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A study of response control on the passive coupling element between two parallel structures

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Abstract. A new structure-vibration-control approach is proposed which uses a passive coupling element between two parallel structures to reduce the seismic response of a system due to earthquake excitation. Dynamic characteristics of the two coupled single-degree-freedom systems subject to stationary white-noise excitation are examined by means of statistical energy analysis (SEA) techniques. Optimal parameters of the passive coupling element such as damping and stiffness under different circumstances are determined with an emphasis on the influence of the structural parameters of the system on the optimal parameters and control effectiveness. Numerical results including the root mean square values of the response due to the filtered white-noise excitation and the time-histories of response to El Centro 1940 NS excitation are presented.

Key words : parallel structure; optimal passive control; seismic response; statistical energy analysis.

1. Introduction

The problem of ensuring the structural integrity of buildings under strong earthquake excitation has always been a challenge for structural engineers. Design for strength alone does not necessarily ensure that the building will respond dynamically in such a way that the comfort and safety of the occupants are maintained. In fact, requirements for strength and safety can be in conflict. Thus, alternative means of increasing the resistance of a structure while maintaining the desired dynamic properties based on the use of various passive, semi-active, active and hybrid control schemes offer great promise (Housner *et al.* 1997).

Base isolation systems have been implemented in civil engineering structures for a number of years because of their simplicity, reliability and effectiveness. Even though it can reduce the inter-story drift and the absolute acceleration of the structure, base isolation induces large base relative displacement, which causes instability in tall structures under wind loads. Thus, base isolation systems are of limited use in seismic control for low or medium buildings (Kelly 1986).

The tuned mass (TMD) or liquid damper system is an another widely used vibration suppressing strategy. The first-mode response of a structure with a TMD tuned to the fundamental frequency of

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the structure can be substantially reduced, which accounts for how TMD systems succeed in reducing wind-excited structural vibrations. But, in general, the higher modal responses may only be marginally suppressed or even amplified, hence, TMD systems are not necessarily effective for seismically induced vibration (Housner *et al.* 1997).

Active or hybrid control may control multiple vibration modes and improve the performance of a passive control scheme. An essential feature of active control systems is that external power is used to effect the control action. This makes such systems vulnerable to power failure, which is highly probable during a strong earthquake. At the same time, active control can destabilize if implementation errors are serious enough, due to neglected dynamics of the implemented systems. In addition, the high cost of active and hybrid systems prevents them from being implemented. (Lee-Glauser *et al.* 1997).

In this study, an innovative method of vibration control for buildings under strong earthquake is presented. The new method of vibration control can be realized by means of installing a passive-energy-dissipation coupling element between two parallel substructures, which takes advantage of the interaction between the parallel substructures to achieve better control effectiveness. In a simpler form, the strategy of the control approach is to remove energy associated with vibration from only one system, the *Primary system* (P). This is done by transferring energy to another system, the *Auxiliary system* (A), and the coupling element by means of interaction between the two systems. In a more complex form, the control strategy is to minimize the total energy of the combined primary-auxiliary system.

The analysis of this primary-auxiliary (PA) system is inherently complex because both primary and auxiliary systems are multi-degree-of-freedom systems and the number of degrees of freedom of the combined system can be prohibitively large. Moreover, resonance effects, non-classical damping, gyroscopic effects and parametric uncertainties would introduce added difficulties (Chen and Soong 1988).

However, important physical insights into complex PA system behavior can be gained by using more simplified procedures while demanding less-detailed response information. One of these approaches is the statistical energy analysis (SEA) (Lyon 1975). It is well known that in the past two decades, SEA has most commonly been applied to the analysis of random vibrations of complex structural systems which consist of two or more simple identifiable substructures under the action of broad-band stationary random (Keane and Price 1987). The analysis of conservatively coupled systems, which are not good representations of practical structures, was studied earlier and some fundamental relationships between power and energy difference were set up (Scharton and Lyon 1968). Recently, the theories of SEA for non-conservatively coupled systems excited by random or correlated forces have been investigated by many researchers (Lai and Soong 1990, Sun and Wang 1996). Definitions of power flow and energy for these systems in their studies are somewhat subjective and contain inconsistencies compared to the original systems (uncoupled systems), thus it is difficult, even impossible, to show the influences of the coupling element on the dynamic response of the uncoupled (original) systems.

In this paper, formulations for time-averaged energy of a PA-system in terms of the structural parameters of P-system and A-system, such as natural frequencies and damping, and the stiffness and damping of coupling elements have been developed so that the optimum design and adjustment of the A-system under different control strategies could be made simply based on minimizing the time-averaged energy of the corresponding objectives. Finally, numerical results about the frequency-response functions and the mean root square values of responses due to filtered white noise excitation as well as the time-histories of responses of a PA system under El-Centro 1940 NS excitation are presented.

2. Basic motion equations and energy formular

Fig. 1 shows the simplest Primary-Auxiliary system consisting of two single-degree-of-freedom systems connected by a coupling spring K_C and a dashpot C_C. The base of the PA system is subjected to a ground acceleration motion $\ddot{X}_g$. The equations of motion for this system can be written as

$$M_P\ddot{X}_P(t)+(C_P+C_C)\dot{X}_P(t)+(K_P+K_C)X_P(t)-K_CX_A(t)-C_C\dot{X}_A(t)=-M_P\ddot{X}_g(t) \quad (1a)$$

$$M_A\ddot{X}_A(t)+(C_A+C_C)\dot{X}_A(t)+(K_A+K_C)X_A(t)-K_CX_P(t)-C_C\dot{X}_P(t)=-M_A\ddot{X}_g(t) \quad (1b)$$

where M, K and C denote mass, stiffness and damping, respectively. The subscripts, P and A, refer to P-system and A-system. It is assumed that the earthquake ground motion is Gaussian and broad band white noise with spectral density S_{gg} spanning the frequencies of the P-A system (Feng and Mita 1995).

The displacement responses X_P and X_A can be obtained from Eq. (1)

$$X_P(t)=\frac{\alpha_P}{D}\ddot{X}_g \qquad X_A(t)=\frac{\alpha_A}{D}\ddot{X}_g \quad (2)$$

where

$$\alpha_P=-[(i\omega)^2+i\omega(\Delta_A+\Delta_C)+(\omega_A^2+\omega_C^2)],\quad \alpha_A=-[(i\omega)^2+i\omega(\Delta_P+\Delta_C)+(\omega_P^2+\omega_C^2)] \quad (3a, b)$$

$$\begin{aligned}D=&(i\omega)^4+(i\omega)^3(\Delta_P+\Delta_A+\Delta_C)+(i\omega)^2(\omega_P^2+\omega_A^2+\omega_C^2+\Delta_P\Delta_A+\Delta_P\Delta_{CA}+\Delta_A\Delta_{CP})\\&+(i\omega)[\Delta_P(\omega_A^2+\omega_{CA}^2)+\Delta_A(\omega_P^2+\omega_{CP}^2)+\Delta_{CP}\omega_A^2+\Delta_{CA}\omega_P^2]+(\omega_P^2\omega_A^2+\omega_P^2\omega_{CA}^2+\omega_A^2\omega_{CP}^2)\\&=a_4(i\omega)^4+a_3(i\omega)^3+a_2(i\omega)^2+a_1(i\omega)+a_0\end{aligned} \quad (3c)$$

The other parameters are defined as follows:

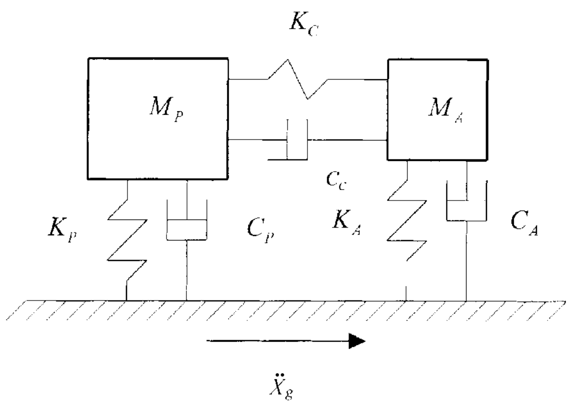

Fig. 1 Two parallel structures connected by a passive coupling element under ground acceleration

bandwidth of P-system: $\Delta_P = \eta_P \omega_P = C_P / M_P$;

bandwidth of A-system: $\Delta_P = \eta_A \omega_A = C_A / M_A$;

coupling damping parameters: $\Delta_{CP} = C_C / M_P$, $\Delta_{CA} = C_C / M_A$, $\Delta_C = \Delta_{CP} + \Delta_{CA}$;

natural frequency of P-system: $\omega_P = \sqrt{K_P / M_P}$;

natural frequency of A-system: $\omega_A = \sqrt{K_A / M_A}$;

coupling stiffness parameters: $\omega_{CP} = \sqrt{K_C / M_P}$, $\omega_{CA} = \sqrt{K_C / M_A}$, $\omega_C = \sqrt{\omega_{CP}^2 + \omega_{CA}^2}$;

It can be shown using Fourier transform methods that the time-averaged total relative energy of the P-system is

$$\bar{E}_P = M_P \langle \dot{X}_P^2(t) \rangle = \frac{1}{2\pi} M_P \int_{-\infty}^{\infty} S_{\dot{X}_P \dot{X}_P(\omega)} d\omega = \frac{1}{2\pi} M_P S_{gg} \int_{-\infty}^{+\infty} \frac{(i\omega\alpha_P) \cdot (i\omega\alpha_P)^*}{D \cdot D^*} d\omega \tag{4}$$

where

$$\begin{aligned}(i\omega\alpha_P) \cdot (i\omega\alpha_P)^* &= -(i\omega)^6 + [(\Delta_A + \Delta_C)^2 - 2(\omega_A^2 + \omega_C^2)](i\omega)^4 - (\omega_A^2 + \omega_C^2)(i\omega)^2 \\ &= b_0(i\omega)^6 + b_1(i\omega)^4 + b_2(i\omega)^2 + b_3\end{aligned} \tag{5}$$

Substituting Eqs. (3c) and (5) into Eq. (4) leads to integrals of the forms (Cremer and Heckl 1973)

$$\begin{aligned}&\int_{-\infty}^{\infty} \frac{b_0\omega^6 + b_1\omega^4 + b_2\omega^2 + b_3}{(a_4\omega^4 + a_3\omega^3 + a_2\omega^2 + a_1\omega + a_0) \cdot (a_4\omega^4 - a_3\omega^3 + a_2\omega^2 - a_1\omega + a_0)} \cdot d\omega \\ &= \pi i \frac{b_0(a_1a_2 - a_0a_3) + a_4(a_3b_2 - a_1b_1) + (a_1a_4 - a_2a_3)a_4b_3/a_0}{a_4(a_4a_1^2 + a_0a_3^2 - a_1a_2a_3)}\end{aligned} \tag{6}$$

Application of this formula to Eq. (4) gives, after some manipulation,

$$\bar{E}_P = -\frac{M_P S_{gg}}{2} \cdot \frac{a_1a_2 - a_0a_3 + (\omega_A^2 + \omega_C^2)^2 a_3a_4 - [2(\omega_A^2 + \omega_C^2) - (\Delta_A + \Delta_C)^2]a_1a_4}{a_4(a_4a_1^2 + a_0a_3^2 - a_1a_2a_3)} \tag{7}$$

Similarly, the time-averaged relative energy of the A-system can be obtained

$$\begin{aligned}\bar{E}_A &= M_A \langle \dot{X}_A^2(t) \rangle = \frac{1}{2\pi} M_A \int_{-\infty}^{\infty} S_{\dot{X}_A \dot{X}_A(\omega)} d\omega = \frac{1}{2\pi} M_A S_{gg} \int_{-\infty}^{\infty} \frac{\alpha_A \cdot \alpha_A^* \cdot \omega^2}{D \cdot D^*} d\omega \\ &= -\frac{M_A S_{gg}}{2} \cdot \frac{a_1a_2 - a_0a_3 + (\omega_P^2 + \omega_C^2)^2 a_3a_4 - [2(\omega_P^2 + \omega_C^2) - (\Delta_P + \Delta_C)^2]a_1a_4}{a_4(a_4a_1^2 + a_0a_3^2 - a_1a_2a_3)}\end{aligned} \tag{8}$$

The total relative vibration energy of the P-A-system can be easily obtained by adding Eqs. (7) and (8)

$$\bar{E} = -\frac{M_P S_{gg}}{2} \cdot \frac{\left(1 + \frac{1}{\mu}\right)(a_1a_2 - a_0a_3) + \left[(\omega_A^2 + \omega_C^2)^2 + \frac{1}{\mu}(\omega_P^2 + \omega_C^2)^2\right]a_3a_4}{a_4(a_4a_1^2 + a_0a_3^2 - a_1a_2a_3)}$$

$$-\frac{\left[2\left(\omega_A^2+\frac{1}{\mu}\omega_P^2+\left(1+\frac{1}{\mu}\right)\omega_C^2\right)-(\Delta_A+\Delta_C)^2-\frac{1}{\mu}(\Delta_P+\Delta_C)^2\right]a_1 a}{a_4(a_4 a_1^2+a_0 a_3^2-a_1 a_2 a_3)} \tag{9}$$

where

$\mu=\frac{M_P}{M_A}$ is the mass ratio of P-system to A-system.

3. Optimum parameters

3.1. Control criteria

The structural control criteria depend on different dynamic loads and response quantities of interest. Minimizing the relative displacement or absolute acceleration of the controlled structures has always been considered as the control objective (Warburton 1982). However, for a strong earthquake, the priority of a structural control objective is to reduce story drift to protect the structure itself. Therefore, the objective of the proposed approach is to reduce the relative displacements between adjacent stories of the controlled system from those which occur for the uncontrolled system. Structural relative vibrational energy provides an upper bound for the absolute values of story drift (Hayen and Iwan 1994). If we can reduce the relative vibrational energy of structures, the vibration response of the structure can be controlled with greater certainty.

There are two forms of excitation time-history for which simple expressions for optimum parameters can be derived, namely steady-state harmonic and random with a white noise spectral density. In this paper, the authors obtain the optimum parameters, which minimize the time-averaged relative vibrational energy of the P or PA systems subjected to white-noise excitation since the stationary white noise is a reasonable simulation of earthquake ground accelerations.

3.2. Optimization

The time-averaged relative vibrational energies $\bar{E}_P$ and $\bar{E}$ of the PA system shown in Fig. 1, when subjected to white-noise excitation of the power spectral density S_{gg}, can be obtained from Eqs. (7) and (9), respectively.

The optimizing condition for the strategy to minimize the relative energy of P-system is

$$\frac{\partial\bar{E}_P}{\partial\beta_2}=0;\ \frac{\partial\bar{E}_P}{\partial\Delta_C}=0 \tag{10}$$

where $\beta_2=\frac{K_C}{K_P}$ and $\Delta_C=\Delta_{CP}+\Delta_{CA}$.

And the optimizing condition for the strategy to minimize the total energy of the PA system is

$$\frac{\partial\bar{E}}{\partial\beta_2}=0;\ \frac{\partial\bar{E}}{\partial\Delta_C}=0 \tag{11}$$

In order to simplify the computation and to decouple the equations for optimization, the dampings of the P system and A system are assumed to be zero. For the optimum design of the passive coupling element, the simplified expressions of the time-averaged energies of P system, A system and PA system can be obtained by substituting $\Delta_P=\Delta_A=0$ into Eqs. (7) to (9)

$$\bar{E}_P=\frac{M_P\cdot S_{gg}}{2}\cdot\frac{\omega_P^2\cdot\Delta_C\cdot[\mu(1+\mu)(1-\beta_1^2)^2-2\mu(1+\mu)^2(1-\beta_1^2)\beta_2+(1+\mu)^4\beta_2^2]+\Delta_C^3\cdot(1+\mu)(\mu+\beta_1^2)}{\omega_P^2\cdot\Delta_C^2\cdot\mu\cdot(1-\beta_1^2)^2} \tag{12a}$$

$$\bar{E}_A=\frac{M_P\cdot S_{gg}}{2}\cdot\frac{\omega_P^2\cdot\Delta_C\cdot\frac{1+\mu}{\mu}\cdot[(1-\beta_1^2)^2-2(1+\mu)(\beta_1^2-1)\beta_2+(1+\mu)^3\beta_2^2]+\Delta_C^3\cdot\frac{1+\mu}{\mu}\cdot(\mu+\beta_1^2)}{\omega_P^2\cdot\Delta_C^2\cdot\mu\cdot(\beta_1^2-1)^2} \tag{12b}$$

$$\bar{E}=\bar{E}_P+\bar{E}_A=\frac{M_P\cdot S_{gg}\cdot(1+\mu)}{2\mu^2}\cdot\frac{\omega_P^2\cdot\Delta_C\cdot[(1+\mu^2)(1-\beta_1^2)^2-2(1+\mu)^2(\mu-1)(1-\beta_1^2)\beta_2+(1+\mu)^4\beta_2^2]+\Delta_C^3\cdot(1+\mu)(\mu+\beta_1^2)}{\omega_P^2\cdot\Delta_C^2\cdot\mu\cdot(\beta_1^2-1)^2} \tag{12c}$$

where

$\beta_1=\dfrac{\omega_A}{\omega_P}$ is the frequency ratio of A system to P system and

$\beta_2=\dfrac{K_C}{K_P}$ is stiffness ratio of passive coupling element to P system.

Table 1 lists the optimal parameter values obtained by combining Eqs. (10) to (12). The current optimization study is based on the assumption that the external excitation is represented by a stationary white noise, and the optimal parameter values do not only depend on the mass ratio μ of P system to A system, but also the natural frequency ratio β_1 of A system to P system. If the A system is considered as a single mass system, i.e., $\beta_1=0$, the A system with optimal stiffness and

Table 1 The optimum parameters of coupling element

Cases	Stiffness parameter β_{2-opt}	Damping parameter Δ_{C-opt}
Strategy 1	$\dfrac{\mu(1-\beta_1^2)}{(1+\mu)^2}$	$\omega_P\sqrt{\dfrac{\mu(1-\beta_1^2)^2}{(1+\mu)(\mu+\beta_1^2)}}$
Strategy 2	$\dfrac{(\mu-1)(1-\beta_1^2)}{(1+\mu)^2}$	$\omega_P\sqrt{\dfrac{(1+\mu^2)(1-\beta_1^2)^2-2(1+\mu^2)(\mu-1)(1-\beta_1^2)\beta_2+(1+\mu)^4\beta_2^2}{(1+\mu)(\mu+\beta_1^2)}}$

Note: Strategy 1 represents minimizing the relative energy of P-system; Strategy 2 represents minimizing the total relative energy of P-A-system damping becomes the standard TMD system. Thus, the optimal parameter expressions in Table 1 are suitable for more general conditions.

For a given P-system: M_P=1.50×10^5 kg, ω_P=10.55 rad/sec, Δ_P=0.422 1/sec, the dynamic behavior of A system is fully characterized by specifying the parameters: mass ratio μ, natural frequency ratio β_1 and damping ratio $\xi_1=\Delta_A/\Delta_P$. To clearly demonstrate the influence of structural parameters of PA system on the optimal parameters of the coupling element and the optimal control effectiveness, special emphasis is paid to control Strategy 1 and two control performance indexes are defined:

$$R_1=\frac{\langle X_P\rangle_{\text{Controlled}}}{\langle X_P\rangle_{\text{Uncontrolled}}} \quad \text{and} \quad R_2=\frac{\langle X_A\rangle_{\text{Controlled}}}{\langle X_A\rangle_{\text{Uncontrolled}}} \tag{13}$$

where $\langle X_P\rangle_{\text{Controlled}}$ and $\langle X_A\rangle_{\text{Controlled}}$ denote root mean square values of relative displacement of P system and A system with optimum stiffness and damping of the coupling element. The expressions $\langle X_P\rangle_{\text{Uncontrolled}}$ and $\langle X_A\rangle_{\text{Uncontrolled}}$ denote root mean square values of relative displacement of P system and A system without control.

Fig. 2 shows the influence of structural parameters of PA system on the optimum stiffness and damping values of the coupling element. When the natural frequency of A system is equal to that of P system, the optimum stiffness and damping values of the coupling element are equal to zero. When the natural frequency of A system is larger than that of P system, only the optimum stiffness is equal to zero, and the mechanism of the control strategy is to dissipate the energy by the coupling element. If the natural frequency of A system is smaller than that of P system, there exists simultaneously optimum stiffness and damping values which increase as the frequency ratio β_1 becomes smaller. The optimum stiffness also depends largely on the mass ratio μ, it increases as μ decreases (i.e. the mass of A system increases); on the other hand, the mass ratio μ has hardly any influence on the optimum damping value. That is to say, as the mass ratio μ and natural frequency ratio β_1 decrease, the interaction between P system and A system through the coupling element becomes stronger, and the control strategy is to remove energy associated with vibration from one system in a optimum way.

Fig. 3 shows the indexes R_1 and R_2 at various mass ratio μ values, natural frequency ratio β_1 values and damping ratio ξ_1 values. Figs. 3(a) to 3(d) show that the optimal control effectiveness depends largely on the natural frequency ratio β_1 of the A system to P system. The control

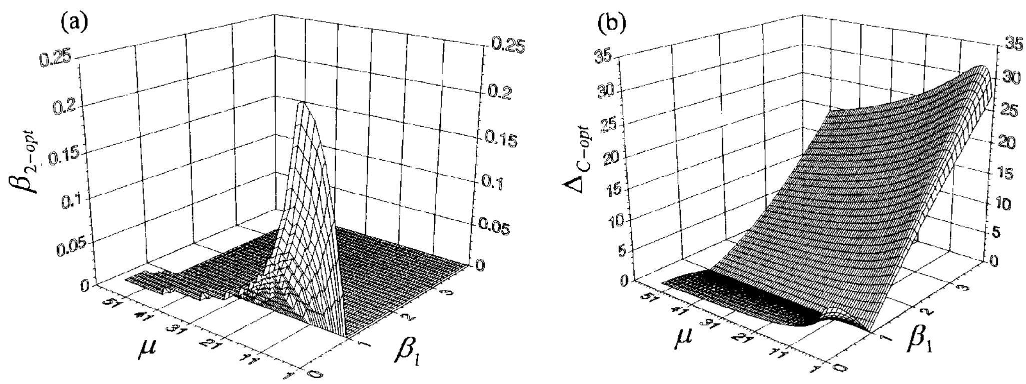

Fig. 2 Optimal parameters of coupling element with different mass ratio μ and natural frequency ratio β_1: (a) Optimal stiffness parameter β_{2-opt}; (b) Optimal damping parameter Δ_{C-opt}

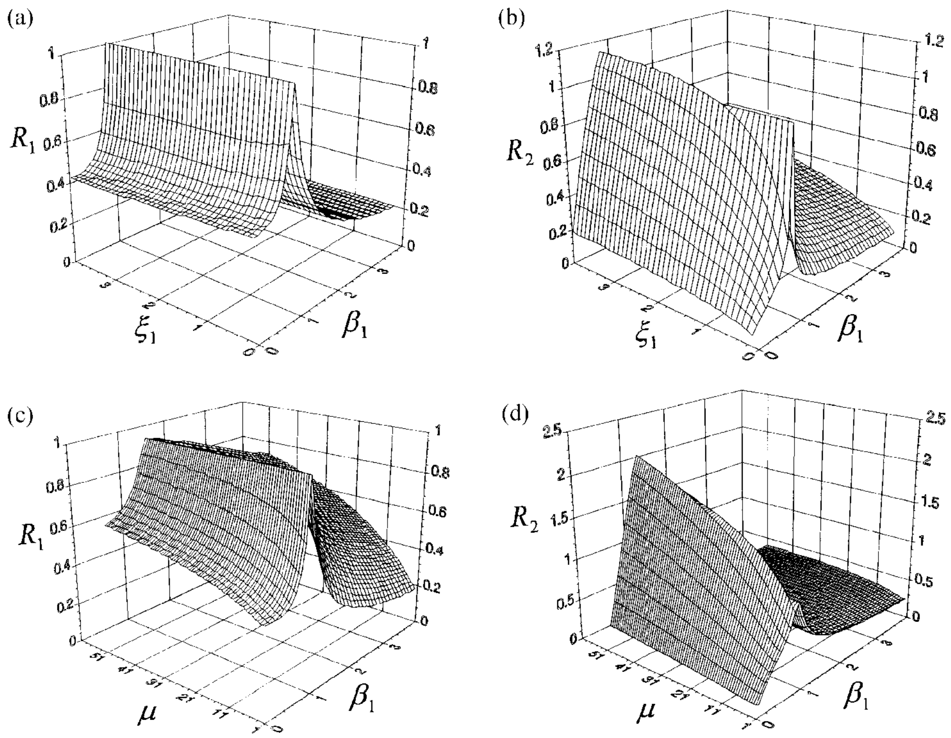

Fig. 3 Structural control indexes R_1 and R_2 with different mass ratio μ, natural frequency ratio β_1 and damping ratio ξ_1: (a) and (b) μ=5/3; (c) and (d) ξ_1=1

effectiveness of the two passive control strategies increases as the natural frequency of A system is far away from that of P system. Figs. 3(a) to 3(b) show that the control effectiveness can be improved by increasing the damping of A system when its natural frequency is near that of P system. We can see from Figs. 3(c) to 3(d) that increasing the mass of A system can effectively reduce the vibration response in both P system and A system. This highlights the importance of choosing a suitable A system for reducing the vibration responses in both P system and PA system besides optimally designing the passive coupling element.

4. Numerical examples

To illustrate the performance of the proposed control, the seismic responses of two different examples PA system with an optimum coupling element as shown in Fig. 1 were numerically simulated and compared with that of a separated P-system and A-system without the coupling element. The mass, natural frequency and damping of P-system are the same as those in the previous section. The structural parameters of A-system are determined by giving the following dimensionless parameters:

Example 1: μ=2.0, β_1=0.5, ξ_1=1.0.

Example 2: μ=5/3, β_1=1.428, ξ_1=1.428.

The optimum values of stiffness and damping of the coupling element can be obtained according

Table 2 The optimum values of stiffness and damping of the coupling element

	Stiffness parameter β_{2-opt}		Damping parameter ξ_{2-opt}	
	Strategy 1	Strategy 2	Strategy 1	Strategy 2
Example 1	0.17578	0.07031	4.0151	5.6782
Example 2	0.0	0.0	4.0010	6.02354

Note: optimum damping parameter $\xi_{2-opt}=C_{C-opt}/C_P$

Table 3 Comparison between eigenvalues of system with and without control

	Uncontrolled	Control Strategy 1	Control Strategy 2
Example 1	5.275±0.211i	7.375±0.9557i	4.971±2.509i
	10.55 ±0.211i	11.34 ±1.725i	9.932±1.108i
Example 2	10.55 ±0.211i	10.73 ±1.074i	10.98 ±1.448i
	15.07 ±0.3014i	14.64 ±1.692i	14.18 ±2.252i

to the formulas in Table 1 and are listed in Table 2.

It is instructive to observe the control effect on structural behavior. The system eigenvalues resulting from the application of the passive coupling element are compared with those without control in Table 3. Only Control Strategy 1 increases the damping in both modes with obvious changes in the natural frequencies of Example 1. In the other cases, the main effect of the passive control strategies is to significantly increase the damping while only slightly altering the natural frequencies, and consequently, the associated stiffnesses.

The frequency transfer functions between the relative displacements of P system and A system in different control strategies and the ground acceleration are compared with those in the uncontrolled case in Fig. 4. Figs. 4(a) and 4(c) show that dramatic reduction at the frequency response of the relative displacement of P system is achieved by the optimum passive coupling element. The relative displacement of A system can also be reduced to some extent from Figs. 4(b) and 4(d). For Example 2, there is no coupling stiffness between P system and A system, only the responses of P system and A system at the corresponding natural frequencies can be appropriately reduced in Figs. 4(c) and 4(d). However, in Example 1, the resonant peaks of PA system can be dramatically shifted and reduced because of the existence of optimum stiffness β_{2-opt} from Figs. 4(a) and 4(b).

In the above section, earthquake ground acceleration has been considered as stationary white noise with broad band frequency components. However, Fourier analyses of existing strong-motion accelerograms reveal that the Fourier amplitude spectra are not constant with frequency even over a limited band (Housner 1955). The Kanai-Tajimi model for ground acceleration, which has the ability to simulate ground resonance in a very simple way, has been used very widely in the analysis of engineering structures under earthquake excitation (Lin and Yan 1987). The ground acceleration is idealized as a stationary random process by passing a Gaussian white noise process through a second-order filter with frequency transfer function of the form

$$S(\omega)=S_0\left[\frac{\omega_g^4+4\omega_g^2\zeta_g^2\omega^2}{(\omega^2-\omega_g^2)+4\omega_g^2\zeta_g^2\omega^2}\right] \tag{14}$$

where ζ_g, ω_g and S_0 are parameters depending on the earthquake magnitude, ground resonance

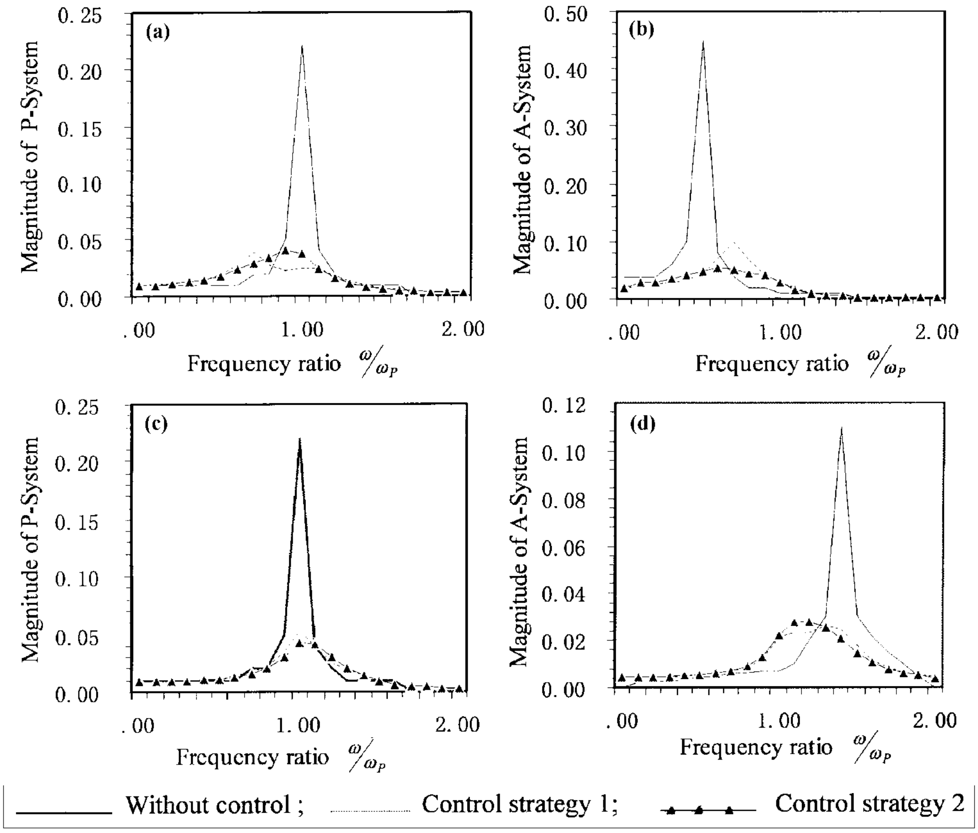

Fig. 4 Comparison of frequency transfer function: (a) and (b) Example 1; (c) and (d) Example 2

frequency, and the attenuation of seismic waves in the ground. Three groups of parameters of earthquake excitation, which represent three different types of soil conditions: firm, mid-firm and soft, used in this example are:

Group 1: ζ_g=0.60, ω_g=15.6 rad/s, S_0=4.8×10^{-3} m^2/s^3
Group 2: ζ_g=0.50, ω_g=10.55 rad/s, S_0=4.8×10^{-3} m^2/s^3
Group 3: ζ_g=0.30, ω_g=3.14 rad/s, S_0=4.8×10^{-3} m^2/s^3

Table 4 The root mean square values of relative displacements of P-system and A-system

Earthquake Excitation	Example	P-system (cm)			A-system (cm)		
		Noncontrol	Control 1	Control 2	Noncontrol	Control 1	Control 2
I	1	2.347	0.997	1.104	3.960	1.987	1.579
	2		1.087	1.162	1.374	0.822	0.790
II	1	2.510	1.084	1.205	4.415	2.275	1.766
	2		1.103	1.203	1.056	0.754	0.704
III	1	0.499	0.468	0.452	2.865	1.036	1.135
	2		0.356	0.366	0.208	0.185	0.180

Table 5 Peak responses subject to El Centro 1940 NS earthquake excitation

		Example 1			Example 2		
		uncontrolled	Control 1	Control 2	uncontrolled	Control 1	Control 2
P-System	Relative Dis. (cm)	3.04	2.55	2.65	3.04	2.28	2.34
	Absolute Acc. (gal)	339.41	225.31	235.03	339.41	293.07	299.59
A-System	Relative Dis. (cm)	12.99	5.25	4.94	1.56	1.67	1.56
	Absolute Acc. (gal)	362.83	282.78	194.29	355.10	291.69	282.28

Where the process intensity is chosen to be S_0=4.8×10^{-3} m^2/s^3, which represents the intensity of the *NS* component of the 1940 El Centro earthquake (Clough and Penzien 1975). The root mean square values of relative displacement of P system and A system under the three groups of earthquake excitation are shown in Table 4. Table 4 tells us that different degrees of response reduction for P system and A system can be achieved by selecting optimum stiffness and damping of the coupling element under different kinds of earthquake excitation. Especially, under relatively firm soil conditions (Group 1 and 2) these strategies are more effective in reducing the response of P system, and this reduction is over 50%. In general, Control Strategy 1 is more effective in reducing the response of P system, while the Control Strategy 2 is better for vibration reduction of A system.

The peaks of seismic response of structures are the key parameters for structural safety. Table 5 lists the peak values of the responses of P system and A system including relative displacement and absolute acceleration. The input ground acceleration is the El Centro 1940 NS earthquake with normalized peak acceleration of 140.0 gal. At the same time, Fig. 5 and Fig. 6 provide a comparison of the relative displacements and absolute accelerations of P system and A system

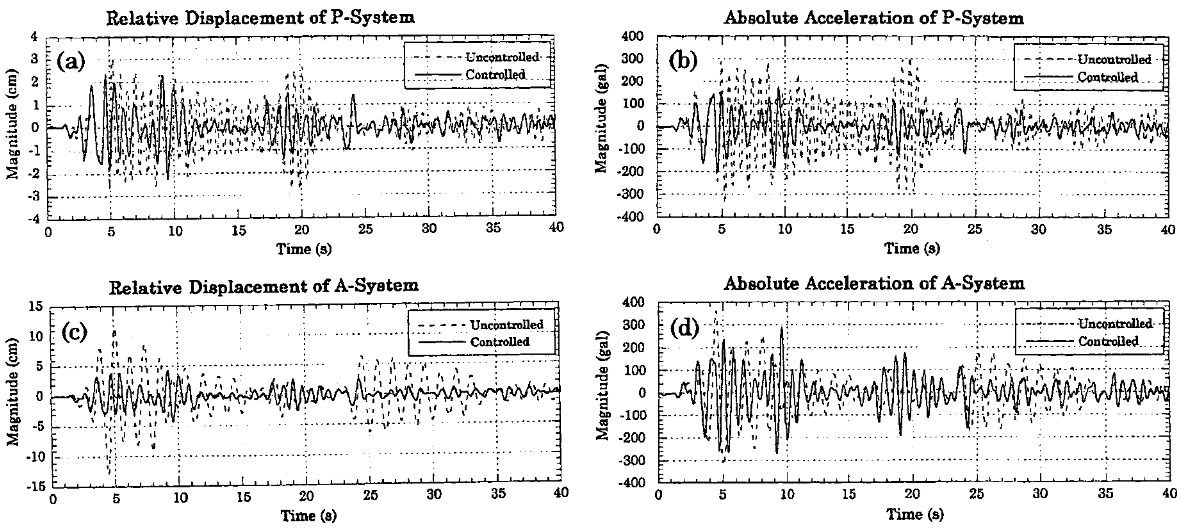

Fig. 5 Comparison of time histories of responses of P-system and A-system in Example 1: (a) Relative displacement of P-system; (b) Absolute acceleration of P-system; (c) Relative displacement of A-system; (d) Absolute acceleration of A-system

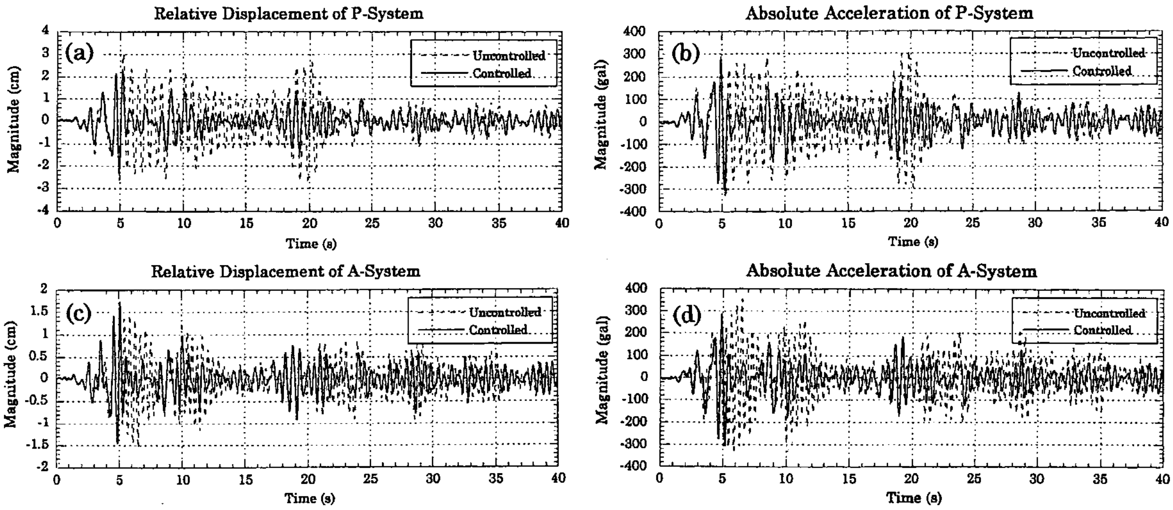

Fig. 6 Comparison of time histories of responses of P-system and A-system in Example 2: (a) Relative displacement of P-system; (b) Absolute acceleration of P-system; (c) Relative displacement of A-system; (d) Absolute acceleration of A-system

under Control Strategy 1 and those without control. The control effectiveness of this strategy for different objectives is clearly demonstrated in Table 5 and Figs. 5 and 6. The rigid A system (in Example 2) is more effective in reducing the relative displacement of P system than the flexural A system (in Example 1), there is an appropriate 25% reduction. On the other hand, the flexural A system is helpful in reducing the absolute acceleration of P system, a reduction amounts to approximately 35%. However, the absolute acceleration of A system in both examples can be reduced to some degree.

5. Conclusions

A new vibration-control method using the interaction of two parallel systems to reduce the vibration response of the system by means of a coupling element was proposed. In its simplest form, the strategy is to remove energy out of the primary system to another system (the auxiliary system and coupling system) as much as possible by choosing the optimum interaction element. In a more complex form, the control strategy is to minimize the total vibration energy of the P-A-system due to earthquake load.

Firstly, the optimum values of stiffness and damping of coupling element for different objectives were derived for a simplified model of the PA system which consists of two coupled single-degree-of-freedom oscillators. The general expressions of optimum stiffness and damping include the mass ratio of P system to A system, and the natural frequency ratio of A system to P system. Then, the influence of structural parameters of PA system such as mass ratio μ, frequency ratio β_1 and damping ratio ξ_1 on the optimum parameters and control effectiveness was discussed in detail. Finally, numerical results including the root mean square values of relative displacement of PA system subjected to filtered white-noise ground excitation and time-histories of relative displacement and absolute acceleration of PA system due to El Centro 1940 NS excitation were presented to demonstrate the effectiveness of this strategy in controlling structural vibration responses under earthquake excitation.

The effectiveness of this strategy depends on the determination of parameters of the coupling element and the structural parameters of PA system. For a suitable PA system, the proposed vibration control method can be quite robust and effective for the vibration of the P system or the total vibration of PA system. The control effectiveness increases as the mass of A system increases and the natural frequency of A system is further away from the P system. A rigid A system is more useful to decrease the relative displacement of P system, while a flexural A system is more effective in reducing the absolute acceleration of P system.

Compared to a conventional TMD system, this control strategy can achieve a dramatic reduction for the vibration of the P system because of the larger mass, more suitable damping and natural frequency of the A system. It does not change the characteristics of the original systems, therefore, it does not weaken the stability of systems under wind load, which has always been the challenge for base-isolation systems. Compared with semi-active or active control systems, it takes advantage of the interaction between the P system and the A system to achieve better control with no power requirements.

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References

Chen, Y. and Soong, T.T. (1988), "State-of-the-art review: Seismic response of secondary systems", *Engineering Structure*, **10**(4), 218-228.

Clough, R.W. and Penzien, J. (1975), *Dynamics of Structures*, McGraw-Hill, Inc.

Cremer, L. and Heckl, M. (1973), *Structure Borne Sound*, New York: Springer Verlag.

Feng, M.Q. and Mita, A. (1995), "Vibration control of tall building using mega sub-configuration", *J. of Engrg. Mech., ASCE*, **121**(10), 1082-1088.

Hayen, J.C. and Iwan, W.D. (1994), "Response control of structural systems using active interface damping", *Proceedings of First World Conf. on Struct. Control*, Los Angeles, California, **1**, 23-32.

Housner, G.W. (1955), "Properties of strong ground motion earthquakes", *Bull. Seismol. Soc. Am.*, **53**(3), 197-218.

Housner, G.W. etc. (1997), "Structural control: Past, present and future", *J. of Eng. Mech.*, **123**(9).

Keane, A.J. and Price, W.G. (1987), "Statistical energy analysis of strongly coupled systems", *J. of Sound & Vib.*, **117**, 363-386.

Kelly, J.M. (1986), "Aseismic base isolation: Review and bibliography", *Soil Dynamics and Earthquake Engineering*, **5**(3), 202-216.

Lai, M.L. and Soong, T.T. (1990), "Statistical energy analysis of primary secondary structural systems", *J. of Engrg. Mech., ASCE*, **116**(11), 2400-2413.

Lee-Glauser, G.J., Ahmadi, G. and Horta, L.G. (1997), "Integrated passive/active vibration absorber for

multistory buildings", *J. of Struct. Engrg., ASCE*, **123**(5).
Lin, Y.K. and Yan, Y. (1987), "Evolutionary Kanai-Majimi earthquake models", *J. of Engrg. Mech., ASCE*, **113**(8), 197-218.
Lyon, R.H. (1975), *Statistical Energy Analysis of Dynamical Systems: Theory and Applications*, MIT Press, Cambridge, Massachusetts.
Scharton, T.D. and Lyon, R.H. (1968), "Power flow and energy sharing in random vibration", *J. of The Acoustical of America*, **43**(6), 1332-1343.
Sun, J.C., Wang, C. and Sun, Z.H. (1996), "Power flow between three series coupled oscillators", *J. of Sound and Vib.*, **189**(2), 215-229.
Warburton, G.B. (1982), "Optimum absorber parameters for various combination of response and excitation parameters", *Earthquake Engrg. and Struct. Dyn.*, **10**(3), 381-401.