Structural Engineering and Mechanics, Vol. 9, No. 6 (2000) 557-568

DOI: http://dx.doi.org/10.12989/sem.2000.9.6.557

RELTSYS: A computer program for life prediction of deteriorating systems

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Abstract. As time-variant reliability approaches become increasingly used for service life prediction of the aging infrastructure, the demand for computer solution methods continues to increase. Effcient computer techniques have become well established for the reliability analysis of structural systems. Thus far, however, this is largely limited to time-invariant reliability problems. Therefore, the requirements for time-variant reliability prediction of deteriorating structural systems under time-variant loads have remained incomplete. This study presents a computer program for RELiability of Time-Variant SYStems, RELTSYS. This program uses a combined technique of adaptive importance sampling, numerical integration, and fault tree analysis to compute time-variant reliabilities of individual components and systems. Time-invariant quantities are generated using Monte Carlo simulation, whereas time-variant quantities are evaluated using numerical integration. Load distribution and post-failure redistribution are considered using fault tree analysis. The strengths and limitations of RELTSYS are presented via a numerical example.

Key words: deteriorating systems; fail-safe structures; importance sampling; life prediction; Monte Carlo simulation; numerical integration; random variables; time-variant reliability.

1. Introduction

As life-cycle cost analysis methods gradually become required for maintenance planning decisions of infrastructure systems (Yanev 1996, Frangopol *et al.* 1997, 1998), the need for predicting the reliability of deteriorating structures continues to increase. When load and/or resistance vary with time, time-variant reliability analysis has to be used to predict the service life of structures. In the past decade, time-variant reliability methods have been developed for deteriorating structures under time varying loads (Wen and Chen 1989, Mori and Ellingwood 1993). Recently, these methods have also been applied to deteriorating bridges (Enright 1998, Enright and Frangopol 1998a, b).

Computing the time-variant reliability of deteriorating structures is not a trivial task. For example, hundreds of millions of computations may be required to predict the reliability of structures consisting of only a few (say, three to five) members (Enright 1998). Since the limit states cannot generally be expressed in closed form, computationally efficient algorithms based on the first or second order reliability method cannot be used (Melchers 1987). Instead, one must resort to numerical integration or simulation. Ordinary Monte Carlo methods can be used but are impractical due to the large number of computations required for accurate results. Variance reduction using

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importance sampling is a solution to this problem, because the number of simulations can be reduced while maintaining the same level of accuracy as compared to ordinary Monte Carlo simulation. Several importance sampling techniques are available (e.g., adaptive, direct, modified search, updating, among others). However, with the exception of adaptive techniques, most of these methods use gradient search algorithms to identify the important regions in which the limit states must be expressed in closed form (Dey and Mahadevan 1998). Adaptive importance sampling is therefore a viable alternative to ordinary Monte Carlo simulation for the prediction of time-variant reliability.

This study presents a computer program for time-variant reliability analysis of general deteriorating structural systems. The computer program for RELiability of Time-Variant SYStems, RELTSYS, uses a combined technique of adaptive importance sampling, numerical integration, and fault tree analysis to compute time-variant reliabilities of components and systems. Time-invariant quantities are generated using Monte Carlo simulation, whereas time-variant quantities are evaluated using numerical integration. Load distribution and post-failure redistribution are considered using fault tree analysis.

2. Time-variant system reliability

Consider the first-failure (also called weakest-link) system of m deteriorating members subjected to a Poisson live load process with intensity S_1 shown in Fig. 1, where the live load LL and dead load DL are denoted by S_1 and S_2, respectively. The time-variant failure probability of this system under S_1 can be expressed as (Mori and Ellingwood 1993):

$$P_f(t_L)_{ser} = \underbrace{\int_0^\infty \cdots \int_0^\infty}_{m-fold} [1 - \exp\left(-\lambda_{S_1} t_L \cdot \{1 - \frac{1}{t_L}\int_0^{t_L} F_{s_1}[\min_{i=1}^{m}\left(\frac{r_i \cdot g_i(t)}{c_i}\right)]dt\}\right)] \cdot f_{\underline{R}_0}(\underline{r})d\underline{r} \quad (1)$$

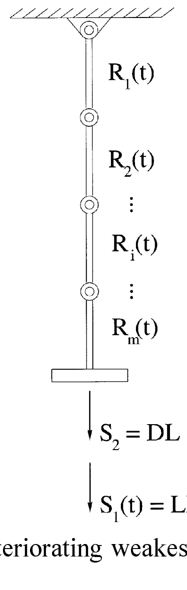


Fig. 1 General deteriorating weakest-link structural system

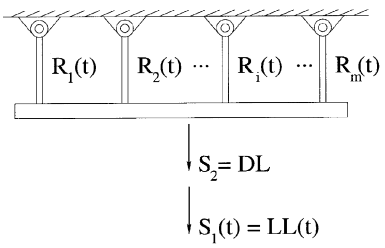


Fig. 2 General deteriorating fail-safe structural system

where S_1=time-variant (live) load, λ_{S_1} and F_{S_1} are the mean load occurrence rate and the cumulative distribution function of S_1, respectively, $g_i(t)$=resistance degradation function for member i, c_i=structural action coefficient for member i, and $f_{\underline{R}_0}(\underline{r})$=joint probability density function of the initial strength of the members in the system.

For service life predictions of deteriorating fail-safe systems, the post-failure material behavior and load sharing characteristics of members have to be considered. The post-failure behavior coefficient η_i can be used to represent the post-failure resistance of a member of a fail-safe system. For perfectly brittle behavior, η_i is assigned a value of 0, and the member carries no load after it has failed. For perfectly ductile behavior, η_i is assigned a value of 1, and the failed member continues to support a load equal to its resistance.

An idealized deteriorating m-member fail-safe system subjected to a Poisson live load process with intensity S_1 is shown in Fig. 2 (Enright and Frangopol 1998b). The cumulative-time failure probability of the system in Fig. 2 can be expressed as (Enright and Frangopol 1998b):

$$P_f(t_L)_{par} = \underbrace{\int_0^{\infty} \cdots \int_0^{\infty}}_{m-fold} F(t_L|\underline{R}_0=\underline{r}) \cdot f_{\underline{R}_0}(\underline{r})d\underline{r} \tag{2}$$

where

$$F(t_L|\underline{R}_0=\underline{r})=1-\exp\left(-\lambda_{S_1} t_L\left\{1-\frac{1}{t_L}\int_0^{t_L} F_{S_1}\left(\min_{k=1}^{m!}\left[\max_{i=1}^{m}\left(\frac{r_i \cdot g_i(t)}{RSF_i^d}+\sum_{j=1}^{l} \eta_j \cdot r_j \cdot g_j(t)\right)\right]\right)dt\right\}\right) \tag{3}$$

$P_f(t_L)_{par}$ represents the probability of failure of the parallel fail-safe system over a duration $(0, t_L)$, S_1 is time varying live load, λ_{S_1} and F_{S_1} are the mean occurrence rate and the cumulative distribution function of S_1, respectively, $g_i(t)$ is the resistance degradation function for member i (i.e., fraction of initial strength of member i remaining at time t), $f_{\underline{R}_0}(\underline{r})$ is the joint probability density function of the initial strength of the members in the system, $m!$=number of cut-sets (i.e., failure paths), RSF_i^d=resistance sharing factor of member i in the damaged state $(DS)_k^{ql}$ where q is the sequence of l failed members, $0 \leq l < m$, η_j=post-failure behavior coefficient of member j, $0 \leq \eta_j < 1$, and $g_j(t)$=resistance degradation function of member j.

3. Adaptive importance sampling

The failure probability P_f of a general structural system can be estimated using importance sampling (Melchers 1987, 1989):

$$P_f = \underbrace{\int \cdots \int}_{D} f_{\underline{X}}(\underline{x})d\underline{x} = \underbrace{\int \cdots \int}_{all\ x} I[\ ]f_{\underline{X}}(\underline{x})d\underline{x} = \int \ldots \int I[\ ]\frac{f_{\underline{X}}(\underline{v})}{h_{\underline{V}}(\underline{v})} \cdot h_{\underline{V}}(\underline{v})d\underline{v} \quad (4)$$

where I [ ]=indicator function (i.e., I [ ]=1 if $\underline{X}$ is in the failure domain D, I [ ]=0 if $\underline{X}$ is not in the failure domain), $f_{\underline{X}}(\ )$ is the probability density function in $\underline{X}$, and $h_{\underline{V}}(\ )$ is the importance sampling function. An estimator of failure probability is defined as (Melchers 1987):

$$\hat{P}_f = \frac{1}{n} \sum_{k=1}^{n} \left\{ I[\ ] \frac{f_{\underline{X}}(\hat{\underline{v}}_k)}{h_{\underline{V}}(\hat{\underline{v}}_k)} \right\} \quad (5)$$

where $n = n_{monte}$=number of trials (i.e., number of samples used for Monte Carlo simulation), and $\hat{\underline{v}}_k$=vector of sample values taken from the importance sampling function $h_{\underline{V}}(\ )$. Importance sampling can be used to solve (1) and (2), provided that an importance sampling function can be identified. The optimal importance sampling function can be expressed in terms of the failure probability P_f (Melchers 1987):

$$h_{\underline{V}}(\underline{v}) = \frac{I[\ ] \cdot f_{\underline{X}}(\underline{x})}{P_f} \quad (6)$$

Although P_f must be known to evaluate (6), it can be approximated by $\hat{P}_f$ which is obtained initially using Monte Carlo simulation. RELTSYS uses an adaptive importance sampling scheme suggested by Mori and Ellingwood (1993) in which the importance sampling function is computed using intermediate failure probability estimates obtained during the simulation procedure. For example, consider one of the random variables v_i of the importance sampling vector $\underline{v}$. The mean value of v_i, μ_{V_i}, can be expressed as:

$$\mu_{V_i} = \frac{1}{P_f} \int \ldots \int \frac{v_i \cdot I[\ ] \cdot f_{\underline{X}}(\underline{v})}{h_{\underline{V}}(\underline{v})} \cdot h_{\underline{V}}(\underline{v})d\underline{v} \quad (7)$$

Comparing (4) and (5), it follows that, from (7), an estimator for μ_{V_i} can be expressed as follows:

$$\hat{\mu}_{V_i} = \frac{1}{\hat{P}_f} \cdot \frac{1}{n} \sum_{k=1}^{n} \left\{ \frac{\hat{v}_{ki} \cdot I[\ ] \cdot f_{\underline{X}}(\hat{\underline{v}}_k)}{h_{\underline{V}}(\hat{\underline{v}}_k)} \right\} \quad (8)$$

where $\hat{\mu}_{V_i}$=estimator for μ_{V_i}, $n = n_{monte}$=number of trials, $\hat{P}_f$=(intermediate) estimate of failure probability (5), and $\hat{\underline{v}}_k$=vector of sample values taken from the importance sampling function $h_{\underline{V}}(\ )$.

4. Overview of RELTSYS

The flowchart of the RELTSYS computer code is shown in Fig. 3. This program consists of four main modules: (a) importance sampling, (b) numerical integration, (c) fault tree analysis, and (d) adaptive algorithm. The initial mean value of the importance sampling function vector is user defined, and is included as part of the input data file. The importance sampling function coefficient of variation (COV) vector is set to an initial value which is proportional to the value of the initial resistance COV vector. Covariance matrices and their associated determinants and inverses are computed for the importance sampling function and initial resistance using orthogonal transformation (Melchers 1987) and general matrix solver algorithms (Press *et al.* 1992).

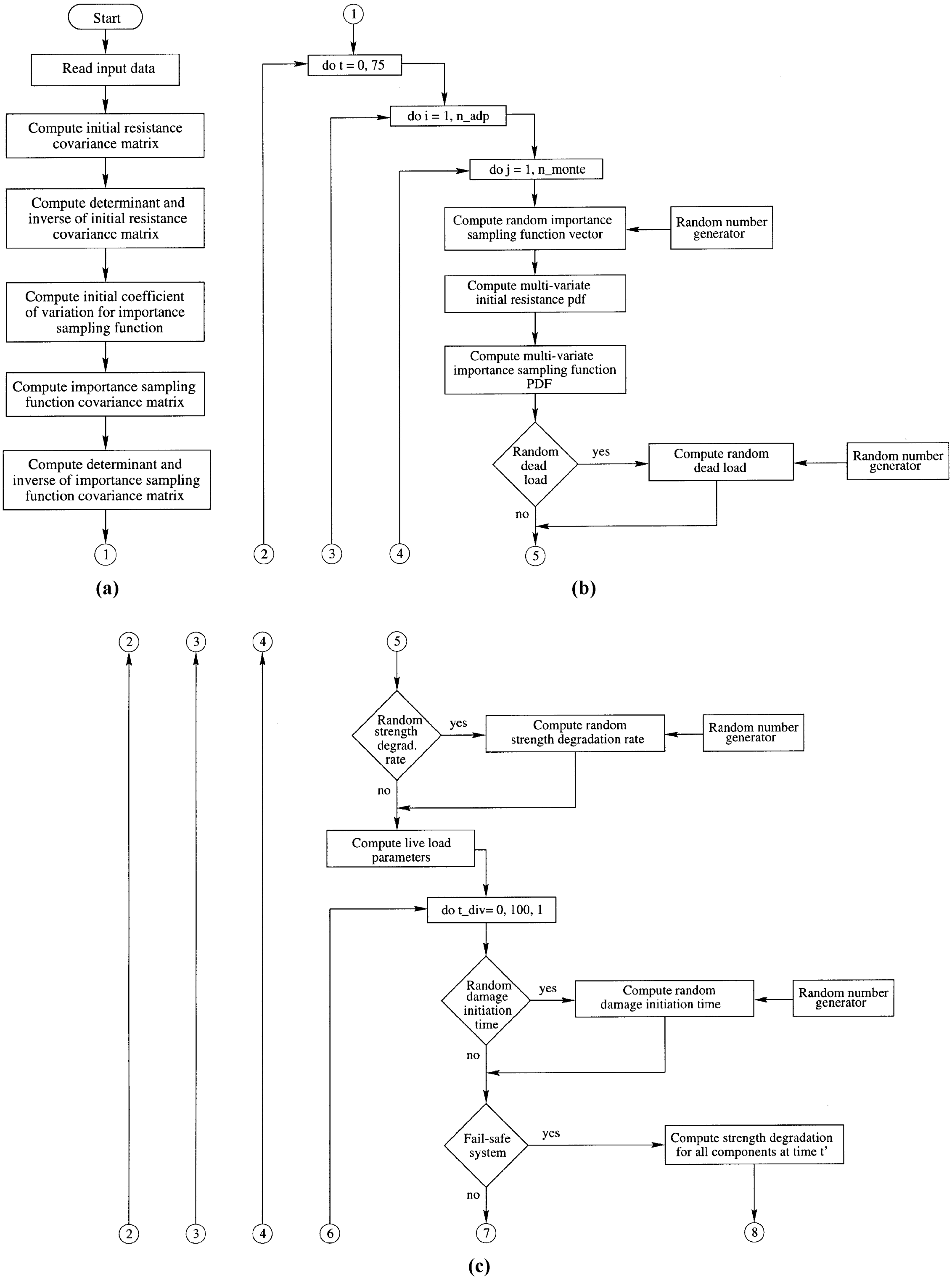
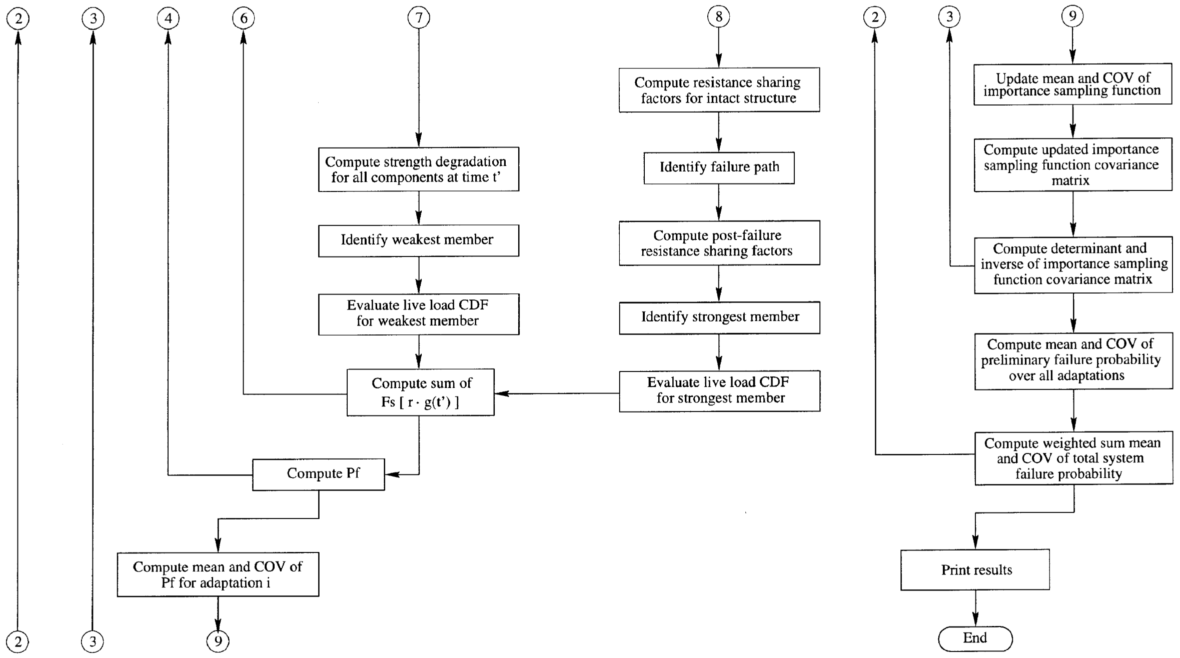


Fig. 3 Flowchart for the RELTSYS computer program: (a) Part 1 of 5; (b) Part 2 of 5; (c) Part 3 of 5



(d) (e)

Fig. 3 Flowchart for the RELTSYS computer program: (d) Part 4 of 5; (e) Part 5 of 5

Using the initial values for the main descriptors of the importance sampling function, an intermediate estimate for the system failure probability is computed using Monte Carlo simulation. The time-invariant random importance sampling function vector $\hat{\underline{v}}_k$ is created by the inverse transformation method (Fig. 4, adapted from Melchers 1987) using a random number generator, and the multi-variate initial resistance and importance sampling probability density functions ($f_{\underline{R}0}(\ )$ and

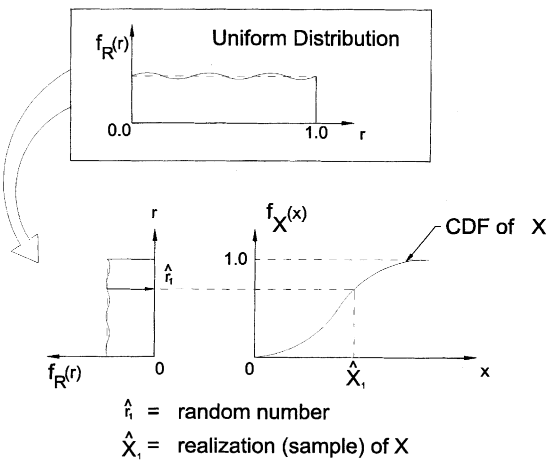


Fig. 4 Inverse transformation method of obtaining random samples in RELTSYS (adapted from Melchers 1987)

$h_{\underline{V}}(\ )$, respectively) are evaluated at the values contained in $\hat{\underline{v}}_k$. Additional time-invariant random variables, such as the dead load and strength degradation random variables are also generated using the inverse transformation method.

The expression $\int_0^{t_L} F_{S_1}(\cdot)\,dt$ is evaluated using numerical integration. At each time increment, the normalized resistance $r_{i,\ norm}$ of each member is computed based on the value of the resistance degradation function $g_i(t)$, the deterministic initial resistance value from the time-invariant initial resistance vector r_i, a structural action coefficient or resistance sharing factor $c_i = RSF_i^d$, and the post-failure behavior coefficient η_j.

The normalized resistance of the overall structure is dependent on the system classification. For first-failure systems, the normalized resistance of the system $r_{n,ff}$ is equal to the minimum value of normalized resistance for all members in the system:

$$r_{n,ff} = \min_{i=1}^{m}\left(\frac{r_i \cdot g_i(t)}{c_i}\right) \tag{9}$$

For fail-safe systems, the normalized resistance of the system $r_{n,fs}$ is computed using fault tree analysis. Resistance sharing factors RSF_i^d are computed based on the stiffnesses of the individual members. The failure path is identified, and the normalized post-failure resistance is computed for individual members considering the failure sequence and associated load redistribution. The normalized resistance of the system is computed by performing a complete fault tree analysis as follows:

$$r_{n,fs} = \min_{k=1}^{m!}\left[\max_{i=1}^{m}\left(\frac{r_i \cdot g_i(t)}{RSF_i^d} + \sum_{j=1}^{l} \eta_j \cdot r_j \cdot g_j(t)\right)\right] \tag{10}$$

Numerical integration is performed for each of the n_{monte} samples generated during Monte Carlo simulation. Once the combined Monte Carlo/numerical integration procedure is complete, the mean and COV of the intermediate estimate of the system failure probability are computed. The importance sampling function and associated covariance matrix are updated based on the mean value of the intermediate failure probability estimate. The combined procedure of Monte Carlo simulation, numerical integration, and fault tree analysis is repeated for the number of adaptations n_{adp} defined by the user. The numbers of simulations and adaptations are selected such that sufficient accuracy is obtained.

The estimator of the total system failure probability $\hat{P}_{ftot}$ depends on a main failure probability estimate, $\hat{P}_{fmain}$, which is obtained using the most recently updated value for the importance sampling function, and a preliminary failure probability estimate, $\hat{P}_{fpre}$, which is an average of all of the intermediate failure probability estimates obtained during preliminary analysis (Mori and Ellingwood 1993):

$$\hat{P}_{ftot} = (1 - \hat{w})\hat{P}_{fmain} + \hat{w}\hat{P}_{fpre} \tag{11}$$

where $\hat{w}$ = weighting factor for combining $\hat{P}_{fpre}$ and $\hat{P}_{fmain}$. The weighting factor $\hat{w}$ depends on the estimated variances of $\hat{P}_{fpre}$ and $\hat{P}_{fmain}$ (see Mori and Ellingwood 1993 for details). Both the mean $E(\hat{P}_{ftot})$ and coefficient of variation $V(\hat{P}_{ftot})$ of $\hat{P}_{ftot}$ are provided in the RELTSYS output file. Users can adjust n_{monte} and n_{adp} so that sufficient accuracy is obtained (i.e., adjust n_{monte} and n_{adp} such that $V(\hat{P}_{ftot}) \leq V^*$, where V^* is the maximum allowable value for computational accuracy (e.g.,

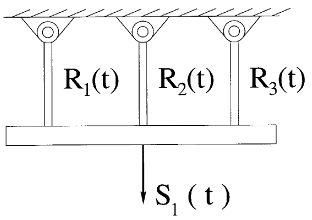


Fig. 5 Numerical example: 3-member deteriorating parallel system

V^*=0.01)).

5. Numerical example

Consider the deteriorating three-member fail-safe system shown in Fig. 5. The element linking the vertical members is assumed to be perfectly rigid and constrained to remain horizontal so that the axial deformations of the members are equal. Initially (i.e., at time t=0), the means and coefficients of variation of the members are assumed, respectively, as follows: $E(R_1)=E(R_2)=E(R_3)=$ 200 and $V(R_1)=V(R_2)=V(R_3)$=0.12 (see Table 1). Also, the initial resistances are assumed to be statistically independent (i.e., ρ_R=0). Furthermore, load and resistance are independent variables. Under the Poisson point process load $S_1(t)$ shown in Fig. 5, Eq. (2) can be used for time-variant reliability analysis of the deteriorating fail-safe system. The members are under environmental attack, and the resistance of each member is a time-variant quantity which can be expressed as:

$$R(t)=R_0 \cdot g(t) \tag{12}$$

where $R(t)$=time-variant resistance, R_0=initial resistance, and $g(t)$=resistance degradation function. Many degradation functions ($0 \leq g(t) \leq 1$) are possible (Enright 1998).
For example:

$$g(t)=1-k_1 \cdot t + k_2 \cdot t^2 \tag{13}$$

where t=elapsed time since degradation initiation, and k_1 and k_2 are random variables. Mean values of k_1 and k_2 and the damage initiation time T_I for members 1, 2, and 3 are indicated in Table 2. The coefficients of variation of these three parameters are set to zero (i.e., $V(k_1)=V(k_2)=V(T_I)$=0, where $V(\cdot)$=coefficient of variation). For details regarding the development of the resistance degradation function and the main descriptors of the random variables k_1 and k_2 in Eq. (13), see Enright (1998) and Enright and Frangopol (1998c). System failure is defined as failure of all members in the system in the interval (0, t_L].

In Fig. 6, failure probabilities associated with the three member system (Fig. 5, Tables 1 & 2) are

Table 1 Load and resistance random variables

Variable (1)	Description (2)	Mean value (3)	Coefficient of variation (4)	Density distribution (5)
R_i^0	Initial resistance of member i	200.0	0.12	Lognormal
LL^0	Initial live load	250.0	0.19	Normal

Table 2 Resistance degradation random variables

Member (1)	$E(k_1)$ (2)	$E(k_2)$ (3)	$E(T_I)$ (4)
1	0.0	0.0	–
2	0.0005	0.0	10.0
3	0.005	0.0	5.0

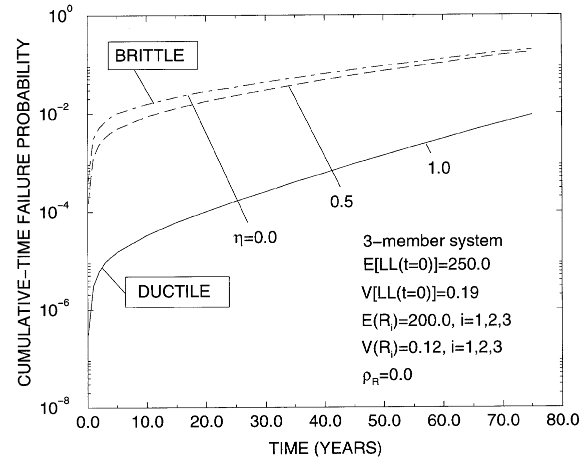


Fig. 6 Failure probability versus time for 3-member system

shown versus time. Failure probability increases with time, regardless of the post-failure material behavior η of the members in the system. It can be observed that η has a significant influence on the system failure probability, with the largest percentage differences occurring between η=0.5 and

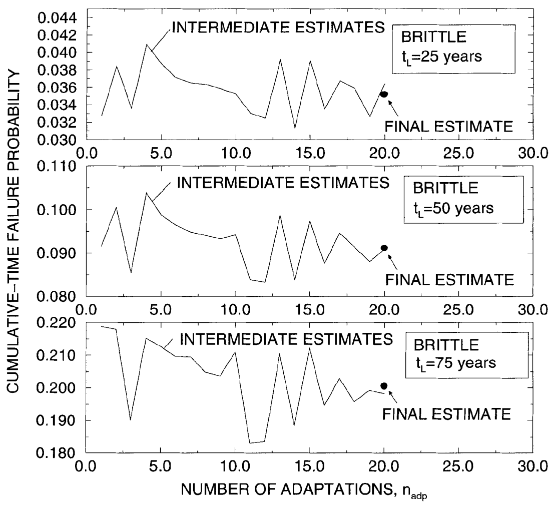


Fig. 7 Failure probability versus number of adaptations for 3-member brittle system

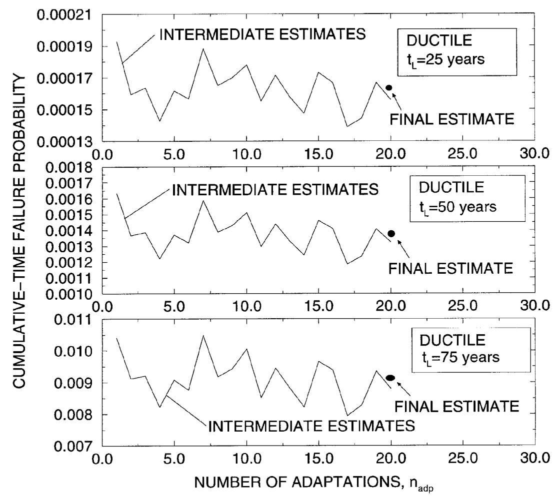


Fig. 8 Failure probability versus number of adaptations for 3-member ductile system

1.0. Further details regarding the influences of various factors (e.g., load/resistance/strength degradation random variables, correlation, material behavior) can be found in Hendawi and Frangopol (1994), Enright (1998), and Enright and Frangopol (1998b).

The influence of the number of adaptations used in the importance sampling on system failure probability estimates is shown in Figs. 7 and 8, for perfectly brittle (η=0) and perfectly ductile (η= 1) material behavior, respectively. In these figures, system failure probability is shown versus the number of adaptations n_{adp} for t_L=25, 50, and 75 years, with n_{monte}=1000. It can be observed that (a) the failure probability values differ significantly for the various evaluation times and material behaviors considered, and (b) the effect of n_{adp} on the cumulative-time failure probabilities is very similar (on a percentage basis).

6. Strengths and limitations of RELTSYS

The adaptive importance sampling/numerical integration technique used in RELTSYS for computing failure probability estimates is mainly based on work reported by Melchers (1989) and Mori and Ellingwood (1993). Combined with fault tree analysis (as described in Enright 1998), this technique allows for the efficient computation of failure probabilities of both first-failure and fail-safe systems. For system reliability analysis, the number of simulations required by RELTSYS is several orders of magnitude lower than that required for ordinary Monte Carlo simulation. For example, failure probabilities for fail-safe systems consisting of five uncorrelated members can be obtained using RELTSYS with n_{monte}=5,000 and n_{adp}=40, with a relatively small computational error (i.e., coefficient of variation of system failure probability, $V\,[P_f(t_L)_{par}]$, less than 0.02). Over one hundred million simulations may be required to achieve the same level of accuracy using ordinary Monte Carlo simulation.

One of the limitations of RELTSYS is related to the initial selection of the importance sampling function. The importance sampling function currently used is based on the multi-variate initial resistance probability density function. This importance sampling function appears to be sufficient for time-variant reliability predictions for systems in which live load and resistance are the only random variables. When additional random variables must be considered (e.g., dead load, strength degradation rate, corrosion initiation time, among others), the number of Monte Carlo simulations generally must be increased by one or more order(s) of magnitude for each additional random variable to obtain accurate results.

Time-variant reliability predictions using RELTSYS are based on a specific relationship among load and resistance. As stated previously in the description of Eqs. (1) and (2), load and resistance must be independent quantities. Furthermore, for system reliability computations, load effects in all of the members must be perfectly correlated. Therefore, RELTSYS does not have the ability to predict the reliabilities of systems in which load and resistance are not independent, and/or load effects in individual members are less than perfectly correlated. It does, however, have the ability to compute reliability of systems in which initial resistances are partially correlated.

Another limitation of RELTSYS is the requirement for user-defined control execution parameters. The total number of simulations n_{sim} required for time-variant system reliability results is the product of the number of Monte Carlo simulations n_{monte} and the number of adaptations n_{adp} (i.e., n_{sim}=$n_{monte} \cdot n_{adp}$). Currently, values for n_{monte} and n_{adp} must be specified by the user. Since the total number of simulations required for accurate system reliability predictions is dependent on a variety of factors (e.g., number of members in the system, total number of random variables, system behavior, post-failure material behavior, correlation among initial member resistances, among others), the user may under- or over- estimate the required values for n_{monte} and n_{adp}. However, the user can adjust these values for sufficient accuracy based on the value of $V(\hat{P}_{ftot})$ provided in the RELTSYS output file. An algorithm can easily be added to the program to adjust n_{monte} and n_{adp} based on the coefficient of variation of failure probability to achieve satisfactory results, eliminating the need for user-defined values of n_{monte} and n_{adp}.

7. Conclusions

RELTSYS is a general computer program which can be used to predict the time-variant reliability of structural systems. It uses a combined technique of adaptive importance sampling, numerical integration, and fault tree analysis to compute time-variant reliabilities of individual components and systems. This approach reduces the number of simulations required for accurate results by several orders of magnitude as compared to ordinary Monte Carlo simulation.

RELTSYS can be applied to a wide variety of structural systems (e.g., first-failure, fail-safe systems). It is currently limited to structures in which load and resistance are statistically independent random variables, and load effects within individual members are perfectly correlated. This can be a severe limitation for structures whose resistance is dependent on load magnitude or sequence (e.g., creep, fatigue, earthquake). RELTSYS also has the ability to compute reliability of systems in which initial resistances are partially correlated.

As illustrated in this study, RELTSYS can be used to predict the life of deteriorating systems under load, resistance, and damage uncertainties. It can also be combined with finite element analysis to predict the time-variant reliability of general structures (see Enright 1998 for details). In either case, time-variant reliability analysis is performed for deteriorating structures which includes the influences of time-variant loads, time-variant resistance, and post-failure load redistribution.

Acknowledgements

The work described in this paper was conducted by the first writer as part of his Ph.D. studies at the University of Colorado under the supervision of the second writer. The partial financial support of the National Science Foundation through grants CMS-9319204, CMS-9506435, and CMS-9522166 is gratefully acknowledged. The cooperation of Professors Bruce Ellingwood and Yasuhiro Mori is also gratefully acknowledged.

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