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Application of Kelvin's approach for material structure of CNT: Polynomial volume fraction law

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Abstract. In this piece of work, carbon nanotubes motion equations are framed by Kelvin's method. Employment of the Kelvin's method procedure gives birth to the tube frequency equation. It is also exhibited that the effect of frequencies is investigated by varying the different index of polynomial function. By using volume fraction for power law index, the fundamental natural frequency spectra for two forms of single-walled carbon nanotubes are calculated. The influence of frequencies against length-to-diameter ratios with varying power law index are investigated in detail for these tubes. Throughout the computation, it is observed that the frequency behavior for the boundary conditions follow as; clamped-clamped, simply supported-simply supported and these frequency curves are higher than that of clamped-free curves. Computer software MATLAB is utilized for the frequencies of single-walled carbon nanotubes.

Keywords: material structure; Kelvin's approach; CNT; fraction law

1. Introduction

Since the discovery of carbon nanotubes (CNTs), has become very important and interest of research due to considerable observation and research publications every year. CNTs have a variety of uses and applications in potential looking fields, some of which are charge detectors, electronics, communication, composite materials, biotechnology, environment, energy storage, chemical, and optical (Iijima, 1991). Therefore, in order to effectively use of CNTs in each of these fields, it is important that their vibration characteristics are examined. In past decade, vibrations of carbon nanotubes have been studied extensively in various more realistic model such as beam and ring (Vodenitcharova and Zhang, 2003, Hsu *et al.* 2008). It is very difficult to observe and identify for studying the vibrational characteristics but a new hybrid technique, which links atomistic and continuum approaches, has also gained ground. Recently, a large number of researchers devoted their time and efforts to scrutinize mechanical and vibrational properties of nanoscale tubes. The nonlinear forced vibration of carbon nanotubes has seldom been observed. However, this issue is very crucial due to the widespread application of the forced nonlinear vibration carbon nanotubes in many practical instruments.

The nonlocal elasticity introduced by Eringen (2002) becomes a turning point as small scale effect was inculcated in to fundamental equations as simply material parameter. Donnell (1996) and Flügge (1962) have been two substantial shell theories practiced extensively in study of static and dynamic characteristics of CNTs. Flügge shell theory takes promising place to generate remarkably accurate developments to examine the CNTs. The existence of long range interactions in materials is the basic reason of application of nonlocal theory. The first ever work presented on use of nonlocal elasticity was by Peddieson *et al.* (2003). Prominent computational competence and accuracy makes nonlocal models an attractive choice for further advancements in field. Wang *et al.* (2006) introduced new modeling for vibration of CNTs and to find the critical buckling strain and tube thickness. Natuski *et al.* (2006) carried out the vibration analysis of nested CNTs in elastic matrix. Flügge shell theory again had been engaged to establish administrative shell equations while proposed method was wave propagation. Ru *et al.* (2004) studied the vibration of single walled carbon nanotubes (SWCNTs) using the classical Euler-Bernoulli beams. It is concluded that the results are gained from this theory are similar to the effects of van der Waals forces. Natuski *et al.* (2007) investigated single and double-walled CNTs filled with fluids by adopting wave propagation approach. Flügge shell theory was proposed to form governing equations of motion for CNTs. Lee and Chang (2008) analyzed the vibration mode shape and frequency of fluid-filled SWCNTs. It is found that mode shape and frequency are influenced significantly by the nonlocal parameters. Hussain and Naeem (2017) examined the frequencies of armchair tubes using Flügge's shell model. The effect of length and thickness-to-radius ratios against fundamental natural frequency with different indices of armchair tube was investigated. Ke *et al.* (2009) investigated free nonlinear vibrations of double-walled CNT and applied differential quadrature technique to derive frequency equations. On the other side, for length scale coefficient and soft elastic medium with embedded carbon nanotube, the nonlocal frequencies are comparatively lower. It is also found that

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the frequencies of the nonlocal model at different stages of temperature are higher than the nonlocal with same temperature. Eringen nonlocal theory and Von-Karman geometry were fully studied by Yang *et al.* (2010). Hussain and Naeem (2018) used Donnell's shell model to calculate the dimensionless frequencies for two types of single-walled carbon nanotubes. The frequency influence was observed with different parameters. Selmi and Bisharat, (2018) studied the Aluminum alloy (Al-alloy) reinforced with single walled carbon nanotubes (SWNT), which represents an important industrial application. Different beam theories (BT) was applied to investigate functionally graded (FG) beams made of Al-alloy reinforced with randomly oriented, straight and long SWNT. The Rayleigh-Ritz method is used to estimate the beam frequencies. Bisen *et al*. (2018) investigated the natural fiber (Luffa cylindrica fiber) reinforced epoxy composite and their structural responses (frequency and deflection) have been computed experimentally and numerically first time using the corresponding experimental elastic properties. Selmi (2019) investigated the effectiveness of single walled carbon nanotubes in improving the dynamic behavior of cracked Aluminium alloy, Al-alloy, beams by using a method based on changes in modal strain energy. Mechanical properties of composite materials are estimated by the Eshelby-Mori-Tanaka method. Dihaj *et al*. (2018) studied the transverse free vibration of chiral double-walled carbon nanotube (DWCNTs) embedded in elastic medium by the non-local elasticity theory and Euler Bernoulli beam model. The governing equations are derived and the solutions of frequency are obtained. Fatahi-Vajari *et al*. (2019) studied the vibration of single-walled carbon nanotubes based on Galerkin's and homotopy method. This work analyses the nonlinear coupled axial–torsional vibration of single-walled carbon nanotubes (SWCNTs) based on numerical methods. Two-second order partial differential equations that govern the nonlinear coupled axial–torsional vibration for such nanotube was derived. Kolahchi and Cheraghbak, (2017) studied with the nonlocal dynamic buckling analysis of embedded microplates reinforced by single-walled carbon nanotubes (SWCNTs). The material properties of structure are assumed viscoelastic based on Kelvin–Voigt model. Agglomeration effects are considered based on Mori–Tanaka approach. The elastic medium is simulated by orthotropic visco-Pasternak medium. The use of wave propagation approach (WPA) is important for the study of nanostructures to develop a new formulism with different theories. In this approach, eigenvalue form is developed with the help of axial modal function in matrix representation. With the help of computer software MATLAB, frequencies of SWCNTs are extracted. The formulation of WPA is given by Zhang *et al.* (2001), a brief yet simple explanation first time. Kiani (2010) studied the single-walled carbon nanotubes (SWCNTs) as a promising delivery nanodevices for a diverse range of applications, however, little is known about their dynamical interactions with moving nanoscale particles. Kolahchi (2017) investigated the bending, buckling and buckling of embedded nano-sandwich plates based on refined zigzag theory (RZT), sinusoidal shear deformation theory (SSDT), first order shear deformation theory (FSDT) and classical plate theory (CPT). In order to present a realistic model, the material properties of system are assumed viscoelastic using Kelvin–Voigt model. Mohammadimehr and Alimirzaei (2017) investigated the buckling, and free vibration analysis of tapered functionally graded carbon nanotube reinforced composite (FG-CNTRC) micro Reddy beam under longitudinal magnetic field using finite element method (FEM). It is noted that the material properties of matrix is considered as Poly methyl methacrylate (PMMA). Bilouei *et al.* (2016) used as concrete the most usable material in construction industry it's been required to improve its quality. Nowadays, nanotechnology offers the possibility of great advances in construction. For the first time, the nonlinear buckling of straight concrete columns armed with single- walled carbon nanotubes (SWCNTs) resting on foundation is investigated. The column is modelled with Euler-Bernoulli beam theory. Sánchez-Portal (1999) conducted a research on ab initio calculations is presented on the structural, elastic, and vibrational properties of single-wall carbon nanotubes with different radii and chiralities. These properties are obtained using an implementation of pseudopotential-density-functional theory, which allows calculations on systems with a large number of atoms per cell. Bilouei *et al.* (2016) and Zamanian *et al.* (2017) studied the buckling behavior of concrete columns with nanofiber reinforced polymer and SiO_2 nano-particles. By using the strain-displacements, Hamilton's principles and Mori- Tanka approach, the governing equation was derived. Numerical results were presented with the variation of elastic foundations. The results presented here may provide a useful design for nanostructures. In another study the viscoelastic effects of the medium were also studied using Kelvin model for the medium surrounding microtubules (MTs) but for the MTs the same classical orthotropic elastic shell model was used (Safeer *et al.* 2019). Many researchers directly used the classical theory for the structure of CNTs (He *et al.* 2005, Hu *et al.* 2007, Gibson *et al.* 2007, Ghavanloo *et al.* 2010, Yoon *et al.* 2002). Kolahchi *et al.* (2016a) concerned with the dynamic stability response of an embedded piezoelectric nanoplate made of polyvinylidene fluoride (PVDF). In order to present a realistic model, the material properties of nanoplate are assumed viscoelastic using Kelvin–Voigt model. The visco-nanoplate is surrounded by viscoelastic medium which is simulated by orthotropic visco-Pasternak foundation. The PVDF visco-nanoplate is subjected to an applied voltage in the thickness direction. Ansari and Ajori (2014) conducted the synthesizing inorganic nanostructures such as boron nitride nanotubes (BNNTs). The torsional vibration behavior of boron-nitride nanotubes (BNNTs) is explored on the basis of molecular dynamics (MD) simulation. Arani and Kolahchi (2016) used a concrete material in construction industry it's been required to improve its quality. Nowadays, nanotechnology offers the possibility of great advances in construction. For the first time, the nonlinear buckling of straight concrete columns armed with single-walled carbon nanotubes (SWCNTs) resting on foundation is investigated in the present study. The column is modelled with EulerBernoulli and Timoshenko beam theories. The characteristics of the equivalent composite being determined using mixture rule. The foundation around the column is simulated with spring and shear layer. Batou *et al.* (2019) studied the wave propagations in sigmoid functionally graded (S-FG) plates using new Higher Shear Deformation Theory (HSDT) based on two-dimensional (2D) elasticity theory. The current higher order theory has only four unknowns, which mean that few numbers of unknowns, compared with first shear deformations and others higher shear deformations theories and without needing shear corrector. Moreover, Benguidiab *et al.* (2014) explored the features of zigzag double-walled CNT. A comprehensive research presented by Salvatore Brischetto (2015) to analyze the vibration characteristic of double-walled CNT by considering shell continuum model. The findings of article were evolved around effects of van der Waals interaction in terms of frequency ratio. Zamanian *et al.* (2017) considered the use of nanotechnology materials and applications in the construction industry. However, the nonlinear buckling of an embedded straight concrete columns reinforced with silicon dioxide (SiO2) nanoparticles is investigated in the present study. The column is simulated mathematically with Euler-Bernoulli and Timoshenko beam models. Agglomeration effects and the characteristics of the equivalent composite are determined using Mori-Tanaka approach. The foundation around the column is simulated with spring and shear layer. Arani *et al.* (2016) used the nonlinear buckling of SWCNTs and the mixture rule was employed for buckling analysis of embedded CNTs with Euler and Timoshenko beam model. The influence of geometrical parameter and elastic foundation with different boundary conditions was investigated. Faleh *et al.* (2020) presented the pulse load effects on forced transient vibrations of porous crystalline shells. A crystalline material contains many voids inside it and also there are nano-size grains which define the material character. Ahmed *et al.* (2019) concerned with post-buckling investigation of nano-scaled beams constructed from porous functionally graded (FG) materials taking into account geometrical imperfection shape. Hence, two types of nanobeams which are perfect and imperfect have been studied. Kolahchi *et al.* (2017) studied the dynamic buckling of sandwich nano plate (SNP) subjected to harmonic compressive load based on nonlocal elasticity theory. The material properties of each layer of SNP are supposed to be viscoelastic based on Kelvin-Voigt model. In order to mathematical modeling of SNP, a novel formulation, refined Zigzag theory (RZT) is developed. Furthermore, the surrounding elastic medium is simulated by visco-orthotropic Pasternak foundation model in which damping, normal and transverse shear loads are taken into account. Sharma *et al.* (2019) studied the functionally graded material using sigmoid law distribution under hygrothermal effect. The Eigen frequencies are investigated in detail. Frequency spectra for aspect ratios have been depicted according to various edge conditions. Motezaker and Eyvazian (2020) deals with the buckling and optimization of a nanocomposite beam. The agglomeration of nanoparticles was assumed by Mori-Tanaka model. The harmony search optimization algorithm is adaptively improved using two adjusted processes based on dynamic parameters. The governing equations were derived by Timoshenko beam model by energy method. The optimum conditions of the nanocomposite beam- based proposed AIHS are compared with several existing harmony search algorithms. Chemi *et al*. (2018) determined the nonlocal critical buckling loads of chiral double-walled carbon nanotubes embedded in an elastic medium, the nonlocal Timoshenko beam theory. The solution for the nonlocal critical buckling loads is obtained using governing equations of the nonlocal theory. Kolahchi and Bidgoli (2016) presented a model for dynamic instability of embedded single-walled carbon nanotubes (SWCNTs). SWCNTs are modeled by the sinusoidal shear deformation beam theory (SSDBT). The modified couple stress theory (MCST) is considered in order to capture the size effects. The surrounding elastic medium is described by a visco-Pasternak foundation model, which accounts for normal, transverse shear, and damping loads. The motion equations are derived based on Hamilton's principle. Mehar *et al*. (2017a, b) studied the frequency response of FG CNT and reinforced CNT using the simple deformation theory, finite element modeling, Mori-Tanaka scheme. They investigated a new frequency phenomenon with the combination of Lagrange strain, Green-Lagrange, for double curved and curved panel of FG and reinforced FG CNT. The characteristics of sandwich and grades CNT was found with labeling the temperature environ. The thermoelastic frequency of single shallow panel was determined using Mori-Tanake formulation. The researches of these authors have opened new frequency spectra for other material researchers. Madani *et al*. (2016) presented vibration analysis of embedded functionally graded (FG)-carbon nanotubes (CNT)-reinforced piezoelectric cylindrical shell subjected to uniform and non-uniform temperature distributions. The structure is subjected to an applied voltage in thickness direction which operates in control of vibration behavior of system. Salah *et al*. (2019) examined a simple four-variable integral plate theory to investigate thethermal buckling properties of functionally graded material (FGM) sandwich plates. The proposed kinematics considers integral terms which include the effect of transverse shear deformations. Kolahchi *et al*. (2016b) investigated the nonlinear dynamic stability analysis of embedded temperature-dependent viscoelastic plates reinforced by single-walled carbon nanotubes (SWCNTs). The equivalent material properties of nanocomposite are estimated based on the rule of mixture. For the carbon-nanotube reinforced composite (CNTRC) visco-plate, both cases of uniform distribution (UD) and functionally graded (FG) distribution patterns of SWCNT reinforcements are considered. The surrounding elastic medium is modeled by orthotropic temperature-dependent elastomeric medium. The viscoelastic properties of plate are assumed based on Kelvin–Voigt theory. Narwariya *et al*. (2018) presented the vibration and harmonic analysis of orthotropic laminated composite plate. The response of plate is determined using Finite Element Method. The eight noded shell 281 elements are used to analyze the orthotropic plates and results are obtained so that the right choice can be made in applications such as aircrafts, rockets, missiles, etc. to reduce the vibration amplitudes. Motezaker and Kolahchi (2017a) investigated the Seismic response of the concrete column covered by nanofiber reinforced polymer (NFRP) layer. The column is modeled using sinusoidal shear deformation beam theory (SSDT). Mori-Tanaka model is used for obtaining the effective material properties of the NFRP layer considering agglomeration effects. Using the nonlinear strain-displacement relations, stress-strain relations and Hamilton\'s principle, the motion equations are derived. Behera and Kumari (2018) conducted first time, an exact solution for free vibration of the Levy-type rectangular laminated plate considering the most efficient Zig-Zag theory (ZIGT) and third order theory (TOT). The plate is subjected to hard simply supported boundary condition (Levy-type) along x axis. Motezaker and Kolahchi (2017b) presented the dynamic analysis of a concrete pipes armed with Silica ($ SiO_2 $) nanoparticles subjected to earthquake load. The structure is modeled with first order shear deformation theory (FSDT) of cylindrical shells. Mori-Tanaka approach is applied for obtaining the equivalent material properties of the structure considering agglomeration effects. Rouhi and Ansari (2012) executed the axial buckling of double-walled CNT subject to various layer-wise conditions by using Rayleigh-Ritz based upon nonlocal Flügge shell theory. Their study showed that the number of different layer-wise boundary conditions dominates the choice of values for nonlocal parameter. Kolahchi *et al.* (2017) focused with general wave propagation in a piezoelectric sandwich plate. The core is consisted of several viscoelastic nanocomposite layers subjected to magnetic field and is integrated with viscoelastic piezoelectric layers subjected to electric field. The piezoelectric layers play the role of actuator and sensor at the top and bottom of the core, respectively. Ansari and Rouhi (2013) summarized the effect of small scale, geometrical parameter and layer-wise end conditions of double-walled CNT by adopting Flügge shell model (FSM). They depicted that the continuum model considering the nonlocal effect compels the short double-walled CNT more flexible. Motezaker *et al.* (2020) presented the present research post-buckling of a cut out plate reinforced through carbon nanotubes (CNTs) resting on an elastic foundation. Material characteristics of CNTs are hypothesized to be altered within thickness orientation which is calculated according to Mori-Tanaka model. For modeling the system mathematically, first order shear deformation theory (FSDT) is applied and using energy procedure, the governing equations can be derived. Recently some researcher used different methods for nonlinear modeling (Shamshirsaz *et al.* 2020, Fanian (2019a, b,c), Sadoughifar *et al.* 2020, Zhang *et al.* 2019, Kocal and Akbarov, 2019, Kar *et al.* 2018, Torabi and Ansari 2018, Şimşek, 2011, Karami and Farid, 2015, Ghadiri *et al.* 2015, Hayati *et al.* 2017, Lee *et al* 2009). Recently Hussain and Naeem (2019a,b, 2020a) and performed the vibration of SWCNTs based on wave propagation approach and Galerkin's method. They investigated many physical parameters

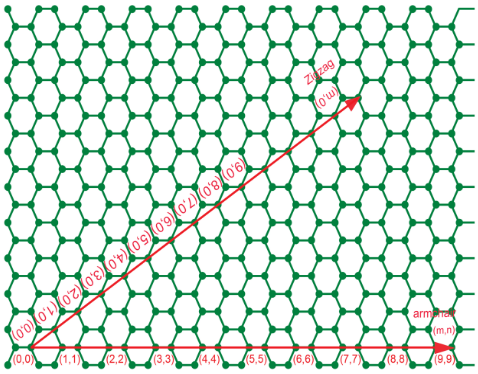

Fig. 1 Hexagonally description of armchair and zigzag on the graphene sheet

for the rotating and non-rotating vibrations of armchair, zigzag and chiral indices. Moreover, the mass density effect of single walled carbon nanotubes with in-plane rigidity have been calculated for zigzag and chiral indices.

In present paper, vibrations of SWCNTs for zigzag index (19, 0) and armchair index (17, 17) have been analyzed with specified conditions. We developed a new model from the combination of the Kelvin's model with wave propagation approach (WPA). The governing equation has been developed for the vibrations of SWCNTs considering the polynomial function with volume fraction. Effects of aspect ratio with varying the index of polynomial function are fully investigated for fundamental natural frequency. It has been shown that frequency curves decrease as an increment in the aspect ratio and increases on increasing the index of polynomial function. The frequency curves for C-F are lower throughout the computation than that of C-C and SS-SS curves.

2. Materials and methods

In fact, CNTs are kinds of rolled graphene sheets, and the rolling manner shows the basic properties of the tube, and that is actually the main reason for the extraordinary feature of the CNTs (Georgantzinos *et al.*, 2009). These cylindrical structures have many fascinating and valuable properties which have several potential applications in different fields (O'Connel, 2006). The armchair and zigzag carbon nanotubes are usually presented by (m, n) and (m, 0), respectively.

The geometry of CNT is shown in Fig. 2. Where (N, S, Q) are the stress resultants and (M) is the moment. The thermal expansion causes pre-stress, which is neglected due to the reference temperature. We arrive at the dynamic equilibrium equations:

$$\begin{cases} \dfrac{\partial N_\alpha}{\partial \alpha} + \dfrac{\partial S_\beta}{\partial \beta} + \kappa = \rho h R \dfrac{\partial^2 u}{\partial t^2} \\ \dfrac{\partial N_\beta}{\partial \beta} + \dfrac{\partial S_\alpha}{\partial \alpha} + Q_\beta = \rho h R \dfrac{\partial^2 v}{\partial t^2} \\ \dfrac{\partial Q_\alpha}{\partial \alpha} + \dfrac{\partial Q_\beta}{\partial \beta} + N_\beta = \rho h R \dfrac{\partial^2 w}{\partial t^2} \end{cases} \tag{1}$$

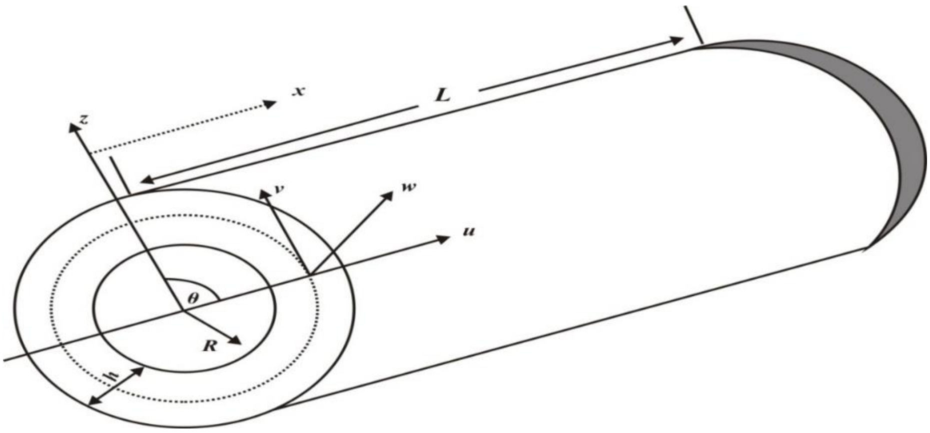

Fig. 2 Geometry of SWCNTs

$$\begin{cases} \dfrac{\partial M_{\alpha\beta}}{\partial \alpha} + \dfrac{\partial M_{\beta}}{\partial \beta} - RQ_{\beta} = 0 \\ \dfrac{\partial M_{\beta\alpha}}{\partial \beta} + \dfrac{\partial M_{\alpha}}{\partial \alpha} - RQ_{\alpha} = 0 \end{cases} \tag{2}$$

where ρ is the mass density.

Using Kelvin model and equations (1) and (2) we get Kelvin-like shell model.

The obtained model is as follows

$$\begin{aligned} &\left[\frac{\partial^2}{\partial \alpha^2} + k_2(1+c^2)\frac{\partial^2}{\partial \beta^2}\right]u + \left[(\mu_1 + k_2)\frac{\partial^2}{\partial \alpha \partial \beta}\right]v \\ &\quad + \left[6 + \frac{\partial}{\partial \alpha} + c^2\left(k_2\frac{\partial^3}{\partial \alpha \partial \beta^2} - \frac{\partial^3}{\partial \alpha^3}\right)\right]w \\ &\quad = \frac{\rho h R^2[1-\nabla^2]}{K}\frac{\partial^2 u}{\partial t^2} \end{aligned} \tag{3}$$

$$\begin{aligned} &\left[(\mu_1 + k_2)\frac{\partial^2}{\partial \alpha \partial \beta}\right]u + \left[k_2(1+3c^2)\frac{\partial^2}{\partial \alpha^2} + k_1\frac{\partial^2}{\partial \beta^2}\right]v \\ &\quad + \left[k_1\frac{\partial}{\partial \beta} - c^2(\mu_1 + 3k_2)\frac{\partial^3}{\partial \alpha^2 \partial \beta}\right]w \\ &\quad = \frac{\rho h R^2[1-\nabla^2]}{K}\frac{\partial^2 v}{\partial t^2} \end{aligned} \tag{4}$$

$$\begin{aligned} &\left[\mu_1\frac{\partial}{\partial \alpha} - c^2\left(\frac{\partial^3}{\partial \alpha^3} - k_2\frac{\partial^3}{\partial \alpha \partial \beta^2}\right)\right]u \\ &+ \left[k_1\frac{\partial}{\partial \beta} - c^2(\mu_1 + 3k_2)\frac{\partial^3}{\partial \alpha^2 \partial \beta}\right]v \\ &+ \left[\left(1+\frac{1}{c^2}\right)k_1 + \frac{\partial^4}{\partial \alpha^4} + k_1\frac{\partial^4}{\partial \beta^4} + 2\ k_1\frac{\partial^2}{\partial \beta^2} + (2\mu_1 + 4k_2)\frac{\partial^4}{\partial \alpha^2 \partial \beta^2}\right]c^2 w \\ &+ \frac{R^2}{K}(1-\nabla^2)\left[Ew + \eta\frac{\partial w}{\partial t}\right] = -\frac{\rho h R^2[1-\nabla^2]}{K}\frac{\partial^2 w}{\partial t^2} \end{aligned} \tag{5}$$

where $K = \frac{E_1 h}{1-\mu_1\mu_2}$, medium has stiffness E, and the viscosity of the medium is η.

Here, for the wave solutions of SWCNTs, the solutions of system of equations (3 ~ 5) for axisymmetric waves is given by (Wang and Gao, 2016)

$$\begin{cases} u(\alpha,t) = U e^{ik\left(\alpha - \frac{vt}{R}\right)} \\ v(\alpha,t) = V e^{ik\left(\alpha - \frac{vt}{R}\right)} \\ w(\alpha,t) = W e^{ik\left(\alpha - \frac{vt}{R}\right)} \end{cases} \tag{6}$$

where U, V and W are the amplitudes of waves along the direction of x, y and z respectively, the dimensionless wave vector in the longitudinal direction is $k = \frac{\pi m R}{L}$, in longitudinal direction m, v is the half axial wave number and wave phase velocity.

Substituting equation (6) in system of equations (3 ~ 5) and simplifying, in matrix form, we get the following system

$$[M^{(1)}(k,v)]_{3\times 3}\begin{bmatrix} U \\ V \\ W \end{bmatrix} = [0 \quad 0 \quad 0]^T \tag{7}$$

For the nontrivial solution of above equation, we have

$$Det\left[M^{(1)}(k,v)\right] = 0 \tag{8}$$

2.1 Polynomial function for volume fraction

The volume fraction V_{cnt} for CNTs with polynomial law for the material property is expressed as (Shen 2009):

$$V_{cnt} = \left[\frac{z}{h} + 0.5\right]^{\aleph} V_{tcnt} \tag{9}$$

The value power law exponent $\aleph$ remained between zero and infinity.

3. Results and discussion

In this section, the versatile numerical technique Kelvin's method has been used in current study to investigate the vibration of armchair and zigzag CNTs with different index of polynomial function. For the convergence rate of CNTs, the non-dimensional frequency enumerated in the current work, i.e., using Kelvin's method are happened to be in a good consistency along with the so-called exact results furnished by Loy *et al.* (1997) as shown in Table 1. There is once again comparison of CNT with Gonclaves *et al.* (2006) as shown in Table 2. The proposed model based on Kelvin's method can incorporate in order to accurately predict the acquired results of material data point. Tables 3-5 show the frequency comparison of armchair and zigzag CNTs with varying different exponent of polynomial law. In these tables, frequencies value are tabulated for three different boundary conditions. It is observed that on enhancing the values of *L: d*, frequencies decrease for three boundary conditions. On increasing the exponent of polynomial law, the frequency values increase. The armchair values for C-C (See Table 3), SS-SS (See Table 4) and C-F (See Table 5) are greater than that of zigzag values with same conditions.

Table 1 Convergence of Kelvin's method frequencies Loy *et al.* (1997)

	Method	$\aleph$ = 1	2	3	4
C-C	Loy *et al.* (1997)	0.032885	0.01393	0.02267	0.04221
	Present	0.034878	0.01405	0.02272	0.04227
S-S	Loy *et al.* (1997)	0.016101	0.00938	0.02211	0.04209
	Present	0.016102	0.00938	0.02211	0.04227
C-S	Loy *et al.* (1997)	0.023974	0.00822	0.00584	0.00871
	Present	0.024721	0.00828	0.00585	0.00871

Table 2 Convergence of Kelvin's method frequencies (Gonclaves *et al*. 2006)

	Method	γ = 8	9	10	11
CNT	(Gonclaves *et al*. (2006).	280.940	288.71	318.4	363.33
	Present	279.145	287.54	317.43	362.87

Table 3 Frequency comparison of C-C armchair and zigzag CNTs versus length: diameter with index of polynomial law

Length: diameter	(11, 11) $\aleph = 2.5$	$\aleph = 3$	$\aleph = 3.5$	(13,0) $\aleph = 2.5$	$\aleph = 3$	$\aleph = 3.5$
10.26	132.57	241.57	242.81	68.575	68.748	68.846
13.89	130.67	238.34	238.94	50.654	50.781	50.854
17.49	129.82	236.89	237.49	40.227	40.329	40.387
21.06	129.37	236.12	236.72	23.408	33.492	33.540
24.66	129.10	235.66	236.25	28.531	28.603	28.644

Table 4 Frequency comparison of SS-SS armchair and zigzag CNTs versus length: diameter with index of polynomial law

Length: diameter	(11, 11) $\aleph = 2.5$	$\aleph = 3$	$\aleph = 3.5$	(13,0) $\aleph = 2.5$	$\aleph = 3$	$\aleph = 3.5$
10.26	240.97	241.58	241.92	66.035	66.202	66.297
13.89	238.01	238.61	238.95	48.778	48.901	48.971
17.49	236.68	237.28	237.62	38.737	38.835	38.891
21.06	235.98	236.57	236.91	32.171	32.252	3.252
24.66	235.55	236.15	236.48	27.474	27.544	27.544

Table 5 Frequency comparison of C-F armchair and zigzag CNTs versus length: diameter with index of polynomial law

Length: diameter	(11, 11) $\aleph = 2.5$	$\aleph = 3$	$\aleph = 3.5$	(13,0) $\aleph = 2.5$	$\aleph = 3$	$\aleph = 3.5$
10.26	240.39	241.00	241.34	63.495	63.656	63.747
13.89	237.69	238.29	238.63	46.901	47.020	47.087
17.49	236.48	237.08	237.42	37.248	37.342	37.395
21.06	235.84	236.43	236.77	30.933	31.011	31.056
24.66	235.45	236.04	236.38	26.418	26.484	256.522

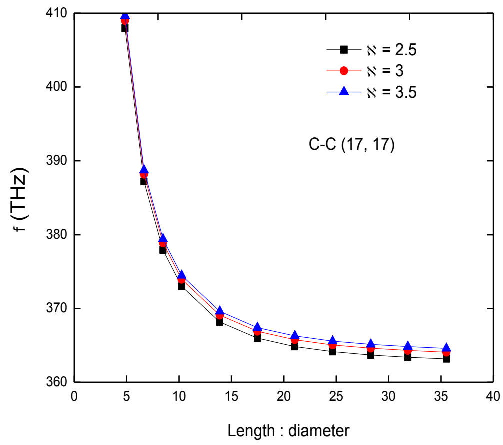

Fig. 3 Effect of C-C SS-SS and C-F armchair (17, 17), frequencies versus length: diameter

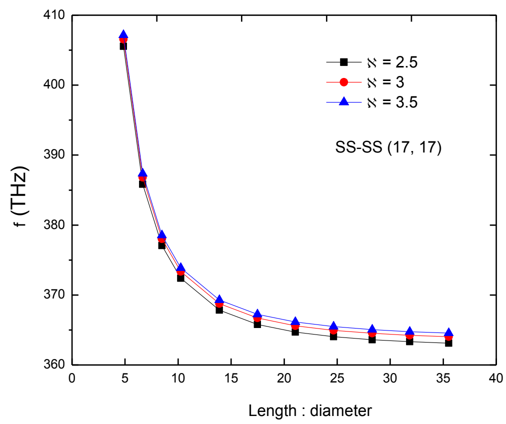

Fig. 4 Effect of C-C SS-SS and C-F armchair (17, 17), frequencies versus length: diameter

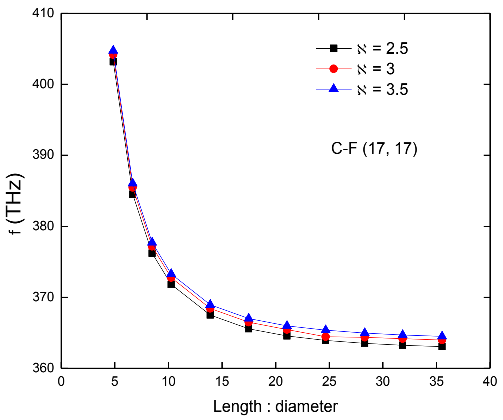

Fig. 5 Effect of C-C SS-SS and C-F armchair (17, 17), frequencies versus length: diameter

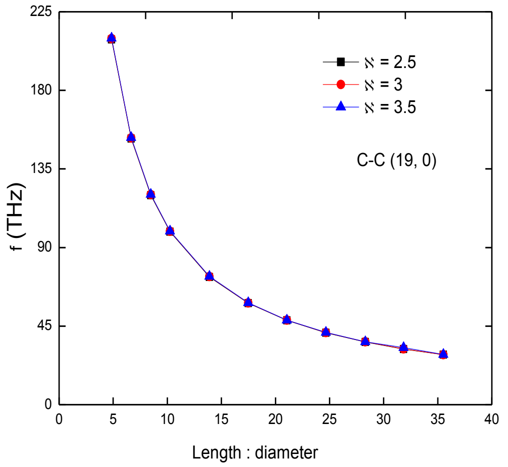

Fig. 6 Effect of C-C SS-SS and C-F Zigzag (19, 0), frequencies versus length: diameter

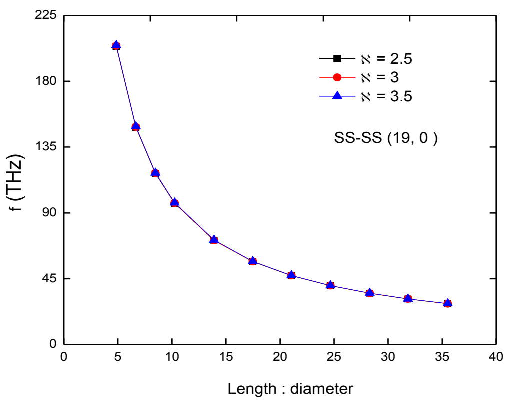

Fig. 7 Effect of C-C SS-SS and C-F Zigzag (19, 0), frequencies versus length: diameter

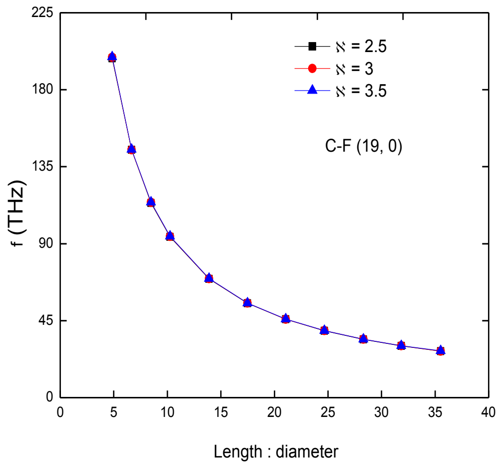

Fig. 8 Effect of C-C SS-SS and C-F Zigzag (19, 0), frequencies versus length: diameter

The fundamental natural frequencies of armchair (17, 17) and zigzag (19, 0) SWCNTs with specified boundary conditions can be obtained using Kelvin's model based on wave propagation approach. Figs. 3-5 shows the frequency response of armchair and zigzag versus length: diameter for different values of power law index $\aleph = 2.5, 3, 3.5$. The results for armchair and zigzag SWCNTs are investigated for different computation with boundary conditions (BCs). The fundamental natural frequencies of armchair (17, 17), and zigzag (19, 0) for C-C, SS-SS and C-F conditions are investigated. It is found that from these figures that the FNF outcomes of C-C frequencies are higher than SS-SS, C-F for varying length: diameter. The frequencies increase on increasing the power law index. Variation of frequencies versus length for indices (19, 0), with BCs has been plotted in graph of Figs. 6-8. The value of fundamental frequency decreases on increasing the length of the tube. In addition, when the tube length: diameter increases from 5~10, the frequency decreases rapidly, while for the length: diameter (= 10 ~ 15), the frequency is gently parallel. In present result, the frequencies are significant at length: diameter (= 35). It shows that natural frequencies decrease as length: diameter is increases, for these boundary conditions.

It can also be reported from Figs. 3 8, that the outcomes of frequency response of armchair with prescribed boundary conditions are higher than that of zigzag frequency values. The variations of power law index $\aleph$ for frequencies (Hz) of carbon nanotubes for BCs have been sketched in graphs. As evidenced by these figures, the fundamental natural frequencies would slightly increase by increasing power law index $\aleph$ for different BCs. For initial values, the gap between frequency curves are little significant and for bigger values of length: diameter, the frequency curves moderately pronounced. It can take into account that the armchair SWCNTs (17, 17) with C-C condition have the prominent and highest frequencies and other BCs followed as SS-SS and C-F. It is also concluded that the frequency curves with changing the values of length: diameter of C-F boundary condition are the lowest outcomes.

4. Conclusion

In this paper, vibrations of armchair and zigzag CNTs have been investigated for the distribution of polynomial function. Here the Kelvin's method has been applied to derive the tube frequency equation. For clamped-clamped conditions, variations frequencies are higher than that of other conditions. The obtained results show that by increasing aspect ratio of carbon nanotubes, frequency value decreases for all boundary conditions. It is noted that with higher aspect ratio, the boundary conditions have a momentous influence on vibration of CNT. It can be concluded that frequencies increased by increasing the power law index. Stability of a carbon nanotubes depends highly on these aspects of material. More the carbon nanotubes material sustains a load due to physical situations, the more carbon nanotubes is stable. Any predicted fatigue due to burden of vibrations is evaded by estimating their dynamical aspects. For future research, such model can be established for the vibration of rotating carbon nanotubes with ring supports.

Declaration of Conflicting Interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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