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Design of sliding-type base isolators by the concept of equivalent damping

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Abstract. One problem with base isolators of the sliding type is that their dynamic responses are nonlinear, which cannot be solved in an easy manner, as distinction must be made between the sliding and non-sliding phases. The lack of a simple method for analyzing structures installed with base isolators is one of the obstacles encountered in application of these devices. As an initial effort toward simplification of the analysis procedure for base-isolated structures, an approach will be proposed in this paper for computing the equivalent damping for the resilient-friction base isolators (R-FBI), based on the condition that the sum of the least squares of errors of the linearized response with reference to the original nonlinear one is a minimum. With the aid of equivalent damping, the original nonlinear system can be replaced by a linear one, which can then be solved by methods readily available. In this paper, equivalent damping curves are established for all ranges of the parameters that characterize the R-FBI for some design spectra.

Key words: base isolator; damping; equivalent damping; resilient-friction base isolator; sliding.

1. Introduction

Base isolators with sliding devices have become an effective tool for reducing the earthquake forces transmitted to the superstructure, because they are rather insensitive to the frequency contents and amplitudes of ground excitations, compared with base isolators of the other types, such as the rubber bearings. However, to solve the dynamic response of structures installed with sliding-type base isolators is not always easy, as distinction must be made between the sliding and non-sliding phases in the time-history response, each of which is governed by a different set of equations (Yang *et al.* 1990). It is the transition between the two phases that makes the problem a nonlinear one, which cannot be solved by structural engineers using methods readily available, such as the method of modal superposition. Partly because of this, the use of base isolators has not evolved as a mature tool in the design of structures against earthquakes. As an initial effort toward simplification of the analysis procedure for base-isolated structures, an approach will be proposed in this paper for rendering the original nonlinear system involving sliding and non-sliding phases to a linear one using the concept of equivalent damping, based on the condition that the sum of the least squares of errors of the linearized response with reference to the original nonlinear one is a minimum.

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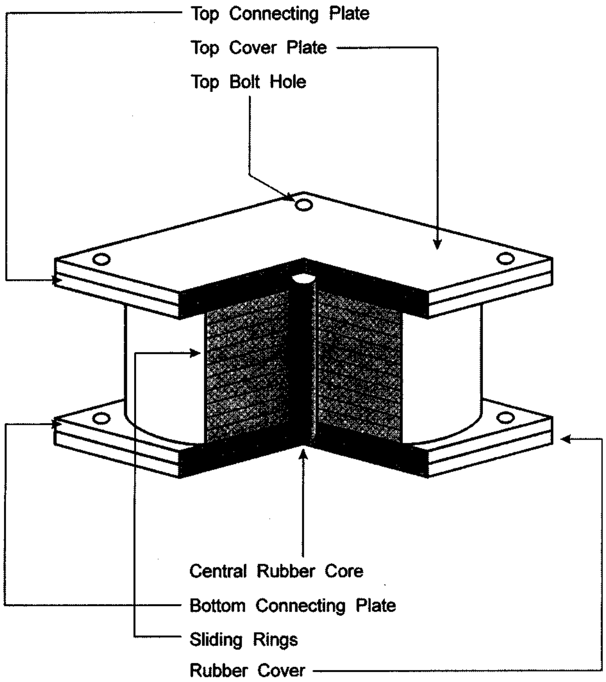


Fig. 1 Resilient-friction base isolator

For the present purposes, the resilient-friction base isolator (R-FBI) is taken as the vehicle of illustration. The R-FBI system, proposed by the University of Utah, is composed of a set of flat rings which can slide on each other with a central rubber core and/or peripheral rubber cores (Fig. 1). When under the earthquake excitation, the interfacial friction force acts in parallel with the elastic force in the rubber. This type of isolator combines the beneficial effect of friction damping with that of the resiliency of rubber (Mostaghel and Khodaverdian 1987). In this paper, it will be demonstrated that for a specific design spectrum, equivalent damping curves can be established for all ranges of the parameters that characterize the isolator. With these curves made available, a nonlinear base-isolated structure can be replaced by a linear one, which can then be analyzed by structural engineers using methods that are readily available for linear problems.

2. Concept of fictitious spring

For a structure that is excited horizontally and allowed to slide on the ground, two phases can be identified for its time-history response. In the non-sliding phase, the base shear of the system is smaller than the frictional resistance, and the foundation raft of the structure moves with the ground. Obviously, the structure can be treated as it were fixed on the ground in this phase. Whenever the base shear of the structure exceeds the frictional resistance, the structure will switch to the sliding phase. To describe the motion of the structure in sliding, the mass of the foundation raft placed over the isolator should be considered in addition to that of the structure. This will result in a system with one more degree of freedom (DOF) compared with that in the non-sliding phase. Although the behavior of the structure in each individual phase is linear, the transition of the structure between the two phases is nonlinear, with different numbers of DOFs required for each phase.

To overcome the numerical difficulties brought by phase transition, it was suggested that the sliding mechanism be replaced by a fictitious spring, with its stiffness set to zero and a large number for the sliding and non-sliding phases, respectively, in assembling the equations of motion for the base-isolated structure. For the non-sliding phase, the static frictional force acting on the contact surfaces is computed as if it were generated by the fictitious spring. However, when the structure switches to the sliding phase, the dynamic frictional force acting on the sliding surfaces is computed not as the fictitious spring force, but as the product of the coefficient of friction by the total normal force of the structure, which remains constant throughout the process of sliding (Yang *et al.* 1990). By so doing, a system that contains different numbers of DOFs for describing the sliding and non-sliding phases can be treated by equations of the same form. Nevertheless, decision must be made concerning the transition between the two phases.

3. Equations of motion

The R-FBI device consists of only a single sliding surface. Referring to the schematic model for the base-isolated system in Fig. 2, we let m, c, and k respectively denote the mass, damping, and stiffness of the superstructure, M the mass of the foundation raft, K and C the stiffness and damping of the isolator, and k_f the stiffness of the fictitious spring replacing the sliding surface. The equations of motion for the base-isolated system shown in Fig. 2 can be written as:

(a) non-sliding phase:

$$\begin{bmatrix} m & 0 \\ 0 & M \end{bmatrix} \begin{Bmatrix} \ddot{U}_1 \\ \ddot{U}_2 \end{Bmatrix} + \begin{bmatrix} c & -c \\ -c & c+C \end{bmatrix} \begin{Bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{Bmatrix} + \begin{bmatrix} k & -k \\ -k & k+K+k_f \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = \begin{Bmatrix} -m\ddot{X}_g \\ -M\ddot{X}_g \end{Bmatrix} \tag{1}$$

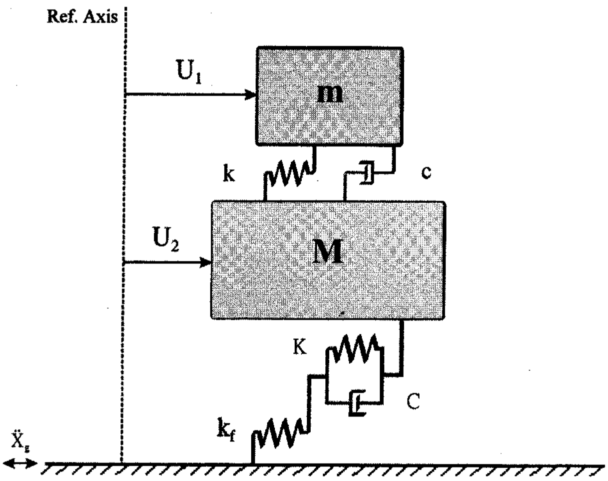


Fig. 2 Mathematical model for R-FBI

(b) sliding phase:

$$\begin{bmatrix} m & 0 \\ 0 & M \end{bmatrix} \begin{Bmatrix} \ddot{U}_1 \\ \ddot{U}_2 \end{Bmatrix} + \begin{bmatrix} c & -c \\ -c & c+C \end{bmatrix} \begin{Bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{Bmatrix} + \begin{bmatrix} k & -k \\ -k & k+K \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = \begin{Bmatrix} -m\ddot{X}_g \\ -M\ddot{X}_g - \mathrm{sgn}(\dot{U}_2)\mu_d g(m+M) \end{Bmatrix} \quad (2)$$

where $\ddot{X}_g$ is the ground acceleration, g the acceleration of gravity, μ_d the coefficient of dynamic friction, $\mathrm{sgn}(\dot{U}_2)$ the sign of $\dot{U}_2$, and U_1 and U_2 respectively denote the displacement of the superstructure and the foundation raft relative to the ground. As can be seen, it is the term with $\mathrm{sgn}(\dot{U}_2)$ that makes the entire system nonlinear.

The condition for the structure to remain in the non-sliding phase is: $|\ddot{X}_g| \le \mu_s g$ and $\dot{U}_2 = 0$, where μ_s is the coefficient of static friction. By using a large stiffness k_f for the fictitious spring, the elongation of the fictitious spring will be negligibly small. The static frictional force F_s existing between the foundation raft and the ground during this phase is equal to the spring force, which can be computed as the product of the stiffness by elongation of the fictitious spring.

On the other hand, the condition for the structure to remain in the sliding phase is: $|\ddot{X}_g| > \mu_s g$ or $\dot{U}_2 \neq 0$. Regardless of the fact that the fictitious spring force is zero due to the use of zero spring constant, $k_f = 0$, the dynamic frictional force F_d should be computed as the product of the frictional coefficient by the total normal force of the structure according to Coulomb's law of friction, and treated as if it were generated by the fictitious spring. The procedures for conducting the step-by-step time-history analysis based on the system equations in Eqs. (1) and (2) using the Newmark β method with constant average acceleration, along with the formulas for detecting the transition time based on the extrapolation schemes, have been presented in Yang *et al.* (1990). It is not the purpose herein to recapitulate all such procedures.

4. Linearization by equivalent damping

As can be seen from Eq. (2), the nonlinearity of the base-isolated system originates mainly from the term containing $\mathrm{sgn}(\dot{U}_2)\mu_d g(m+M)$ in the equations of motion for the sliding phase. By the method of stochastic equivalent linearization (Constantinou and Tadjbakhsh 1984), this term can be replaced by a term defined as the product of an equivalent damping C_e by the velocity $\dot{U}_2$ of the foundation raft relative to the ground. The solution of the original nonlinear system is then reduced to that of an equivalent linearized system. Here, the error e brought by linearization is

$$e = C_e \dot{U}_2 - \mathrm{sgn}(\dot{U}_2)\mu_d g(m+M) \quad (3)$$

Theoretically, both $\dot{U}_2$ and $\mathrm{sgn}(\dot{U}_2)$ are continuous functions of time t. However, in the step-by-step time-history analysis, both $\dot{U}_2$ and $\mathrm{sgn}(\dot{U}_2)$ appear in discrete form, that is,

$$\dot{U}_2 = \dot{U}_2(t_i) = \dot{X}_i$$
$$\mathrm{sgn}(\dot{U}_2(t)) = \mathrm{sgn}(\dot{U}_2(t_i)) = \mathrm{sgn}(\dot{X}_i) \quad (4)$$

Accordingly, Eq. (3) can be rewritten,

$$e_i = e(t_i) = C_e \dot{X}_i - \mathrm{sgn}(\dot{X}_i)\mu_d g(m+M) \quad (5)$$

The errors can be minimized by making the sum of their squares a minimum. Define the sum of the squares of errors as

$$E=\sum_{i=1}^{n}(e_i)^2=\sum_{i=1}^{n}[C_e\dot{X}_i-\mathrm{sgn}(\dot{X}_i)\mu_d g(m+M)]^2$$
$$=\sum_{i=1}^{n}[C_e^2\dot{X}_i^2-2\dot{X}_iC_e\mathrm{sgn}(\dot{X}_i)\mu_d g(m+M)+\mu_d^2g^2(m+M)^2] \tag{6}$$

Let the derivative of E with respect to C_e equal zero,

$$\frac{\partial E}{\partial C_e}=0 \tag{7}$$

or

$$\sum_{i=1}^{n}[C_e\dot{X}_i^2-2\dot{X}_i\mathrm{sgn}(\dot{X}_i)\mu_d g(m+M)]=0 \tag{8}$$

from which it can be solved

$$C_e=\frac{\sum_{i=1}^{n}\dot{X}_i\mathrm{sgn}(\dot{X}_i)\mu_d g(m+M)}{\sum_{i=1}^{n}\dot{X}_i^2} \tag{9}$$

This is exactly the formula for computing the equivalent damping. By the concept of equivalent damping, the original nonlinear equation in Eq. (2) can be approximately linearized as

$$\begin{bmatrix} c & -c \\ -c & c+C+C_e\end{bmatrix}\begin{bmatrix} m & 0 \\ 0 & M\end{bmatrix}\begin{Bmatrix}\ddot{U}_1\\ \ddot{U}_2\end{Bmatrix}+\begin{bmatrix} c & -c \\ -c & c+C+C_e\end{bmatrix}\begin{Bmatrix}\dot{U}_1\\ \dot{U}_2\end{Bmatrix}+\begin{bmatrix} k & -k \\ -k & k+K\end{bmatrix}\begin{Bmatrix}U_1\\ U_2\end{Bmatrix}=\begin{Bmatrix}-m\ddot{X}_g\\ -M\ddot{X}_g\end{Bmatrix} \tag{10}$$

Since the system has been linearized, there is no need to consider the transition between the sliding and non-sliding phases. For the present purposes, both the original nonlinear equations, i.e., Eqs. (1) and (2), and the equivalent linearized equations, i.e., Eq. (10), will be solved by the Newmark β method with constant average acceleration. These results can then be substituted into Eq. (9) to yield the equivalent damping.

In this study, the equivalent damping curves will be established for some specific design spectra for all ranges of the parameters that characterize the R-FBI. With these curves made available for a specific design spectrum, structural engineers can pick up from these curves a proper value of equivalent damping for the R-FBI to be installed on the base of the structure, and then proceed with analysis of the base-isolated structure using methods that are suitable for linear systems. One advantage with this approach is that there is virtually no need to solve the original nonlinear problems involving the transition of non-sliding and sliding phases, while the analysis of base-isolated structures can basically be conducted using methods that are readily available.

5. Artificial earthquakes

As was stated previously, the objective of this paper is to establish the equivalent damping curves for the R-FBI devices with respect to a specific (acceleration) design spectrum. To this end, the time-history ground acceleration $\ddot{X}_g$, or the so-called artificial earthquake, corresponding to the specified spectrum should first be created, which serves as the input to the original system equations given in Eqs. (1) and (2), and the linearized ones in Eq. (10). The following is a summary of the procedure, following basically that of Kaul (1978), for creating the ground acceleration $\ddot{X}_g(t)$ from the design spectrum.

The equation of motion for a single DOF system under the excitation $\ddot{X}_g(t)$ is

$$\ddot{X}(t) + 2\omega\zeta\dot{X}(t) + \omega^2 X(t) = -\ddot{X}_g(t) \tag{11}$$

where $X(t)$ is the time-history response, ω the natural frequency of vibration, and ζ the damping ratio of the single-DOF system. Let $\ddot{X}_a(t)$ denote the absolute acceleration of the system, i.e., $\ddot{X}_a(t) = \ddot{X}(t) + \ddot{X}_g(t)$. Also, let t_m denote the time at which $|\ddot{X}_a(t)|$ reaches its maximum, that is,

$$F(\omega) = \ddot{X}_a(t_m) \tag{12}$$

For a specific damping ratio ζ, the occurrence time t_m for the peak response will depend on the frequency ω, and so will $F(\omega)$. The absolute acceleration spectrum $S(\omega)$ is then

$$S(\omega) = |F(\omega)| \tag{13}$$

For the single-DOF system subject to the ground excitation $\ddot{X}_g(t)$, the absolute acceleration $\ddot{X}_a(t)$ can be written as

$$\ddot{X}_a(t) = \int_0^\infty \ddot{X}_g(\tau) h(t-\tau)\, d\tau \tag{14}$$

where $h(t)$ is the impulse response function for the absolute acceleration of the system.

Now, let the acceleration spectrum be changed by an amount $\delta F(\omega)$, which may also be interpreted as the difference between the target spectrum and the computed spectrum to be discussed later on. Then for an acceleration spectrum $F(\omega) + \delta F(\omega)$, the ground acceleration $\ddot{X}_g(t)$ should be replaced by $\ddot{X}_g(t) + \delta\ddot{X}_g(t)$, the absolute acceleration $\ddot{X}_a(t)$ by $\ddot{X}_a(t) + \delta\ddot{X}_a(t)$, and the occurrence time t_m for the peak response by $t_m + \delta t_m$. With reference to Fig. 3, the change $\delta F(\omega)$ in $F(\omega)$ can be written as

$$\delta F(\omega) = \ddot{X}_a(t_m + \delta t_m) + \delta\ddot{X}_a(t_m + \delta t_m) - \ddot{X}_a(t_m) \tag{15}$$

By the use of Eq. (14), the preceding equation can be approximated as

$$\delta F(\omega) = \dddot{X}_a(t_m)\,\delta t_m + \int_0^\infty \delta\ddot{X}_g(\tau)\, h(t_m - \tau)\, d\tau \tag{16}$$

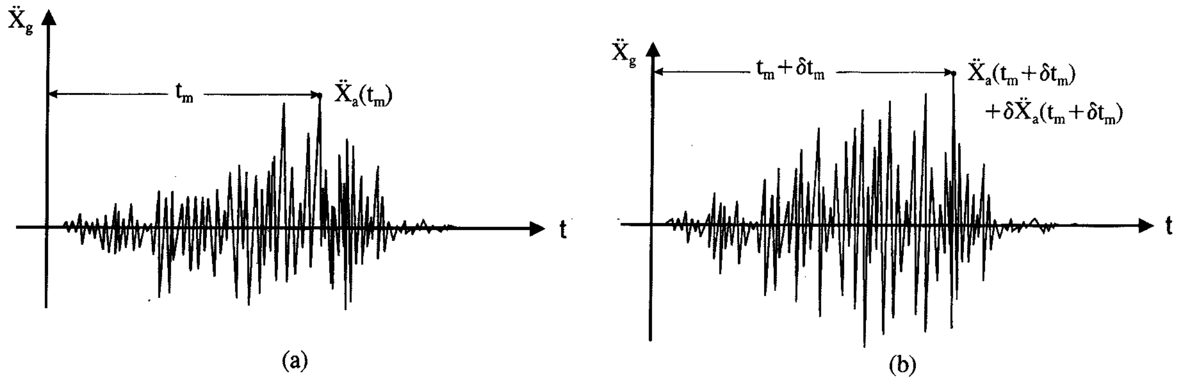


Fig. 3 Occurrence of peak response for acceleration spectrum: (a) $F(\omega)$; (b) $F(\omega)+\delta F(\omega)$

Since the absolute acceleration $\ddot{X}_a(t)$ attains its extremum at $t = t_m$, its first derivative must vanish at $t=t_m$. Thus, the preceding equation reduces to

$$\delta F(\omega) = \int_0^\infty \delta\ddot{X}_g(\tau)\, h(t_m - \tau)\, d\tau \tag{17}$$

Here, it can be seen that assuming the occurrence time t_m for the peak response to remain unaltered, the change $\delta F(\omega)$ in response spectrum can be computed as a result of a small change in ground acceleration $\delta\ddot{X}_g(t)$.

Conversely, if the occurrence time t_m for the peak response is known and a small change $\delta F(\omega)$ in response spectrum is desired, the corresponding change in $\ddot{X}_g(t)$ can be obtained by inversion of the integral in Eq. (17). This forms the basis for creating the artificial earthquake, consistent with the given spectrum. In general, it is assumed that the target spectrum $S_t(\omega)$ is to be matched at n discrete points ω_i, i=1, 2, 3, ..., n, i.e.,

$$\delta F(\omega_i) = \int_0^\infty \delta\ddot{X}_g(\tau)\, h_i(t_{m_i} - \tau)\, d\tau; \quad i = 1, 2, \ldots, n \tag{18}$$

in which it should be noted that the impulse function is evaluated when the frequency ω_i takes the value ω_i. Further, it is assumed that the change $\delta\ddot{X}_g(t)$ in ground acceleration can be expressed as a linear combination of n known linearly independent functions $f_j(t)$,

$$\delta\ddot{X}_g(t) = \sum_{j=1}^{n} a_j f_j(t) \tag{19}$$

where a_j are the coefficients to be determined. Substituting Eq. (19) into Eq. (18) yields the following simultaneous equations:

$$\sum_{j=1}^{n} A_{ij} a_j = b_i; \qquad i = 1, 2, \ldots, n \tag{20}$$

where

$$\begin{aligned} A_{ij} &= \int_0^\infty f_j(\tau)\, h_i(t_{m_i} - \tau)\, d\tau \\ b_i &= \delta F(\omega_i) \end{aligned} \tag{21}$$

Here, the terms b_j should be interpreted as the deviations of the computed spectrum from the target spectrum evaluated at the discrete points. Since A_{ij} and b_j are known, Eq. (20) can be solved to yield a_j, and the change $\delta\ddot{X}_g(t)$ in earthquake acceleration can be computed accordingly using Eq. (19). It follows that the earthquake excitation $\ddot{X}_g(t)$ can be iteratively updated until the target spectrum is matched by the computed spectrum within a prescribed degree of tolerance. It should be noted that using the above procedure, one needs to start from an initial estimate of the earthquake excitation $\ddot{X}_g(t)$ and the corresponding spectrum $S(\omega)$; the latter can be computed using conventional integration procedures, say, by the Newmark β method. In this study, the number of matching points is taken as n=50.

For the present purposes, the design spectra shown in Figs. 4 and 5 for the hard and soft soils, respectively, in Taiwan with 2 and 5 percent damping are considered (Chen 1997). Corresponding to these spectra, the artificial earthquakes generated using the scheme described above have been plotted in Figs. 6 and 7, which will be used as the ground excitations in the numerical studies to follow.

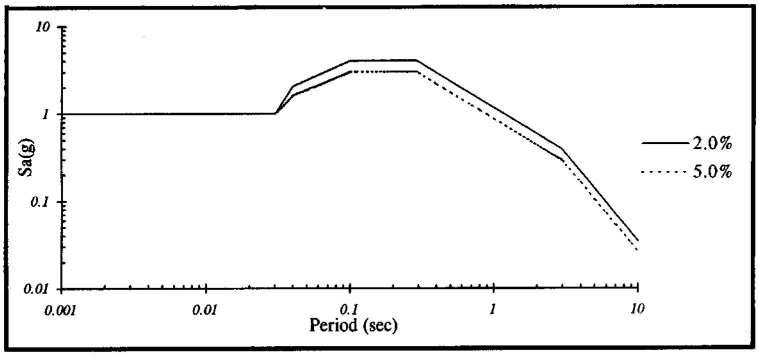


Fig. 4 Design spectra for hard soil: (a) 2% damping: (b) 5% damping

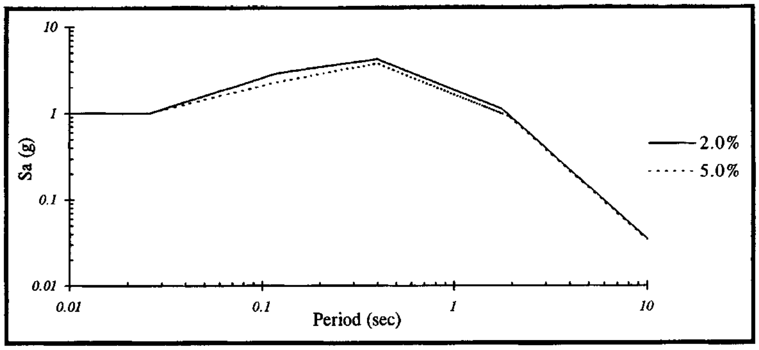


Fig. 5 Design spectra for soft soil: (a) 2% damping: (b) 5% damping

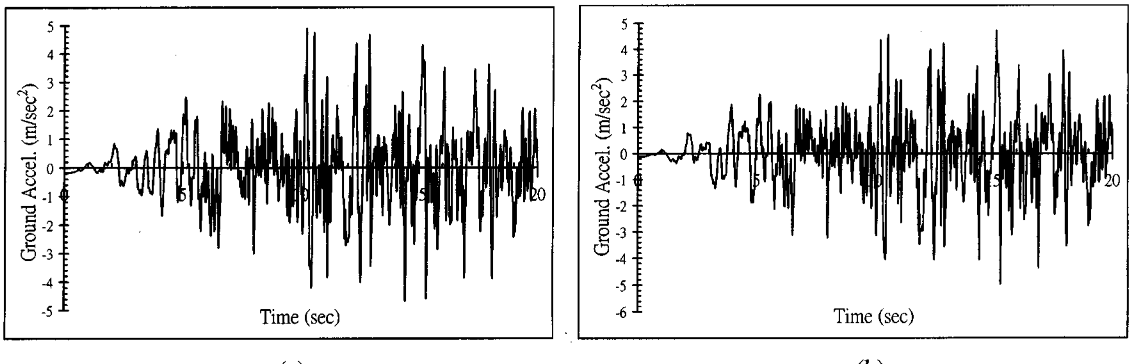
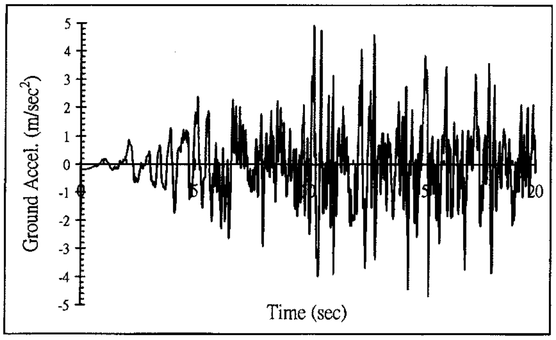


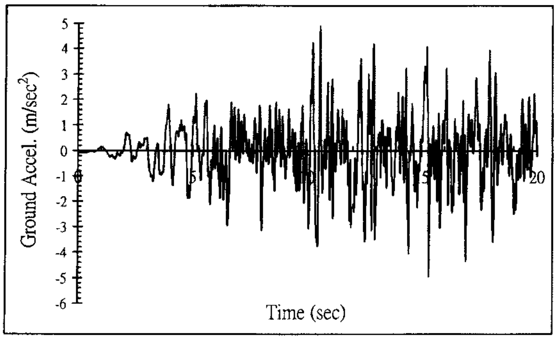
Fig. 6 Artificial earthquakes for hard soil: (a) 2% damping; (b) 5% damping

6. Numerical applications

Consider a single-DOF base-isolated system with the frequency of vibration specified as ω_s=10 rad/s for the superstructure, and with the following properties for the isolator: ω_i=5 rad/s, ζ_i=5%, static and dynamic coefficients of friction μ_s=μ_d=0.04. In addition, the followings are used in the time-history analysis: Δt=0.001 s, k_f=1,000 k. For the present purposes, the mass ratio is defined as



(a)



(b)

Fig. 7 Artificial earthquakes for soft soil: (a) 2% damping; (b) 5% damping

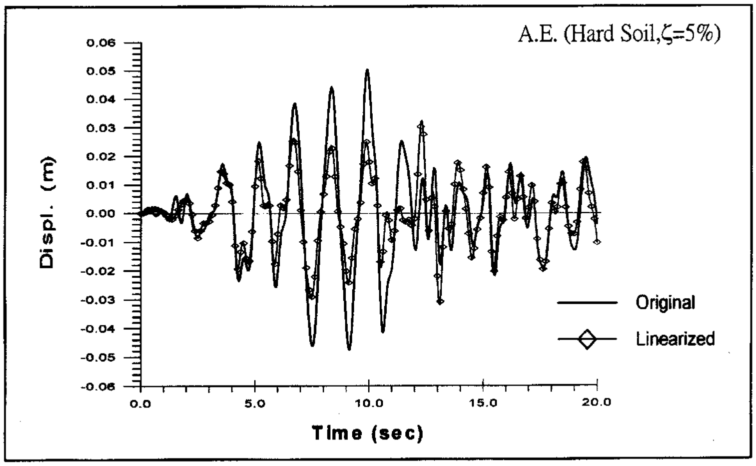


Fig. 8 Relative displacement of superstructure (hard soil, ζ=5%)

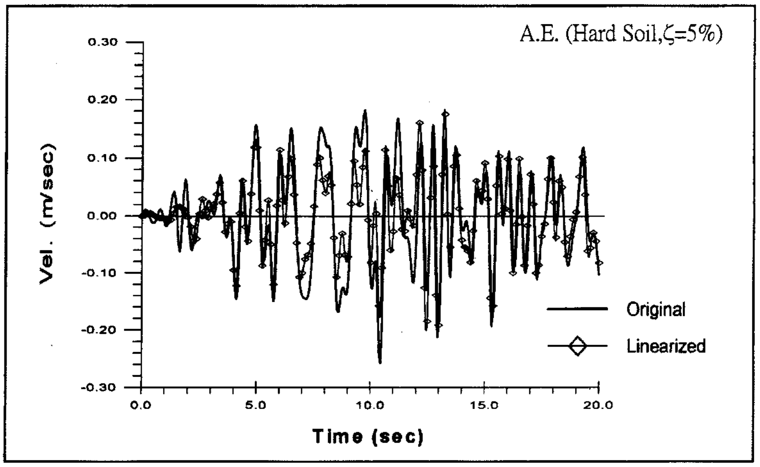


Fig. 9 Relative velocity of superstructure (hard soil, ζ=5%)

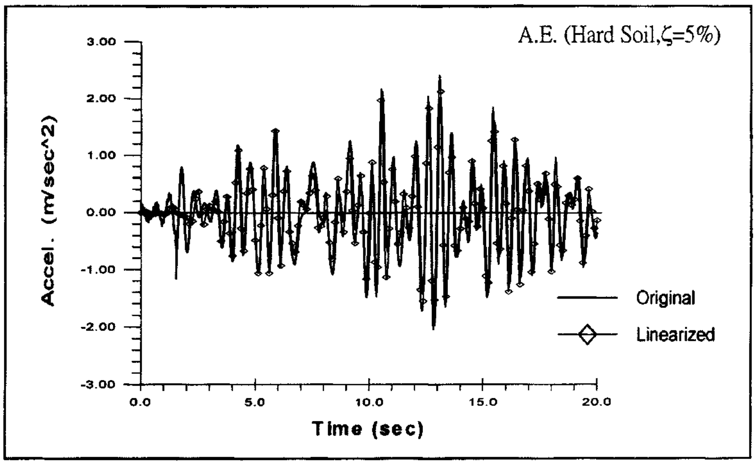


Fig. 10 Relative acceleration of superstructure (hard soil, ζ=5%)

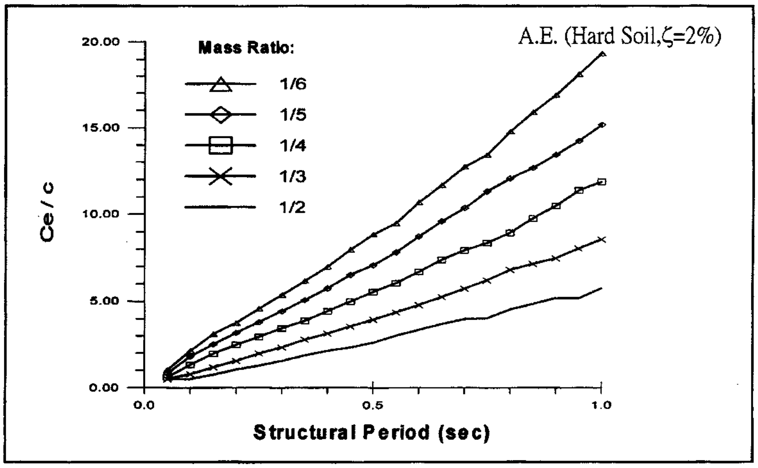


Fig. 11 Equivalent damping for F-RBI (hard soil, ζ=2%)

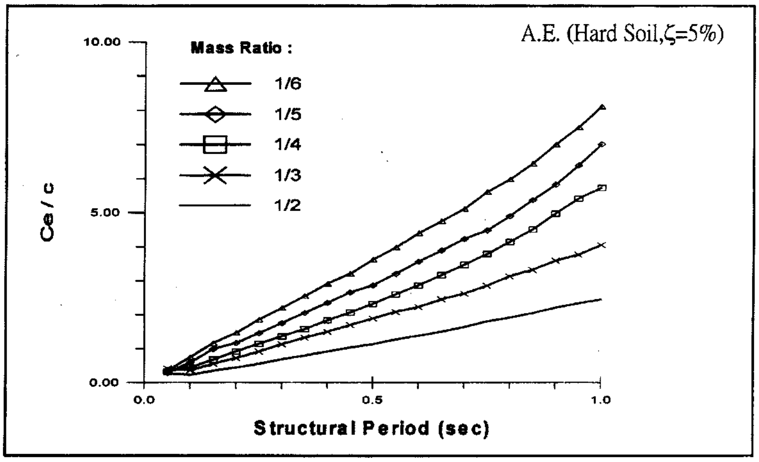


Fig. 12 Equivalent damping for F-RBI (hard soil, ζ=5%)

$$\eta = \frac{m}{m + M} \tag{22}$$

and throughout the analysis, a value of η=1/3 is selected.

As an illustration, for the case of hard soils with 5% of damping, the responses computed for the displacement, velocity, and acceleration of the superstructure relative to the ground using the present linearized procedure have been compared with those of the original nonlinear system in Figs. 8-10. From these figures, it is seen that although the displacements calculated from the linearized theory shows certain deviations from that of the original nonlinear system, good agreement has been achieved for the velocity and acceleration calculated from the two theories. From an engineer's point of view, it is believed that the great simplification in method of analysis brought by the concept of equivalent damping can outweigh the slight loss in accuracy. The equivalent damping curves constructed for both the hard and soft soils, with 2% and 5% of damping, and for various mass ratios η have been plotted in Figs. 11-14, which appear to be

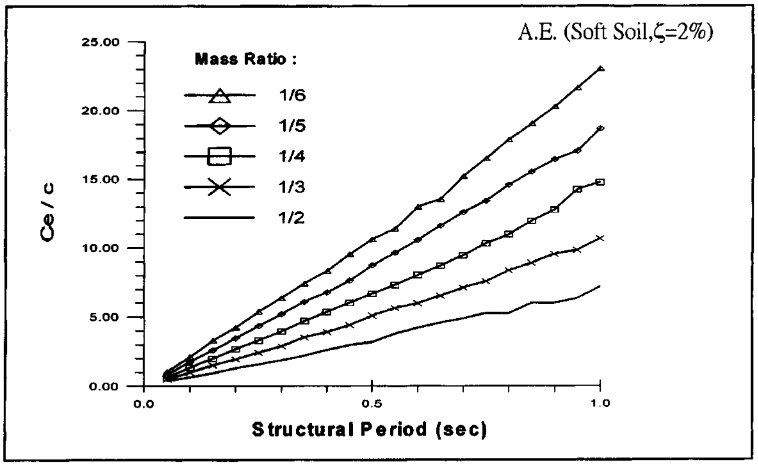


Fig. 13 Equivalent damping for F-RBI (soft soil, ζ=2%)

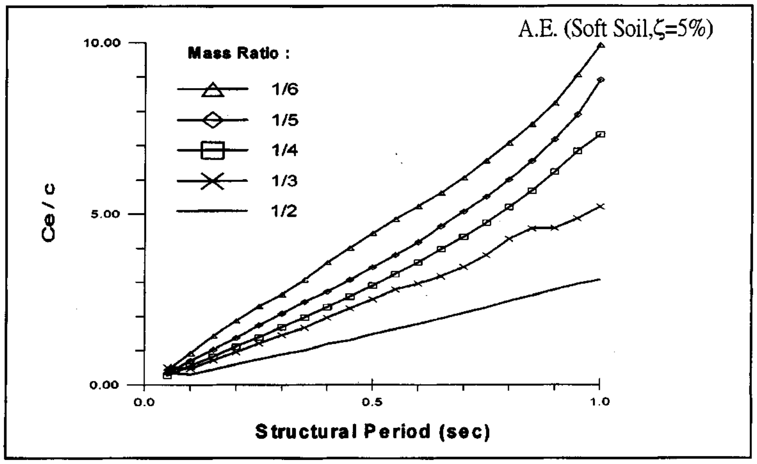


Fig. 14 Equivalent damping for F-RBI (soft soil, ζ=5%)

more or less linear within the practical range of application.

As the equivalent damping curves have been made for the R-FBI for some specific design spectra, given the properties of the structure and the F-RBI, structural engineers can interpolate from these curves the equivalent damping for the particular isolation device considered, and proceed with calculation of the structural responses using methods that are suitable for linear problems. Such an approach has the advantage that there is no need to deal with the original nonlinear dynamic problem using the time-history integration technique, which may not be readily available to structural engineers.

7. Conclusions

By the concept of equivalent damping, the original nonlinear dynamic problem of a base-isolated system involving non-sliding and sliding phases can be approximated by a linear one. In this paper, equivalent damping curves have been constructed for structures installed with resilient-friction base isolators (R-FBI) for some specific design spectra. As the equivalent damping can be found from the curves established specifically for the R-FBI devices with respect to a design spectrum, structural engineers can proceed with analysis of the base-isolated structures using the equivalent linear systems, with methods that are readily available. Although the replacement of the original nonlinear system by a linear one may result in some approximation for the displacement of the superstructure, it is believed that, from an engineer's point of view, the great simplification in method of analysis can outweigh the slight loss in accuracy, which is beneficial concerning promotion of the R-FBI devices in aseismic engineering.

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