Structural Engineering and Mechanics, Vol. 8, No. 1 (1999) 85-102
DOI: http://dx.doi.org/10.12989/sem.1999.8.1.085

The use of eccentric beam elements in the analysis of slab-on-girder bridges

Tommy H.T. Chan† and Jeffrey H.F. Chan‡

Department of Civil and Structural Engineering, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong

Abstract. With the advent of computer, the finite element method has become a most powerful numerical method for structural analysis. However, bridge designers are reluctant to use it in their designs because of its complex nature and its being time consuming in the preparation of the input data and analyzing the results. This paper describes the development of a computer based finite element model using the idea of eccentric beam elements for the analysis of slab-on-girder bridges. The proposed method is supported by a laboratory test using a reinforced concrete bridge model. Other bridge analytical schemes are also introduced and compared with the proposed method. The main aim of the comparison is to prove the effectiveness of the shell and eccentric beam modelling in the studies of lateral load distribution of slab-on-girder bridges. It is concluded that the proposed finite element method gives a closer to real idealization and its developed computer program, SHECAN, is also very simple to use. It is highly recommended to use it as an analytical tool for the design of slab-on-girder bridges.

Key words: bridge deck analysis; eccentric beam model; finite element methods; grillage analogy method; lateral load distribution; rigid link element; semi-continuum method; slab-on-girder bridge.

1. Introduction

Slab-on-girder bridges are one of the common structural forms for bridge superstructures. Nowadays, bridge engineers have many bridge analytical schemes available in hand for the analysis of this bridge type, e.g., grillage analogy method (West 1973), the semi-continuum method (Mufti *et al.* 1997) and finite element methods (Zienkiewiez 1989). These methods are either not able to truly represent the actual behaviour of a bridge, e.g., the grillage analogy method, or too complicated to use as a design tool, e.g., finite element methods. Because of its simplicity in use, the grillage analogy method is generally adopted by many engineers for bridge design. The output from a finite element method is difficult to be analyzed and much time will be spent on the preparation of the input file. Although there exist many pre- and post- processors for the use of finite element methods, the number of such processors for the design of slab-on-girder bridges is comparatively much less than that for the design of other structures.

Chan and Chan (1994) made use of the idea of eccentric beam elements and formulated a finite element method which is simple to use and yet it can model a slab-on-girder bridge more realistically when comparing with the semi-continuum method or grillage analogy method. It is

† Ph.D.
‡ Research Student

highly recommended to use the method in the design of slab-on-girder bridges. This paper describes the formulation of such finite element method for the analysis of slab-on-girder bridges. Emphasis is made on the studies of the load distribution of this type of bridges. The investigation includes laboratory work and theoretical studies. A reinforced concrete slab-on-beam bridge model was casted for the present study. In addition, virtual bridge models within the practical ranges as stated by Bakht and Moses (1988) were also used. The results from the proposed method were compared with those from the experiments and other traditional tools for bridge analyses. An example used by Mounir *et al.* (1997) was employed to illustrate that the proposed method could produce results similar to other finite-element modelling techniques.

2. Shell and eccentric beam idealization

The shell and eccentric beam idealization are based on finite element methods (Zienkiewiez 1989). Even within the scope of finite element methods, there are different idealizations for slab-on-girder bridges to study its lateral load distribution, e.g., Imbsen and Nutt (1978), Brockenbrough (1986), Hays *et al.* (1986). Memari and West (1990), Tarhini and Frederick (1992) and Bishara (1993). Although the number of idealizations are numerous, they can generally be categorized into four types as shown in Fig. 1. Usually, shell elements or brick elements are used to model the slab, and beam elements or shell elements are used to model the girders. As the neutral axis of each girder is different from that of the slab, rigid links are generally used to connect the girder elements to the slab model.

Obviously, the idealization of shell and eccentric beam modelling is simpler than the traditional

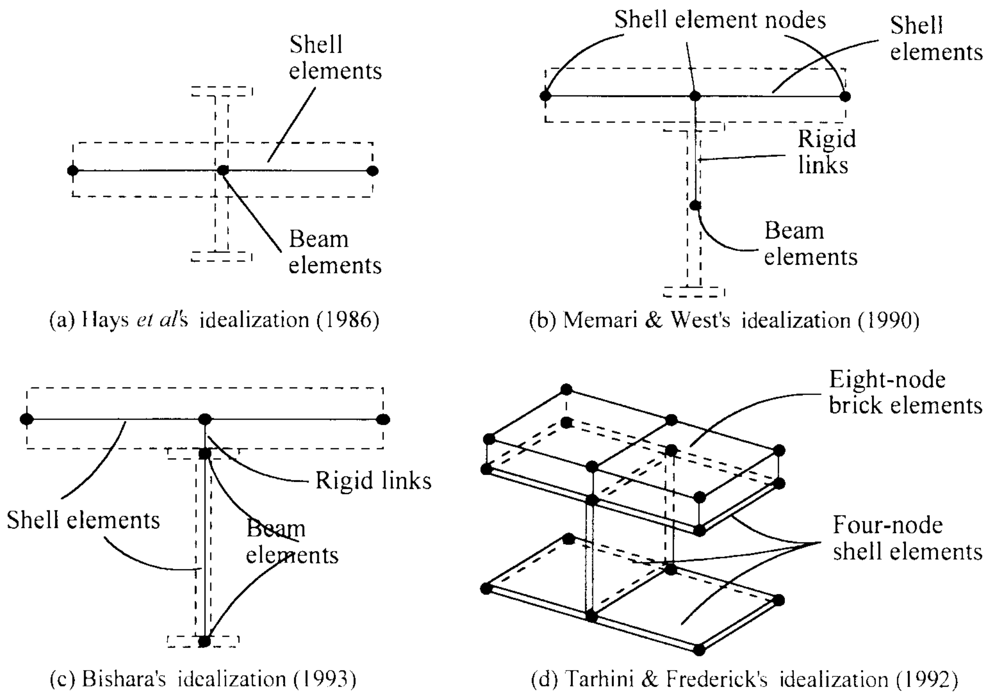


Fig. 1 Typical finite element idealizations for slab-on-girder bridges

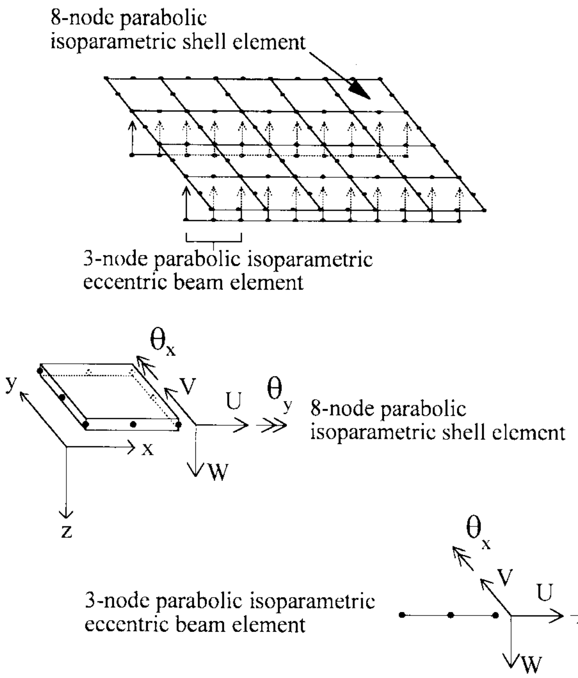


Fig. 2 Idealization of proposed modelling (shell and eccentric beam modelling)

finite element idealizations. The idea of using eccentric beam elements was pioneered by Miller (1980). It can be applied to a plate-beam system in which the beams are placed at an eccentricity from the middle plane of the plate. These structures include composite beams with plate as top flange or eccentric stiffeners on a base plate. However, it is seldom used for the analysis of slab-on-girder bridges in the past literatures.

The proposed finite element idealization of the bridge model is shown in Fig. 2. The slab is modelled as 8-node parabolic isoparametric shell elements with five degrees of freedom at each node. The girders are modelled using 3-node parabolic isoparametric eccentric beam elements with also five degrees of freedom at each node, and no slipping is assumed between the slab and the girders. It can be shown that the most important advantage of using the proposed modelling-*Shell and Eccentric Beam Modelling*, when compared with other common finite element modelling for this structural form, is its simplicity of omitting the intermediate rigid link elements between the shell elements and beam elements, and yet retaining the material variation and sectional properties of the slab and the girders.

2.1. Element stiffness matrix of shell elements

The shell element used can be considered as a combination of two parts: the *Plate Bending Part* and the *Membrane Part* as shown in Fig. 3. It is capable of simulating both in-plane (plane stress) and out-of-plane (plate bending) deformation. Hinton and Owen (1977) treated plane stress element and plate bending part as two separated elements. In this study, the elements were modified so as the effect of plate bending and plane stress were combined in a flat shell element. A simple rectangular flat shell element can be obtained by superimposing the plate bending part and the membrane part. The elasticity matrix of the shell element, $[D_s]$, is therefore expressed as:

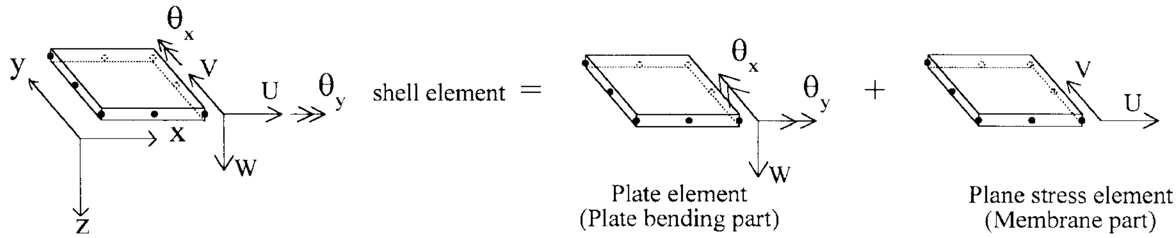


Fig. 3 Composition of the shell element

$$[D_s] = \begin{bmatrix} [D_{sp}] & 0 & 0 \\ 0 & [D_{sb}] & 0 \\ 0 & 0 & [D_{ss}] \end{bmatrix} \tag{1}$$

where $[D_{sp}]$ of order 3×3, $[D_{sb}]$ of order 3×3, and $[D_{ss}]$ of order 2×2 are respectively the elasticity matrices for membrane, bending, and shearing parts of the shell element.

The size of the element stiffness matrix depends upon the number of nodes in the element and the number of degrees of freedom per node. In the present model, there are five degrees of freedom at each node. Therefore the size of the element stiffness matrix is 40×40. The element stiffness matrix linking nodes i and j can then be calculated as:

$$[K_{shell}^{ij}] = \int_A [B_i^s]^T [D_s] [B_j^s] dA \tag{2}$$

where $[B_k^s]$ is the strain matrix of node k of the shell element. The formulation of $[D_{sp}]$, $[D_{sb}]$, $[D_{ss}]$ and $[B_k^s]$ could be obtained in any standard text book on finite element methods, e.g., Bathe (1996).

The integration of the stiffness terms are carried out numerically (Zienkiewiez 1989). In the present study, the self-developed program, SHECAN, can provide a choice of 2×2 mesh or 3×3 mesh of Gauss points for integration.

It is noted that there are better shell elements (Bathe 1996) which can be used to analyze very complex shell geometries and stress distributions. The general formulation for these shell elements can model curved shells as well. However, when such shell elements are used to model the slab of a slab-on-girder bridge, their behaviour is similar to the shell element proposed in this study. The formulation of the adopted shell element can be improved so as not to exhibit shear and membrane locking. The displacement-based formulation has the disadvantage that the lower-order elements lock as a result of spurious shear strains, and when the elements are curved, also because of spurious membrane strain. Indeed, the least order of interpolation that should be used is a cubic interpolation of displacements leading to 16-node quadrilateral shell elements. In order to circumvent the locking behaviour, a mixed interpolation is proposed by Bathe (1996). Fortunately, as all the shell elements used to model the deck of a slab-on-girder bridge in the present project are straight and not curved, and the thickness of the slab is rather thin when compared to the length of the shell elements, the effect of shear and membrane locking is not significant in this study.

2.2. Element stiffness matrix of beam/eccentric beam element

The girders are modelled using three-node parabolic isoparametric beam elements with original six degrees of freedom at each node. The element stiffness matrix of this beam element, $[K^e_{beam}]$, can be expressed in terms of its elasticity matrix, $[D_b]$, and a typical sub-matrix linking node i and j can then be expressed in the following form:

$$[K^{ij}_{beam}] = \int_0^L [B^b_i]^T [D_b] [B^b_j] \, dx \tag{3}$$

where $[B^b_k]$ is the strain matrix of node k of the three-node beam element. Then formulation of $[D_b]$ and $[B^b_k]$ can also be found in any standard text book on finite element methods, e.g., Bathe (1996).

When this beam element is used to model an eccentric stiffener, its nodal displacements may be regarded as 'slave' variables while those of the shell elements will behave as 'master' variables. The beam nodal displacements can therefore be expressed in terms of the shell nodal displacements using a transformation matrix (Miller 1980). As the shell element nodal rotational displacement about the z-axis is equal to zero at all nodes, the last column and row of the transformation matrix can be discarded and the transformation equation then becomes:

$$\begin{Bmatrix} u_i \\ v_i \\ w_i \\ \theta_{ix} \\ \theta_{iy} \end{Bmatrix}_{beam} = \begin{bmatrix} 1 & 0 & 0 & -e & 0 \\ 0 & 1 & 0 & 0 & -e \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ w_i \\ \theta_{ix} \\ \theta_{iy} \end{Bmatrix}_{shell} \tag{4}$$

where u_i, v_i and w_i are the nodal translational displacements in the x, y and z directions respectively and θ_{ix}, θ_{iy} are the nodal rotational displacements about the x and y axes respectively. Eq. (4) can be written as:

$$\{d^b_i\} = [T]\{d^s_i\} \tag{5}$$

Similarly, the nodal load vector on the eccentric beam, $\{P_{beam}\}$, can be transformed into equivalent nodal loads on the shell element, $\{P_{shell}\}$, by the equation:

$$\{P_{shell}\} = [T]^T \{P_{beam}\} \tag{6}$$

The member stiffness equation in terms of the stiffness matrix of the beam element, $[K^e_{beam}]$, is:

$$\{P_{beam}\} = [K^e_{beam}]\{d^b_i\} \tag{7}$$

Substituting $\{P_{beam}\}$ of the above equation into Eq. (6), it gives:

$$\{P_{shell}\} = [T]^T [K^e_{beam}]\{d^b_i\} \tag{8}$$

Combining Eqs. (5) and (8), it gives

$$\{P_{shell}\} = [T]^T [K^e_{beam}][T]\{d^s_i\} \tag{9}$$

The stiffness matrix for the eccentric beam, $[K^e_{eccent}]$, is therefore expressed as:

$$[K^e_{eccent}] = [T]^T [K^e_{beam}][T] \tag{10}$$

The stiffness matrix of the stiffener element can be transformed from the stiffener beam nodes to the corresponding shell mid-surface nodes. This stiffness matrix thus obtained is superimposed on the shell stiffness matrix at appropriate locations.

3. Computer program SHECAN

Based on the above proposed bridge modelling, a computer program SHECAN is written. SHECAN, which is an acronym derived from SHell and ECcentric beam ANalysis, includes the procedures of setting up and assembling the various element stiffness matrices and element load vectors, solving the simultaneous equations, and finally obtaining the nodal displacements and giving the output of stresses. It can also output the moment in each girder at any cross-section for the study of lateral load distribution of a bridge. At any cross section, one girder unit means one beam element plus two shell elements. Taking moment about the node level of the shell elements, the moment per each girder unit, M_t, is calculated as:

$$M_t = M_s + M_b + e \times F_b \tag{11}$$

where e is the eccentricity, F_b is the beam nodal axial force, M_s is the total nodal longitudinal moment of the two shell elements and M_b is the beam nodal longitudinal moment.

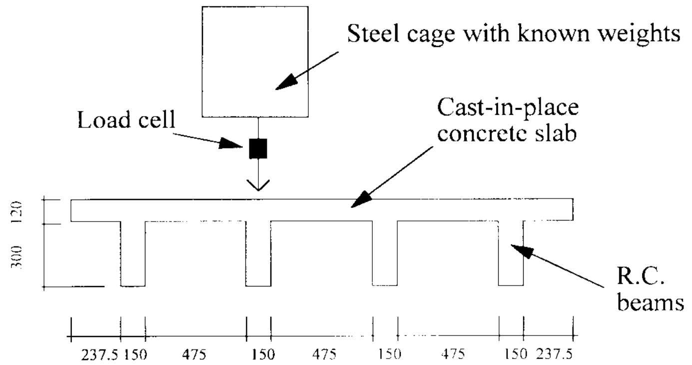
4. Laboratory study

A reinforced concrete slab-on-beam bridge model was casted for the present project. A single point load was applied at various locations of the casted bridge model in order to provide real data for the comparison of load distribution characteristics by various analytical methods.

4.1. Details of the laboratory bridge model

The laboratory bridge model consisted of a cast-in-place 120 mm thick concrete slab on four reinforced concrete beams. The width of the bridge was 2.5 m and the span was 3.0 m. The typical section of the bridge is shown in Fig. 4. The end conditions were assumed simply supported. The torsional parameter α and flexural parameter θ as defined by Bakht & Moses (1988) for the test bridge model were 0.3822 and 0.9263 respectively.

Axial compressive tests using 300 mm $\times$ 150 mm ϕ specimens were carried out in accordance with CS1 (1990) to determine the Young's modulus of the concrete material. In addition, strain gauge readings were collected for the calibration of the stiffness of the bridge model. The calibration procedures were refereed to O'Connor and Chan (1988). The static point load system mentioned below was applied at four locations along the longitudinal center line of the bridge deck. For each loading location, the load was increased from 0 to 25 kN. Mid-span moments were calculated and the plots of bending moment versus the measured mid-span strain obtained. The Young's modulus of the bridge model was calculated as the ratio of the average of the slopes of the plots to the bridge section modulus computed at the level of the strain gauges. The calibrated Young's modulus was found to be 33.7×10^6 kN/m^2.



Note: All dimensions are in mm

Fig. 4 Typical section of test bridge model and loading arrangement

4.2. Load system

As the existing loading frame in the laboratory was not large enough for the bridge deck model, and due to the limited space and resource in the laboratory, it was decided to use a steel cage with known weights as a static point load on the model. The load applied through the cage was measured by a calibrated load cell attached to the cage as shown in Fig. 4. The loading was increased from 0 kN to 25 kN. There were totally sixteen loading positions on the slab of the bridge as shown in Fig. 5.

4.3. Response measurements

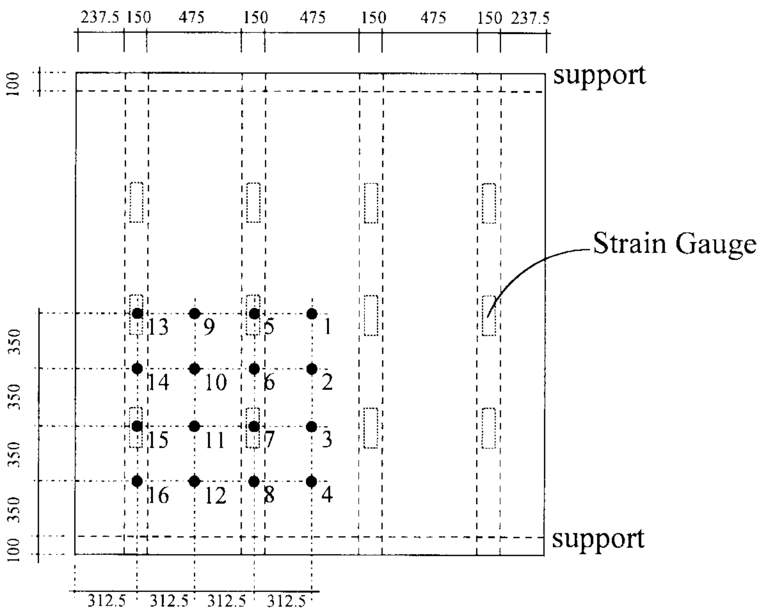
Strains were measured in the longitudinal direction at 1/4, 1/2 and 3/4 spans at the bottom face of each reinforced concrete beams. The locations of the concrete strain gauges are also shown in Fig. 5. It was assumed that the longitudinal strain of any positions between the strain gauges could be interpolated from the readings of corresponding strain gauges. Besides, the strain was assumed zero at the supports.

5. Theoretical study

In order to demonstrate the validity of the proposed modelling in lateral load distribution, various analytical techniques for slab-on-girder bridges were compared with the proposed modelling. The bridge analytical schemes adopted were grillage analogy method, semi-continuum method and finite element method.

5.1. Preliminary study

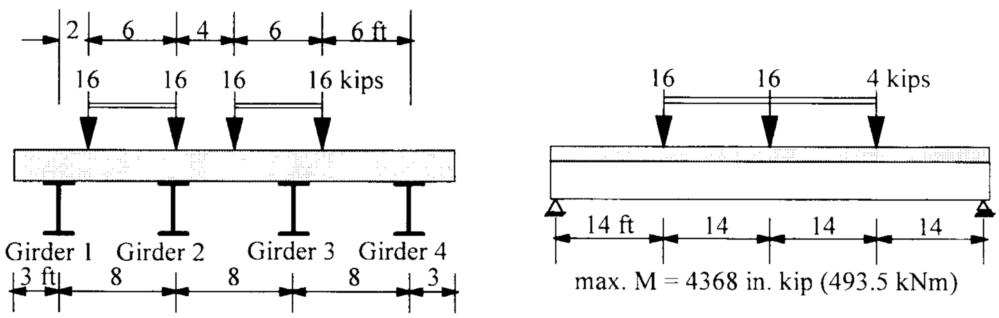
As there exist so many finite element idealizations for slab-on-girder bridges, the present study does not intend to compare the proposed model with all the other finite element models for this type of bridge. Mounir *et al.* (1997) used four finite element models to obtain the distribution factors of a slab-on-girder bridge and compared the computed values with the value specified in



Note:- ● Load Positions

All dimensions are in mm

Fig. 5 Plan view of test bridge model showing the positions of load and strain gauges



(a) Bridge Cross Section and Transverse Position of HS20 Trucks

(b) Longitudinal Position of HS20 Trucks

(Note: 1ft = 0.3048m & 1kip = 4.448kN)

Fig. 6 Bridge cross section and positions of HS20 trucks (after Mounir *et al.* 1997)

AASHTO (1996). The four models (Cases a to d) were similar to the finite element idealizations shown in Fig. 1 (a to d) respectively. The load distribution factor, DF, is a key element in designing bridges. AASHTO (1996) uses DF (girder spacing in ft/5.5) to obtain the moment in an individual element. The selected bridge was 30ft (9.15 m) wide and consisted of a 7.5 in (190 mm) reinforced concrete slab supported by four W 30×160 steel girders spaced at 8 ft (2.45 m). Two AASHTO design trucks (HS20) were positioned simultaneously on the two-lane bridge to produce the maximum moment in girder 2. Fig. 6 shows the bridge cross section and the positions of HS20 trucks. The distribution factors and maximum girder moments for the cases together with that

Table 1 Results from SHECAN and other FEM

	max. Girder Moment (kNm)	DF
Case a	609.6	1.24
Case b	609.6	1.24
Case c	561.3	1.14
Case d	588.2	1.19
SHECAN	605.5	1.23

obtained by SHECAN are shown in Table 1. The load distribution factors were calculated using the maximum moments from the finite-element models divided by maximum moment found in a simply supported beam to a single line of AASHTO design truck as shown in Fig. 6b. It can be seen that SHECAN could produce similar results as the other FEM methods and all the computed distribution factors are less than the AASHTO DF, 1.45, for this bridge.

It can be seen from Table 1 that the results from SHECAN were bounded by results from other models and close to the results from Case a (Hays *et al*'s Idealization 1986) and Case b (Memari & West's Idealization 1990). The idedalizations for Case a and Case b are similar. However Case a assumes the centroid of each girder coincided with the centroid of concrete slab as shown in Fig. 1a and this is different from the actual situation. The finite element idealization by Memari & West (1990) is therefore selected for comparison with other bridge analysis methods in the following study. For the other two finite element idealizations, Case c (Bishara's 1993) and Case d (Tarhini & Frederick's 1992), the moment per each girder is calculated by using the stress distribution results and the accuracy of the results would be comparatively lower. They also need larger quantities of input data and much time will be spent to analyze the results. Hence these two methods are not recommended as analytical tools for the studies of lateral load distributions.

5.2. Virtual bridge models

Five simply supported slab-on-girder virtual bridge models were used for the present study. The material of the bridge models were assumed to be concrete and all bridges consisted of five equal spacing girders. The overall width of each bridge was 12.5 m. The span length and the slab thickness were varied respectively from 15 m to 25 m by 5 m increments and from 150 mm to 250 mm by 50 mm increments. The virtual bridge models, geometry and cross-sectional properties of the girders are shown in Fig. 7.

The practical ranges of the bridge characterizing parameters α (torsional) and θ (flexural) stated by Bakht and Moses (1988) for slab-on-girder bridges are 0.05-0.2 and 0.5-2.0 respectively. The size and dimensions of virtual bridge models were selected in such a way that the corresponding characterizing parameters α and θ were within their practical ranges. Table 2 summarizes the values of α and θ for the five analytical bridge models studied in this section.

5.3. Loading cases

The virtual bridge models discussed above were analyzed individually for four separate loading cases of different loading positions. The four loading positions were considered so as to give both symmetric and asymmetric conditions with respect to the longitudinal and transverse centre lines of the bridge. In each case, a single point load of 100 kN was placed as shown in Fig. 7 and the

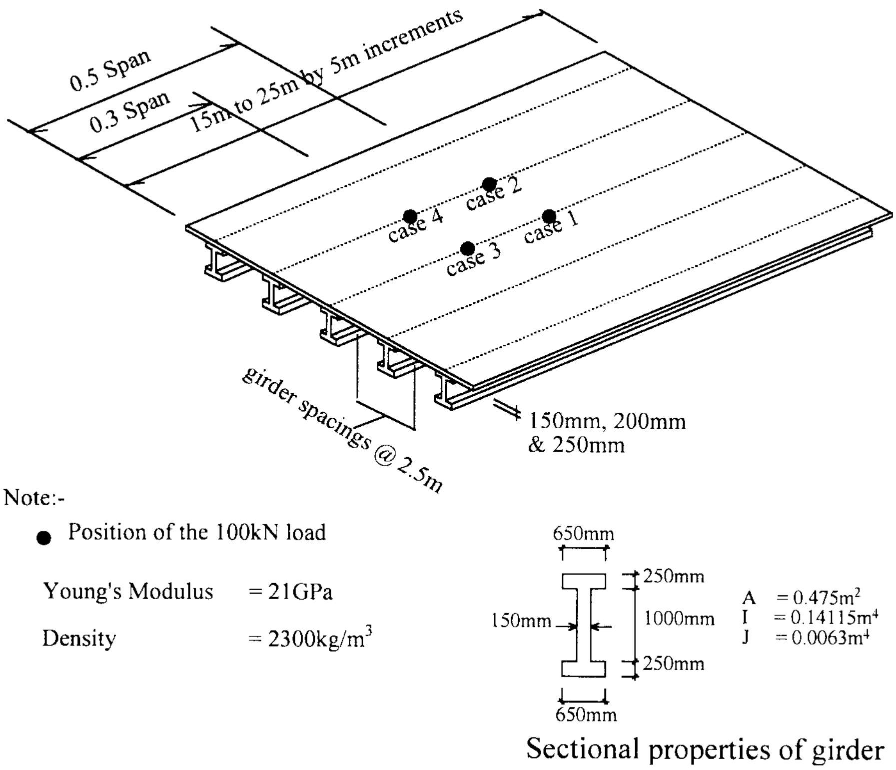


Fig. 7 Virtual bridge model and loading cases

Table 2 Torsional (α) and filexural (θ) parameters of virtual bridge models

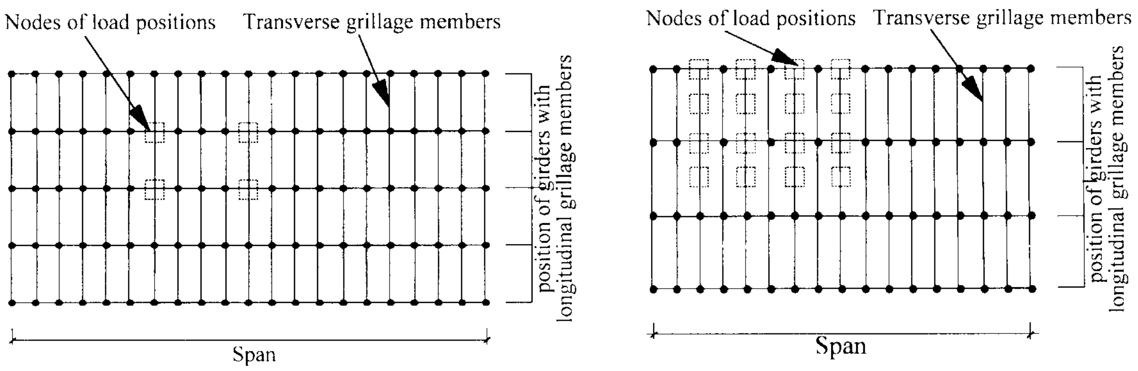
Span	Slab Thickness	α	θ
15 m	200 mm	0.1375	1.5496
20 m	200 mm	0.1375	1.1622
25 m	200 mm	0.1375	0.9297
20 m	150 mm	0.1559	1.4015
20 m	250 mm	0.1409	1.0070

cases are separately for symmetrical and unsymmetrical loading conditions. The study for these four positions was sufficient to ensure the generality of the results.

6. Modelling schemes for bridge models

6.1. By grillage analogy method

Commercial computer program microSTRAN-3D™ (Engineering Systems 1987) was used to generate the grillage beams system. Fig. 8 shows the idealized grillage layout for the virtual bridge models and the laboratory bridge model.



(a) Virtual Bridge Model (b) Laboratory Bridge Model

Fig. 8 Grillage discretization scheme (by MicroSTRAN 3D)

6.2. By Semi-continuum method

Computer program SECAN (Mufti *et al.* 1997) which is based on the semi-continuum method was used to analyze the bridge models. Unlike other methods for bridge deck analysis, no special discretization scheme is needed for a slab-on-girder bridge. It only requires the dimensions and the properties of the bridge models for the formatted input file.

6.3. By Memari & West's modelling

Commercial finite element package SAP90™ (Wilson and Habibullah 1992) was used to generate Memari & West's modelling, and the discretization scheme used to analyze the virtual bridge models is shown in Fig. 9. The element nodal moments of the beam elements, shell elements and the element nodal forces of the shell elements were computed. The mid-span moment per girder was calculated manually by taking moment about the node level of beam elements.

6.4. By shell and eccentric beam modelling

Program SHECAN was used to generate Shell and Eccentric Beam Modelling. For the case of virtual bridge models, the discretization scheme used to analyze the virtual bridge models is shown in Fig. 10. It can be seen that the nodes of the eccentric beam elements were directly attached to the nodes of the shell elements because of the 'master and slave' relationship. Therefore there was no need to generate the beam elements and rigid link elements at the input phase as other finite element methods.

7. Discussion of results

7.1. Laboratory bridge model

As stated before, in the laboratory work, a 25 kN point load was applied individually at loading

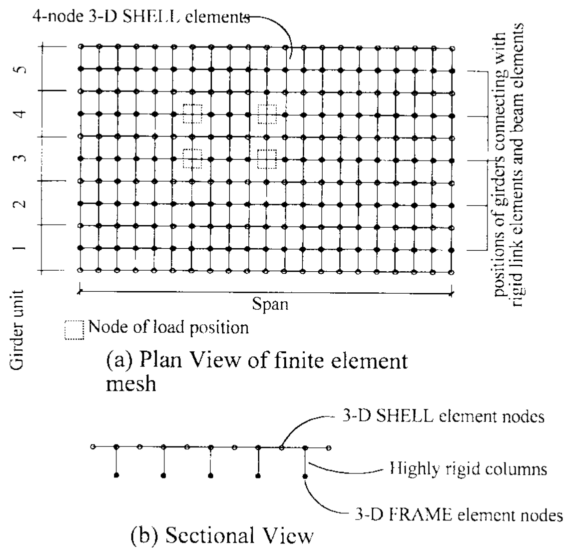


Fig. 9 Memari & West's discretization scheme for virtual bridge model (by SAP90)

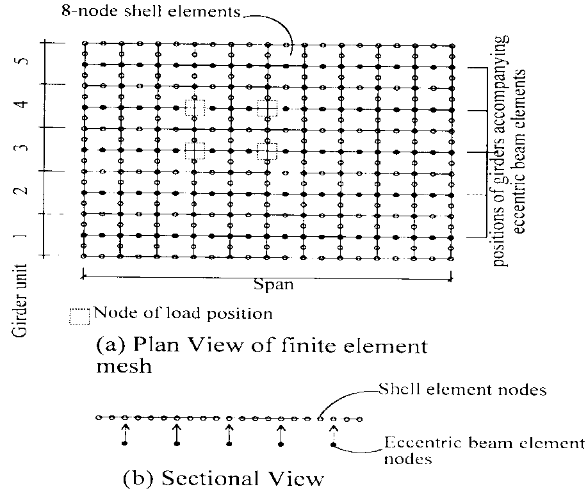


Fig. 10 Shell and eccentric beam discretization scheme for virtual bridge model (by SHE-CAN)

positions 1 to 16 of the reinforced concrete bridge model to explore the nature of lateral load distribution. The readings at various strain gauge locations were recorded for each case. Chan (1996) gives a detailed report to the results of each loading case with a comparison with the results from various analytical schemes. Fig. 11 shows four typical plots of the longitudinal bending moment at the transverse section of corresponding loading position versus each beam girder number for loading positions 3, 7, 9, 13. The plots show results from the laboratory test together with those obtained from the four analytical schemes. For clarity, the values of the ratio (BM Ratio) of the bending moments obtained from the four schemes to the experimental results are shown in Table 3.

It is noted that some of the absolute values of the BM ratio are much larger than 1. This is because the corresponding measured moments are very small. As a reference, the values of the corresponding moments measured from the laboratory test are also given in Table 4.

In the study, it showed that the results from the eccentric beam model were generally closest to the laboratory results, and the second was those from the Memari & West's model followed by the results using the semi-continuum method and the grillage analogy. The semi-continuum method and the grillage analogy gave almost the same results.

For almost all load cases considered, the semi-continuum and the grillage analogy methods always gave larger bending moments than the test results indicating these two methods overestimated the results. It should be noted that the girder carrying most of the distributed bending moment was being considered in design. Therefore, the grillage analogy method and the semi-continuum method are conservative for design propose. From another point of view, the transverse medium of the laboratory bridge model idealized by these two methods was generally less stiff than the actual case, i.e., these two methods distributed less loading to adjacent girders than the actual case.

When compared to the laboratory results, the Memari & West's model gave less bending moments than the test results for most of the cases considered. In other words, Memari & West's

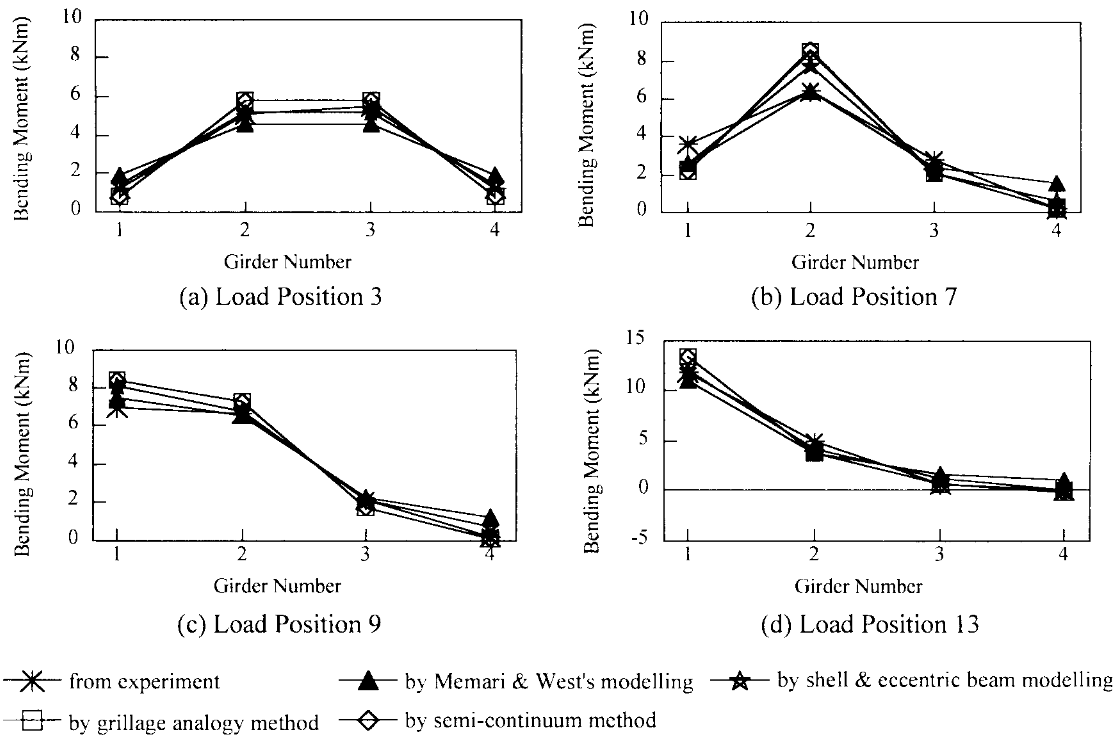


Fig. 11 Selected results of test bridge model

Table 3 BM ratio for various schemes

Girder No.	Position 3				Position 7			
	1	2	3	4	1	2	3	4
M & W's EFM	1.49	0.89	0.84	1.49	0.73	1.00	0.85	7.41
SHECAN	1.08	1.01	0.95	1.09	0.70	1.20	0.76	3.09
Grillage	0.63	1.12	1.06	0.63	0.61	1.32	0.76	1.27
SECAN	0.63	1.12	1.06	0.63	0.60	1.32	0.76	1.20

(a) Load at positions 3 & 7

Girder No.	Position 9				Position 13			
	1	2	3	4	1	2	3	4
M & W's EFM	1.07	0.99	1.07	9.36	0.94	0.77	2.21	103.
SHECAN	1.17	1.01	0.99	4.74	1.01	0.84	1.77	13.8
Grillage	1.22	1.10	0.82	0.40	1.14	0.75	0.82	-20.9
SECAN	1.22	1.10	0.82	0.29	1.14	0.75	0.80	-22.2

(b) Load at positions 9 & 13

model gives underestimated results. It may therefore be unsafe for design propose in using this finite element model. The transverse medium of the test bridge model idealized by this method was generally stiffer than the actual case and distributed less loading to adjacent girders.

Table 4 Longitudinal BM (in kNm) at the transverse section of the loading position

Girder No.	1	2	3	4
Position 3	1.28	5.12	5.45	1.28
Position 7	3.64	6.46	2.79	0.21
Position 9	6.91	6.65	2.10	0.13
Position 13	11.81	4.94	0.70	0.01

The bending moments calculated by SHECAN did not always give underestimated results as those given by the Memari & West's model. For almost all the cases studied, the results of SHECAN were in between the results of the Memari & West's finite element model and the grillage/semi-continuum methods. This implies that the stiffness of transverse medium of the test bridge model idealized by SHECAN was between those of the Memari & West's finite element model and the grillage/semi-continuum methods.

7.2. The virtual bridge models

The results of SHECAN for the virtual bridge models were compared to the results from other analytical schemes. The plots of the longitudinal mid-span bending moment carried by each girder unit versus the girder number for the four loading cases are shown in Figs. 12 to 15 respectively. In general, the patterns of lateral distribution analyzed by various methods are approximately the same.

The results of the semi-continuum method agreed very well to the grillage analogy method for all cases. This is expected as a relatively fine grillage mesh had been used in the idealization of the bridge models.

The transverse media for the models of SHECAN and Memari & West are shell elements interconnected at their nodal points. Likewise, the transverse media for the semi-continuum model and the grillage model are infinite and finite number of transverse beams respectively. The results showed that the transverse media of the shell elements of the finite element methods, both the

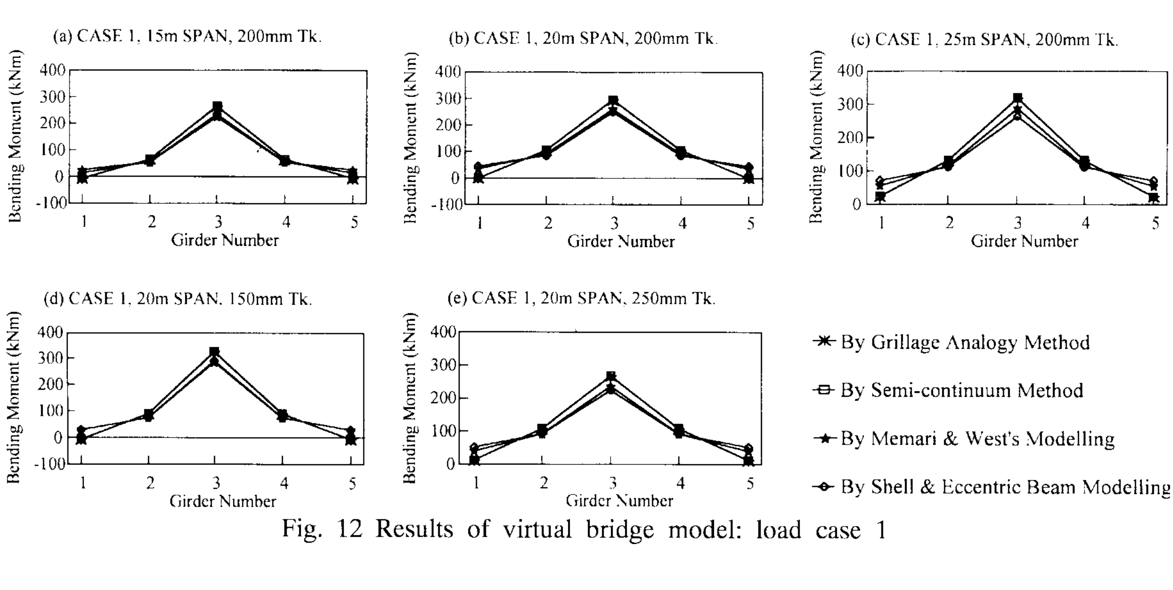


Fig. 12 Results of virtual bridge model: load case 1

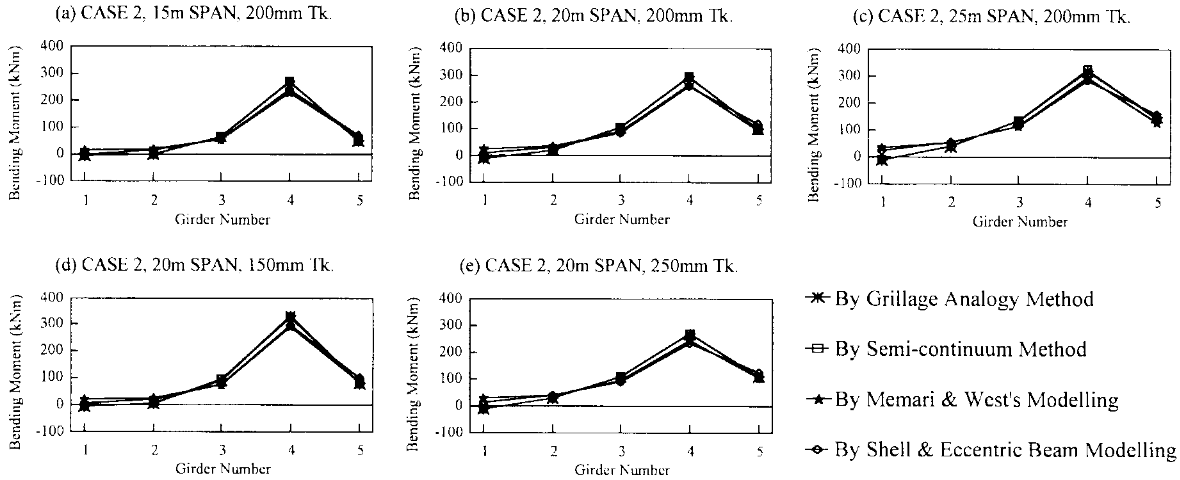


Fig. 13 Results of virtual bridge model: load case 2

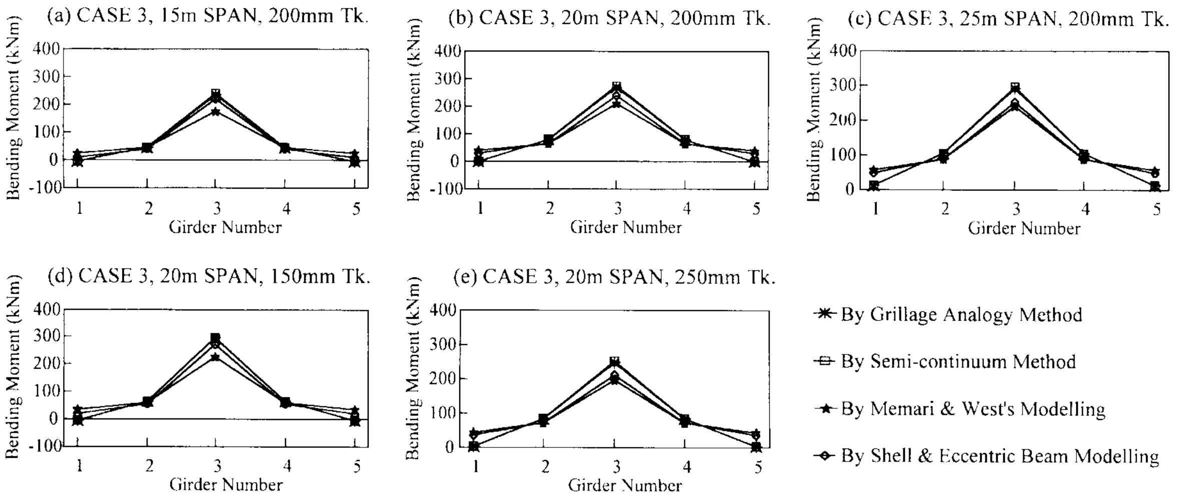


Fig. 14 Results of virtual bridge model: load case 3

shell and eccentric beam and Memari & West's models, were stiffer than that of the transverse beams of the semi-continuum and the grillage analogy methods. In other words, finite element models distributed the loading more evenly among the girders.

For the cases of loading at mid-span (load cases 1 & 2), the results of SHECAN were very close to the results from the Memari & West's model. For the case of loading at three-tenth span (load case 3 & 4), the results of SHECAN were in between the results from the Memari & West's model and the results from the grillage analogy/semi-continuum methods.

8. General comments on bridge analysis software

Because of its simplicity in use and comprehensibility, the grillage analogy method may be at present the most popular computer aided method for the bridge deck analysis in a bridge design.

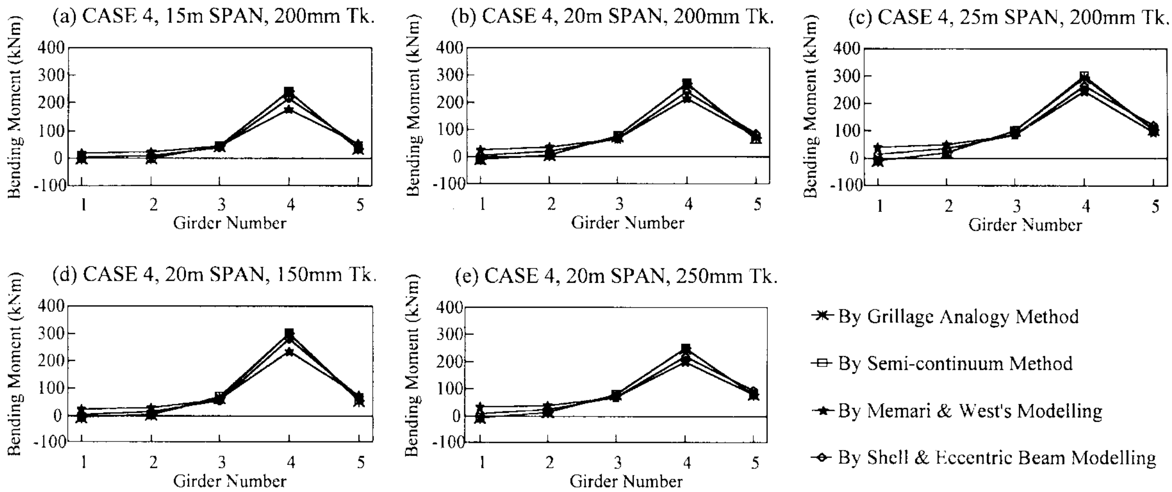


Fig. 15 Results of virtual bridge model: load case 4

In addition, it is inexpensive to implement and can be operated on most personal computers. Nearly all the commercial structural analysis programs have a very simple input format for the generation of grillage analogy model. The most difficult part may be the calculation of equivalent sectional properties of various grillage members. For loading data, these commercial structural analysis packages usually provide various loading types including concentrated or distributed loads applied to a member. However, some difficulty may arise when a load is not directly acted on a grillage member, e.g. in the middle of a grid.

The program SECAN, using the semi-continuum method, requires a very simple input file. It only needs the girder spacings, the modulus of elasticity, the shear modulus, the moment of inertia and the torsional inertia of each girder as the girder input data and the thickness, the modulus of elasticity, the shear modulus and the equivalent shear area of the slab as the slab input data. The data input format of SECAN are shown in Mufti *et al.* (1997). The program SHECAN, which has an automatic mesh generation routine for slab-on-girder bridges, requires an input file as simple as that of SECAN. SHECAN can also output the moment for each girder unit at any cross-section for the study of the lateral load distribution of a bridge. The total moment calculated at mid-span gave about 1% difference to that of the simple beam model.

Compared to other methods, Memari & West's model, using a commercial finite element packages, is the most time consuming and tedious method in the preparation of input file and interpretation of output files. In the input phase, it has to generate all the shell elements, rigid links elements and beam elements. In the output phase, the user has to calculate the longitudinal moment of each girder unit manually. Therefore, this method is seldom adopted by an engineer in the analysis for a slab-on-girder bridge.

Compared with the semi-continuum method or the grillage analogy method, the finite element method using shell and eccentric beam modelling is closer to the real case of a slab-on-girder bride. When we comparing with other finite element methods, the shell and eccentric beam modelling has its simplicity of leaving the intermediate elements, e.g., rigid links, but retaining the material variation and sectional properties of the slab and the girders. This 'simplicity' leads to much time saving in the preparation of input file and the analysis of output results. In addition, by the program SHECAN, the moment per each girder unit need not be calculated manually as in other finite element packages.

In the discretization of Memari & West's model by a commercial finite element packages, e.g., SAP90™, there exists some limitation in the application of point load. Only nodal loads of shell elements are provided. Therefore, the user has to consider how to convert an applied load to equivalent nodal loads. The program SHECAN does not have this limitation, as the load can be applied anywhere within a shell element.

9. Conclusions

(1) The finite element model proposed in this study, shell and eccentric beam modelling, can be used efficiently and accurately in the analysis of lateral load distribution of slab-on-girder bridges. The performance of the proposed method is well supported when compared with other analytical methods and experimental results.

(2) The set of results of SHECAN using the shell and eccentric beam modelling is closest to the experimental results, and the second is the set of results from the Memari & West's idealization followed by the results of the semi-continuum method and the grillage analogy method.

(3) The transverse media of the shell elements of the finite element methods are stiffer than that of the semi-continuum method and grillage analogy method.

(4) The semi-continuum method and grillage analogy method give almost the same results.

(5) The shell and eccentric beam modelling has its simplicity of omitting intermediate elements when compared to other finite element methods.

(6) The computer programs SECAN using semi-continuum method and SHECAN using the shell and eccentric beam finite element method are the easiest to use. The input file for SHECAN is as simple as that for the SECAN. In addition, SHECAN can compute and output the distributed bending moment per each girder unit. Memari & West's model by any commercial finite element package is the most tedious method in the preparation of input file and interpretation of output files.

References

American Association of State Highway and Transportation Officials (AASHTO) (1996), *Standard Specifications for Highway Bridges*, 16th Ed., Washington, D.C.

Bakht, B. and Moses, F. (1988), "Lateral distribution factors for highway bridges", *Journal of Structural Engineering*, **114**(8), 1785-1803.

Bathe, K.J. (1996), *Finite Element Procedures*, Prentice-Hall, Inc., Englewood Cliffs.

Bishara, A.G. (1993), "Wheel load distribution on simply supported skew I-Beam composite bridges", *Journal of Structural Engineering*, **119**(2), 399-419.

Brockenbrough, R.L. (1986), "Distribution factors for curved I-girder bridges", *Journal of Structural Engineering*, **112**(10), 2200-2215.

Chan, H.F. (1996), "Bridge-vehicle model for highway bridge impact studies", MPhil Thesis, Dept. of Civil And Structural Engineering, The H.K. Polytechnic University, Hong Kong.

Chan, T.H.T. and Chan, J.H.F. (1994), "A finite element model on load distribution", *Development in Short and Medium Span Bridge Engineering '94*, edited by Aftab A. Mufti, B. Bakht, & L.G. Jaeger, CSCE.

CSI (1990), "Construction Standard: Testing Concrete Vol. 1", Hong Kong Government.

Engineering Systems Pty Limited. (1987), *microSTRAN-3D User's Manual*, Watford, England.

Hays, C.O., Sessions, L.M. and Berry, A.J. (1986), "Further studies on lateral load distribution using EFA", *Transportation Research Record 1072*, Transportation Research Board, Washington, D.C.

Hinton, E. and Owen, R.J. (1977), *Finite Element Programming*, Academic Press, Inc.

Imbsen, R.A. and Nutt, R.V. (1978), "Load distribution study on highway bridges using STRUDL FEA", *Proceedings of Conference on Computing in Civil Engineering, ASCE*, New York, 639-654.

Memari, A.M. and West, H.H. (1990), "Computation of bridge design forces from influence surfaces", *Computers & Structures*, **38**, 547-556.

Miller, R.E. (1980), "Reduction of the error in eccentric beam modelling", *International Journal for Numerical Methods in Engineering*, **15**, 575-582.

Mounir, E.M., Tarhini, K.M., Frederick, G.R. and Tayar, C. (1997), "Finite-element analysis of steel girder highway bridges", *Journal of Bridge Engineering*, **2**(3), 83-87.

Mufti, A.A., Bakht, B. and Jaeger, L.G. (1997), *Bridge Superstructures, New Developments*, Islamabad, National Book Foundation.

O'Connor, C. and Chan, T.H.T. (1988), "Wheel loads from bridge strains: laboratory studies", *Journal of Structural Engineering*, **114**(8), 1724-1740.

Tarhini, K.M. and Frederick, G.R. (1992), "Wheel load distribution in I-girder highway bridges", *Journal of Structural Engineering*, **118**(5), 1285-1294.

West, R. (1973) "The use of a grillage analogy for the analysis of slab and pseudo-slab bridge decks", Research Report 21, Cement and Concrete Association, London.

Wilson, E.L. and Habibullah, A. (1992), *Structural Analysis Users Manual and SAP90 Computer Programs for the Finite Element Analysis of Structures*, Computers and Structures Incorporated.

Zienkiewiez, O.C. (1989), *Finite Element Method*, McGraw-Hill Book Company (UK) Limited.