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Identification of damage using natural frequencies and system moments

S. Hassiotis†

Department of Civil and Environmental Engineering, University of South Florida, Tampa, FL 33620, U.S.A.

Abstract. A method is presented to find the location and magnitude of damage in a structure using data from dynamic tests. The test data include a combination of natural frequency measurements, taken before and after the occurrence of damage, and response measurements taken after damage. An algorithm is developed to identify localized increases in the flexibility of the structural members. Increases in flexibility are attributed to damage. The algorithm uses the sensitivity of the flexibility matrix to changes in the natural frequencies of the structure to identify the damage. A set of under-determined equations is solved using an objective function which is derived from measurements of the system moments. Damage ranging from 10 to 60% increase in the flexibility of a member was successfully identified in a 50 d.o.f. structure, using a small number of natural frequency and velocity measurements.

Key words: inverse identification, damage, dynamic measurement, system moments.

1. Introduction

Research into the area of structural identification has originated in the fields of aeronautical and mechanical engineering. The demands in such fields are to identify the model of a system given either response data or modal data, and to use the identified model to predict the full dynamic response. An aspect of this area of research is the identification of a spatial model (the distribution of stiffness, mass and damping in the system matrices). Solutions to this problem have been obtained assuming that the identified system is a perturbation of the original spatial model (Collins *et al.* 1974, Baruch 1978, Chen and Garba 1980, Kabe 1985, Kuo and Wada 1987). In such work, dynamic-test data are used to update the structural parameters of a finite element model such that the model reproduces measurements of modal parameters, or response. This procedure has been adopted and developed to assess damage (or changes in the structural parameters) using dynamic data (Cawley and Adams 1979, Hassiotis and Jeong 1995).

Natural frequencies depend on the global properties of a system and thus, can be used with frequency-domain identification procedures to find the location and the magnitude of changes in the stiffness or flexibility. However, used alone, these data cannot provide reliable results especially because several combinations of damage in the structure can produce the same changes in the natural frequencies. Cawley and Adams (1979) were among the first to use an incomplete

† Assistant Professor

set of measured natural frequencies to identify the location and provide a rough estimate of structural damage. All possible elements of a structure were checked individually to obtain the location of the damage, which makes its implementation to larger structures and multiple damages difficult. Another drawback is that the magnitude of the stiffness change is given only in a qualitative sense. Hassiotis and Jeong (1993, 1995) introduced an optimization algorithm to identify both the location and the magnitude of single or multiple damage using changes in the natural frequencies. The algorithm was used to detect damage in a 90 d.o.f frame, and was found to give good results if the number of damaged members is kept below three. Pabst and Hagedorn (1993) used the changes in the frequencies to identify the location of a single crack on a cantilever beam.

If a set of mode shapes is known in addition to the natural frequencies, the identification problem becomes more robust. Most work in this area uses measurements of eigenvalues and eigenvectors for off-line identification of stiffness (Stubbs *et al.* 1990, Smith and Beattie 1991, Lindner *et al.* 1993, Sheinman 1993, Kaouk and Zimmerman 1994, Lim and Kashangaki 1994, Liu 1995). An on-line identification procedure to find the severity and location of damage using natural frequencies and mode shapes has been developed by Tsou and Shen (1994) using neural networks. A principal disadvantage in these approaches is their dependence on a set of measured mode shapes. Accurate measurements of mode shapes is practically impossible to obtain.

Several investigators approached the problem of damage identification in the time domain. In most cases off-line techniques have been developed (Beck and Katafygiotis 1992, Koh *et al.* 1992). An algorithm for on-line identification of damage using time-response was introduced by Lin *et al.* (1990). These methods can only give an approximate location of the damage. If a possible function of the time-dependent stiffness in each member can be supplied by the investigator, the degradation of the stiffness can also be found. However, it is impractical to assume that such functions can be supplied for every member of the structure.

In summary, a robust algorithm to identify damage with easily accessible measurements does not exist. Traditionally, the time-domain and the frequency-domain identification procedures have been seen or developed as rivals. However, neither procedure can be used alone to identify damage. In this paper, time-domain and frequency-domain data will be used together to find the changes in the flexibility of the structure.

1.1. Eigenvalue sensitivity analysis

The equation of motion of an undamped mechanical system is given by

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{K}\boldsymbol{q} = \boldsymbol{F}\cdot\boldsymbol{u} \tag{1}$$

where $\boldsymbol{M}$ is an $n\times n$ mass matrix, $\boldsymbol{K}$ is an $n\times n$ stiffness matrix, $\boldsymbol{q}$ is an n-vector of nodal displacements, $\boldsymbol{F}$ is an $n\times l$ matrix which indicates the location of the force input, and $\boldsymbol{u}$ is an $l\times 1$ vector of forcing functions.

The assumption of a harmonic solution for $\boldsymbol{q}$ leads to the eigenvalue problem

$$(\boldsymbol{K} - \lambda_i \boldsymbol{M})\phi_i = 0 \tag{2}$$

where the eigenvalue λ_i is the square of the circular natural frequency ω_i, and the eigenvector ϕ_i is the corresponding natural mode shape of the system.

In terms of the flexibility matrix, $\boldsymbol{R}=\boldsymbol{K}^{-1}$, the eigenvalue problem can be written as:

$$(\frac{1}{\lambda_i}\boldsymbol{I} - \boldsymbol{RM})\phi_i = 0 \tag{3}$$

Damage of the original structure is assumed to cause changes of the flexibility matrix by an amount $\delta\boldsymbol{R}$. It is further assumed that the damage is not accompanied by a change in mass. The eigenvalue problem of the damaged structure is given by

$$(\frac{1}{\lambda_i + \delta\lambda_i}\boldsymbol{I} - (\boldsymbol{R} + \delta\boldsymbol{R})\boldsymbol{M})\phi_i = 0 \tag{4}$$

where it is assumed that the eigenvalues change by $\delta\lambda$. Although the eigenvectors also change due to the damage by an amount $\delta\phi$, we will not use such change in the identification procedure. We assume herein that data is available on the natural frequencies only (The errors introduced by the changes in the eigenvectors have been used in the past to provide an optimality criterion (Hassiotis and Jeong 1995).

Eq. (4) can be written as

$$\{\frac{1}{\lambda_i} - \frac{\delta\lambda_i}{\lambda_i(\lambda_i + \delta\lambda_i)}\}\boldsymbol{I}\,\phi_i - \boldsymbol{RM}\,\phi_i - \delta\boldsymbol{RM}\,\phi_i = 0 \tag{5}$$

and, by using Eq. (3), it can be simplified to

$$\frac{-\delta\lambda_i}{\lambda_i(\lambda_i + \delta\lambda_i)}\boldsymbol{I}\,\phi_i - \delta\boldsymbol{RM}\,\phi_i = 0 \tag{6}$$

The orthogonality condition of the eigenvectors can be used to find the sensitivity equations that relate the changes of the natural frequencies to the parameters of the structure. First, Eq. (6) is multiplied by $\phi_j^T\boldsymbol{M}$

$$\phi_j^T\boldsymbol{M}\,\phi_i\,g_i - \phi_j^T\boldsymbol{M}\,\delta\boldsymbol{RM}\,\phi_i = 0 \tag{7}$$

where the term that carries the natural frequency information is defined by

$$g_i = \frac{-\delta\lambda_i}{\lambda_i(\lambda_i + \delta\lambda_i)} \tag{8}$$

Then, the orthogonality condition

$$\phi_i^T\boldsymbol{M}\,\phi_j = \delta_{ij} \tag{9}$$

is used to get

$$g_i = \phi_i^T\boldsymbol{M}\,\delta\boldsymbol{RM}\,\phi_i \tag{10}$$

The change in the global flexibility matrix, $\delta\boldsymbol{R}$, can be expanded as a linear combination of the changes in the flexibility matrices of each element, $\boldsymbol{R}_j^e$,

$$\delta\boldsymbol{R} = \sum_{j=1}^{n_e}\boldsymbol{R}_j^e\,\delta r_j \tag{11}$$

where n_e is the total number of elements in the structure. By substituting Eq. (11) into Eq. (10) we get the sensitivity equations as

$$g_i = \sum_{j=1}^{n_e} \phi_i^T \boldsymbol{M R}_j^e \boldsymbol{M} \phi_i \, \delta r_j \tag{12}$$

or

$$\boldsymbol{D} \, \boldsymbol{\delta r} = \boldsymbol{g} \tag{13}$$

where $\boldsymbol{g}$ is an m-vector of eigenvalue data, $\boldsymbol{\delta r}$ is an n_e-vector of the unknown changes in the flexibility, $\boldsymbol{D}$ is an $m \times n_e$-matrix of elements $\boldsymbol{D}(i, j)=\phi_i^T \boldsymbol{M R}_j^e \boldsymbol{M} \phi_i$.

The simultaneous equations relate the change of the flexibilities of each element to the changes in the frequencies of the structure. If $\boldsymbol{g}$ is available through measurements, the solution of Eq. (13) yields the changes in the element flexibilities. Only a small number of natural frequencies can usually be measured, ($m<n$), which renders Eq. (13) underdetermined. Since an infinity of solutions can satisfy them, they can be solved uniquely only by introducing an optimality criterion.

1.2. Moments of the impulse response as optimality criteria

The optimality criterion needed can be derived from several approaches. In general, least squares formulations and error minimization lead to a minimization of a criterion in the form:

$$\begin{gathered} \text{minimize } \frac{1}{2} \, \boldsymbol{\delta r}^T \, \boldsymbol{Q} \, \boldsymbol{\delta r} + \boldsymbol{\delta r}^T \boldsymbol{c} \\ \text{subject to } \boldsymbol{D} \, \boldsymbol{\delta r} = g \\ \text{and } \boldsymbol{\delta r} \geq 0 \end{gathered} \tag{14}$$

Here, $\boldsymbol{c}$ is a given n_e-vector, and $\boldsymbol{Q}$ is a given positive definite $n_e \times n_e$-matrix. The inequality constraint is derived from physical reasoning. In general, damage does not produce a decrease in flexibility. The problem is a quadratic programming problem with linear equality and inequality constraints. If $\boldsymbol{Q}$ is positive definite, this problem is strictly convex and a unique solution for $\boldsymbol{\delta r}$ can be found using efficient algorithms that have been developed in the field of linear and nonlinear programming.

In this paper, an optimality criterion is developed using the response of the system after damage, as this is represented by the moments of the system. By defining a state vector $x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$, Eq. (1) can be written as

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{15}$$

where

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ M^{-1}F \end{bmatrix}$$

$$C = [H \;\; J]$$

Here $\boldsymbol{H}$ is an $s \times n$-matrix of zeroes and ones to indicate the location of the s measurements in displacement, $\boldsymbol{J}$ is a $s \times n$-matrix of zeroes and ones to indicate the location of the s measurements in velocity.

The moments of the impulse response at the measurement points are defined by

$$\boldsymbol{M}_i = \int_0^\infty t^i \boldsymbol{W}(t) dt \quad i = 0, 1, 2, \cdots$$

where

$$\boldsymbol{W}(t) = Ce^{A^t}\boldsymbol{B}$$

is the impulse response matrix of the system at the measurement points (that is, $\boldsymbol{W}_{ij}$ is the response at output i due to an impulse excitation at input j). Therefore, measurements of the impulse-response taken at several points in the structure can be used to obtain the system-moment data.

In theory, the system moments are related to the state-space matrices of the system as follows

$$\boldsymbol{M}_i = \boldsymbol{C}\boldsymbol{A}^{-i}\boldsymbol{B} \quad i = 0, 1, 2, \cdots \tag{16}$$

For i=0 we obtain the first parameter as

$$M_o = JM^{-1}F \tag{17}$$

This parameter does not contain the flexibility matrix $\boldsymbol{R}$ and will not be used in the derivation of the optimality criterion. The second moment, for i=1, is given by

$$M_1 = CA^{-1}B \tag{18}$$

Since $\boldsymbol{A}^{-1}$ can be evaluated as

$$A^{-1} = \begin{bmatrix} \boldsymbol{0} & -\boldsymbol{K}^{-1}\boldsymbol{M} \\ \boldsymbol{I} & \boldsymbol{0} \end{bmatrix} \tag{19}$$

moment $\boldsymbol{M}_1$ can be written as

$$M_1 = -HRF \tag{20}$$

The assumption herein is that $\boldsymbol{M}_1$ for a lightly damped structure can be measured after the occurrence of damage. In such cases, a possible optimality criterion can be derived to minimize the difference between the moment measured after damage, $\boldsymbol{M}_1$, and the one obtained from the model of the damaged structure, $-\boldsymbol{H}(\boldsymbol{R}+\delta\boldsymbol{R})\boldsymbol{F}$:

$$\text{minimize } \delta\boldsymbol{R} \; \| \boldsymbol{M}_1 + \boldsymbol{H}(\boldsymbol{R}+\delta\boldsymbol{R})\boldsymbol{F} \|^2 \tag{21}$$

where $\| \cdot \|$ denotes the Frobenius norm of the expression. The optimization problem becomes

$$\text{minimize } \delta r_i \; \| \boldsymbol{M}_1 + \boldsymbol{HRF} + \sum_{k=1}^{ne} \boldsymbol{HR}_k^e \delta r_k \boldsymbol{F} \|^2 \tag{22}$$

By defining $\boldsymbol{U}=\boldsymbol{HRF}$ ($s \times l$ matrix) and $\boldsymbol{V}_k=\boldsymbol{HR}_k^e\boldsymbol{F}$ ($s \times l$ matrix) the Frobenius norm can be rewritten as

$$\sum_{i=1}^{s} \sum_{j=1}^{l} \{M_1(i, j) + U(i, j) + \sum_{k=1}^{ne} V_k(i, j)\, \delta r_k\}^2 \tag{23}$$

or

$$\sum_{i=1}^{s}\sum_{j=1}^{l} M_1^2(i,j) + U^2(i,j) + \left(\sum_{k=1}^{n_e} V_k(i,j)\,\delta r_k\right)^2 + 2M_1(i,j)\,U(i,j)$$
$$+\,2M_1(i,j)\sum_{k=1}^{n_e} V_k(i,j)\,\delta r_k + 2U(i,j)\sum_{k=1}^{n_e} V_k(i,j)\,\delta r_k \tag{24}$$

By dropping the constant term, this can be simplified to

$$\sum_{i=1}^{s}\sum_{j=1}^{l}\sum_{p=1}^{n_e}\sum_{k=1}^{n_e} V_k(i,j)\,\delta r_k\,V_p(i,j)\,\delta r_p + \sum_{i=1}^{s}\sum_{j=1}^{l}\sum_{k=1}^{n_e} 2\{M_1(i,j)V_k(i,j)$$
$$+\,U(i,j)\,V_k\,(i,j)\}\,\delta r_k \tag{25}$$

Eq. (25) is equivalent to:

$$\delta r^T c + \frac{1}{2}\,\delta r^T Q\,\delta r \tag{26}$$

where, $c_k = \sum_i^s \sum_j^l M_1(i,j)\,V_k(i,j) + U(i,j)\,V_k(i,j)$ and $Q(p,k) = \sum_i^s \sum_j^l V_k(i,j)\,V_p(i,j)$. This optimization criterion can be used for the solution of the optimization problem 14. It can be shown that Q is a positive definite matrix, hence, the optimization problem is a convex quadratic optimization.

2. Assessment of damage in frame

To assess the damage in a structure, we first assume that an analytical model exists that describes the system before the damage. Using this model, we calculate a set of natural frequencies. Then, a known decrease in stiffness is induced, referred here as the "actual" damage, and the natural frequencies and the moments of the structure are calculated. These are input as data into the optimization algorithm, and the "predicted" damage is calculated. The International Mathematical and Statistical Libraries (IMSL 1987) were used to solve the eigenvalue and the optimization problems.

The proposed algorithm was used to assess damage in a 50 d.o.f. mass-spring system, shown in Fig. 1. A variable stiffness is assumed through the system, and the value of the stiffness of each element is given in Table 1. The mass is taken as unity. This system produces a set of eigenvalues, as shown in Table 2. If the stiffness in any of the elements changes, the eigenvalues also change. A set of eigenvalues of a damaged state is also given in Table 2.

The correct identification of damage depends on (1) the number of damaged members, and degree of damage; (2) the amount and type of data available through measurements; and (3) the location of the measurements. Figs. 2 to 11, summarize the general trend of the solution to the damage identification problem, using the algorithm described herein. Although a stiffness decrease of up to 60% is used in this example, the algorithm is capable of predicting a stiffness decrease of up to 90%. If such damage exists in just a few elements, the assumption that the damaged structure is within a small perturbation of the undamaged structure is not violated and the algorithm arrives at the correct identification. However, if such a high degree-of-damage exists in most of the elements, the small-perturbation assumption is violated and the algorithm might not arrive at acceptable answers.

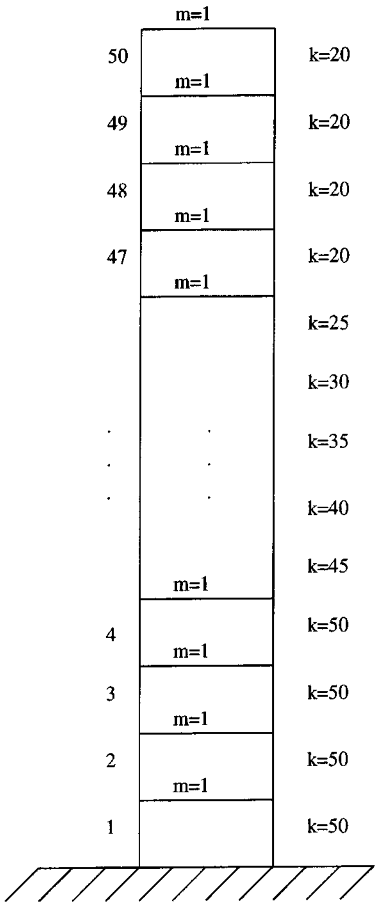

Fig. 1 50-story, 50 d.o.f. frame

Table 1 Stiffness-distribution of 50 d.o.f. system

Elements	Stiffness
1-5	50
6-10	45
11-15	40
16-25	35
36-45	30
46-50	20

Fig. 2 shows the prediction of damage if the stiffness of every element in the 50 d.o.f. structure is reduced anywhere from 10 to 60%. Successful identification of such extended damage requires that the response of the structure is measured at every element.

Less data is needed if the reduction of the stiffness occurs in a few elements only. In what follows, the structure is damaged at six locations by a given percent decrease in the element stiffness: Elements 2 (30%), 8(10%), 15(40%), 22(50%), 34(20%), and 50(60%). A combination of data that includes measurements of the natural frequencies before and after damage plus

Table 2 Eigenvalues of 50 d.o.f. spring-mass structure

Mode	Undamaged Eigenvalues	Damaged Eigenvalues
1	0.366E-01	0.343E-01
2	0.285E+00	0.276E+00
3	0.774E+00	0.733E+00
4	0.150E+01	0.142E+01
5	0.243E+01	0.230E+01
6	0.365E+01	0.339E+01
7	0.506E+01	0.482E+01

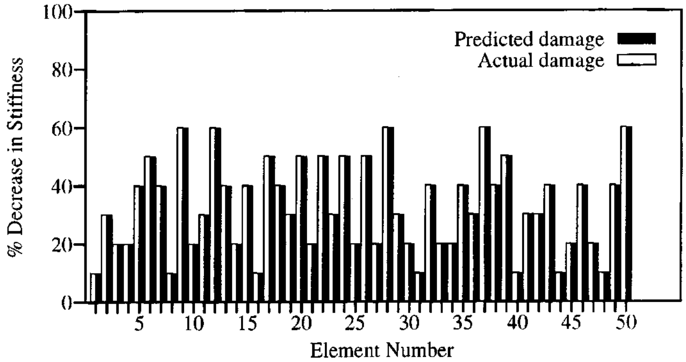

Fig. 2 Identification of damage using: 1 nat. frequency, 1 force input at d.o.f 50, 50 displacement measurements (d.o.f. 1, 2, ..., 50)

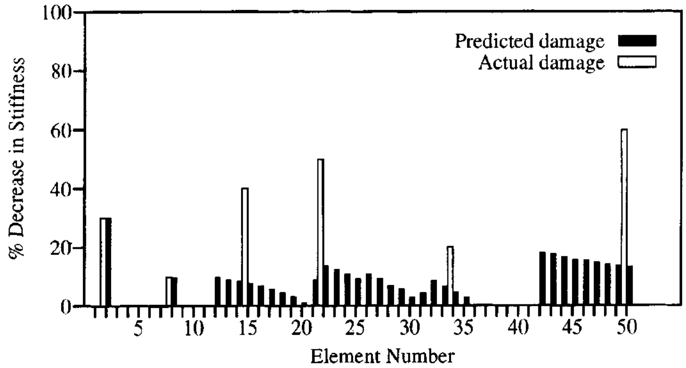

Fig. 3 Identification of damage using: 1 nat. frequency, 15 displacement measurements (d.o.f. 1, 2, 7, 8, 10, 11, 20, 21, 30, 31, 40, 41, 50)

displacement records taken at several locations of the structure after damage are needed to identify the stiffness reductions. The eigenvalues of this state of damage are shown in Table 2.

Fig. 3 shows that 15 displacement records and the change in the first natural frequency do not constitute enough data to find the damage of the six elements. If the change in three natural frequencies is known in addition to the system moments, the damage in most elements can be found, as seen in Fig. 4, in which, damage in Element 34 was erroneously attributed to element

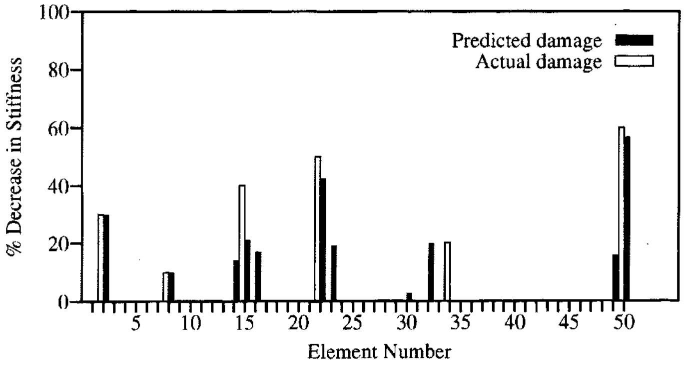

Fig. 4 Identification of damage using: 3 nat. frequencies, 15 displacement measurements (d.o.f. 1, 2, 7, 8, 10, 11, 20, 21, 30, 31, 40, 41, 50)

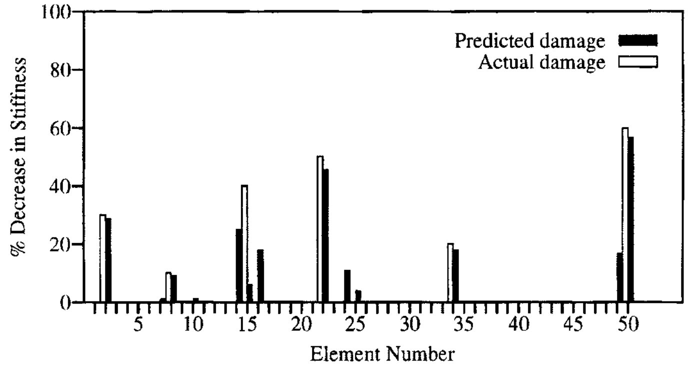

Fig. 5 Identification of damage using: 7 nat. frequencies, 15 displacement measurements (d.o.f. 1, 2, 7, 8, 10, 11, 20, 21, 30, 31, 40, 41, 50)

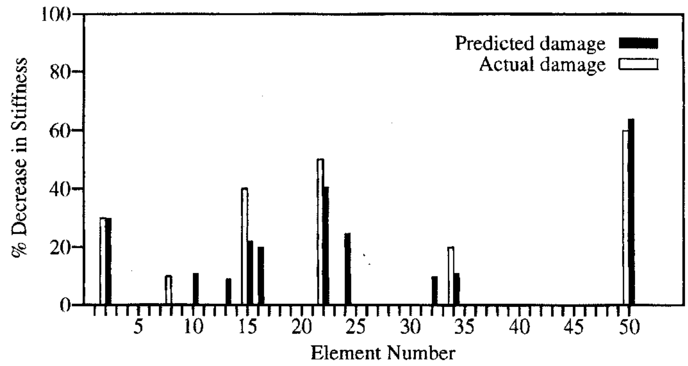

Fig. 6 Identification of damage using: 7 nat. frequencies, 10 displacement measurements (d.o.f. 1, 2, 10, 11, 20, 21, 30, 31, 40, 41)

32. If data on seven natural frequencies is available, damage in all elements is found successfully, as shown in Fig. 5. The increase in the data was not enough to eliminate any spurious damage that is part of the optimal solution.

Keeping the number of natural frequencies to seven and reducing the number of displacement

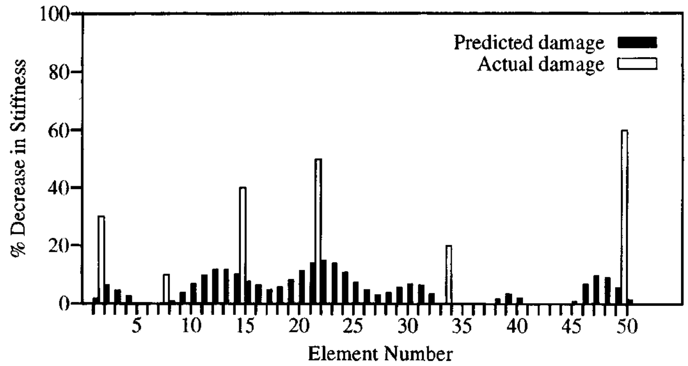

Fig. 7 Identification of damage using: 7 nat. frequencies, no displacement measurements

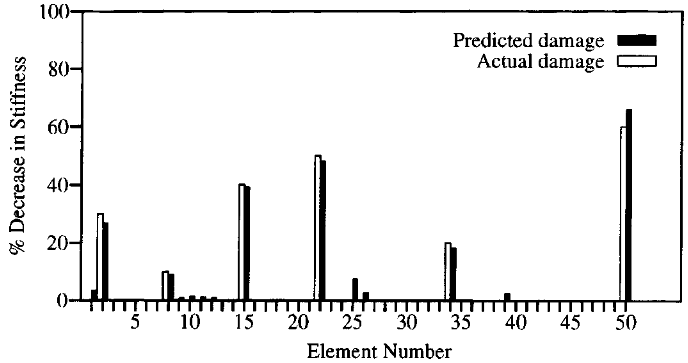

Fig. 8 Identification of damage using: 7 nat. frequencies, 15 displacement measurements (d.o.f. 1, 2, 3, ..., 15)

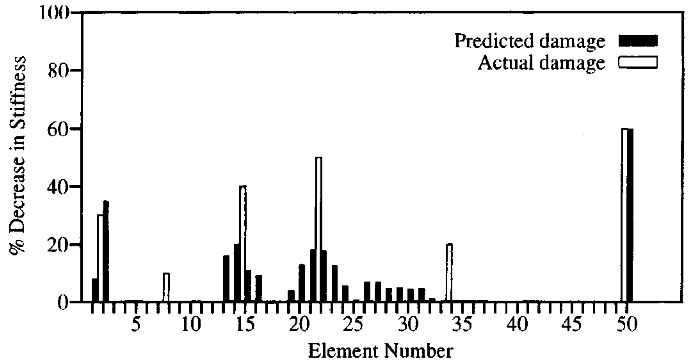

Fig. 9 Identification of damage using: 7 nat. frequencies, 15 displacement measurements (d.o.f. 36, 37, ..., 50)

measurements to 10, also hurts the identification, as shown in Fig. 6 where the damage of Element 8 is erroneously attributed to Element 10. The use of natural frequencies alone fails to identify the damage of the six elements, as seen in Fig. 7.

The location of the displacement measurements affects the identification process significantly. Figs. 5, 8, and 9 can be compared to show that, in this particular problem, the best identification

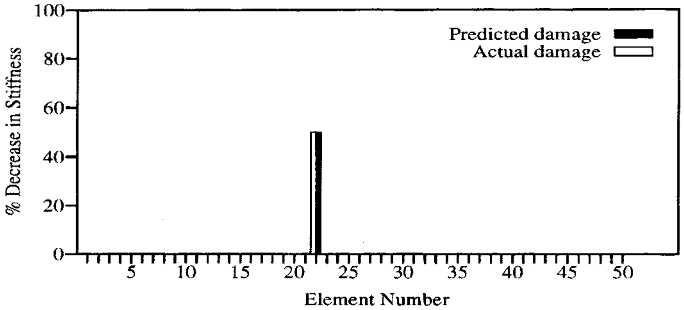

Fig. 10 Identification of damage using: 1 nat. frequency, 22 displacement measurements (d.o.f. 1, 2, ..., 22 OR 20, 21, ..., 42, OR 1, 2, 3, 10, 11, 12, ..., 50)

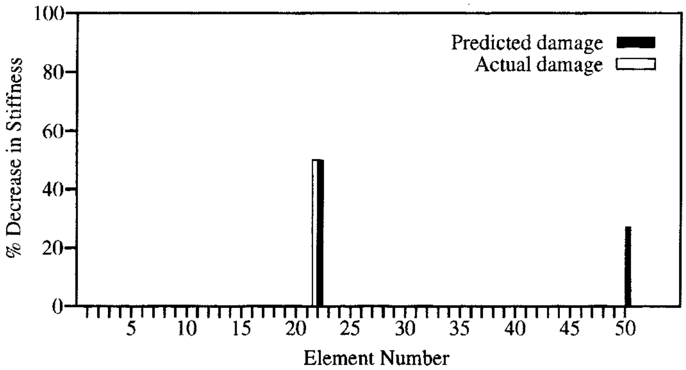

Fig. 11 Identification of damage using: 3 nat. frequencies, 2 displacement measurements (d.o.f. 1, 2)

was obtained when the 15 displacement measurements were known at degrees-of-freedom one to 15. It can be seen that the identification fails if the 15 measurements are assumed to be at degrees-of-freedom 36 to 50.

The advantage of using both, natural frequencies and displacement measurements, is accentuated in the results shown in Figs. 10 and 11. To identify the single damage of Element 22, it requires either (1) 1 natural frequency plus 20 displacement measurements or (2) 3 natural frequencies plus 2 displacement measurements.

In summary, the quality of the damage identification is a function of the amount of data that can be included in the solution. The identification process arrives at excellent results when a dynamic measurement exists for every element. In such cases, the number of unknown δk is equal to the number of measurements, and the optimal solution is very close to the exact solution (for either single or multiple damage sites). However, such a large number of measurements cannot be expected to be taken from a real structure. Fortunately, a much smaller amount of data is needed for the exact identification of damage in one element. To find damage when it occurs in more than one element, we must increase the amount of information in the problem. Erroneous solutions or spurious damage can be eliminated with the inclusion of more and more data. The amount of data that is needed is problem-specific and depends mainly on (1) the number of damaged sites that are expected to exist in the structure, and (2) the accuracy of identification that is needed.

3. Conclusions

An identification algorithm has been developed to find the location and the magnitude of damage in multi-degree-of-freedom structures. The algorithm is a quadratic optimization problem that depends on the relationship of the parameters of the structure to (1) the natural frequencies of the structure, and (2) the moments of the impulse response of the structure.

The identification algorithm was tested for its ability to predict the damage in a 50 d.o.f. spring-mass structure. Parametric studies on such structure lead to the following conclusions:

- The combination of two types of measurements, natural frequencies and response measurements, has contributed in the development of an improved algorithm for the identification of damage.
- If damage occurs in a single location, a very limited amount of data can locate and quantify it.
- The number of measurements needed to find the damage depends on the number of elements that are damaged. If damage occurs in namy locations, the amount of data needed for correct identification increases. At the limit, if damage is so widespread as to affect every element in the structure, it can be identified only with a complete number of measurements taken at every node of the structure.
- The location of the displacement measurements plays a significant role in the correct identification of the damage. For the structure that was presented herein, measurements that were taken close to the base delivered more information into the problem than measurements taken towards the free end.

Current work to improve this algorithm is being conducted in three areas: (1) the proper inclusion of data noise; (2) the optimal location of measurements; and (3) the optimal combination of measurements.

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Notations

The following symbols are used in this paper:

$\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}$	= matrix coefficients of state vector equations
$\boldsymbol{D}$	= matrix relating changes in stiffness to changes in eigenvalues
$\boldsymbol{F}$	= matrix of force-input locations
$\boldsymbol{H}_2$	= third Markov parameter
J	= optimality criterion
$\boldsymbol{K}$	= stiffness matrix
$\boldsymbol{K}^e$	= element stiffness matrix in global coordinates
$\boldsymbol{M}$	= mass matrix
$\boldsymbol{Q}$	= weighting matrix in quadratic optimization
$\boldsymbol{c}$	= vector in linear term of quadratic optimization
$\boldsymbol{q}$	= vector of nodal displacements
$\boldsymbol{r}$	= vector of residuals
$\boldsymbol{u}$	= vector of forcing functions
$\boldsymbol{x}$	= state vector
$\delta\boldsymbol{K}$	= change of the stiffness matrix
$\delta\boldsymbol{M}$	= change of the mass matrix
$\delta\boldsymbol{k}$	= vector of element stiffness changes
$\delta\lambda$	= vector of the changes in the eigenvalues
$\delta\phi$	= change of an eigenvector
λ	= vector of the eigenvalues of the structure
ϕ	= an eigenvector of the structure
ω	= natural frequency of the structure