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A method of optimum design based on reliability for antenna structures

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Abstract. A method of optimum design based on reliability for antenna structures is presented in this paper. By constructing the equivalent event, the formula is derived for calculating the reliability of reflector accuracy of antenna under the action of random wind load. The optimal model is developed, in which the cross sectional areas of member are treated as design variables, the structure weight as objective function, the reliability of reflector accuracy and the strength or stability of structural elements as constraints. The improved accelerated convergence gradient algorithm developed by the author is used. The design results show that the method in this paper is feasible and effective.

Key words: antenna structure; reflector accuracy; strength and stability; reliability constraints; optimum design; improved accelerated convergence gradient algorithm.

1. Introduction

Since the concept of homologous design of parabolic antenna structure proposed by Von Hoerner in 1976, many papers about optimum design of antenna structures have been published, such as Levy and Melosh *et al.* (1976). These works have played a significant role in the design of antenna structures. However, since the restrictions on the theory and method in structure design at that time, in those early papers the loads applied on the antenna structure are considered to be determinate, for example, with the self-weight of structure and given invariable wind speed as well as temperature gradient being as designing loads of antenna structure. Apparently, this is out of accord with real case. Usually, an antenna structure is being acted by various random loads inevitably during working time. So it is needed to study the optimum design of the antenna structure based on reliability.

In recent decades, the development of theory of reliability has attracted attention to the reliability of antenna structures. In 1985 Wang studied the reliability of antenna surface accuracy, which is affected by the random factors in structural manufacture and assembly. However, for the large antenna structures without cover, the wind undoubtedly is the most main random load affecting the reflector accuracy. So in 1990 Chen analyzed the reliability of antenna reflector

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accuracy under the action of random wind load. Based on the early studies by the author (Chen Dai and Sun 1990, Chen and Duan 1994, Fen and Moses 1986), Enevoldsen and Sorensen 1994, a method of optimum design of antenna structure based on reliability is proposed in this paper.

2. Reliability analysis of antenna structures

2.1. Structural analysis under the action of random loads

Supposed that the two kinds of determinate and random loads act upon an antenna structure simultaneously during working time, so that the finite element equation of the structure can be divided into two parts, i.e., the ones of determinate and random

$$[K]\{\delta\}=\{L\} \tag{1a}$$
$$[K]\{\Delta_R\}=\{L_R\} \tag{1b}$$

where $[K]$ is the stiffness matrix of structure; $\{L\}$, $\{\delta\}$ are the determinate load vector and the displacement vector of the structure respectively; $\{L_R\}$, $\{\Delta_R\}$ are the random load vector and the random displacement vector of the structure respectively.

Analysis shows that for the antenna structures without cover the main random load comes from the wind pressure. According to Sachs (1978) and authors studies (Chen, Dai and Sun 1990), if geometric dimensions of an antenna structure is not too large, the wind pressure that acts upon the structure in each sample is approximately submitted to a fully correlated Normal distribution, that is

$$\{L_R\}\sim N[E(\{L_R\}),\ D(\{L_R\})]=N[E(\{L_R\}),\ V_L^2E^2(\{L_R\})] \tag{2a}$$
$$\rho(L_{Ri},L_{Rj})=1 \quad (i,j=1,2,\cdots,m) \tag{2b}$$

where $E(\cdot)$, $D(\cdot)$ indicate the expectation and variance operator respectively; V_L is the variation coefficient of the random wind pressure load vector; $\rho(L_{Ri}, L_{Rj})$ is the correlative coefficient of load component L_{Ri} and L_{Rj}; m is the number of components in the load vector $\{L_R\}$.

From Eqs. (1a) and (1b) the determinate and random displacement vectors of the structure can be written as

$$\{\delta\}=[K]^{-1}\{L\} \tag{3a}$$
$$\{\Delta_R\}=[K]^{-1}\{L_R\} \tag{3b}$$

Furthermore, according to the relationship between node displacement and element stress, the determinate and random vectors of stress of eth element in the structure, $\{S^e\}$ and $\{S_R^e\}$, can be expressed as

$$\{S^e\}=[D^e][T^e]\{\delta^e\} \quad (e=1,2,\cdots,n_e) \tag{4a}$$
$$\{S_R^e\}=[D^e][T^e]\{\Delta_R^e\} \quad (e=1,2,\cdots,n_e) \tag{4b}$$

where $[D^e]$ is the relation matrix of eth element between the node displacement and element stress; $[T^e]$ is the coordinate transform matrix of eth element; $\{\delta^e\}$ and $\{\Delta_R^e\}$ are respectively the determinate and random vector of node displacement of eth element; n_e is the number of elements in the structure.

It is well known that the distribution of linear transformation of the Normal variable is invariant.

If the considered structure belongs to elastic system, it can be deduced that all the response random vector $\{\Delta_R\}$ and $\{S_R^e\}$ $(1, 2, \cdots, n_e)$ have the same distribution type as the load $\{L_R\}$, that is

$$\{\Delta_R\} \sim N[E(\{\Delta_R\}),\ \ D(\{\Delta_R\})] \tag{5a}$$

$$\rho(\Delta_{Ri}, \Delta_{Rj}) = 1 \quad (i, j = 1, 2, \cdots, m) \tag{5b}$$

where
$$E(\{\Delta_R\}) = [K]^{-1}E(\{L_R\}), \quad \sqrt{D(\{\Delta_R\})} = [K]^{-1}V_L E(\{L_R\})$$

$$\{S_R^e\} \sim N[E(\{S_R^e\}),\ \ D(\{S_R^e\})] \quad (e = 1, 2, \cdots, n_e) \tag{6a}$$

$$\rho(S_{Ri}^e, S_{Rj}^e) = 1 \quad (i, j = 1, 2, \cdots, m_e : e = 1, 2, \cdots, n_e) \tag{6b}$$

where
$$E(\{S_R^e\}) = [D^e][T^e]E(\{\Delta_R^e\}), \quad \sqrt{D(\{S_R^e\})} = [D^e][T^e]\,V_L E(\{\Delta_R^e\})$$

m_e is the number of stress components in eth element.

2.2. Reliability analysis of reflector accuracy of antenna

The reliability of reflector accuracy R_r is the probability that the mean square value of the displacements of the reflector along the electrical axis of antenna, $U_{rms}{}^2$, does not exceed its given design value under the action of loads $\{L\}$ and $\{L_R\}$, that is

$$R_r = \text{Prob}\{U_{rms}^2 \le U^{*2}\} \tag{7}$$

$$U_{rms}^2 = \frac{1}{n_r}\sum_i^{n_r}(\Delta_{iz} + \delta_{iz})^2 \tag{8}$$

where U^{*2} is the given design accuracy, its value depends on Ruze (1952) formula; δ_{iz}, Δ_{iz} are the determinate and random variable of the displacements of ith node on the reflector in direction z (that is the direction of antenna electric axis); n_r is the number of nodes on the reflector.

Through the proceeding derived in Chen, Dai and Sun (1990), the formula of accuracy reliability (7) can be expressed as the following equivalent formula:

$$R_r = \text{Prob}\{\chi^2(1) \le \chi_{rms}^{*2}\} = F_{\chi^2(1)}(\chi_{rms}^{*2}) \tag{9}$$

$$\chi_{rms}^{*2} = \frac{1}{n_r}\sum_i^{n_r}\left[\frac{U^* - (\mu_{iz} + \delta_{iz})}{V_L\,\mu_{iz}}\right]^2 \tag{10}$$

where $\chi^2(1)$ is the χ^2 distribution for one degree-of-freedom, that is noncentral distribution; $F_{\chi^2(1)}(\cdot)$ is the probability distribution function of variable $\chi^2(1)$; μ_{iz} is the mean value of variable Δ_{iz}.

2.3. Reliability analysis of strength and stability for structural elements

Since the antenna structures are complex and highly redundant ones, so far it is exceedingly difficult to analyze the systematic reliability of structure precisely. Therefore the reliability analysis for strength and stability of the antenna structure is same as the conventional structure analysis, i.e., the reliability analyzing is directed against each structural element. That is to say, the structural optimization based on reliability in this paper belongs to the component order level of reliability design.

After the allowable stresses are given and the mean value and variance of stress variables are obtained, according to the theory of stress-strength interference in structural reliability (Thoft-Christensen and Murotsu 1986), the reliability of strength and stability of element e can be expressed as:

$$R_S^e = \text{Prob}\{\eta^e S_a^e - (S_R^e + S^e) \geq 0\} \quad (e = 1, 2, \cdots, n_e) \tag{11}$$

where, S^e, S_R^e are respectively the determinate and random stresses of element e; S_a^e is the allowable stress of element e; η^e is the strength-stability coefficient of eth element (bar), its value is determined by following formula.

$$\eta^e = \begin{cases} 1 & \text{when } e\text{th element (bar) is stretched} \\ \exp\left\{\dfrac{-(\lambda^e)^2}{20000}\right\} & \text{when } e\text{th element (bar) is compressed} \end{cases} \tag{12}$$

where λ^e is the ratio of slenderness of the eth element (bar).

From formula (6a) it is known that all the random variables of stress, S_R^e, are submitted to Normal distribution, that is

$$S_R^e \sim N[(E(S_R^e),\ \ D(S_R^e)] = N[E(S_R^e),\ \ V_L^2 E^2(S_R^e)] \quad (e = 1, 2, \cdots, n_e) \tag{13}$$

If all the allowable stress S_a^e (e=1, 2, $\cdots$, n_e) are also random variables, according to general treatment in structural engineering, then S_a^e (e=1, 2, $\cdots$, n_e), are considered as submitting to normal distribution, i.e.,

$$S_a^e \sim N[E(S_a^e),\ \ D(S_a^e)] = N[E(S_a^e),\ \ V_{S_a^e}^2 E^2(S_a^e)] \quad (e = 1, 2, \cdots, n_e) \tag{14}$$

where $V_{S_a^e}$ the variation coefficient of random variable S_a^e.

Hence, by means of the theory of second-moment in structural reliability (Thoft-Christensen and Murotsu 1986), the reliability of strength and stability for eth element, i.e., formula (11), can be rewritten as:

$$R_S^e = \Phi(\beta^e) \tag{15}$$

$$\beta^e = [\eta^e E(S_a^e) - (E(S_R^e) + S^e)] \cdot [(\eta^e)^2 V_{S_a^e}^2 E^2(S_a^e) + V_P^2 E^2(S_R^e)]^{-1/2} \quad (e = 1, 2, \cdots, n_e) \tag{16}$$

where $\Phi(\cdot)$ is the probability distribution function of standard Normal variable; β^e is the reliability index of eth element.

If all the allowable stress S_a^e (e=1, 2, $\cdots$, n_e) are deterministic constants, then the reliability index in the formula (16) can be expressed as:

$$\beta^e = [\eta^e S_a^e - (E(S_R^e) + S^e)] \cdot [V_P E(S_R^e)]^{-1} \quad (e = 1, 2, \cdots, n_e) \tag{16a}$$

3. The mathematical model and method of reliability optimum design

3.1. Mathematical model

For a parabolic antenna, especially for large and high-precision antenna, the electromagnetic performance depends on the shape accuracy of the reflector. So, if a reflector of antenna is not satisfied with the demands of accuracy, it will lose its significance. Consequently, in order to ensure the fine electromagnetic performance of antenna, the strict demands of its reflector accuracy under operation are generally put forward. In addition being an engineering structure, it also should be ensured that it would not be damaged in the worst load case. Due to this feature of antenna structure, the optimum design model based on reliability is developed in which the bar cross sectional areas of antenna skeleton are treated as design variables, the weight of the structure is minimized and meanwhile, the following constraints are subjected to.

(1) The accuracy reliability of the reflector in the working load case (mean value of wind speed $E(V)$=20 m/s).

(2) The reliability of strength and stability of the structural elements in the worst load case (mean value of wind speed $E(V)$=50 m/s).

(3) The low boundary of variable $A_{\min}$.

The mathematical model can be expressed as:

$$\text{find} \quad \bar{A} = (A_1, A_2, \cdots, A_n)^T \tag{17}$$

$$\min \quad W(\bar{A}) = \sum_i^n A_i \rho_i \sum_j l_{ij} \tag{18}$$

$$\text{s.t.} \quad R_r^* - R_r(\bar{A}) \le 0 \quad \text{(in the working load case)} \tag{19}$$

$$R_s^* - \min_{e=1,n_e} \{R_S^e(\bar{A})\} \le 0 \quad \text{(in the worst load case)} \tag{20}$$

$$A_{\min} - A_i \le 0 \quad (i = 1, 2, \cdots, n) \tag{21}$$

where n is the number of design variables; A_i, ρ_i, $\sum_j l_{ij}$ are the cross sectional area, the material density and the total length of ith class design variables, respectively; R_r^* is the given value of accuracy reliability of the reflector; R_S^* is the given value of strength or stability reliability of the weakest element; $A_{\min}$ is the given low boundary value of the design variables.

3.2. Optimization method

According to the former experience, the accuracy condition of reflector is usually the most important design demand (Levy *et al.* 1976, Wang and Li 1981, Ye 1984). So that, the constraint of accuracy reliability, Eq. (19), is always taken as the tightest constraint in optimum design. In this situation, we solve the problem with the criterion method. First the Lagrange function corresponding with the problem is constructed, that is

$$L(W, \lambda_1, \lambda_2) = \sum_i^n A_i \rho_i \sum_j l_{ij} + \lambda_1 (R_r^* - R_r) + \lambda_2 (A_{\min} - A_i) \tag{22}$$

where λ_1 and λ_2 are Lagrange multipliers.

Then the optimal criterion can be derived by Kuhn-Tucker condition as

$$\frac{\partial R_r}{\partial A_i} = \frac{1}{\lambda_1} (\rho_i \sum_j l_{ij} - \lambda_2) = \Lambda \quad (i = 1, 2, \cdots, n) \tag{23}$$

This criterion indicates that when the structure is satisfied with the constraints of accuracy reliability and simple bound, and its weight is minimum, the variability of accuracy reliability caused by changing each design variables will converge to a same constant Λ uniquely.

Considering that R_r is quadratic function of the displacement random variables, and usually is directly proportional to design variables A_i (1, 2, $\cdots$, n), we set up the following iterative formula in terms of improving the accelerated convergence gradient algorithm.

$$A_i^{(K+1)} = A_i^{(K)} \left[\frac{\partial R_r^{(K)}}{\partial A_i^{(K)}} \middle/ \sqrt{\frac{1}{n} \sum_i^n \left(\frac{\partial R_r^{(K)}}{\partial A_i^{(K)}} \right)^2} \right]^{C_1(K)} \cdot \left[\frac{R_r^*}{R_r^{(K)}} \right]^{C_2(K)} \tag{24}$$

$$A_i^{(K+1)} \geq A_{\min} \quad (i = 1, 2, \cdots, n)$$

where: $\frac{\partial R_r^{(K)}}{\partial A_i^{(K)}} \middle/ \sqrt{\frac{1}{n} \sum_i^n \left(\frac{\partial R_r^{(K)}}{\partial A_i^{(K)}} \right)^2}$ is the ratio between sensitivity of variable of ith class design variable and the root mean square of sensitivity of all variables, this term gives the revival direction of A_i at iteration step K. $R_r^*/R_r^{(K)}$ is the ratio between given accuracy reliability and the accuracy reliability at iteration step K, this term gives the revival step size of A_i at iteration step K. $C_1(K)$, $C_2(K)$ both are the accelerated convergence exponents at iteration step K, their value that taking in the iteration procedure can be refereed to (Fen and Moses 1984).

The mathematical action of iterative Eq. (24) is to revise the ith class design variable in terms of its sensitivity percentage occupied in all variable sensitivity.

Due to that the accuracy reliability $R_r(\bar{A})$ is implicit and nonlinear compound function of designing vectors, here we obtain the sensitivities $\partial R_r^{(K)}/\partial A_i^{(K)}$ (i=1, 2, n) by means of numerical method. That means, the differential is replaced by difference approximately. Of cause, the sensitivity of reliability-based also can be estimated by analytic or semi-analytic method, but they are rather complex (Bjerager and Krenk 1989).

3.3. The terminating condition of iteration

According to the meaning of optimal criterion (23), apparently, the terminating condition of iteration can be established as:

$$1 - \delta_o \leq \frac{\partial R_r^{(K)}}{\partial A_i^{(K)}} \middle/ \sqrt{\frac{1}{n} \sum_i^n \left(\frac{\partial R_r^{(K)}}{\partial A_i^{(K)}} \right)^2} \leq 1 + \delta_o \quad (i = 1, 2, \cdots, n) \tag{25}$$

where δ_o is the terminating iteration error for the reliability sensitivity.

In addition to Eq. (25), the other two terminating conditions of iteration also can be established in terms of the objective function and the constraint function of accuracy reliability. They are

$$| W^{(K+1)} - W^{(K)} | / W^{(K)} \leq \varepsilon_w \tag{26a}$$

$$| R_r^{(K+1)} - R_r^* | / R_r^* \leq \varepsilon_r \tag{26b}$$

where ε_w, ε_r is the terminating iteration error for the objective function and accuracy reliability constraint respectively.

When the iteration is stopped, the reliability constraints of strength and stability are needed to be checked. If one of them is not satisfied, we shall regulate some design variables in concern, until Eqs. (20) and (21) are also satisfied.

4. Examples

According to the preceding theoretical analysis and the formally derivation, the corresponding computer program is developed in FORTRAN language. Several structures are calculated with the program and the results are satisfactory. Two examples are given below.

Example 1. Four-bar space truss (see Fig. 1)

The structural parameters are: elasticity modules $E=10^7$(psi); material density $\rho=0.1$ (lb/in^3) and allowable stretching and compression stresses of all bars which are same constant, they are S_a^e ($\pm$) =25000 (psi). The load case is that the node 1 is acted by the correlative Normal random load whose variation coefficient $V_L=0.2$. The vector of load mean values $[E(L_{1x}), E(L_{1y}), E(L_{1z})]^T=[10\text{K}, 20\text{K}, -60\text{K}]^T$. The design variables are the cross section of four bars, that is $\bar{A}=(A_1, A_2, A_3, A_4)^T$. The constraint conditions are:

$$R_r = \text{Prob}\{\delta_{1z} \le 0.3 \text{ in}\} \ge R_r^* = 0.95$$

$$\min_{e=1,4} \{R_S^e\} \ge R_S^* = 0.9999$$

$$A_i \ge A_{\min} = 0 \quad (i = 1, 2, 3, 4)$$

The calculating converged after six iterations. The iterative procedure and optimum design

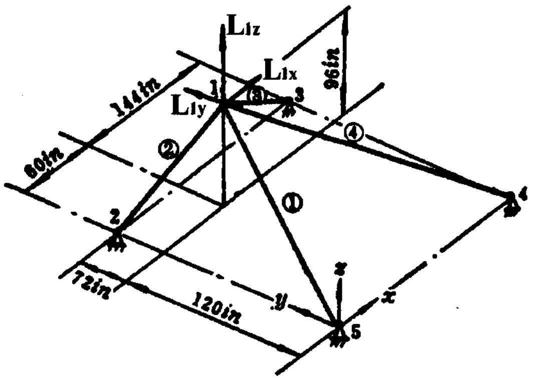

Fig. 1 Four-bar space truss

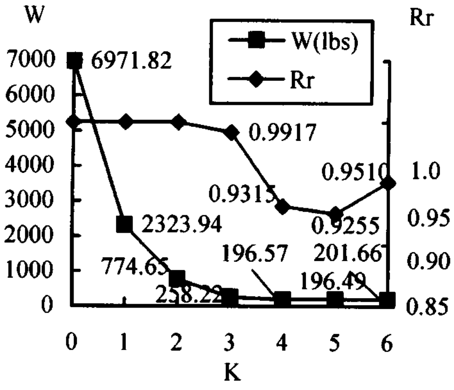

Fig. 2 The iterative procedure for four-bar space truss

Table 1 The optimum design results of four-bar space truss

Variables (in^2)	A_1	A_2	A_3	A_4	W(lb.)	R_r	$\min\{R_S^e\}$	Iteration number
Initial value	100	100	100	100	6971.8	1.0000	1.0000	0
Khan *et al.* 1979's results	0.001	3.651	0.769	2.759	121.5	0.1521	0.5120	6
Authors results	1.948	4.603	3.104	2.354	201.7	0.9510	1.0000	6

results are shown in Fig. 2 and Table 1.

In Table 1 the reliability results of (Khan *et al.* 1979) is obtained by the common optimization method in which the mean value of random load is taken as a constant load. By comparison of the two results in Table 1, it is clear that for the same mathematical model, the design results are very different between the reliability optimization and the common optimization, in other words, the optimal results of common optimization is usually an unfeasible solution for the optimal design based on reliability.

Example 2. 8 m-diameter parabolic antenna structure (see Fig. 3)

This is a rotator parabolic antenna with 8 m-diameter. The skeleton of antenna is a space bar system, which consists of twelve radiating beams, four circular beams and lots of oblique-jackstay. The reflector is made of 4mm-thickness aluminum plate. Due to the symmetry, only one fourth of its parts is drawn out in Fig. 3.

The structural parameters are: $E=2.1\times10^6$ (kg/cm^2); $\rho=7.8\times10^{-3}$ (kg/cm^3); the allowable stretching stress of every bar $S_a^e(+)=2100$ (kg/cm^2), while the allowable compression stress $S_a^e(-)$ depends on the stability condition of *e*th element (bar). The center of the reflector is 15m high above ground level. The given index of reflector accuracy $U^*=0.85$(mm).

The design variables are the cross section of all bars. Suppose that all the sections are circular with internal diameter of 20mm whereas the external diameters are not equal to each other. Because of the symmetry of the structure all the design variables can be merged into twelve classes. The classified numbers are seen in Fig. 3.

The load case is that: when the axis of antenna is located in horizontal attitude, it is acted simultaneously by the self-weight of structure and correlative Normal random wind pressure with variation coefficient $V_L=0.2$ (in general, this is one of the worst loading cases for parabolic antenna structure). The constraint conditions are as follows:

(1) When the mean value of wind speed at the standard height (10m above ground level) $E(V_0)=$ 20 (m/s), the working probability of the reflector accuracy, R_r, is not less than 0.95, i.e.,

$$R_r = \text{Prob}\{U_{rms} \le 0.85\text{mm}\} \ge R_r^* = 0.95 \quad \text{(in the working load case)}$$

(2) When the mean value of wind speed at the standard height $E(V_0)=50$(m/s), the safety probability of the weakest element in the structure is not less than 0.99, i.e.,

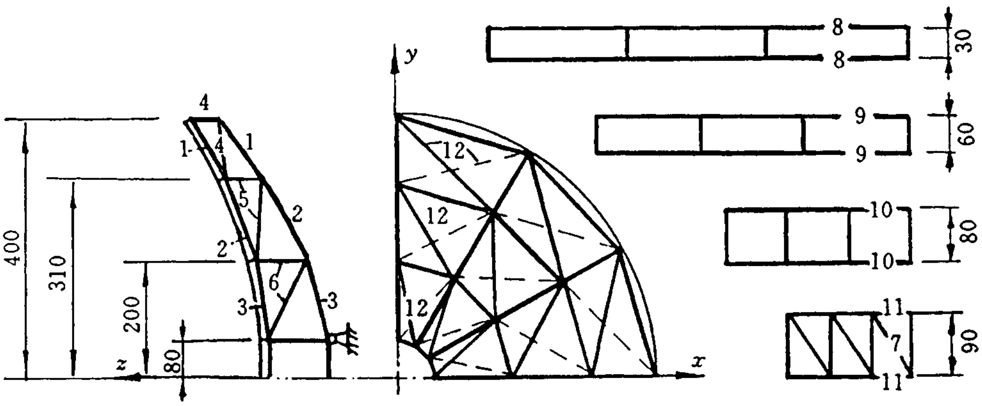

Fig. 3 8m-diameter parabolic antenna structure

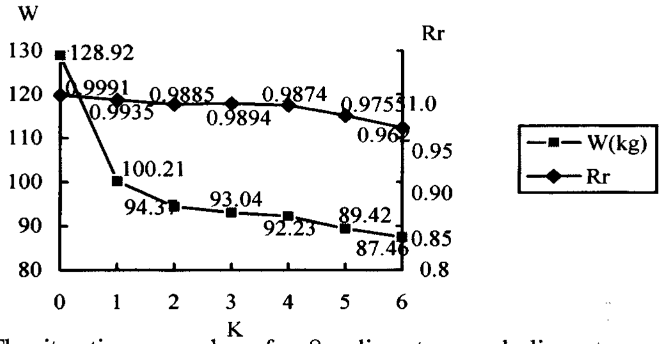

Fig. 4 The iteration procedure for 8m-diameter parabolic antenna structure

Table 2 The optimum design results of 8m-diameter parabolic antenna structure

Variable (cm^2)	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}
Initial values	1.5	2.0	3.0	1.0	1.5	1.5	3.0	1.0	1.5	2.0	3.0	1.0
Design results	0.5796	1.7473	2.6018	0.5	0.5	1.5879	0.6427	1.4928	1.8665	0.5	0.5	0.5

Structure's index	W (kg)	R_r	$\min\{R_S^e\}$	Iteration number
Initial values	128.92	0.99914	1.00000	0
Design results	87.47	0.96199	0.99132	6

$$\min_{e=1,n_e} \{R_S^e\} \geq R_S^* = 0.99 \quad \text{(in the worst load case)}$$

(3) The cross section of all bars is not less than 0.5 (cm^2), i.e.,

$$A_i \geq A_{\min} = 0.5(\text{cm}^2).$$

Because that the center of reflector is much higher than ground level, we can neglect the ground condition effect on distribution of wind pressure. The distribution curves of the wind pressure on the reflector can be then regarded as one family of concentric circles. Therefore, only the structure with one-fourth parts is calculated. The calculation converged after 6 iterations. The Fig. 4 and Table 2 give the iteration procedure and the optimum design results respectively.

5. Conclusions

The computational examples show that:

(1) For the high-precision antenna structure, the optimal solution that is satisfied with the accuracy reliability, generally speaking, can also be satisfied with the reliability constraints of strength and stability of the structural elements. Thus, it seems more reasonable to treat the accuracy reliability as the tightest constraint for antenna structure.

(2) When structure is acted by random load, if one takes the mean value of random load as the design load and optimizes the structure with a common optimization method, the obtained optimal solution is no more feasible solution in such case. However, the optimal design results based on reliability can provide the optimal feasible solution for the structure.

(3) The optimization method and the computer program designed in this paper is suitable not

only for antenna structure, but also for other engineering structures in which the displacement reliability is tightest constraint, such as example 1.

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