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Nonlinear analyses of axisymmetric reinforced concrete structures

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Abstract. A modified constitutive model has been developed for nonlinear finite element analysis of axisymmetric reinforced concrete structures, and then implemented into an existing finite element program. By comparing with the experimental results available, the present model has been validated.

Key words: Nonlinear analysis; finite element method; axisymmetric structure; reinforced concrete.

1. Introduction

In the past decades, a great deal of advanced researches on reinforced concrete structures was reported. At the same time, a great number of large-scale reinforced concrete (RC) structures such as nuclear power plants and LNG (liquefied natural gas) tanks have been constructed. Although the design of RC structures has been continuously improved, there are still many problems remained, especially in the case of huge RC structures. The importance of these kinds of structures makes it necessary to thoroughly understand their behavior. For large-scale reinforced concrete structures of axisymmetric configuration, the axisymmetric loading condition is one of the major loading conditions in practical design. An example of such structures is a massive bottom slab structure of storage tank for nuclear power reactors, liquefied natural gas, in which its design concept in some of the existing codes is based on the flexural and shear behavior of linear members rather than on those of axisymmetric ones (Japan Society of Civil Engineers 1991, Japan Gas Association 1979). Such design concept was often verified through model tests of such large-scale structures. For example, the formula of the ultimate shear capacity of linear members, which is used for the shear design of the axisymmetric RC structure, is mostly based on the experimental results. Based on some test result of circular slab (1985), formulae were proposed to evaluate the shear capacity of a model of circular slab. To make the practical design more rational and economic, it is indispensable to thoroughly understand behavior of such massive bottom slab structures. Unfortunately, it is impossible to obtain the ultimate capacity of the slab by experiments in real scale due to unacceptable expenses.

One of the competitive analytical procedures, i.e. finite element method becomes attractive. The basic requirement for carrying out an accurate nonlinear FEM analysis of reinforced concrete structure is to obtain reliable constitutive models for reinforced concrete materials. A literature review reveals that there exist a number of two-dimensional constitutive models and a few three-

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dimensional constitutive models. However, there doesn't exist an axisymmetric constitutive model for the RC materials (Mackerle 1994). Some researches have been done to develop analytical methods to predict the nonlinear behavior of the RC slabs based on experimental results. Due to the complicated characteristics, the overall behavior of a RC slab, especially for an axisymmetric case, are influenced by various parameters. Researchers have proposed several methods to estimate the behavior of such kind of structures in different aspects. Cheung, *et al.* (1990), developed a layered finite element for material nonlinear analysis of a reinforced concrete slab, by using orthotropic nonlinear elastic concrete model proposed by Darwin. The Newton-Raphson iteration method and the relaxation technique are used to ensure convergence of nonlinear solution. Rangan (1989) recommended simplified methods to predict the behavior of a RC slab by considering the redistribution of internal forces between the cracking load and ultimate load. Shehata *et al.* (1989) presented an analytical model based on experimental results to evaluate punching resistance of an axisymmetric reinforced concrete slab subjected to concentrated loads. The model has the advantages of describing the slab behavior beyond cracking stage and estimating both punching and flexural capacities of such slabs. Failure criteria are expressed in terms of the limiting strains of the limit state at which tension is developed in the critical region of concrete. Zhou & Jian (1991) proposed a finite element method to imitate the failure behavior of punching for a RC slab, stresses and deformation under punching failure were obtained. Huovinen (1990) concentrated on the study of the effect of stirrups in punching shear of reinforced concrete slabs. The performance of the stirrups was analyzed by carrying out series of loading tests on slabs. The test results indicate that stirrup was very effective to improve the punching shear resistance of RC slabs. Some experiments for RC circular slab have been made by Iwaki *et al.* (1985), and several empirical formulae were proposed to estimate the ultimate load of the RC slab with axisymmetric configuration, but there was no prediction of the nonlinear load-deformation behavior, and the study was limited to a relatively small scale structure. Okamura and Maekawa (1991) established a general nonlinear finite element analysis of two-dimensional reinforced concrete structures for an arbitrary static loading history. Their constitutive models of reinforced concrete materials were developed from four models namely, the concrete compression, tension stiffening, shear transfer and the reinforcing bar. The concept is a smeared crack RC model expressing a macroscopic behavior of reinforced concrete structures. The reliability of their proposed model was well verified by experimental results. In addition, Lan (1994) modified the Okamura and Maekawas model in order to tackle the problem of 2-dimensional RC structures with multi-directional reinforcements.

This study aims to develop an axisymmetric reinforced concrete constitutive model for a nonlinear finite element analysis and implement it into an existing computer code, and then to verify the model through comparison with experimental results available.

2. Theoretical model

Based on a smeared crack model dealing macroscopically with cracks and reinforcing bars by expressing the average stress and average strain relationships in a RC element, Okamura and Maekawa established a constitutive model for the cracked concrete, and then developed nonlinear finite element program, called WCOMR, to analyze the reinforced concrete panel structure (Okamura and Maekawa 1991). The reliability of Okamura and Maekawa's constitutive model was verified with a large number of experimental results. This model was modified to consider the multi-directional reinforcing bars of arbitrary number and at arbitary directions (Lan 1994). In this study, the modified Okamura and Maekawa's constitutive model is applied to axisymmetric RC structures. A modified constitutive model has been developed to meet the axisymmetric stress state.

2.1. Model before cracking

For the constitutive model of axisymmetric reinforced concrete element in the elastic range, i.e., before cracking of concrete, the material is assumed to follow linear elastic constitutive model. The stress-strain relation for a linear elastic axisymmetric material can be expressed as

$$\begin{Bmatrix} \sigma_{rr} \\ \sigma_{zz} \\ \tau_{rz} \\ \sigma_{\theta\theta} \end{Bmatrix} = \frac{1}{(1+\nu)(1-2\nu)} \begin{bmatrix} (1-\nu)E_c & \nu E_c & 0 & \nu E_c \\ & (1-\nu)E_c & 0 & \nu E_c \\ Symmetry & & \frac{(1-2\nu)}{2}E_c & 0 \\ & & & (1-\nu)E_c \end{bmatrix} \begin{Bmatrix} \varepsilon_{rr} \\ \varepsilon_{zz} \\ \gamma_{rz} \\ \varepsilon_{\theta\theta} \end{Bmatrix} \tag{1}$$

where σ_{rr}, σ_{zz}, τ_{rz} are axial stresses in r-direction (radial) and z-direction (vertical), and shear stress in rz plane; ε_{rr}, ε_{zz}, γ_{rz} are corresponding strains; $\sigma_{\theta\theta}$, $\varepsilon_{\theta\theta}$ are stress and strain in θ-direction (hoop) (see Fig. 1); E_c, ν are the Young's modulus and Poisson's ratio of concrete. It is noted that the stiffnesses of reinforcing bars are excluded in the above equation due to their negligible effects in comparison with those of concrete.

2.2. Model after cracking

In the present model, after cracking in either of the three directions, i.e., radial, vertical or hoop direction, the material is assumed to behave separately in the plane of radial and vertical direction and in the hoop direction. In other words, it is assumed that there is no coupling effect between the behavior in the plane of radial and vertical direction and that in the hoop direction (see Fig. 1). Based on the above assumptions, the constitutive model for axisymmetric cracked RC element can be divided into two independent models, i.e., two-dimensional model in the plane of radial and vertical direction (σ_{rr}, σ_{zz}, τ_{rz}, in Fig. 1), and one-dimensional model in the hoop direction ($\sigma_{\theta\theta}$, in Fig. 1).

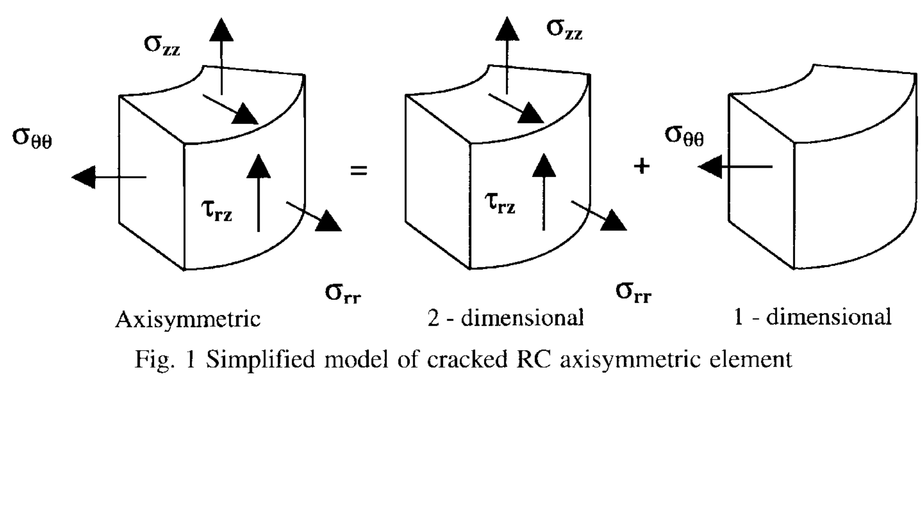

Fig. 1 Simplified model of cracked RC axisymmetric element

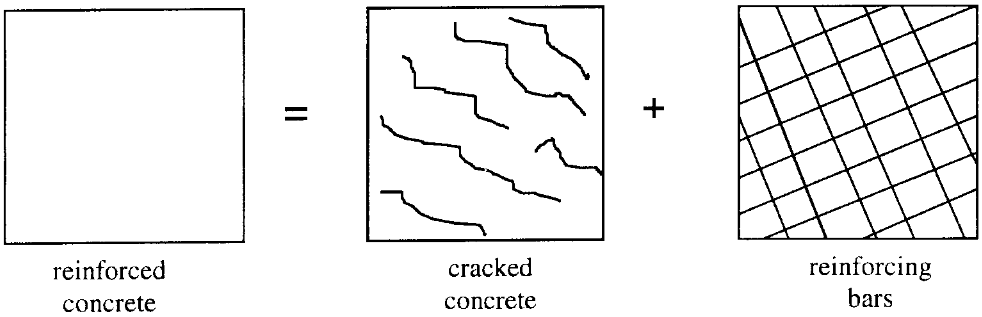

Fig. 2 Composition of reinforced concrete (RC) in-plane element model

2.2.1. Two-dimensional model

For two-dimensional model, the constitutive model developed by Okamura and Maekawa (1991) is adopted. In their model, the cracked RC element has been constructed by combining the constitutive law for cracked concrete and that for reinforcing bars, in which the relationships between the average stress and the average strain are given. In the constitutive law for the cracked concrete, there are three independent models namely :

- Tension stiffening model for tensile stress-strain in the direction perpendicular to the crack
- Compression model for compressive stress-strain in the direction parallel to the crack
- Shear transfer model for shear stress-strain along the crack

The details of formulation of the constitution law can be found (Okamura & Maekawa 1991).

2.2.2. One-dimensional model

For one-dimensional model, the original constitutive laws based on uniaxial test of RC element under tension and compression are employed as follows :

1) Uniaxial tensile stress-strain relationship

From Okamura *et al.* (1991), Shima *et al.* (1987), Izumo (1989), when cracks occur in the RC element under uniaxial tension, the average stress-strain relationship for the bar in concrete has been formulated by assuming the tensile stress distribution of the bar as a cosine curve, and the tension stiffening model of the concrete. By combining the average stress-strain relation of concrete and bar, the stress-strain relation of reinforced concrete can be obtained.

2) Uniaxial compressive stress-strain relationship

In Okamura *et al.* (1991), Maekawa *et al.* (1983), Okamura (1991), the relationship between the average stress and strain for cracked concrete under compression is expressed by

$$\sigma'_c = E_0 K(\varepsilon'_c - \varepsilon'_p) \tag{2}$$

where

$$E_0 = \frac{2f''_c}{\varepsilon'_{c0}} \tag{3}$$

$$K = \exp\left[-0.73\left(\frac{\varepsilon'_c}{\varepsilon'_{c0}}\right)\left\{1 - \exp\left(-1.25\frac{\varepsilon'_c}{\varepsilon'_{c0}}\right)\right\}\right] \tag{4}$$

$$\varepsilon'_p = \varepsilon'_c - \varepsilon'_{c0}\left(\frac{20}{7}\right)\left\{1 - \exp\left(-\frac{0.35\varepsilon'_c}{\varepsilon'_{c0}}\right)\right\} \tag{5}$$

in which E_0, K, ε'_p are called initial tangential elastic constant, fracture parameter, plastic compressive strain, respectively;

f''_c : uniaxial compressive strength of concrete;

f'_c : cylinder compressive strength of concrete;

ε'_{c0} : compressive strain corresponding to f''_c, ε'_{c0}=0.15%-0.25%;

σ'_c, ε'_c : compressive stress and strain of concrete;

For the model of bar, the compressive stress-strain relationship of bare bar is used. Similarly, by combining the models of concrete and bar, the model of RC can be established.

3. Experimental verification of the constitutive model

3.1. Test on circular slabs under a uniformly distributed load

The test of circular slab subjected to a uniformly distributed load, performed by Iwaki *et al.* (1985) is selected to check the reliability of the proposed constitutive models for axisymmetric reinforced concrete structures. Dimension and reinforcement arrangement of the tested slabs is illusrated in Fig. 3, and the material properties are tabulated in Table 1. Reinforcement ratio of a vertical stirrup p_v for each specimen is varied, i.e. p_v=0, 0.1%, 0.2%, 0.4% and 0.65% while the reinforcement ratios in the radial and hoop direction are fixed. The D6 reinforcing bars are used for both radial and hoop reinforcements and the D3 reinforcing bar are used for stirrups. The specimens are simply supported by a circular edge and an upward uniformly distributed load is applied on the

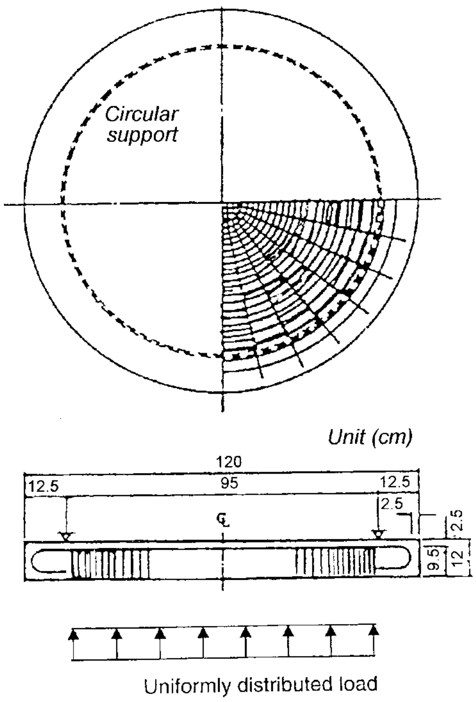

Fig. 3 Tested circular slab under a uniformly distributed load (from Iwaki *et al.* 1985)

Table 1 Material properties

Specimen	f'_c (kgf/cm^2)	f_y (kgf/cm^2)	Reinforcement ratio (%)		
			p_r	p_θ	p_v
			1d from support	1d from support	1.5d from support
A02	240	3500	0.39	1.42	0
SG2D6	254	3500	0.39	1.42	0.1
SG2D5	254	3500	0.39	1.42	0.2
SG2D4	256	3500	0.39	1.42	0.4
SG2D3	250	3500	0.39	1.42	0.65

Notes : f'_c: compressive strength of concrete
f_y : yield strength of reinforcing bar
p_r : reinforcement ratio in radial direction
p_θ : reinforcement ratio in hoop direction
p_v : reinforcement ratio in vertical direction (stirrups)

region inside the circular support.

3.2. Numerical Procedure

The above-mentioned tested circular slab was discretized into a four-node axisymmetric finite element mesh, and due to symmetry, only half of the slab is analyzed. Fig. 4 shows the FE mesh and loading condition. It is noted that in this study, the uniaxial tensile strength of concrete was approximated by f_t=0.5 $f_c^{2/3}$ (Japan Society of Civil Engineers 1991), and the reduction factor for that value was set to be 0.5 due to the size effect of concrete. In addition, the reduction factor for the yield strength of reinforcing bars (D6, D3) was set to be 0.85 due to a small diameter of bar. These reduction factors are determined based on a trial-and-error calculation, i.e. the concrete tensile strength and the steel yield strength were treated as the parameters in the analyses, and then those values which give the best fit to the experimental results were obtained.

In the present analysis, a non-iterative procedure using a load control method was used to obtain a

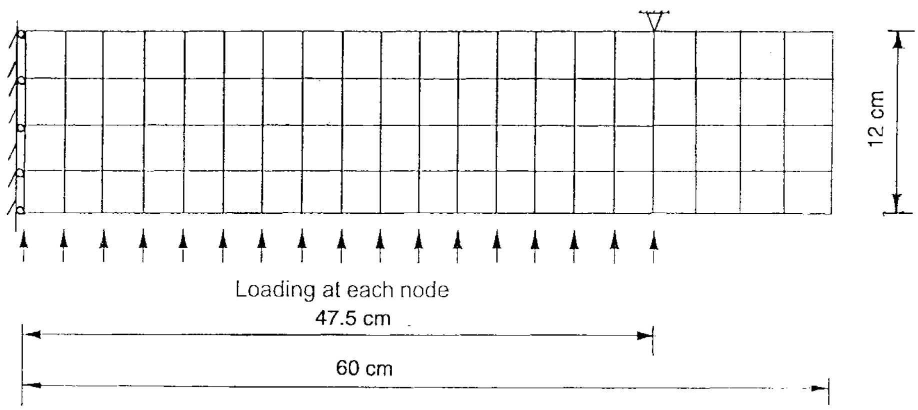

Fig. 4 FE mesh and loading condition of the circular slab

complete load-displacement curve for each specimen. In fact, the iterative procedures were tried, but the convergence failed immediately after the first yielding of the bar. The reason has not been clarified yet, however, it might be explained by the fact that due to very complicated cracking patterns almost in the whole span of the specimen, the degree of nonlinearity for the structure becomes very high, and this leads to the difficulty in obtaining converged solution. There exist a numbers of algorithms, such as line-search algorithm, i.e. arc-length method, to deal with sudden non-linearity. However, in this research, only force control method was adopted for simplicity.

3.3. Comparison between experimental and numerical results

Nonlinear analyses of the RC circular slab for five cases of different ratios of stirrups by using WCOMR with the proposed constitutive models for axisymmetric reinforced concrete structures were conducted. Compared with the test results, the load-displacement curves for specimen, i.e. A03 (p_v=0) and SG2D6 (p_v=0.1%) are shown in Figs. 5 and 6 respectively. Nonlinear behaviors of the tested specimens are predicted reasonably well, although the initial stiffnesses of the load-displacement curve in Fig. 5 and 6 are different between the numerical and experimental results due to an over-estimation of the elastic modulus of RC slab. The yielding point predicted by the numerical analysis of each case agrees well with test results.

Table 2 tabulates the ultimate loads measured and predicted, as can be seen, all predicted loads are lower than the measured ones. The differences between the measured and predicted results are in the range of 4.4% to 16.5% for the specimens with varied stirrup ratios from 0% to 0.65%.

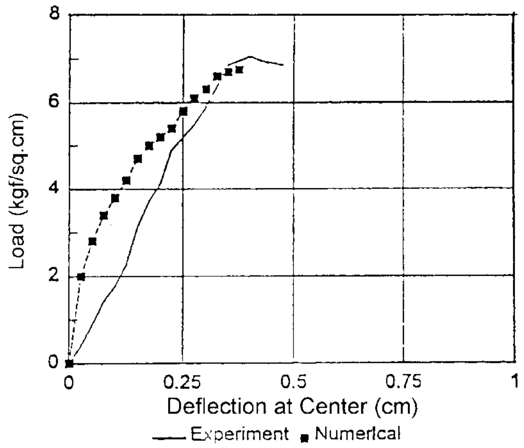

Fig. 5 Comparison of the load-displacement for the specimen A03

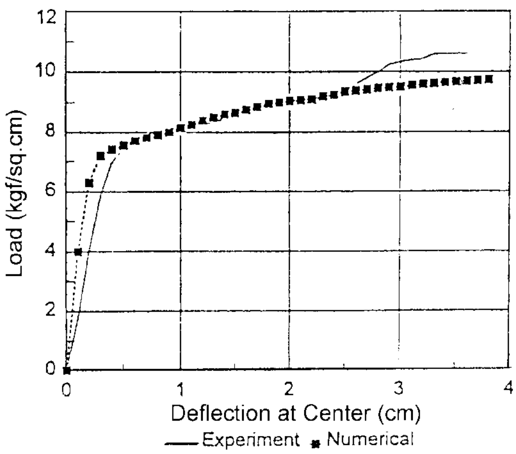

Fig. 6 Comparison of the load-displacement for the specimen SG2D6

Table 2 Comparison of the ultimate loads for different cases of the reinforcement ratio of stirrup

No. of Specimen	A03 (p_v=0)	SG2D6 (p_v=0.1%)	SG2D5 (p_v=0.2%)	SG2D4 (p_v=0.4%)	SG2D3 (p_v=0.65%)
Experimental (kgf/sq.cm)	7.06	10.6	12.74	12.78	13.23
Numerical (kgf/sq.cm)	6.75	9.80	10.6	10.81	11.05
Difference (%)	4.4	7.5	16.8	15.4	16.5

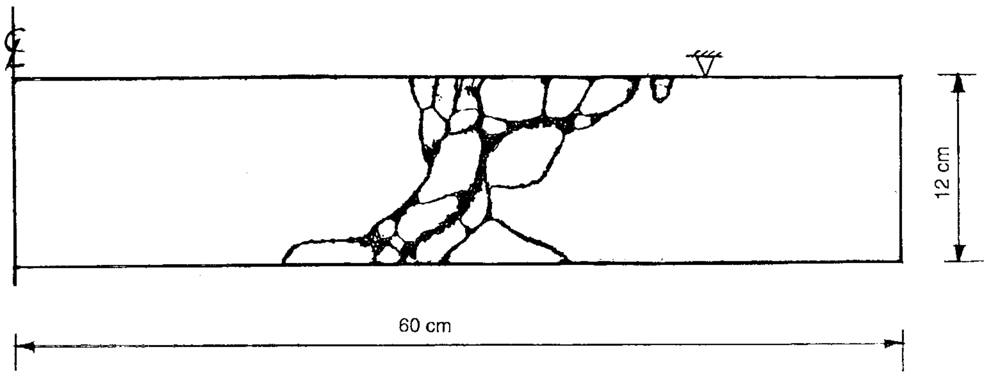

Fig. 7 Crack pattern of experiment at ultimate state for the specimen SG2D6 (from Iwaki *et al.* 1985)

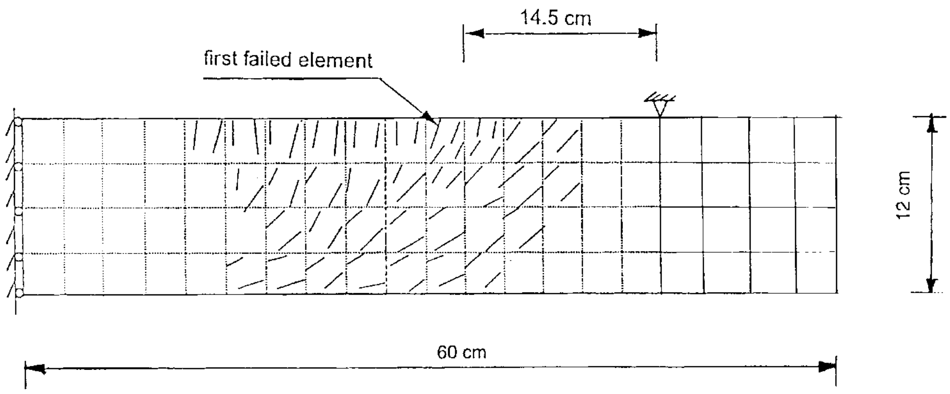

Fig. 8 Crack pattern of numerical analysis at ultimate state for the specimen SG2D6

Although the difference becomes larger while the stirrup ratio is increased with the stirrup ratio, there exists a reasonable good agreement between the predicted and the measured failure load. One contributing factor to those differences could be that the hardening portion of the reinforcing bar model was excluded in the analyses.

Figs. 7 and 8 show the cracked patterns observed in the test and predicted in the numerical analysis for specimen SG2D6. As shown in Figs. 7 and 8, the crack pattern obtained in the present numerical results has the same tendency as the tested results. Numerical results show that the shear punching failure is the predominant failure pattern, which is identical with the test report, and the first failure element located at 14.5 cm from the support on the upper surface as shown in Fig. 8, coincides well with that obtained from the experimental result (14.25 cm from the support on the upper surface). Therefore, from the above comparison between the experimental and numerical results, it can be concluded that the present analytical procedure can be used to simulate the behavior of the tested RC circular slabs subjected to a uniformly distributed load up to the failure of the structures.

4. Conclusions

In this study, a new constitutive model for axisymmetric RC element has been established by combining independent models, namely two-dimensional model in the radial and vertical plane, and one-dimensional in the hoop direction, and using the Okamura and Maekawa's constitutive models. The existing nonlinear finite element program WCOMR was modified to the proposed axisymmetric RC element. By comparing with the test results of circular slabs under a uniformly distributed load with different stirrup ratios, the new constitutive model has been validated.

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