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Comparative studies of double- and triple-layer space trusses

Ahmed El-Sheikh†

Department of Civil Engineering, University of Dundee, Dundee DD1 4HN, U.K.

Abstract. In some space truss applications, particularly those with large spans, the choice of a triple-layer system might prove more cost effective than the more commonly used double-layer solution. However, there are currently no clear guidelines as to which system would be more competitive for intermediate span lengths. In this paper, comparisons in terms of the weight, stiffness and number of joints and members are made between the two system types and presented in order to simplify the choice process for the designer. The comparisons are carried out using an approximate analysis technique that is explained in this paper, and checked to be reasonably accurate and suitable for the preliminary design of space trusses.

Key words: space trusses; approximate analysis; design; competitiveness.

1. Introduction

Since the beginning of their commercial use four decades ago, the popularity of space trusses has increased, especially for large open areas with few or no intermediate supports. Over the years, they have become known for their pleasing appearance, lightweight, easy fabrication and rapid erection. Hundreds of successful space truss applications now exist all over the world covering stadiums, public halls, exhibition centres, aeroplane hangers and many other buildings.

The majority of space truss applications employ systems of the double-layer grid type. However, triple-layer space trusses are also in use, particularly in covering very large spans, and where double-layer trusses would need heavy members and could be less economical. The choice between a double-layer and a triple-layer system is usually easy in small and very large span applications, but is not as straightforward in applications with intermediate spans. For instance, a shift from a double-layer to a triple-layer floor system would result in the following advantages (refer to Fig. 1):

- The structure would have smaller chord member forces due to the typical reduction in chord panel size, and hence growth in the number of chord members employed. If the structure was also allowed a depth increase, as is usually the case, the chord member forces of the triple-layer structure would further reduce.
- The compression chord and diagonal members would be less likely to buckle due to the combined decrease in their effective length and internal forces.
- With the typical increase in the number of chord members, the structural integrity of the

† Ph.D.

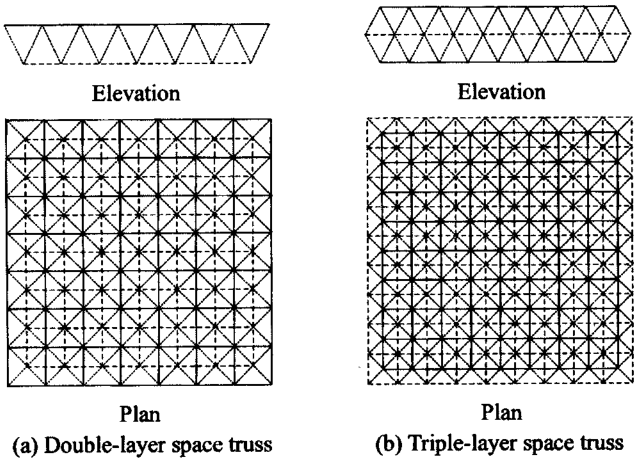

Fig. 1 Typical double- and triple-layer space trusses

truss would be less dependent on a small number of critical compression members (El-Sheikh 1997).

- The members and nodes would typically be of smaller size, and hence easier to manufacture and assemble.

However, consideration must also be given to the following disadvantages usually associated with triple-layer systems:

- A triple-layer space truss uses significantly more members and nodes; a consistent feature that can affect the structure's cost competitiveness.
- A triple-layer system uses a larger floor depth, leading to a taller structure subjected to higher wind loads, and requiring more cladding.

Further, while triple-layer space trusses are known to have a high degree of statical indeterminacy (El-Sheikh 1997), (about 33% in square-on-square systems), double-layer alternatives have a considerably reduced indeterminacy degree in the order of 15-25% (Affan 1987). Although there is conflicting evidence on whether having a high degree of indeterminacy is beneficial (Schmidt *et al.* 1976), overall it seems that the merits of a higher degree of indeterminacy outweigh the drawbacks (El-Sheikh 1997).

Another issue is related to the sensitivity of both types of systems to member geometric imperfections, due to length discrepancies or initial out-of-straightness. In two earlier studies (El-Sheikh 1995, 1997), it has been identified that triple-layer space trusses commonly have a lower sensitivity to imperfections than their double-layer counterparts. This was apparent in notably less strength reductions and less ductility losses associated with member imperfections in triple-layer trusses.

The purpose of the work presented in this paper is to carry out comparative parametric studies on both types of space truss systems in order to assist future designs concerned with space truss applications. The focus in this work is on the weight (steel consumption), the stiffness and the number of joints and members. The paper uses a simplified design technique based on two approximate analysis methods presented earlier in El-Sheikh (1996). The accuracy of the equations presented is assessed in this paper and found suitable for the comparative studies conducted, as well as the preliminary design of one-way and two-way space trusses.

2. Approximate analysis technique for one-way space trusses

One-way space trusses, such as those used in bridge applications or where the aspect ratio exceeds 2 : 1, can be analysed while only considering the members running between the supports (El-Sheikh 1996).

2.1. One-way double-layer space trusses:

For the double-layer space truss with two opposite lines of supports shown in Fig. 2:
L_1=length of the bottom chord in direction 1 (main direction)
L_2=length of the bottom chord in direction 2 (secondary direction)
PW_1=panel width in direction 1
PW_2=panel width in direction 2
The length of chord members can be calculated as:

$$\text{Total length of top members in direction } 1 = (L_1 + PW_1)\left(\frac{L_2}{PW_2} + 2\right),$$

$$\text{Total length of top members in direction } 2 = (L_2 + PW_2)\left(\frac{L_1}{PW_1} + 2\right),$$

$$\text{Total length of bottom members in direction } 1 = L_1\left(\frac{L_2}{PW_2} + 1\right), \text{ and}$$

$$\text{Total length of bottom members in direction } 2 = L_2\left(\frac{L_1}{PW_1} + 1\right).$$

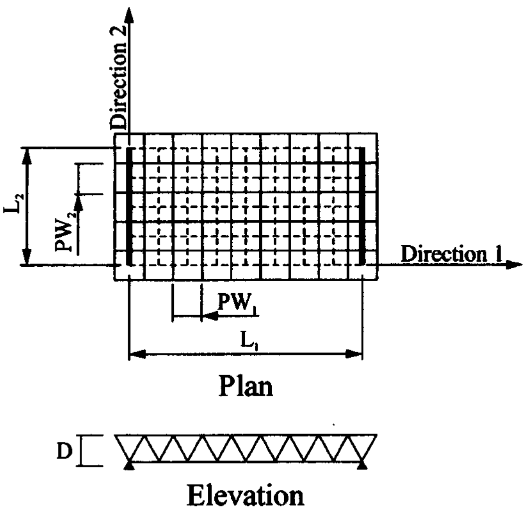

Fig. 2 Layout of a one-way double-layer space truss

Assuming that the total factored surface load acting on the truss is W, the maximum total bending moment in direction 1 is:

$$M = \frac{W(L_2 + PW_2)L_1^2}{8}.$$

By dividing this moment by the truss depth, D, and number of chord members in direction 1, the chord member forces are obtained as:

$$\text{Maximum force in one top chord member in direction } 1 = \frac{W(L_2 + PW_2)L_1^2}{8D\left(\dfrac{L_2}{PW_2} + 2\right)}, \text{ and}$$

$$\text{Maximum force in one bottom chord member in direction } 1 = \frac{W(L_2 + PW_2)L_1^2}{8D\left(\dfrac{L_2}{PW_2} + 1\right)}.$$

These forces can then be divided by the maximum material stress relevant in each situation, to obtain the chord member cross-sectional areas (i.e., in tension members, consideration is made of the yield stress, σ_y, while in compression members, the buckling stress, σ_p, for a typical slenderness ratio of around 80, is considered). Further, due to the much smaller chord member forces in the secondary direction, these chord members were sized assuming that their maximum forces would be about 10% of the forces in the main direction, and no reduction in size below this level was allowed in these members due to practical reasons (Schmidt 1972).

Eventually, the weight of the chord members is obtained as:

Weight of top chord members $= WT$

$$= \left\{ \left[(L_1 + PW_1) \cdot \left(\frac{L_2}{PW_2} + 2 \right) \right] + 0.1 \left[(L_2 + PW_2) \cdot \left(\frac{L_1}{PW_1} + 2 \right) \right] \right\} \frac{W(L_2 + PW_2)L_1^2 \gamma}{8D\left(\dfrac{L_2}{PW_2} + 2\right)\sigma_p}, \text{ and}$$

Weight of bottom chord members $= WB$

$$= \left\{ \left[L_1 \left(\frac{L_2}{PW_2} + 1 \right) \right] + 0.1 \left[L_2 \left(\frac{L_1}{PW_1} + 1 \right) \right] \right\} \frac{W \cdot PW_2 \cdot L_1^2 \gamma}{8D\,\sigma_y}$$

where γ=weight density of truss material.

Further, since the total load on the truss is: $W(L_1+PW_1)\cdot(L_2+PW_2)$, the vertical load on each diagonal member connected to a support (which are the most highly loaded diagonals)

$$= W(L_1 + PW_1)PW_2/4,$$

hence the maximum resultant compression force in a diagonal

$$= W(L_1 + PW_1)PW_2 \frac{\sqrt{\left(\frac{PW_1}{2}\right)^2 + \left(\frac{PW_2}{2}\right)^2 + D^2}}{4D}.$$

By assuming a slenderness ratio of around 120, the buckling stress of the diagonals (σ_p') can be obtained, leading finally to the weight of the diagonals being:

$$WD = 4\left(\frac{L_1}{PW_1} + 1\right)\left(\frac{L_2}{PW_2} + 1\right)\left[\left(\frac{PW_1}{2}\right)^2 + \left(\frac{PW_2}{2}\right)^2 + D^2\right]\frac{W(L_1 + PW_1)PW_2\gamma}{4D\,\sigma_p'}.$$

Therefore, the total weight of the one-way double-layer space truss is $WT+WB+WD$.

Finally, by analogy with a beam, the stiffness of the one-way structure (total load required to produce a unit central sag) can be obtained as $\frac{384EI_1}{5L_1^3}$, where I_1=second moment of area of chord members in direction 1.

2.2. One-way triple-layer space trusses

The analysis of one-way triple-layer space trusses is quite similar to that of double-layer trusses described above. And therefore, by analogy with double-layer trusses and reference to Fig. 3, the weights of top chord, bottom chord and diagonal members of a one-way triple-layer space truss can be obtained as:

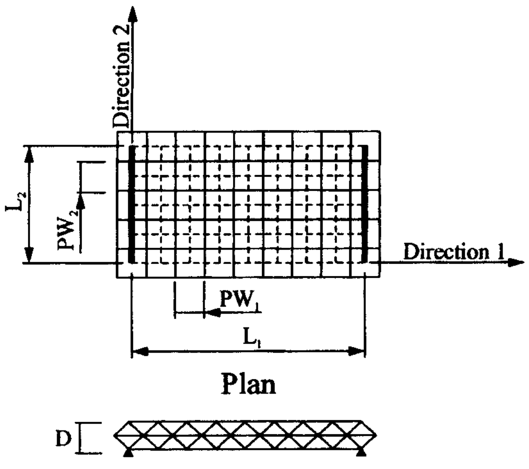

Fig. 3 Layout of a one-way triple-layer space truss

Weight of top chord members = WT

$$= \left\{ \left[L_1 \left(\frac{L_2}{PW_2} + 1 \right) \right] + 0.1 \left[L_2 \left(\frac{L_1}{PW_1} + 1 \right) \right] \right\} \frac{WL_2 L_1^2 \gamma}{8D \left(\frac{L_2}{PW_2} + 1 \right) \sigma_p}$$

Weight of bottom chord members = WB

$$= \left\{ \left[L_1 \left(\frac{L_2}{PW_2} + 1 \right) \right] + 0.1 \left[L_2 \left(\frac{L_1}{PW_1} + 1 \right) \right] \right\} \frac{WL_2 L_1^2 \gamma}{8D \left(\frac{L_2}{PW_2} + 1 \right) \sigma_y}$$

Weight of diagonal members = WD = $\left(\frac{L_1}{PW_1} + 1 \right) \left(\frac{L_2}{PW_2} + 1 \right) (PW_1^2 + PW_2^2 + D^2) \frac{WL_1 PW_2 \gamma}{D \sigma_p'}$

Further, as an approximation that has been proven reasonably accurate in several FE analyses (as will be explained later), it can safely be assumed that the cross-sectional area of truss middle chord members is equal to about 10% that of the top chord members in the main direction. Therefore, the weight of the middle chord can be approximated as:

$$WM = \left\{ (L_1 + PW_1) \cdot \left(\frac{L_2}{PW_2} + 2 \right) + (L_2 + PW_2) \cdot \left(\frac{L_1}{PW_1} + 2 \right) \right\} \frac{WL_2 L_1^2 \gamma}{80D \left(\frac{L_2}{PW_2} + 1 \right) \sigma_p}$$

leading finally to the total weight of the triple-layer truss being $WT+WB+WD+WM$.

Also, the flexural stiffness (total load required to produce a unit central sagging) is obtained as before in the form: $\frac{384EI_1}{5L_1^3}$, where I_1=second moment of area of all top, bottom and middle chord members in direction 1.

3. Analysis technique for two-way space trusses

3.1. Two-way double-layer space trusses

Use is made in this section of the approximate analysis method presented in El-Sheikh (1996). In this method, the truss is treated as two one-way sub-systems in two perpendicular directions, and the total truss load, W, is divided between the sub-systems according to the truss aspect ratio, $\alpha=L_1/L_2$. According to this method, the load portions in directions 1 and 2, are $W \frac{1}{1+\alpha^4}$ and $W \frac{\alpha^4}{1+\alpha^4}$, respectively. Notice that in this method, it is assumed that L_1 is larger than L_2, leading

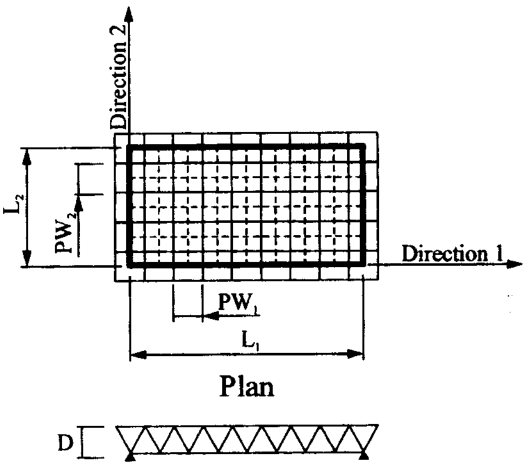

Fig. 4 Layout of a two-way double-layer space truss

to α always greater than or equal to 1.0, and consequently, directions 1 and 2 change status and become the secondary and main direction, respectively.

Now, by following the procedure set out above for one-way trusses and repeating it for the two sub-systems, the total weights of the chord members of the double-layer truss shown in Fig. 4, can be obtained as:

$$\text{Weight of top chord members} = WT = (L_1+PW_1)\left(\frac{L_2}{PW_2}+2\right)\frac{W(L_2+PW_2)L_1^2\gamma}{(1+\alpha^4)\,8D\,\sigma_p\left(\dfrac{L_2}{PW_2}+2\right)}$$

$$+(L_2+PW_2)\left(\frac{L_1}{PW_1}+2\right)\frac{W\alpha^4(L_1+PW_1)L_2^2\gamma}{(1+\alpha^4)\,8D\,\sigma_p\left(\dfrac{L_1}{PW_1}+2\right)} = \frac{W\gamma(L_1^2+\alpha^4L_2^2)}{(1+\alpha^4)\,8D\,\sigma_p}(L_1+PW_1)(L_2+PW_2),$$

$$\text{and Weight of bottom chord members} = WB = L_1\left(\frac{L_2}{PW_2}+1\right)\frac{W(L_2+PW_2)L_1^2\gamma}{(1+\alpha^4)\,8D\,\sigma_y\left(\dfrac{L_2}{PW_2}+1\right)}$$

$$+L_2\left(\frac{L_1}{PW_1}+1\right)\frac{W\alpha^4(L_1+PW_1)L_2^2\gamma}{(1+\alpha^4)\,8D\,\sigma_y\left(\dfrac{L_1}{PW_1}+1\right)} = \frac{W\gamma}{(1+\alpha^4)\,8D\,\sigma_y}[(L_2+PW_2)L_1^3+\alpha^4(L_1+PW_1)L_2^3].$$

Notice that in using the above equations, and for practical reasons, the chord member forces in the secondary direction (with a larger span) should not be allowed to reduce below 10% that of the members in the main direction. This situation would arise for space trusses with aspect ratio, α >1.75.

The weight of diagonal members can then be obtained as before:

$$WD = 4\left(\frac{L_1}{PW_1}+1\right)\left(\frac{L_2}{PW_2}+1\right)\sqrt{\left(\frac{PW_1}{2}\right)^2+\left(\frac{PW_2}{2}\right)^2+D^2}\cdot\frac{\gamma}{\sigma_p{}'}\cdot R_{max},$$

Where R_{max}=maximum resultant force in a diagonal member, taken as the largest of:

$$\frac{W\cdot PW_2(L_1+PW_1)\sqrt{\left(\frac{PW_1}{2}\right)^2+\left(\frac{PW_2}{2}\right)^2+D^2}}{4D(1+\alpha^4)}$$

and

$$\frac{W\cdot\alpha^4\cdot PW_1(L_2+PW_2)\sqrt{\left(\frac{PW_1}{2}\right)^2+\left(\frac{PW_2}{2}\right)^2+D^2}}{4D(1+\alpha^4)}$$

And finally, the total truss weight is calculated as $WT+WB+WD$.

The stiffness of the structure as obtained from the two truss sub-systems should be the same as this was the requirement on which the load division between the two directions was based. Accordingly, the truss flexural stiffness (total load required to produce a unit central sag) can be obtained as $\frac{384(1+\alpha^4)EI_2}{5\alpha^4\cdot L_2^3}$, where I_2=second moment of area of chord members in direction 2

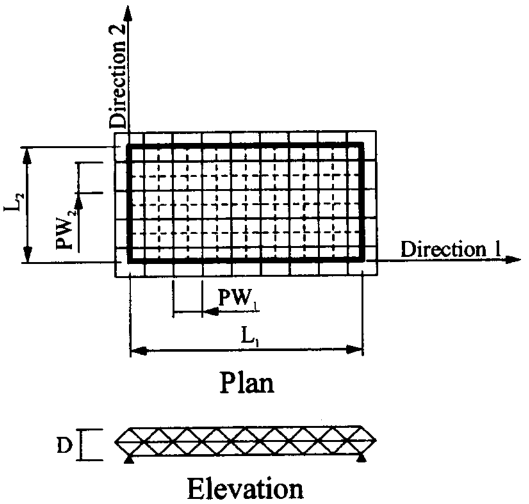

Fig. 5 Layout of a two-way triple-layer space truss

(the main direction).

3.2. Two-way triple-layer space trusses

By analogy with double-layer trusses and reference to Fig. 5 that shows a typical edge-supported space truss with three chord layers, the weight of chord and diagonal members in a general two-way triple-layer truss can be obtained as follows:

$$\text{Weight of top chord members} = WT = L_1\left(\frac{L_2}{PW_2}+1\right)\frac{W\cdot L_2 L_1^2\gamma}{(1+\alpha^4)\,8D\,\sigma_p\left(\frac{L_2}{PW_2}+1\right)}$$

$$+L_2\left(\frac{L_1}{PW_1}+1\right)\frac{W\alpha^4\cdot L_1 L_2^2\gamma}{(1+\alpha^4)\,8D\,\sigma_p\left(\frac{L_1}{PW_1}+1\right)} = \frac{W(L_1\cdot L_2)\gamma}{(1+\alpha^4)\,8D\,\sigma_p}\,[L_1^2+\alpha^4 L_2^2],$$

$$\text{Weight of bottom chord members} = WB = \frac{W(L_1\cdot L_2)\gamma}{(1+\alpha^4)\,8D\,\sigma_y}\,[L_1^2+\alpha^4 L_2^2],$$

Weight of middle chord members = WM

$$= \frac{W(L_1\cdot L_2)\gamma}{(1+\alpha^4)\,80D\,\sigma_p}\left[\frac{L_1(L_1+PW_1)(L_2+2PW_2)}{(L_2+PW_2)}+\frac{\alpha^4 L_2(L_2+PW_2)(L_1+2PW_1)}{(L_1+PW_1)}\right], \text{ and}$$

Weight of diagonal members = WD

$$=8\left(\frac{L_1}{PW_1}+1\right)\left(\frac{L_2}{PW_2}+1\right)\sqrt{\left(\frac{PW_1}{2}\right)^2+\left(\frac{PW_2}{2}\right)^2+\left(\frac{D}{2}\right)^2}\cdot\frac{\gamma}{\sigma_p'}\cdot R_{\max},$$

where $R_{\max}$=maximum resultant force in a diagonal member, taken as the largest of:

$$\frac{W\cdot PW_2\cdot L_1\sqrt{\left(\frac{PW_1}{2}\right)^2+\left(\frac{PW_2}{2}\right)^2+\left(\frac{D}{2}\right)^2}}{4D(1+\alpha^4)}$$

and

$$\frac{W\cdot\alpha^4\cdot PW_1\cdot L_2\sqrt{\left(\frac{PW_1}{2}\right)^2+\left(\frac{PW_2}{2}\right)^2+\left(\frac{D}{2}\right)^2}}{4D(1+\alpha^4)}.$$

Finally, the total truss weight is calculated as the summation of all the individual weights: *WT, WB, WM* and *WD*.

Also, the flexural stiffness of the truss can be obtained as before in the form:

$$\frac{384(1+\alpha^4)EI_2}{5\alpha^4 \cdot L_2^3},$$

where I_2=second moment of area of all top, bottom and middle chord members in direction 2.

4. General comments on approximate technique

The technique presented above is intended to provide a quick and reasonably accurate design of double-layer and triple-layer space trusses. The member designs are based on the largest forces that exist in each chord and in the diagonals. However, the method could easily be upgraded by allowing member size variation within the same member group. This can be done by dividing

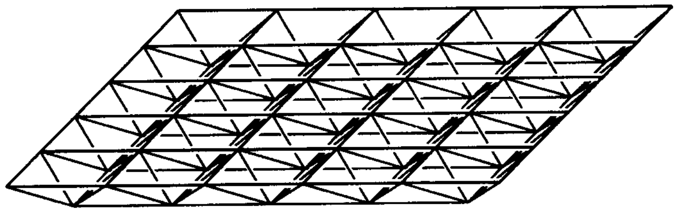

Fig. 6 Typical square-on-square double-layer space truss

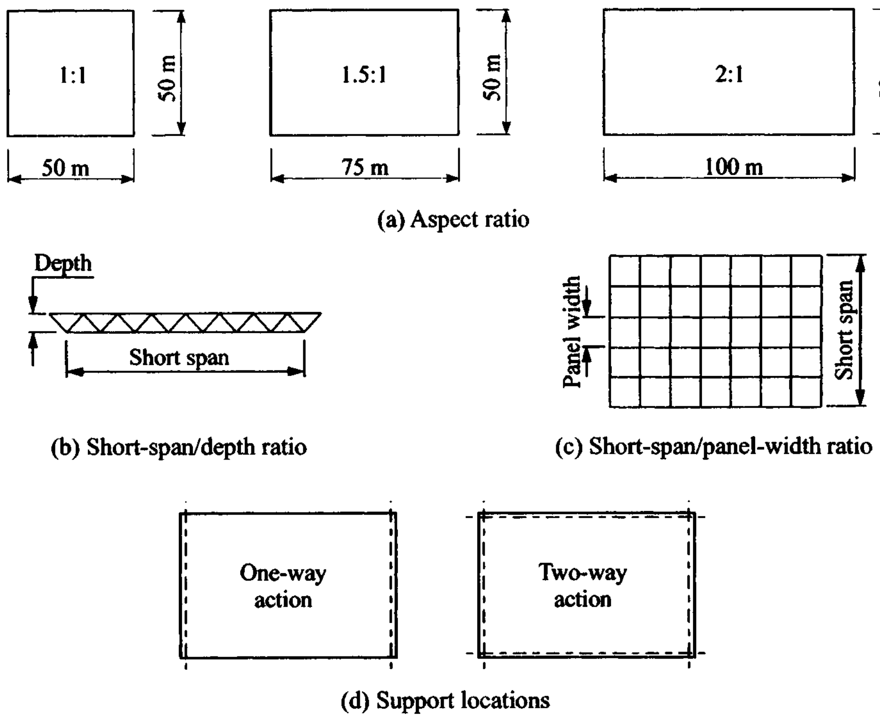

Fig. 7 Parameters considered in parametric study

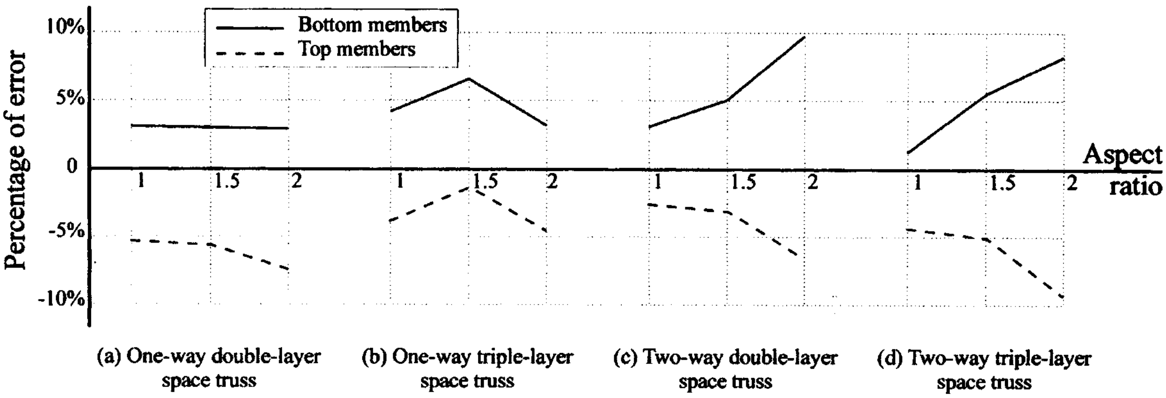

Fig. 8 Assessment of accuracy of approximate technique

each member group into as many sub-groups as desired, and performing a separate design for each sub-group. The practical effects of such an action on overall truss design are currently under investigation.

Further, the technique, as presented, considers only a symmetrical *UDL* loading of the space truss, but could be easily modified to take into account any non-symmetrical *UDL* or concentrated load cases.

5. Parametric study to assess accuracy of analysis technique

As part of this work, a parametric study was conducted to assess the accuracy of the approximate analysis technique detailed above. The study, which included comparisons with finite element analysis (FEA) results, involved several square-on-square space trusses (of the configuration shown in Fig. 6) covering wide variations of the following important parameters (also see Fig. 7):

(a) Truss aspect ratio ($\alpha = L_1/L_2 = 1$, 1.5 and 2),

(b) Short-span/depth ratio (was either 16, 20 or 24),

(c) Short-span/panel-width ratio (was either 12, 16 or 20), and

(d) Truss boundary conditions (both one-way and two-way action situations were considered).

In plan, the space trusses ranged in size between 50 m$\times$50 m and 50 m$\times$100 m, with a depth between 2.083 m and 3.125 m. The trusses were designed twice; using the approximate technique described herein and using ABAQUS finite element package. Designs based on Abaqus results were done while sizing all members in the same chord equally, and the same for all diagonal members in the truss. This was done to ensure compatibility with the approximate technique developed in this work.

The member sizes obtained using the two methods were then compared to assess the accuracy of the approximate technique. The comparisons held (some of which are presented graphically in Fig. 8) showed that in all cases considered, including both double- and triple-layer trusses, the errors associated with the approximate method were below 10% for the main chord members, below 20% for all other member groups and below 14% for the total truss weights. These results demonstrated the reasonable accuracy of the technique and its suitability for at least the preliminary design of space trusses.

The approximate technique was then employed in a number of comparative studies between

Table 1 Weight per unit area (kN/m^2) of double-layer and triple-layer space trusses

Aspect ratio (α)	No. of panels in short direction	$\frac{L_2}{D_{double}}$	One-way trusses						Two-way trusses					
			Double layer	Triple layer, $D_{triple}=D_{double}\times n$					Double layer	Triple layer, $D_{triple}=D_{double}\times n$				
				$n=$ 1.00	$n=$ 1.25	$n=$ 1.50	$n=$ 1.75	$n=$ 2.00		$n=$ 1.00	$n=$ 1.25	$n=$ 1.50	$n=$ 1.75	$n=$ 2.00
1 : 1	12	16	1.104	1.292	1.088	0.960	0.876	0.820	0.727	0.800	0.667	0.582	0.526	0.488
		20	1.272	1.564	1.292	1.120	1.004	0.920	0.855	0.974	0.800	0.688	0.612	0.557
		24	1.456	1.840	1.508	1.292	1.144	1.036	0.991	1.152	0.939	0.800	0.704	0.634
	16	16	1.080	1.204	1.028	0.928	0.864	0.828	0.711	0.756	0.638	0.566	0.520	0.490
		20	1.216	1.436	1.204	1.056	0.960	0.896	0.821	0.911	0.756	0.657	0.591	0.545
		24	1.372	1.680	1.388	1.204	1.076	0.988	0.942	1.071	0.879	0.756	0.671	0.611
	20	16	1.096	1.168	1.016	0.932	0.884	0.856	0.716	0.738	0.631	0.568	0.529	0.506
		20	1.208	1.376	1.168	1.040	0.960	0.908	0.813	0.881	0.738	0.648	0.590	0.550
		24	1.344	1.596	1.332	1.168	1.056	0.980	0.923	1.030	0.851	0.738	0.660	0.607
1.5 : 1	12	16	1.078	1.280	1.075	0.948	0.865	0.811	0.912	1.024	0.860	0.758	0.691	0.648
		20	1.243	1.548	1.280	1.108	0.992	0.911	1.057	1.239	1.024	0.887	0.793	0.729
		24	1.423	1.825	1.493	1.280	1.132	1.026	1.214	1.460	1.195	1.024	0.906	0.821
	16	16	1.063	1.195	1.022	0.919	0.856	0.818	0.899	0.959	0.819	0.698	0.686	0.654
		20	1.197	1.427	1.195	1.050	0.954	0.890	1.019	1.146	0.959	0.842	0.765	0.713
		24	1.350	1.670	1.380	1.195	1.070	0.982	1.154	1.341	1.108	0.959	0.858	0.787
	20	16	1.082	1.160	1.009	0.924	0.875	0.850	0.913	0.932	0.810	0.741	0.702	0.681
		20	1.193	1.369	1.160	1.033	0.952	0.901	1.015	1.101	0.932	0.830	0.765	0.723
		24	1.327	1.590	1.326	1.160	1.050	0.975	1.135	1.280	1.059	0.932	0.844	0.783
2 : 1	12	16	1.065	1.274	1.069	0.943	0.860	0.805	1.045	1.183	0.998	0.884	0.810	0.762
		20	1.229	1.541	1.274	1.103	0.987	0.906	1.203	1.428	1.183	1.028	0.923	0.851
		24	1.407	1.807	1.487	1.274	1.126	1.021	1.375	1.682	1.379	1.183	1.050	0.954
	16	16	1.054	1.191	1.018	0.915	0.852	0.814	1.035	1.104	0.950	0.858	0.804	0.773
		20	1.187	1.423	1.191	1.046	0.950	0.886	1.162	1.315	1.104	0.975	0.889	0.834
		24	1.339	1.665	1.376	1.191	1.066	0.978	1.309	1.535	1.272	1.104	0.992	0.914
	20	16	1.074	1.157	1.006	0.920	0.872	0.847	1.056	1.073	0.939	0.866	0.825	0.807
		20	1.184	1.366	1.157	1.030	0.949	0.898	1.161	1.260	1.073	0.961	0.890	0.846
		24	1.318	1.587	1.323	1.157	1.047	0.972	1.288	1.461	1.222	1.073	0.976	0.910

double- and triple-layer space trusses. The comparisons focused on the weight, the stiffness and the number of joints and members involved in each case. The comparisons were extended to cover reasonably wide variations of truss aspect ratio, span/depth ratio and number of chord panels (as detailed above) in order to include a wide spectrum of practical situations and to provide reliable comparisons between double- and triple-layer trusses. Additionally, cases where the depth of triple-layer trusses were increased to 1.25, 1.50, 1.75 and 2.00 times the depth of corresponding double-layer trusses were considered in order to recognise the fact that triple-layer trusses would commonly have smaller span/depth ratios than double-layer trusses.

6. Weight comparisons

The design technique described above was used to design several one-way and two-way double- and triple-layer space trusses with different aspect ratios, span/depth ratios and number of chord panels. Calculated unit weights (per unit area) are presented in Table 1.

The unit weights presented clearly illustrate a number of points, the most important of which is that, for the same depth, triple-layer trusses were considerably heavier than equivalent double-layer trusses (by 15.1% on average). However, allowing depth increases in triple-layer trusses (in appreciation of the common practice) resulted in a gradual improvement in their competitiveness. Starting from the cases involving depth increases of 25-50%, triple-layer trusses became lighter than their double-layer counterparts. Also, with a depth increase of 100%, triple-layer trusses eventually became 33.6% lighter, on average.

It must be noted, however, that the weight is not the only important factor affecting the cost and competitiveness of space trusses. Other, possibly more significant factors include the stiffness, the number of joints and members and the jointing system used. Nevertheless, space truss weight should be given proper consideration as it represents an important cost item, in addition to other direct effects on the design of columns and foundations supporting the structure.

Other points of significant importance include the following:

- Changing truss aspect ratio did not lead to any significant variation in the unit weights of one-way trusses, as the main span was kept unchanged.
- However, the unit weights of two-way trusses (with two and three layers of chord members) increased with higher aspect ratios, and gradually approached the unit weights of one-way trusses. This finding is consistent with the inherent two-way action of space trusses and the observations made in earlier research regarding their optimum performance in square areas (Makowski 1981, 1984, El-Sheikh and El-Bakry 1995).
- It is evident that the truss unit weights increased in all cases with higher span/depth ratios (approximately inversely proportional to depth).
- There was no overall consistent trend as to the effect of using more chord panels on the weight of both one-way and two-way trusses apart from the fact that the resulting weight changes were always quite small (maximum change was 6%).

7. Stiffness comparisons

Space truss stiffness is used in this paper as a further measure of the efficiency and competitiveness of these systems. In the stiffness comparisons that follow, the focus is limited to the values of flexural stiffness per unit weight, in order to isolate the weight factor from the comparisons and to present a clearer picture of the stiffness to be expected in each case.

The values of truss flexural stiffness (total surface load required to produce a unit central sag) per unit weight for all space trusses considered are presented in Table 2. These values are obtained using the approximate technique described in this paper. From the values given, it is clear that double-layer trusses outperformed their triple-layer equivalents (with the same depth) on a stiffness/weight basis (by 17.2% on average). However, with depth increases, the stiffness/weight values of triple-layer trusses improved gradually and became superior to those of double-layer trusses. On average, the ratios between the stiffness/weight values of double- and triple-layer trusses with D_{triple}=1.0, 1.25, 1.5, 1.75 and 2.0 D_{double}, were 1.21, 0.81, 0.59, 0.46 and 0.38, respectively. These comparisons indicate clearly that the effectiveness and competitiveness of triple-layer space trusses improved considerably with larger overall depths.

Furthermore, the following trends could be seen by inspecting the stiffness/weight values given in Table 2:

- The flexural stiffness per unit weight of one-way trusses (both with two and three layers of chord members) showed no significant change in response to variations in truss aspect ratio.

Table 2 Flexural stiffness per unit weight (1/m) of double-layer and triple-layer space trusses

Aspect ratio (α)	No. of panels in short direction	$\frac{L_2}{D_{double}}$	One-way trusses: Double layer	One-way: Triple layer, $D_{triple}=D_{double}\times n$, $n=$ 1.00	$n=$ 1.25	$n=$ 1.50	$n=$ 1.75	$n=$ 2.00	Two-way trusses: Double layer	Two-way: Triple layer, $D_{triple}=D_{double}\times n$, $n=$ 1.00	$n=$ 1.25	$n=$ 1.50	$n=$ 1.75	$n=$ 2.00
1 : 1	12	16	64.0	50.7	75.4	102.6	130.9	159.6	97.0	81.9	122.9	168.8	218.0	268.7
		20	44.4	33.6	50.7	70.3	91.5	113.8	66.0	53.8	81.9	114.2	150.0	188.2
		24	32.3	23.7	36.2	50.7	66.9	84.3	47.4	37.9	58.2	81.9	108.6	137.7
	16	16	63.9	54.5	79.6	106.1	132.8	158.7	97.3	86.7	128.3	173.6	220.5	267.3
		20	45.4	36.5	54.5	74.4	95.4	116.8	67.3	57.6	86.7	119.7	155.2	192.3
		24	33.6	26.0	39.3	54.5	71.0	88.3	48.9	40.8	62.1	86.7	114.0	143.1
	20	16	62.3	56.2	80.7	105.7	130.0	152.8	95.5	88.9	129.8	173.1	216.7	259.0
		20	45.2	38.1	56.2	75.7	95.7	115.6	67.2	59.5	88.9	121.3	155.6	190.6
		24	33.9	27.4	41.0	56.2	72.4	89.0	49.4	42.4	64.1	88.9	115.8	144.1
1.5 : 1	12	16	63.8	51.2	76.2	103.7	132.5	161.7	68.7	56.9	84.6	115.3	147.4	180.0
		20	44.2	33.9	51.2	71.0	92.5	115.1	47.5	37.6	56.9	79.0	102.9	128.1
		24	32.2	23.9	36.6	51.2	67.5	85.1	34.4	26.7	40.5	56.9	75.0	94.7
	16	16	63.8	54.8	80.1	107.0	134.0	160.3	68.3	60.7	88.8	118.7	148.7	178.1
		20	45.3	36.7	54.8	74.9	96.1	117.8	48.3	40.7	60.7	83.0	106.7	130.6
		24	33.5	26.2	39.6	54.8	71.5	89.0	35.5	29.0	43.9	60.7	79.2	98.7
	20	16	62.2	56.5	81.2	106.4	131.0	154.1	66.6	62.4	89.9	117.8	145.3	171.2
		20	45.1	38.3	56.5	76.1	96.3	116.4	47.9	42.2	62.4	84.2	106.7	128.9
		24	33.8	27.5	41.2	56.5	72.8	89.6	35.7	30.5	45.6	62.4	80.6	99.1
2 : 1	12	16	63.6	51.5	76.6	104.3	133.3	162.7	67.5	55.3	82.1	111.3	141.6	172.1
		20	44.1	34.0	51.5	71.3	93.0	115.8	46.9	36.7	55.3	76.5	99.3	123.4
		24	32.1	24.0	36.7	51.5	67.9	85.6	34.2	26.0	39.6	55.3	72.8	91.6
	16	16	63.7	55.0	80.4	107.4	134.6	161.1	66.8	59.3	86.3	114.5	142.6	169.7
		20	45.3	36.8	55.0	75.2	96.5	118.3	47.7	39.9	59.3	80.7	103.2	125.8
		24	33.4	26.2	39.7	55.0	71.7	89.3	35.2	28.5	42.9	59.3	77.0	95.6
	20	16	62.1	56.6	81.4	106.8	131.5	154.8	64.7	61.0	87.2	113.6	139.0	162.5
		20	45.1	38.4	56.6	76.3	96.6	116.8	47.1	41.6	61.0	81.9	103.1	123.9
		24	33.7	27.5	41.3	56.6	73.0	89.9	35.3	29.9	44.7	61.0	78.3	96.0

This was due to the main span remaining unchanged with aspect ratio variations.

- On the other hand, the flexural stiffness per unit weight of two-way trusses underwent a gradual reduction with higher aspect ratios, down to levels close to those of one-way trusses.
- Truss flexural stiffness per unit weight decreased progressively with higher span/depth ratios.
- In most cases considered, changing the number of chord panels led to only a small effect on truss stiffness per unit weight. There was also no consistent trend as to how the stiffness/weight values changed.
- Two-way trusses enjoyed much improved stiffness/weight values compared to their one-way counterparts, but this superiority deteriorated gradually with higher aspect ratios. This finding is compatible with the two-way nature of space trusses.

9. Number of joints and members

In most space truss systems, truss members are prepared with member end fittings and joined together using special node connectors. The member end fittings and node connectors are usually sophisticated components that are expensive to produce and hence account for a large percentage of the total cost of the structure (Iffland 1982, Codd 1984). For this reason, the number of joints and members (and hence member end fittings) included is a major consideration in any space truss design.

In this study, the number of joints and members in double- and triple-layer space trusses can be calculated as follows while considering the same notation described above in this paper:

$$\text{Number of joints in double-layer trusses} = \left(\frac{L_1}{PW_1}+1\right)\left(\frac{L_2}{PW_2}+1\right)+\left(\frac{L_1}{PW_1}+2\right)\left(\frac{L_2}{PW_2}+2\right)$$

$$\text{Number of joints in triple-layer trusses} = 2\left(\frac{L_1}{PW_1}+1\right)\left(\frac{L_2}{PW_2}+1\right)+\left(\frac{L_1}{PW_1}+2\right)\left(\frac{L_2}{PW_2}+2\right)$$

$$\text{Number of members in double-layer trusses} = 8\left(\frac{L_1}{PW_1}+1\right)\left(\frac{L_2}{PW_2}+1\right)$$

$$\text{Number of members in triple-layer trusses} = 14\left(\frac{L_1}{PW_1}\cdot\frac{L_2}{PW_2}\right)+13\left(\frac{L_1}{PW_1}+\frac{L_2}{PW_2}\right)+12$$

By applying these equations, the number of joints and members used in all double- and triple-layer trusses considered was calculated, and presented graphically in Figs. 9 and 10. The figures

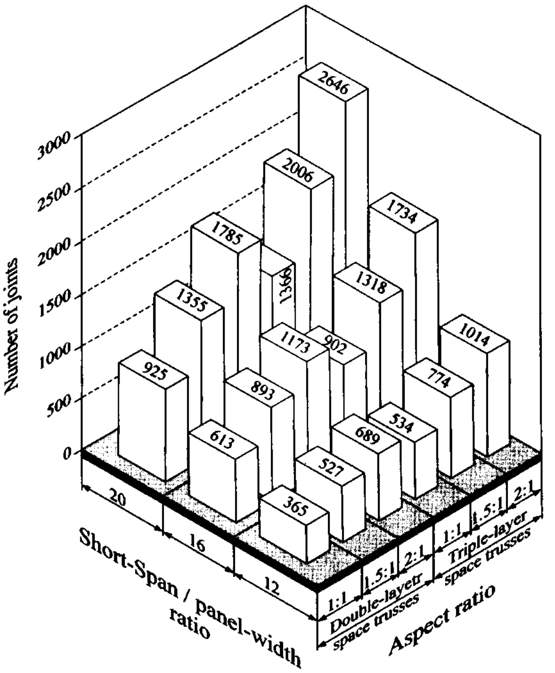

Fig. 9 Number of joints in double- and triple-layer space trusses

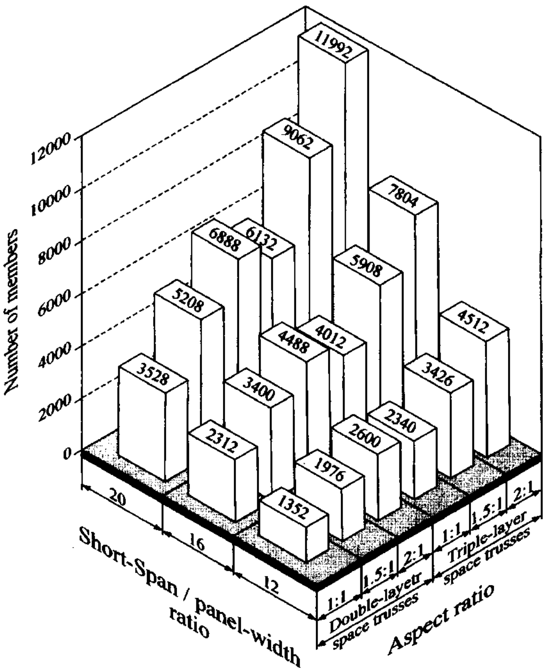

Fig. 10 Number of members in double- and triple-layer space trusses

clearly show that triple-layer trusses typically involve more joints and members, a consideration that should be taken into account when comparing the two systems in space truss designs. From the figures it appears that triple-layer trusses employ an average of 47.4% more joints and 73.7% more members than equivalent double-layer trusses. This finding must, however, be seen in tandem with the fact that the joints and members of triple-layer trusses would typically be of smaller size, and hence could be easier to manufacture and assemble.

10. Conclusions

The study presented in this paper includes the development of an approximate technique for designing one-way and two-way double- and triple-layer space trusses. The technique developed has been assessed in a parametric numerical study and found reasonably accurate with an error margin within 14% for all cases considered. The technique, as presented, considers a symmetrical *UDL* loading of the space truss, but can be easily adjusted to take into account any non-symmetrical *UDL* or concentrated load cases. Equations to predict the stiffness of the truss in each case are also included.

Having established the reasonable accuracy of the design technique, it was then employed in a number of parametric studies to compare double- and triple-layer space trusses in terms of weight of structure, flexural stiffness and number of joints and members. These studies were conducted to provide structural designers with information on the relative performance of the two types of systems, which could be beneficial in choosing which one to employ in a certain application. It must be emphasised, however, that the present work is not claimed to provide all the answers needed for the problem in hand. Rather, it is an attempt to improve understanding and appreciation of the differences between the two types of systems, and further work in this area is still needed. For instance, investigate issues such as behaviour and failure modes, sensitivity to local damage, cost competitiveness, etc., to shed more light on the choice process. However, from the work conducted, and reported in this paper, the following conclusions are drawn:

(1) Double-layer trusses are superior to their triple-layer counterparts (with the same depth) in terms of the weight and flexural stiffness and employ fewer joints and members. This superiority is evident for all cases considered.

(2) Triple-layer truss competitiveness improves progressively on both the weight and stiffness fronts with increasing truss depth. However, the fact that they require more joints and members remains. The cost implication of this point should be considered in all situations. Also, it must be noted that with larger truss depths, the structure is likely to be taller, hence subjected to greater wind forces and needing more cladding.

(3) Due to the two-way nature of space trusses, both double- and triple-layer systems are most effective in square areas with all edges supported. In more rectangular areas and with the loss of supports along two opposite edges, hence creating a one-way action situation, the truss effectiveness decreases significantly, which is evident in having more weight and less stiffness.

(4) In contrast to the effect of changes in the aspect ratio or the span/depth ratio, changing the number of chord panels in a space truss results in only a small effect on truss weight and stiffness.

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