}=Q\delta_{kj}, Q=Q_w \times I_{2n\times 2n}$. Here, Q is the discrete-time process noise covariance matrix and δ_{kj} is the Kronecker delta. In Eq. (9), when describing the active characteristics of the structural system, an additional term, w(k), can be used to represent the uncertainty in a numerical manner. The uncertainty could be a random disturbance, the uncertain parameters, or the error due to the over-simplified numerical model assumption.

Furthermore, the measurement noise is added into the discrete measurement equation. A discretetime statistic model of the measurement vector can be presented as follows

$$Z(k) = HX(k) + v(k) \tag{10}$$

Z(k) is the discrete observation vector. v(k) represents the measurement noise vector and is assumed to be Gaussian white noise with zero mean and the variance, $E\{v(k)v^{T}(k)\}=R\delta_{kj}$, $R=R_{v}\times I_{2n\times 2n}$, R is the discrete-time measurement noise covariance matrix.

3. FIEM combined LQG design

For standard linear quadratic Gaussian problem, the system under control is assumed to be described by the stochastic discrete-time state space equations as below (Lewis *et al.* 1995)

$$X(k) = \Phi X(k-1) + \Lambda U(k-1) + \Gamma w(k-1)$$
(11)

$$Z(k) = HX(k) + v(k) \tag{12}$$

where w(k) and v(k) are zero-mean white noise with variances Q and R, respectively. Generally speaking, the deterministic dynamic input sequence is neglected or assumed to be zeros in conventional

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LQG design. The conventional LQG method is used to find the active control force, U(k) as a function of state, X(k), $i \le k \le N-1$, so as to minimize the performance index

$$J_{i}(U) = E\left\{\frac{1}{2}X^{T}(N)Q_{0}X(N) + \frac{1}{2}\sum_{k=i}^{N-1} [X^{T}(k)Q_{1}X(k) + u^{T}(k)Q_{2}X(k)]\right\}$$
(13)

where $Q_1 \ge 0$, $Q_2 \ge 0$ and $Q_0 \ge 0$ are all symmetric weighting matrices. By using the separation theorem (Kwakernaak *et al.* 1972), the optimal feedback control force vector can be obtained

$$U(k) = -K_r(k)\hat{X}(k) \tag{14}$$

Here the regular gain $K_r(k)$ is given by

$$K_r(k) = \left[\Lambda^T P_2(k+1)\Lambda + Q_2\right]^{-1} \Lambda^T P_2(k+1)\Phi$$
(15)

where $P_2(k)$ is the solution of the discrete-time Riccati equation. The Riccati equation is shown below

$$P_{2}(k) = \Phi^{T} \{ P_{2}(k+1) - P_{2}(k+1)\Lambda [\Lambda^{T} P_{2}(k+1)\Lambda + Q_{2}]^{-1}\Lambda^{T} P_{2}(k+1) \} \Phi + Q_{1}, k \le N$$
(16)

$$P_2(N) = Q_0 \tag{17}$$

Assuming a limiting solution for the Riccati equation exists and is denoted by $P_2(\infty)$, then the corresponding steady-state regulator gain, which is a constant feedback gain

$$K_{\infty} = \left[\Lambda^T P_2(\infty)\Lambda + Q_2\right]^{-1} \Lambda^T P_2(\infty)\Phi$$
(18)

A synthesis active control method is proposed for structural vibration control considering the input disturbance forces. Combining the fuzzy input estimation method with LQG is proposed in this study. The input disturbance forces are estimated by the fuzzy input estimation method for the open loop control, which is used to cancel out the input disturbance forces. By combining the open loop and LQG feedback control laws, the synthesis control method is established as follows:

$$U(k) = -K_r(k)\hat{X}(k) - (\Lambda^T \Lambda)^{-1}\Gamma \hat{F}(k)$$
(19)

Here $\hat{F}(k)$ is the estimated input vector using the fuzzy input estimation method. $\hat{X}(k)$ is the state vector estimated by the Kalman filter with input forces and control forces terms. The equations are shown as follows (Tuan *et al.* 1996)

$$\hat{X}(k/k-1) = \Phi \hat{X}(k-1/k-1) + \Lambda U(k-1) + \Gamma \hat{F}(k-1)$$
(20)

$$P(k/k-1) = \Phi P(k-1/k-1)\Phi^{T} + \Gamma Q \Gamma^{T}$$
(21)

$$\hat{Z}(k) = Z(k) - H[\Phi \hat{X}(k/k-1) + \Lambda U(k-1) + \Gamma \hat{F}(k-1)]$$
(22)

$$S(k) = HP(k/k-1)H' + R$$
⁽²³⁾

$$K(k) = P(k/k-1)H^{T}S^{-1}(k)$$
(24)

$$\hat{X}(k/k) = \hat{X}(k/k-1) + K(k)\hat{Z}(k)$$
 (25)

$$P(k/k) = [I - K(k)H]P(k/k - 1)$$
(26)

In the above Eqs. (20) to (26), the superscript '-' indicates filter estimation. $\hat{X}(k/k-1)$ is the state estimation error covariance, $\hat{Z}(k)$ is the predictor residual, S(k) is the innovation covariance, K(k) is the Kalman gain, $\hat{X}(k/k)$ is the state filter, P(k/k) is the state filter error covariance.

The related recursive least square estimator equations are as follows (Tuan et al. 1996)

$$B_s(k) = H[\Phi M_s(k-1) + I]\Gamma$$
(27)

$$M_{s}(k) = [I - K(k)H][\Phi M_{s}(k-1) + I]$$
(28)

$$K_{b}(k) = \gamma^{-1} P_{b}(k-1) B_{s}^{T}(k) [B_{s}(k) \gamma^{-1} P_{b}(k-1) B_{s}^{T}(k) + S(k)]^{-1}$$
⁽²⁹⁾

$$P_b(k) = [I - K_b(k)B_s(k)]\gamma^{-1}P_b(k-1)$$
(30)

$$\hat{F}(k) = \hat{F}(k-1) + K_b(k) [\overline{Z}(k) - B_s(k)\hat{F}(k-1)]$$
(31)

where $\overline{Z}(k)$ is the bias innovation produced by the measurement noise and input disturbance, $K_b(k)$ is the correction gain. B(k) and M(k) are the sensitivity matrices. γ is the weighting factor. $P_b(k)$ is the error covariance of the input estimation process and $\hat{F}(k)$ is the estimated dynamic inputs.

Some filter parameters must be obtained before the filtering process, such as the state transition matrix of the structural system, Φ , the measurement matrix, H, the discrete-time processing noise covariance matrix, Q and the discrete-time measurement noise covariance matrix, R. The on-line state estimation, $\overline{X}(k/k-1)$ and state estimation error covariance, P(k/k-1) of the filter will be acquired when the observation vector is input immediately after the initial conditions X_0 and P_0 are drawn into the estimator. K(k) gets smaller as the processing noise covariance matrix, Q and the state filter error covariance get smaller according to Eqs. (21) and (24). This indicates that the new measurement is mitigating to the state predicted correction. K(k) gets smaller as the measurement noise covariance matrix, R, gets larger according to Eqs. (23) and (24). That is to say, the measurement error mitigates the state estimation of the estimator. In other words, the Kalman gain K(k) depends on R_v and Q_{w} . The above-mentioned principle is a key problem that the appropriate R_v and Q_w can be chosen in accordance with the system properties and the magnitude of the noise interference in the estimation process. R_v can be chosen in accordance with the precision of the measurement instrument. Q_w can be chosen in accordance with the modular error in the system. The Kalman gain can be slightly corrected with a higher precision measurement instrument, this is, the modular error in the system changes from big to small. For this reason the processing noise covariance can be defined as follows

$$Q_{w}(k+1) = Q_{w}(k) \times 10^{\alpha(k)}$$
(32)

where $\alpha(k)$ is the fuzzy accelerating factor chosen in the interval, [-1, 1]. The estimation precision

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Fig. 2 Membership functions of the fuzzy sets for $\theta(k)$, $\gamma(k)$ and $\alpha(k)$

		Input variable $\theta(k)$						
		ES	VS	SV	MV	LV	VL	EL
Output variables	$\alpha(k)$	EL	VL	LV	MV	SV	VS	ES
	$\gamma(k)$	EL	VL	LV	MV	SV	VS	ES

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Table 1 The fuzzy rule base
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gets better as the $\alpha(k)$ get smaller. Conversely, the estimation precision gets worse as $\alpha(k)$ gets larger.

The weighting factor $\gamma(k)$ is another important parameter that affects the estimation precision in the estimation process. It also plays the role as an adjustable parameter to control the estimator bandwidth or the gain magnitude of the recursive least square estimator. It can operate at each step based on the innovation produced by the Kalman filter. A fuzzy estimator has been proposed based on the fuzzy logic inference system. The processing noise covariance matrix and weighting factor can be adjusted at each step based on the innovation produced by the Kalman filter. The specific membership is defined by using the Gaussian functions shown in Fig. 2. A fuzzy rule base is a collection of fuzzy IF-THEN rules as Table 1. The detailed formulation of this technique can be found in the reference (Lee 2011). A design flow chart of the active vibration control for a beamrotating machinery system using the LQG control technique and fuzzy input estimation method is shown in Fig. 3.



Fig. 3 Flowchart of the FIEM combined with the LQG

4. Results and discussion

To verify the effectiveness and robustness of the presented approach in controlling the vibration force, inputs on the beam-machine are used to evaluate the present control algorithm. The vibration force inputs are generated by the unbalanced rotary machine under nominal operating conditions. The machine-beam system used in the numerical simulations is shown in Fig. 1. The simulation conditions and system parameters are given as follows. The beam structure is modeled as a simple beam composed of two standard steel, $S8 \times 23$ with a total span L=5 m. The cross-section area of the beam at the inertia moment is $I=2\times1.63=3.26$ m⁴. An eccentric rotating machine was mounted upon the middle of the beam span. The weight of the machine is W=71168 kg. The weight of the eccentric rotator is W'=177.92 kg. The machine is excited by the vibratory force generated by an unbalanced member, that is the eccentric distance, $e_0=0.1$ m. The vibration forces of the system is 10% critical damping. The machine vibration forces are harmonic at a constant frequency of 10 Hz. The viscous damping coefficient of the system is 10% critical damping. The machine vibration forces are harmonic at a constant frequency of 10 Hz. The viscous damping coefficient of the system is 10% critical damping. The machine vibration forces are harmonic at a constant frequency of 10 Hz. The viscous damping coefficient of the system is 10% critical damping. The machine vibration forces are harmonic at a constant frequency of 10 Hz.

4.1 Example: periodic sinusoidal vibration force input.

The periodic sinusoidal vibration force input generated by a machine mounted upon the beam structure under nominal operating conditions. This periodic sinusoidal vibration force input is shown as follows

$$F(t) = m' e_0 \overline{\omega}^2 \sin \overline{\omega} t \tag{33}$$

where m = 18 kg, $e_0 = 0.254 \text{ m}$ and $\overline{\omega} = 20 \text{ Hz}$. The initial processing noise variance, $Q_w(0) = 10^3$. The measurement noise variance, $R_v = \sigma^2 = 10^{-16}$, state weighting matrices $Q_0 = Q_1 = Q_s \times I_{1\times 1}$, $Q_s = 1 \times 10^9$, control weighting matrices $Q_2 = Q_c \times I_{1\times 1}$, $Q_c = 1$. By applying the active dynamic reaction which contains noise to

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Fig. 4 Time histories of the relative displacements using the periodic sinusoidal load input (10 Hz)



Fig. 5 The Variance in the Output Variable $\alpha(k)$

the presented control algorithm, the time histories of the beam-machine system responses with and without control are shown in Fig. 4. The FIEM combined with the LQG controller has better effective and precision performance than the conventional LQG controller. Fig. 5 shows that the estimator has great tracking performance for the larger output variable, $\alpha(k)$, chosen to generate larger processing noise variance, $Q_w(k)$, according to Eq. (22). The estimator has great noise reducing effect for the smaller output variable, $\alpha(k)$, chosen to generate smaller processing noise variance, $Q_w(k)$, when the unknown is a steady input system. In a word, the estimation precision gets better as the $\alpha(k)$ get smaller.



Fig. 6 The variance in the fuzzy weighting factor $\gamma(k)$



Fig. 7 The Variance in the Kalman Gain K(k)

Conversely, the estimation precision gets worse as $\alpha(k)$ gets larger. Fig. 6 shows that a smaller weighting factor can be chosen in the fuzzy recursive least square method when a larger unknown is input into the system. Note that the faster the forgetting effect is, the lower the smoothing effect will be, that is, it introduces oscillation. The fuzzy weighting factor $\gamma(k)$ is employed to compromise between the tracking capability upgrade and the loss of estimation precision. The Kalman gain K(k) is plotted for the acquisitions presented in Fig. 7. Fig. 7 shows that at the initial stage, the Kalman gain K(k) increases as the filter opens the bandwidth and acquires the necessary information from the measurements to form the estimates. During this stage, the estimation error covariance is large. When the filter has

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Fig. 8 Step response of LQG and LQG+IE controllers



Fig. 9 The estimation result using the periodic sinusoidal load input

acquired the correct estimate, the estimation error will be small and the Kalman gain K(k) decreases to a smaller value. The proposed control method can quickly converge to steady state. Fig. 8. shows the step response of LQG and LQG+IE controllers. According to Fig. 8, the proposed controller has quicker converged speed than conventional LQG controller. The estimation result of the periodic sinusoidal vibration force input can be obtained and shown in Fig. 9. Fig. 10 shows the overall time histories of the control forces required for the proposed method and LQG method. Moreover, a more rapid input frequency (15 Hz) was taken into account during the simulation process. Fig. 11 displays the estimation



Fig. 10 Time histories of the control forces of a machine-beam system



Fig. 11 The estimation results and time histories of the relative displacements

result of the input force and the time histories of the relative displacements. The time histories of the responses of a machine-beam system with and without control are shown in Fig.12. Fig. 13 shows time

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Fig. 12 Time histories of the relative displacements using the periodic sinusoidal load input (15 Hz)



Fig. 13 Time histories of the relative displacements using the difference input frequency (20 Hz and 25 Hz)

histories of the relative displacements using the difference input frequency (20 Hz and 25 Hz). Further, the proposed controller was demonstrated using the initial processing noise variance, $Q_w(0)=10^3$, the measurement noise variance, $R_v=\sigma^2=10^{-16}$ and input frequency, $\overline{\omega}=20$ Hz. Fig. 14 shows the estimation results and time histories of the relative displacements ($Q_w(0)=10^5$, $R_v=10^{-14}$ and $\overline{\omega}=25$ Hz). According the simulation results, the proposed control method, which combines the FIEM and the LQG controller, is suitable for dealing with the optimal control problem in the time-varying system model.



Fig. 14 The estimation results and time histories of the relative displacements ($Q_w(0)=10^5$, $R_v=10^{-14}$ and $\overline{\omega}$ =25 Hz)

5. Conclusions

In this study, a combination of the fuzzy input estimation method (FIEM) and the linear quadratic Gaussian problem (LQG) was proposed to suppress the vibrations of a machine mounted upon a beam under nominal operating conditions. The proposed controller can inversely estimate the time-varying input force and effectively implement the vibration control in real-time. According to the simulation results, the proposed control method has been successfully applied to reduce the vibrations of a beam-rotating machinery system. The results demonstrate that proposed controller has better properties than the conventional LQG controller. The practicality of this study provides great help in real-time condition monitoring and active vibration control for rotary machines. Also, the technique will be implemented experimentally in the future work.

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