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Numerical investigation on beams prestressed with FRP

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Abstract. This paper aims to make a contribution to understanding which methods apply for structural analysis of beams prestressed with FRP cables. A parametric non-linear numerical analysis of simply supported beams has been performed. In this analysis the shape of the cross-section, the strength of concrete, the material adopted for the cables (steel, GFRP, CFRP), the prestressing system (bonded or unbonded prestressing) and the degree of prestressing were changed to collect a broad range of data which, the author contends, should cover the most frequent types of common practice. The output data themselves and their comparison allow us to suggest some rules that could be adopted when dealing with beams prestressed with these innovatory materials that have an elastic-brittle behaviour.

Key words: fibre reinforced plastic cables; unbonded prestressing; bonded prestressing; structural analysis; load-carrying capacity.

1 Introduction

The overall analysis of a concrete structure can be performed by means of:
– linear analysis
– linear analysis with redistribution
– plastic analysis
– non-linear analysis

This is a well-established custom written in all the codes (see for instance CEB-FIP 1991, CEN 1991, ACI 1995), but among these methods linear analysis is the most frequently adopted in common practice because it is simple and flexible.

As is well known the reliability of this method is established by the static theorem of the limit analysis that states that if the overall cross section behaviour is elasto-plastic, the safety factor (that is the variable loads amplification factor at ultimate) computed by means of a linear structural analysis and non-linear load carrying capacity verification of the cross-sections does not exceed the exact one, or it is on the safe side.

Usually, the plastic behaviour of the cross section is related to the yielding of the steel reinforcement. This means that the designer has to take care that the ultimate limit state of the cross-section is reached after a marked yielding of the reinforcement.

When referring to a structural member reinforced with FRP rebars and cables, yielding is no longer possible because the behaviour of this reinforcement is elastic-brittle. However this means that new criteria are needed to design these structures.

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Nevertheless we must state that yielding of the rebars is not essential for the static theorem to apply. If the overall cross section behaviour is essentially elastic, with a final branch of the moment-curvature diagram where increasing curvatures do not change the internal bending moment, the static theorem still applies whatever the cause of this behaviour happens to be (yielding of the reinforcement or non-linear behaviour of the cross section related for instance to cracking of concrete). This statement justifies the efforts that some research teams are making to design FRP rebars, made by coupling distinct reinforcing fibres, whose overall behaviour looks similar to that of a steel rebar under a monotonic load increase (see for instance Malvar 1994).

This paper aims to contribute to understanding which of the previous methods can be adopted for the analysis of structures reinforced with FRP. The work done consists in a parametric analysis. Two beams of different shape were numerically simulated up to collapse. From one simulation to another the type of reinforcement (steel, CFRP - carbon fibre reinforced plastic, GFRP - glass fibre reinforced plastic), the strength of concrete, the prestressing system and the degree of prestressing were changed to obtain a wide survey of the behaviour of these structural elements.

The numerical analysis avoids all the uncertainties related to a large number of experimental tests that should differ from one another just in one parameter, and it is also cheap. Obviously the computer program adopted has been widely tested by means of comparison with experimental tests to guarantee its reliability.

An analysis, performed by means of the moment-curvature diagrams of a cross-section would keep us from taking into account that the damage of a structural element near ultimate is distributed over a segment of finite length. Moreover, when dealing with unbonded prestressing the cross-section behaviour loses its meaning because of lack of bond between concrete and the cables. In this circumstance only the overall behaviour of the structural element is significant. On the other hand the analysis of hyperstatic structures is complex and the results are often hardly understandable in depth. These are the reasons why it was decided to adopt simple isostatic beams.

2. The numerical algorithm

The numerical algorithm adopted is that developed by Pisani (1996). This algorithm was at first developed to analyse beams prestressed with unbonded tendons, but it can simulate RC or PC beams as well.

The structural analysis includes second order effects (or the equilibrium equations are written referring to the deformed shape of the beam), gross displacements (to exclude approximations related to stresses in the external cables when referring to the bent beam), change in length of the beam owing to compression.

The method has just three limitations:

– cracking is spread over a segment of finite length in the concrete substructure (consequently precast segmental beams, especially if cast with dry joints, are excluded from this analysis);

– shear deformation is neglected both before and after cracking (that is, the beam has the amount of vertical stirrups necessary to resist shear at all loading stages);

– tensile strength of concrete is neglected (this assumption is expected to have no significant effect on the load carrying capacity of the beam, since the contribution of concrete in the tensile zone becomes negligible at high loading levels).

The constitutive laws adopted are those discussed in CEB-FIP (1991) for concrete and steel and in

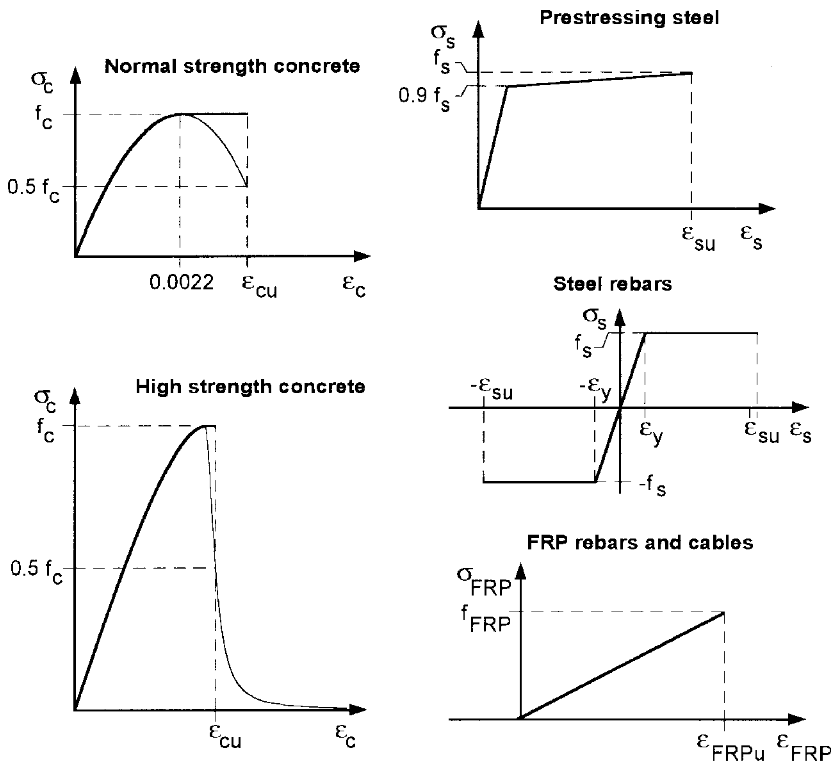

Fig. 1 Constitutive laws of concrete, steel and composite reinforcement

CEB-FIP (1995) for high performance concrete. The constitutive laws adopted for CFRP and GFRP rebars and cables are linear elastic up to collapse (see for instance Erki *et al.* 1993, Machida 1993, Nanni 1993, Agarwal *et al.* 1994).

The shapes of these laws are briefly summarized in Fig. 1. Examining this figure it can be stated that the descending branch of the constitutive laws of concrete have been substituted by horizontal segments. The CEB-FIP Bulletins (1991 and 1995) observe that both for normal strength and for high strength concretes 'the strain softening behaviour is highly dependent on the testing procedure'. It is nevertheless generally assessed that this approximation usually does not lead to significant errors in the evaluation of the overall behaviour of the structure when dealing with normal strength concrete. Some simulations proved that the same statement can be applied to the overall structural behaviour of beams casted with high strength concrete.

Owing to the fact that our aim is to simulate experimental tests, no safety factor was introduced both on the material side and on the action side.

Sixty-six experimental tests, carried out by twelve different research teams (Du *et al.* 1985, Cooke *et al.* 1981, Mattok *et al.* 1971, Harajli 1993, Taerwe *et al.* 1992, Chakrabarti *et al.* 1994, Jerrett *et al.* 1996, Taerwe *et al.* 1997, Billet *et al.* 1954, Sen *et al.* 1994, Niitani *et al.* 1997, Yonekura *et al.* 1997) were reproduced to verify the reliability of the numerical algorithm. These tests include PC, PPC, RC (bonded prestressing, unbonded prestressing, external prestressing) beams cast with normal or high strength concrete and reinforced or prestressed with steel, CFRP and GFRP. Twenty-nine of these tests (or those by Du *et al.* 1985, Cooke *et al.* 1981, Mattok *et al.* 1971, Harajli 1993,

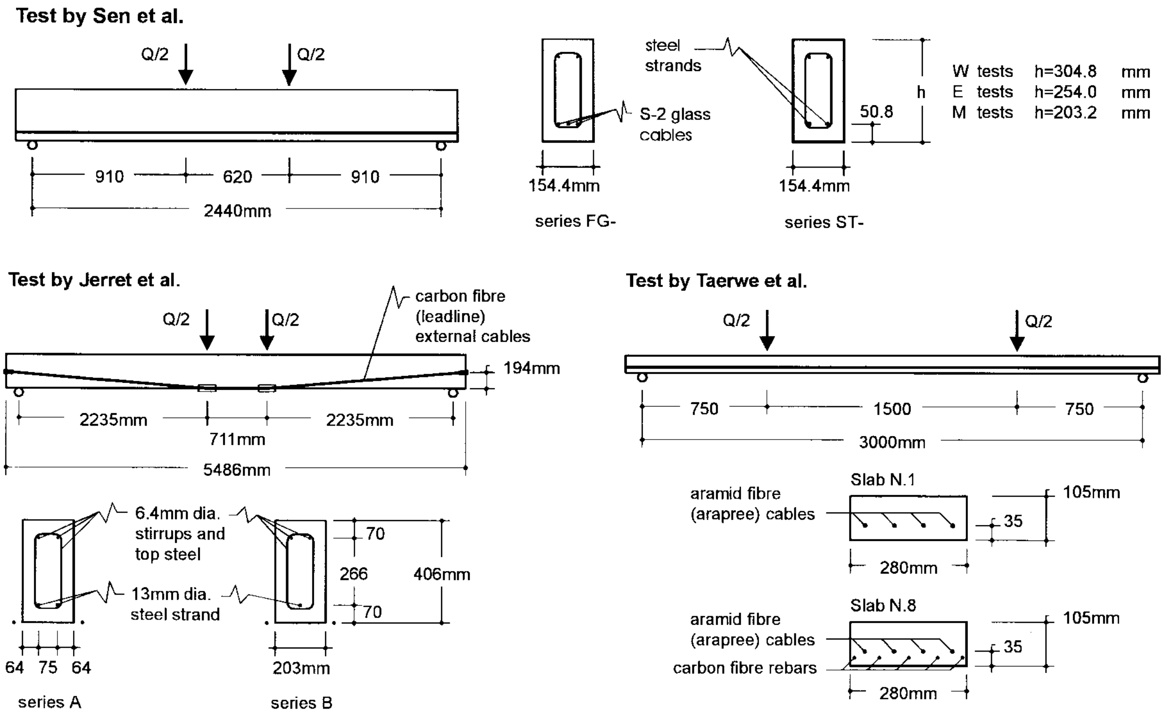

Fig. 2 Specimens of the tests by Jerrett *et al.* (1996), Taerwe *et al.* (1997), Sen *et al.* (1994)

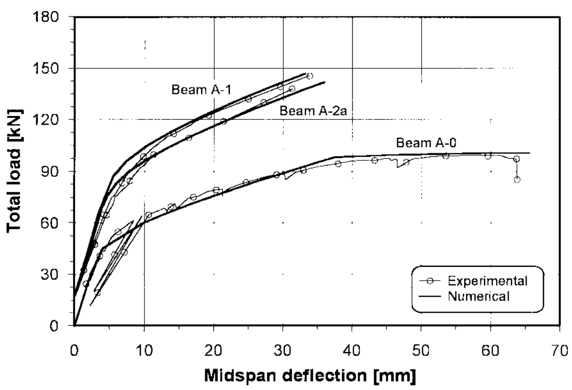

Fig. 3 Tests by Jerrett *et al.* (1996), series A

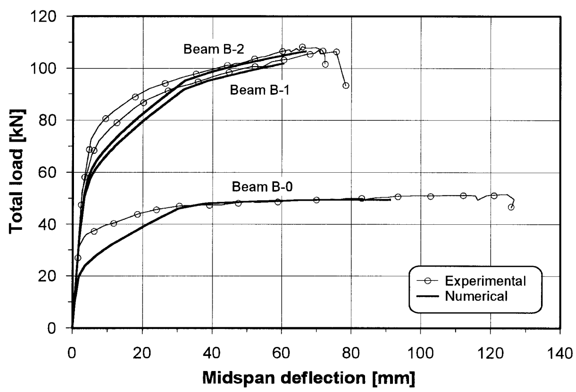

Fig. 4 Tests by Jerrett *et al.* (1996), series B

Taerwe *et al.* 1992, Billet *et al.* 1954) are discussed in Pisani *et al.* (1996), and Pisani (1998). Fig. 3 to Fig. 6 compare the outcome of another twelve tests (or those by Jerrett *et al.* 1996, Taerwe *et al.* 1997, Sen *et al.* 1994, briefly described in Fig. 2) with the numerical simulations.

The numerical output proved to be in accordance with the experimental results at every stage of loading and developed confidence in the computational procedure. Note that the computer program was designed not to allow manipulation of the output, or the input needed is the geometry of the specimen (its shape, its reinforcement, its prestressing cables, the position of the variable load) and

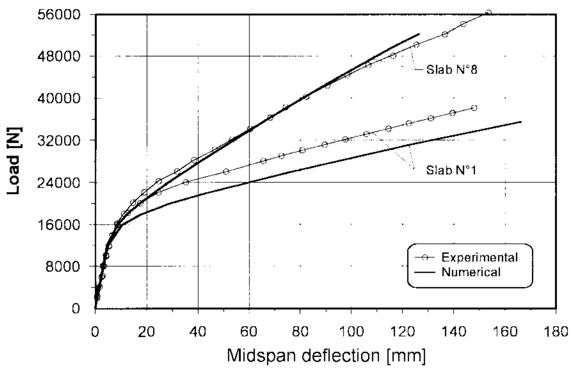

Fig. 5 Tests by Taerwe *et al.* (1997)

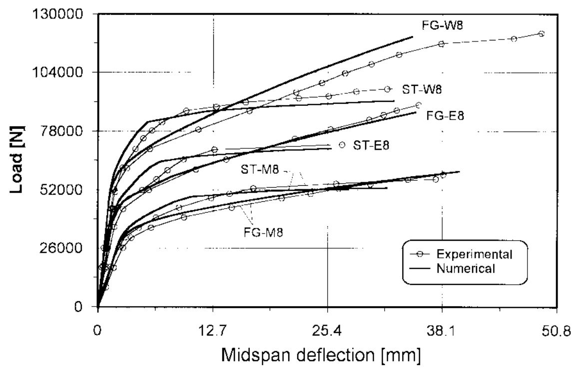

Fig. 6 Tests by Sen *et al.* (1994)

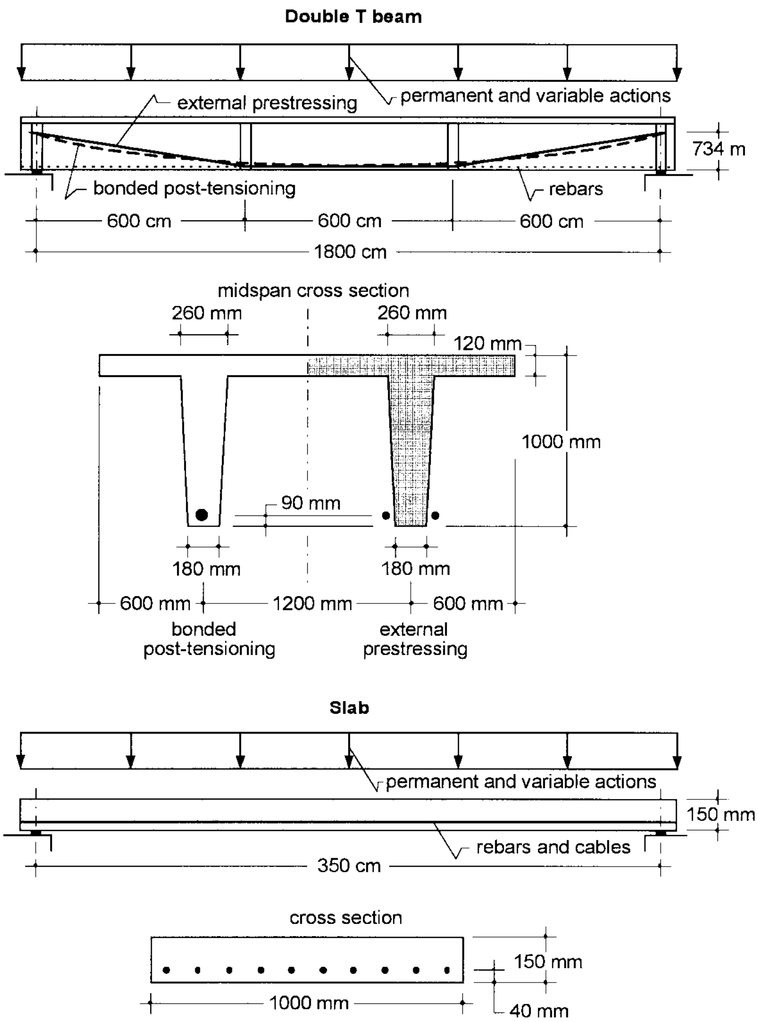

Fig. 7 Shapes of the specimens

Table 1 Mechanical properties of the materials

<table>
<tr><th>Material</th><th>f_o [MPa]</th><th>f_∞ [MPa]</th><th>E [GPa]</th><th>ε_{uo}</th><th>$\varepsilon_{u\infty}$</th></tr>
<tr><td>Normal strengh concrete</td><td colspan="2">38</td><td>34</td><td colspan="2">0.0037</td></tr>
<tr><td>High strength concrete</td><td colspan="2">88</td><td>42</td><td colspan="2">0.0029</td></tr>
<tr><td>Steel rebars</td><td colspan="2">500</td><td>200</td><td colspan="2">0.12</td></tr>
<tr><td>Steel cables</td><td colspan="2">1860</td><td>195</td><td colspan="2">0.06</td></tr>
<tr><td>CFRP rebars and cables</td><td colspan="2">1810</td><td>147</td><td colspan="2">0.0123</td></tr>
<tr><td>GFRP rebars</td><td>690</td><td>483</td><td>42</td><td>0.0164</td><td>0.0115</td></tr>
<tr><td>GFRP cables</td><td>1670</td><td>1170</td><td>51</td><td>0.0327</td><td>0.0229</td></tr>
</table>

Table 2 Reinforcement areas

	Double T beam			Slab		
	PC beam(1)	PPC beam		PC slab(2)	PPC slab	
	A_p [mm^2]	A_p [mm^2]	A_r [mm^2]	A_p [mm^2]	A_p [mm^2]	A_r [mm^2]
Steel	2224	1112	4072	493.5	197.4	1005
CFRP	2294	1143	1238	507.1	202.9	305
GFRP	3548	1768	3246	784.5	313.8	801

(1) Both for external prestressing and bonded post-tensioning
(2) Both for unbonded internal prestressing and bonded pre-tensioning

the parameters, all experimentally measured, that describe the materials behaviour (see Fig. 1). Many other tests, found in the scientific literature, where discarded because of lack of any of these data.

The mean value of the difference gathered in the computation of the load carrying capacity of the sixty-six specimens is −1.3% with a standard deviation equal to 5.2%.

In unbonded prestressing, friction between cables and deviators (or ducts) is neglected, but the previous comparisons with the experimental tests demonstrated that this assumption does not cause significant errors in the analysis.

3. Planning of the parametric analysis

Two specimens were designed. The first one is the double T beam, described in Fig. 7, whose behaviour is ductile when reinforced or prestressed with the amount of steel reinforcement described in Table 2. The second one is a slab that shows a brittle behaviour. Neither of the specimens has compression reinforcement. This choice comes from the observation that GFRP reinforcement has a compressive elastic modulus that is generally small and close to that of concrete (see Building Research Institute 1995), so that ‘FRP reinforcement in the compression zone shall be deemed to provide no compressive resistance in design’ (Canadian Standards Association 1995) whereas the contribution of steel in the compression zone is significant.

It is well known that creep and shrinkage of concrete modify the tensile stress in the steel tendons. The tensile stress in these cables was therefore set to be $\sigma_{pso}\cong 1350$ N/mm^2 immediately after tensioning and grouting and $\sigma_{ps\infty}\cong 1000$ N/mm^2 after long term loading, to simulate both these conditions when the load acting on the beam is its self-weight.

Table 1 quotes the mechanical properties of the materials adopted. Referring to GFRP rebars and cables it has to be mentioned that this material exhibits a marked decrease of strength when subjected to sustained stress (Glaser *et al*. 1983). This decrease is approximately 30% when the long-term stress is about 50% of the short-term strength of the material and consequently both the instantaneous (when the beam is tested immediately after stressing and grouting) and a reduced (70%) strength of GFRP rebars and cables have been adopted.

An equivalence rule is needed to compare the numerical results of beams reinforced with steel or FRP. This work is particularly devoted to the analysis of PC (prestressed concrete) and PPC (partially prestressed concrete) beams. The behaviour of these structures under service loads is independent of the elastic modulus of the reinforcement as they are not cracked (at least when dealing with quasi-permanent actions). If the serviceability limit states are not markedly influenced by the type of reinforcement adopted, the equivalence rule has to be related to the ultimate limit state, or the tensile reinforcement must carry a constant tensile force at ultimate, that is:

$$A_s \cdot f_s = A_{FRP} \cdot f_{FRP} \tag{1}$$

This rule applies both to prestressing cables and, separately, to rebars and leads to the areas described in Table 2 once A_s has been set.

To have a similar behaviour under service loads the same level of prestressing is nevertheless needed, that is:

$$\sigma_{ps\infty} \cdot A_{ps} = \sigma_{pFRP\infty} \cdot A_{pFRP} \tag{2}$$

after long term loading. The tensile stress immediately after stressing has been determined by means of:

$$\sigma_{pFRPo} = \sigma_{pFRP\infty} \cdot \frac{\sigma_{pso}}{\sigma_{ps\infty}} = 1.35 \cdot \sigma_{pFRP\infty} \tag{3}$$

in spite of the fact that, owing to their low elastic modulus, GFRP cables exhibit a lower time dependent loss of prestress than steel cables (whose time dependent loss is nevertheless lower than the one herein considered). The previous equations allow us to determine the stress at midspan (under the self-weight of the beam) in the FRP cables. Note that Eq. (3) leads to a tensile stress σ_{pFRPo} that for GFRP is about 50% of its instantaneous strength, so it matches the assumptions previously introduced.

AFRP (aramid fibre reinforced plastic) cables have not been considered because of their marked relaxation (see for instance ASM 1987, Kaci 1995). Their strength and their modulus of elasticity are anyway in between those of GFRP and CFRP.

4. Beams fully prestressed with unbonded cables

Dealing with unbonded prestressing it was stated that the most realistic way to prestress the double T beam is to adopt external post-tensioning, whereas unbonded internal cables were adopted

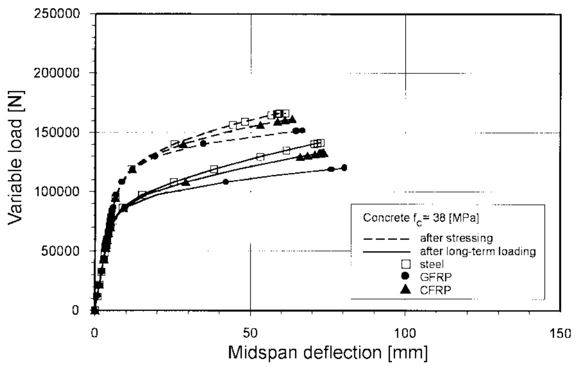

Fig. 8 PC slab, unbonded internal prestressing, $f_c = 38$ N/mm^2

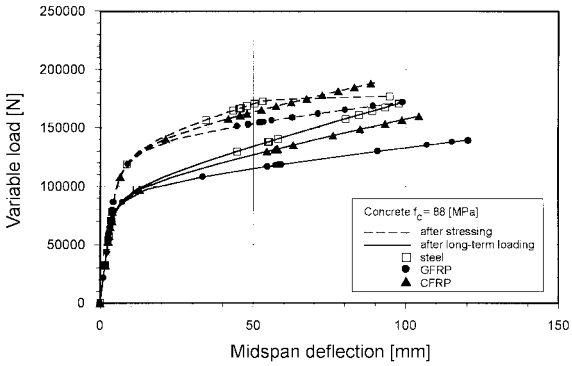

Fig. 9 PC slab, unbonded internal prestressing, $f_c = 88$ N/mm^2

Table 3 PC slab, unbonded internal prestressing

f_c [MPa]	Pretressing material	Time	σ_{pg} [MPa]	σ_{pu} [MPa]	σ_{cu} [MPa]	P_u [kN]	Collapse	Δu [cm]
38	steel	0	1350	1670	38	166.2	concrete	6.1
		∞	1000	1413	38	141.5	concrete	7.3
	GFRP	0	856.8	956.0	38	151.7	concrete	6.7
		∞	629.3	755.2	38	119.9	concrete	8.0
	CFRP	0	1314	1576	38	161.5	concrete	6.3
		∞	973.1	1294	38	133.0	concrete	7.4
88	steel	0	1350	1675	88	177.1	concrete	9.5
		∞	1000	1624	88	171.7	concrete	9.9
	GFRP	0	856.8	1022	88	171.7	concrete	9.9
		∞	629.3	835.5	88	139.3	concrete	12.0
	CFRP	0	1314	1737	88	189.0	concrete	9.0
		∞	973.1	1474	88	159.8	concrete	10.4

for the slab. These assumptions keep us from comparing the behaviour of the two specimens.

The load versus midspan deflection diagrams of the PC slabs are drawn in Figs. 8 and 9. Table 3 quotes a brief summary of the numerical output.

Collapse always occurs because of concrete crushing whereas the stress increase of the cables in the critical zone is reduced owing to the lack of bond, that is the steel cables do not yield, as already stated in many of the experimental tests adopted to verify the reliability of the numerical program. The behaviour of these slabs is therefore bi-linear, with a first branch that represents the uncracked beam (whose stiffness is independent of the elastic modulus of the cables) and a second branch where the beam is cracked around midspan (whose slope is proportional to the elastic modulus of the cables). The only exception to this behaviour refers to the slab made with high performance concrete and steel cables with high initial stress. In this case yielding of the cables occurs and the second branch of the curve is 'beheaded' by a third branch which is almost horizontal.

An increase in the strength of concrete leads to a decrease in the compressed strip in the critical zone at ultimate (when dealing with steel cables after all losses have occurred, at midspan this strip is 23 mm thick when adopting normal strength concrete and 15 mm thick when making use of high performance concrete). In spite of the decrease of the ultimate strain ε_{cu} the different position of the neutral axis means higher curvatures, or higher displacements, higher tensile stress in the cables and hence higher load at ultimate.

In this case the adoption of a more brittle material (high performance concrete) leads to a more ductile (or rather deformable) beam, even though after cracking the specimens made with normal strength concrete show higher midspan deflections than those made with high performance concrete when compared at the same load level.

The broad similarity of the overall structural behaviour of similar specimens prestressed with steel or FRP advises us to adopt for FRP rules similar to those already provided for steel cables.

Since the lack of bond between concrete and the cables leads to a complex hyperstatic problem, not suitable for common practice, the Eurocode (CEN 1994) suggests, for normal buildings, an approximate solution, or it may be assumed that the tensile stress increase in the unbonded steel tendons (with length not exceeding a single span) at ultimate is equal to 100 MPa. Table 3 shows that the more the elastic modulus of the cables decreases, the more their stress increase at ultimate decreases, so that the previous assumption could be modified for FRP as follows:

$$\sigma_{pu}-\sigma_{pg}=100[\mathrm{MPa}]\frac{E_p[\mathrm{GPa}]}{195[\mathrm{GPa}]} \tag{4}$$

Table 4 and Figs. 10 and 11 carry the numerical simulations made for the double T beam prestressed with external tendons.

Because of the lack of contact between the cables and the concrete substructure other than in the anchorages and in the deviators, the shape of the external tendons is piecewise polygonal up to ultimate. The more the specimen herein studied bends, the more the lever arm of the external cables decreases at midspan. As soon as the steel cables yield the specimens collapse because of instability, that is collapse occurs when the strains of the materials are still far from ultimate, or load and

Table 4 Double T PC beam, external prestressing

f_c [MPa]	Pretressing material	Time	σ_{pg} [MPa]	σ_{pu} [MPa]	σ_{cu} [MPa]	P_u [kN]	Collapse	Δu [cm]
38	steel	0	1350	1670	33.3	1061	instability	17.1
		∞	1000	1670	38	964.6	instability	37.5
	GFRP	0	885	1032	38	980.9	instability	30.0
		∞	653	829	38	700.4	concrete	36.7
	CFRP	0	1313	1789	38	1106	concrete	35.1
		∞	973	1525	38	869.4	concrete	41.3
88	steel	0	1350	1670	46.6	1063	instability	16.1
		∞	1000	1670	78	980	instability	36.6
	GFRP	0	885	1087	88	998.2	concrete	41.0
		∞	653	868	88	720	concrete	45.0
	CFRP	0	1313	1810	79.3	1132	cable	35.5
		∞	973	1810	88	960	cable	63.6

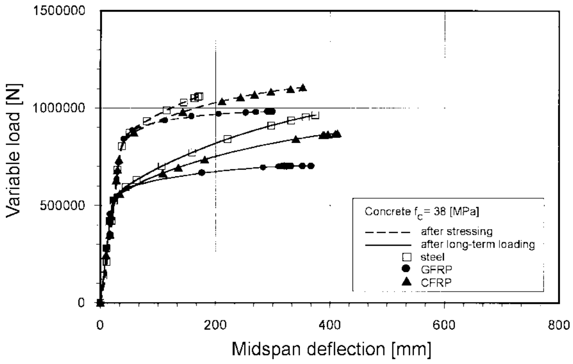

Fig. 10 Double T PC beam, external prestressing, f_c = 38 N/mm^2

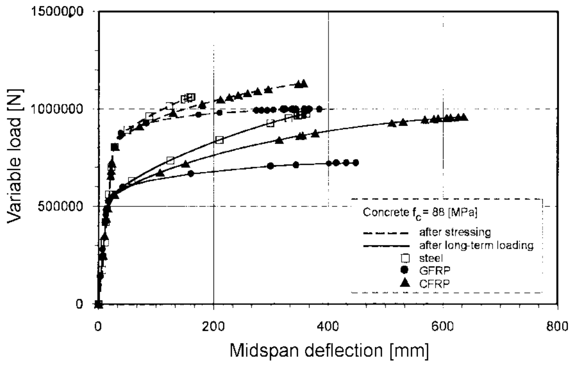

Fig. 11 Double T PC beam, external prestressing, f_c = 88 N/mm^2

deflection at ultimate are independent of the strength of concrete. This effect could be reduced by adopting more deviators. Their number anyway agrees with the suggestions by Virlogeaux (1983).

For the FRP cables behaviour does not provide for yielding, all the curves drawn in Figs. 10 and 11 consist again of a first branch (the uncracked beam) independent of the elastic modulus of the cables and a second branch whose slope mainly depends on the stiffness of the cables (cracked beam). An increase of concrete strength means a slight increase of the load carrying capacity of the specimen that because of its low actual stiffness markedly increases the deflection at ultimate.

It is surprising that the most deformable beam is the one made with the most brittle materials (high performance concrete and CFRP cables). Bending at ultimate and load carrying capacity of the specimens made with FRP are similar, or even better, than those of the beams prestressed with steel cables. This outcome enables us to adopt the same rules already stated by the Eurocode (CEN 1994) for steel cables, that is “if, for simplification, instead a non-linear analysis of the structure as a whole a sectional verification based on a linear analysis is performed, the increase in strain of the prestressing cables should be neglected”.

The lack of a long experience in these technologies (unbonded and external prestressing) suggests not to adopt redistribution unless in a refined non-linear analysis.

The adoption of a linear analysis in these cases is not justified through the static theorem of the limit analysis, but is a consequence of the adoption of a constant (or almost constant) tensile stress in the cables, so that the second branch of all the previous curves is ignored or a protective approximation, suitable for common practice, is adopted (Pisani *et al.* 1996). Note that the slope of these second branches decreases with the elastic modulus of the cables and, therefore, the error gathered when adopting this approximate solution decreases if steel cables are substituted by GFRP ones.

To tell the truth, the stress in the cables increases also in the uncracked range of behaviour but, as stated by Naaman (1990), this stress increase is lower (up to 2/3) than that of a twin beam prestressed with bonded cables that, in turn, is not very significant.

5. Beams fully prestressed with bonded cables

Tables 5 and 6 and Figs. 12 to 15 display the output of the simulations performed on specimens

Table 5 Double *T* PC beam, bonded post-tensioning

f_c [MPa]	Pretressing material	Time	σ_{pg} [MPa]	σ_{pu} [MPa]	σ_{cu} [MPa]	P_u [kN]	Collapse	Δu [cm]
38	steel	0	1339	1860	38	1312	cable	167.9
		∞	984	1860	38	1321	cable	178.1
	GFRP	0	855	1670	37.6	1925	cable	76.1
		∞	628	1170	30.8	1265	cable	49.1
	CFRP	0	1306	1810	20.2	1239	cable	15.3
		∞	989	1810	24.4	1253	cable	26.5
88	steel	0	1361	1860	82.1	1336	cable	164.0
		∞	996	1860	83.1	1337	cable	173.9
	GFRP	0	861	1670	59.0	1952	cable	73.4
		∞	632	1170	41.3	1279	cable	48.0
	CFRP	0	1324	1810	23.8	1249	cable	14.2
		∞	999	1810	30.5	1265	cable	25.5

Table 6 PC slab, bonded pre-tensioning

f_c [MPa]	Pretressing material	Time	σ_{pg} [MPa]	σ_{pu} [MPa]	σ_{cu} [MPa]	P_u [kN]	Collapse	Δu [cm]
38	steel	0	1311	1704	38	169.3	concrete	7.2
		∞	973	1698	38	168.6	concrete	8.3
	GFRP	0	874	1306	38	203.6	concrete	10.6
		∞	622	1138	38	179.3	concrete	13.1
	CFRP	0	1284	1810	37.2	179.6	cable	4.9
		∞	953	1810	38	182.0	cable	8.2
88	steel	0	1320	1733	88	182.5	concrete	10.6
		∞	979	1714	88	180.9	concrete	10.6
	GFRP	0	877	1479	88	248.4	concrete	14.1
		∞	624	1170	83	194.4	cable	12.9
	CFRP	0	1291	1810	55	186.3	cable	4.4
		∞	957	1810	66.7	189	cable	7.4

prestressed with bonded cables.

As is well known, bond increases the efficiency of the cables in the critical zone and therefore the load carrying capacity of these specimens increases with respect to that of the previous ones, the materials and the shapes adopted being the same. In most of the cases considered the cables break, but even when collapse occurs because of concrete crushing the strain in the cables is not far from ultimate.

When dealing with steel cables this outcome means yielding of the reinforcement. This behaviour is marked when dealing with the double T beam, whereas it is not very important in the slab because of its reduced compressive zone that leads to a premature crushing of concrete.

The strain increase between yielding and ultimate (of the steel cables) is anyway enough to verify that the load carrying capacity of the specimens is almost independent of the initial tensile stress in

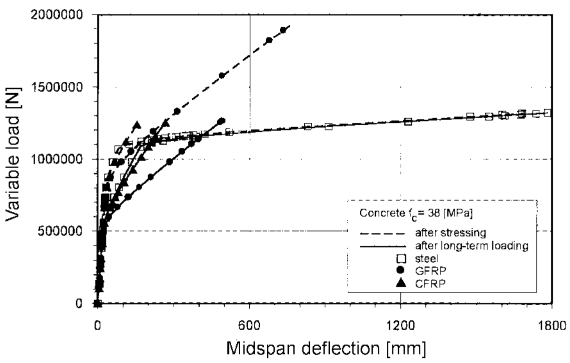

Fig. 12 Double T PC beam, bonded post-tensioning, f_c = 38 N/mm^2

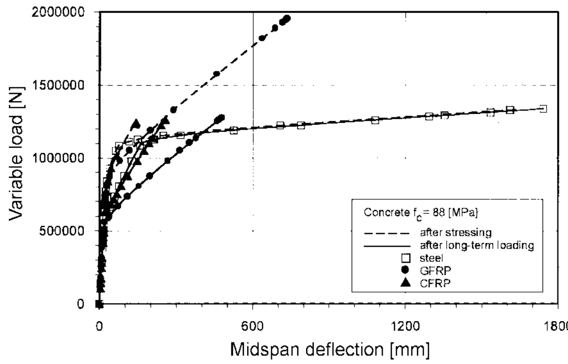

Fig. 13 Double T PC beam, bonded post-tensioning, f_c = 88 N/mm^2

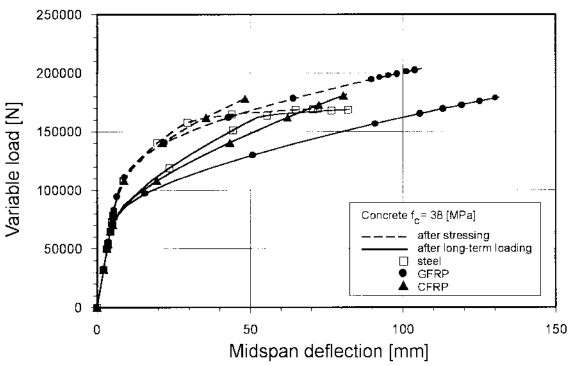

Fig. 14 PC slab, bonded pre-tensioning, f_c = 38 N/mm^2

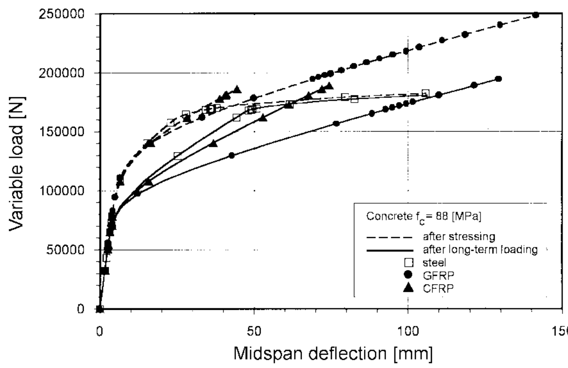

Fig. 15 PC slab, bonded pre-tensioning, f_c = 88 N/mm^2

the cables under the self-weight of the beam, the type of concrete being the same (see for instance Pisani 1998). The small difference gathered depends on the adoption of a hardening constitutive law (for the steel cables) instead of an elasto-plastic one.

The ductile behaviour of the specimens prestressed with steel (particularly that of the double T beam) enables us to adopt the static theorem of the limit analysis and hence allows us to use all the methods for structural analysis mentioned in the introduction.

The specimens prestressed with FRP show a different, more brittle, hardening behaviour which, in a hyperstatic structure, might lead to redistribution but does not match the conditions necessary to adopt the static theorem, or only a non-linear structural analysis should apply to these structures.

To overcome the difficulty related to this analysis, a rule similar to the one adopted for unbonded internal prestressing could be suggested. This rule should allow a linear structural analysis of hyperstatic structures prestressed with bonded FRP cables if the tensile stress increase in the cables is limited, so that the response of the structural element is beheaded by this assumption and its design load carrying capacity is markedly reduced. The rule should take into account that the difference between the load at cracking and the load at ultimate is much higher than that related to unbonded cables and that the load carrying capacity is almost independent of the elastic modulus of the cables. It has nevertheless to be pointed out that a rule like this is not so useful as those that deal with unbonded prestressing. An isostatic beam prestressed with unbonded (internal or external) cables is an hyperstatic structure (made of a RC substructure interacting through the deviators and the anchorages with the cables substructure) whose analysis is complicated and cumbersome. The previous rules markedly simplify the structural analysis and lead to reliable, simple methods suitable for common practice. Most of the traditional (bonded cables) PC structures are isostatic so that only equilibrium equations are needed to determine the internal actions in a cross section. In those cases where prestressing is adopted in hyperstatic structures, a refined structural analysis under service loads is anyway needed to assess the effect of the delayed behaviour of concrete and a non-linear structural analysis at ultimate is just as difficult: as a result the real need for a simplified rule is doubtful.

When examining the behaviour of the specimen prestressed with GFRP cables it can be stated that their load-carrying capacity markedly decreases in time. This outcome is a consequence of the 30% in time strength decrease already stated for the GFRP cables under sustained load. The more the stress at ultimate of the GFRP cables is near their strength, the more this strength decrease is significant in the computation of the load-carrying capacity of the specimen (the area of the cables being the same at tensioning and after long-term loading). The same outcome does not apply to beams prestressed with unbonded cables because there (see Tables 3 and 4) the tensile stress in the cables at ultimate is always lower than the reduced strength (i.e., 70% of the instantaneous strength).

6. Partially prestressed beams

A lot of numerical simulations were performed to control the effect exerted by the adoption of FRP cables and rebars in partially prestressed beams but they all lead to the same conclusion. When no steel reinforcement is adopted the beam behaviour is elastic-hardening, with a great load increase between cracking and ultimate, so that only non-linear structural analysis applies. An example of these computations is briefly summarized in Fig. 16.

The lack of yielding in the FRP reinforcement prevents us from making the best use of the strength of both the cables and the rebars. As a result, the load carrying capacity of the beams reinforced with FRP is much lower than that of similar beams reinforced with steel. This outcome is

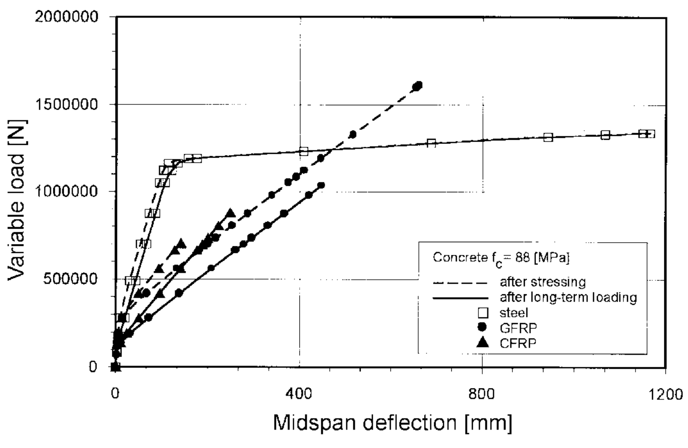

Fig. 16 Double T PC beam, partial (bonded) prestressing, f_c = 38 N/mm^2

emphasized when dealing with CFRP rebars and cables that are made with the same material. In this case the cables break when the rebars are not markedly stressed.

7. Conclusions

The parametric analysis demonstrated that only non-linear analysis should apply when dealing with the investigation at ultimate limit state of hyperstatic structures fully or partially prestressed with FRP bonded cables. It also showed that moment redistribution shall be expected in the analysis whereas the static theorem of the limit analysis does not apply.

The lack of bond in unbonded and external prestressing makes it possible to adopt for FRP cables either a non-linear analysis (redistribution expected), or a linear analysis (that does not take into account redistribution) with simplified rules similar to those already stated for steel cables: a structural linear analysis can be performed if the tensile stress of the external cables is set constant up to ultimate, or the stress increase in unbonded internal cables at ultimate is fixed through Eq. (4).

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Notation

A	area
E	modulus of elasticity
P_u	variable load at ultimate
ε	strain (generic)
σ	stress
σ_{pg}	tensile stress in the prestressing cables when the variable load is not acting
σ_{pu}	tensile stress in the prestressing cables at ultimate
σ_{cu}	maximum compressive stress in concrete at ultimate
Δu	midspan deflection at ultimate (owing to the variable load)
c	concrete
FRP	fibre reinforced plastic

f	strength
p	prestressing cable
r	rebar
s	steel
u	at ultimate
o	imediately after stressing and grouting
∞	after long-term loading