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An investigation of the thermodynamic effect on the response of FG beam on elastic foundation

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Abstract. This study presents an analytical approach to investigate the thermodynamic behavior of functionally graded beam resting on elastic foundations. The formulation is based on a refined deformation theory taking into consideration the stretching effect and the type of elastic foundation. The displacement field used in the present refined theory contains undetermined integral forms and involves only three unknowns to derive. The mechanical characteristics of the beam are assumed to be varied across the thickness according to a simple exponential law distribution. The beam is supposed simply supported and therefore the Navier solution is used to derive analytical solution. Verification examples demonstrate that the developed theory is very accurate in describing the response of FG beams subjected to thermodynamic loading. Numerical results are carried out to show the effects of the thermodynamic loading on the response of FG beams resting on elastic foundation.

Keywords: FGMs beams; Three-dimensional theory; undetermined integral forms; elastic foundation; thermodynamic effect

1. Introduction

Beam-type structures are typically used in various engineering applications such as aerospace, civil and mechanical. In recent decades, functionally graded materials (FGMs) are used in beam forms (Gul *et al*. 2019). It is well known that FGM offers certain opportunities such as uninterrupted variation of the material properties in one or more directions. This makes it possible to have structures with very interesting characteristics such as high resistance to temperature shocks, lower transverse shear stresses and high strength to weight ratio (Barati and Shahverdi 2016, Lal *et al*. 2017, Rezaiee-Pajand *et al* 2018, Faleh *et al*. 2018, Avcar 2019, Ahmed *et al*. 2019, Balubaid *et al*. 2019, Dash *et al*. 2019, Rahmani *et al*. 2020, Kaddari *et al*. 2020). Therefore, understanding the behavior of the beams in various loading conditions is paramount in order to have a reliable design (Ghiasian *et al*. 2015). Stability analysis of beams under thermodynamic effect has great importance in their design process.

Analyze of FGM beams under various loading and considerations have been studied through the past decade by various investigators.

In the literature, many beam theories have been proposed to study the behavior of beam-type structures. The conventional beam theories, including the Euler and Timoshenko beam theories have been extensively used in modeling the static and dynamic behavior of beams (Chen *et al.* 2018).

Caliò and Greco (2013) have studied the free vibration of the Timoshenko beam-columns on the elastic Pasternak foundations using the dynamic stiffness matrix method. Using the Euler–Bernoulli beam theory Fu *et al.* (2012) analyzed the dynamic stability and the thermo-piezoelectric buckling of FG beams subjected to steady heat conduction. Also, Ghiasian *et al.* (2013) studied the static and dynamic buckling of FG beam on non-linear elastic foundation subjected to uniform temperature rise across thickness. Using the Rayleigh-Ritz method, Pradhan and Chakraverty (2013) studied the free vibration response of Euler-Bernoulli and Timoshenko FG beams having different boundary conditions.

It should be noted that the Euler beam theory also called classical beam theory (CBT) ignores shear deformation and applies only to slender beams. Therefore, it underestimates deflection and overestimates buckling load and natural frequencies (Fahsi *et al.* 2019, Nguyen and Nguyen 2015). The Timoshenko beams theory or called the first order beam theory (FSBT) accounts for the shear deformation effect, but requires a shear correction factor. Sakar and Ganguli (2014) studied the free vibration of axially FG clamped Timoshenko beams. Ranjan *et al.* (2019) used first shear deformation theory for thermo-elastic free vibration analysis of functionally graded flat panel with temperature gradient along thickness. Draoui *et al*. (2019) employed also FSDT for static and dynamic behavior of nanotubes-reinforced sandwich plates.

In order to avoid the use of a shear correction factor and

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to take into account the transverse shear deformation for better accuracy, researchers have developed higher order shear theories for the study of FG beams (HSBT). However the efficiency of the HSBT depends on the appropriate choice of displacement field which is an interesting subject that attracted many research (Nguyen and Nguyen 2015, Mehar *et al.* 2018abcd, Dash *et al.* 2018, Ramteke *et al.* 2019, Singh *et al.* 2019, Sahu *et al.*2020ab, Sahla *et al.* 2019, Batou *et al.* 2019, Salah *et al.* 2019, Mehar *et al.* 2019abc and 2020, Mehar and Panda 2020).

Tounsi and his co-workers have proposed several HSDT (higher order shear deformation theories) for the study and analysis of FG structures with or without elastic foundations (Bachir Bouiadjra *et al.* 2013 and 2018, Ait Atmane *et al.* 2017, Chaabane *et al.* 2019, Zarga *et al.* 2019, Berghouti *et al.* 2019, Boutaleb *et al.* 2019, Tlidji *et al.* 2019, Ahmed *et al.* 2019, Bourada *et al.* 2019, Medani *et al.* 2019, Boussoula *et al.* 2020, Tounsi *et al.* 2020, Refrafi *et al.* 2020). Giunta *et al.* (2013) studied the thermal static behavior of FG beams subjected to thermo-mechanical loading on the base of Carrera unified formulation. Şimşek (2010) have compared results of various HSDT of FG beams subjected to vibration. Mantari and Yarasca (2015) developed an efficient 4-unknown quasi-3D hybrid theory for the bending analysis of FG beams.

The behavior of beams on elastic foundations is an important area of research in engineering. It is necessary to take the beam-foundation-soil interaction into account in a simple way to serve properly to the purposes of the application (Avcar and Mohammed 2018). Therefore, many studies have been conducted. Yas *et al.* (2017) examined the free vibration of FG beams resting on Pasternak foundation using the Euler–Bernoulli theory and by means of Generalized Differential Quadrature (GDQ) method. Avcar and Mohammed (2018) presented an analytical solution for the vibration of the FG beam on Pasternak foundation and under different boundary conditions. Duy *et al.* (2014) presented an analytical formulation to obtain eigen solutions of the FG beams resting on elastic foundation. Zhong *et al.* (2016) analyzed force vibration of FG beams resting on elastic foundation under heat conduction. Tossapanon and Wattanasakulpong (2016) studied dynamic analysis of FG beam on two parameters elastic foundation. Bellal *et al.* (2020) investigated the buckling behavior of a single-layered graphene sheet resting on viscoelastic medium via nonlocal four-unknown integral model.

Studies on the thermodynamic behavior of FG structures are very limited. Through the literature we find only the works of Zenkour and Sobhy (2013), Bachiri *et al.* (2018) and Mekerbi *et al.* (2019) as new studies dealing with this behavior. Moreover, there is no work available in the literature related to thermodynamic behavior of FG beam on elastic foundation by employing a quasi-3D theory according to the knowledge of the authors. Addou *et al.* (2019) examined the effects of porosity on dynamic response of FG plates resting on Winkler/Pasternak/Kerr foundation using quasi 3D HSDT. Boukhlif *et al.* (2019) presented a simple quasi-3D HSDT for the dynamics analysis of FG thick plate on elastic foundation. Also, Boulefrakh *et al.* (2019) discussed the effect of parameters of visco-Pasternak foundation on the bending and vibration properties of a thick FG plate. Zaoui *et al.* (2019) employed new 2D and quasi-3D shear deformation theories for free vibration of functionally graded plates on elastic foundations. Karami *et al.* (2019a) investigated the wave propagation of FG anisotropic nanoplates resting on Winkler-Pasternak foundation.

In this paper, thermodynamic behavior of FG beams based on Winkler–Pasternak foundation is studied by using an efficient analytical method. The equations of motion are derived using a quasi-3D theory for the case of plates but modified to make it applicable for the case of beams. The formulation takes both the thermodynamic effect through the thickness of the beam as well as the effect of the elastic foundation.

The mechanical characteristics of the FG beam are assumed to be varied across the thickness according to a simple exponential law distribution of the volume fraction of the constituents. Validation of the present method is demonstrating the good agreement between our results and those available in literature. A detailed parametric study is presented to show the effect of the different parameters on the thermodynamic response of FG beam.

2. Quasi-3D theory for functionally graded beams

Consider a functionally graded (FG) beam of thickness h and length L. The beam is assumed to rest on a Winkler-Pasternak elastic foundation. The mechanical characteristics of the beam are assumed to be varied according to thickness as (Zenkour and Sobhy 2013):

$$P(z) = P_m \, e^{\beta\left(\frac{z}{h}+\frac{1}{2}\right)^{p}}, \qquad \beta = \ln\left(\frac{P_c}{P_m}\right) \tag{1}$$

$P(z)$ is the effective material properties like Young's modulus E, density ρ, and thermal expansion coefficient α. The subscripts m and c refer to metal and ceramic. p is the power law index.

2.1 Kinematics

The displacement field satisfying the conditions of transverse shear stresses (and hence strains) vanishing at (x, y, ±h/2) on the outer (top) and inner (bottom) surfaces of the beam, is given as follows:

$$\begin{aligned} u\,(x,z,t) &= u_0(x,t) - z\,\frac{\partial w_0}{\partial x} + f(z)\phi_x(x,t) \\ w\,(x,z,t) &= w_0(x,t) + g(z)\theta(x,t) \end{aligned} \tag{2}$$

$u_0, w_0, and\ \varphi_x$are the three-unknown displacement of the mid-plane of the beam.

By considering that$\varphi_x = \int \theta(x,t)dx$, the displacement fields mentioned above can be written as follows:

$$u\,(x,z,t) = u_0(x,t) - z\frac{\partial w_0}{\partial x} + k_1 f(z)\int \theta(x,t)dx \tag{3}$$

$$w(x,z,t)=w_0(x,t)+g(z)\theta(x,t)$$

The integrals defined in the above equations shall be resolved by a Navier type method and the displacement fields can be rewritten as:

$$u(x,z,t)=u_0(x,t)-z\frac{dw_0}{dx}+k_1A'f(z)\frac{d\theta}{dx}$$
$$w(x,z,t)=w_0(x,t)+g(z)\theta(x,t) \tag{4}$$

where

$$g(z)=\frac{2}{15}\frac{df}{dz},\ k_1=-\lambda^2;A'=-\frac{1}{\lambda^2};\lambda=\frac{m\pi}{L} \tag{5a}$$

$$k_1=\alpha^2 \tag{5b}$$

The coefficient *A'* is expressed according to the Navier type solution and they are given by:

$$A'=-\frac{1}{\alpha^2}, \tag{6a}$$

And

$$\alpha=\frac{m\pi}{a} \tag{6b}$$

It can be seen that the displacement field in Eqs. (3)-(4) contains only three unknowns u_0, w_0 *and* θ

The shape function $f(z)$ is given as follows:

$$f(z)=z-\frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right) \tag{7}$$

The kinematic relations can be obtained as follows:

$$\varepsilon_x=\varepsilon_x^0-zk_x^b+f(z)k_x^\theta$$
$$\varepsilon_z=g'(z)\varepsilon_z^0 \tag{8}$$
$$\gamma_{xz}=\left(g(z)+k_1A'f'(z)\right)\gamma_{xz}^s$$

where

$$\varepsilon_0=\frac{du_0}{dx}$$
$$k_x^b=-\frac{d^2w_0}{dx^2};$$
$$k_x^\theta=k_1A'\frac{d^2\theta}{dx^2}; \tag{9}$$
$$\gamma_{xz}^s=\frac{d\theta}{dx};$$
$$\varepsilon_z^0=\theta(x,t)\text{ and }g'(z)=\frac{dg}{dz}$$

2.2 *Constitutive relations*

By assuming that the material of FG beam obeys Hooke's law, the stresses in the beam become:

$$\begin{Bmatrix}\sigma_x\\ \sigma_z\\ \tau_{xz}\end{Bmatrix}=\begin{bmatrix}Q_{11}&Q_{13}&0\\ Q_{13}&Q_{11}&0\\ 0&0&Q_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x-\alpha T\\ \varepsilon_z-\alpha T\\ \gamma_{xz}\end{Bmatrix} \tag{10a}$$

$$Q_{11}=\frac{E(z)}{(1-v^2)};\quad Q_{13}=vQ_{11},\quad Q_{66}=\frac{E(z)}{2(1+v)} \tag{10b}$$

where $(\sigma_x,\sigma_z\tau_{xz})$ and $(\varepsilon_x,\ \varepsilon_z,\ \gamma_{xz})$ are the stress and strain components, respectively. $\alpha(z)$ is the coefficient of thermal expansion, and T is the distribution of the temperature load.

2.3 *Equations of motion*

Hamilton's principle is used herein to derive the equations of motion. The principle can be stated in analytical form as (Behera and Kumari 2018, Fenjan *et al*. 2019ab, Faleh *et al*. 2020):

$$0=\int_0^T(\delta U+\delta K-\delta T)dt \tag{11}$$

where δU is the variation of strain energy and δK is the variation of potential energy; δT is the variation of kinetic energy.

The variation of strain energy of the beam can be stated as:

$$\delta U=\int_0^L\int_{-h/2}^{h/2}\left[\sigma_x\delta\varepsilon_x+\sigma_z\delta\varepsilon_z+\tau_{xz}\delta\gamma_{xz}\right]dxdz \tag{12}$$

Substituting Eqs. (8)-(10) into Eq. (12) and integrating through the thickness of the beam, Eq. (12) can be rewritten as

$$\delta U=\int_A\left\{\begin{aligned}&N_x\frac{d\delta u_0}{dx}-M_x^b\frac{d^2\delta w_0}{dx^2}+k_1A'M_x^s\frac{d^2\delta\theta}{dx^2}\\ &+N_z\delta\theta+Q\frac{d\delta\theta}{dx}\end{aligned}\right\}dA \tag{13}$$

where:

$$\begin{Bmatrix}N_x\\ M_x^b\\ M_x^s\end{Bmatrix}=\int_{-h/2}^{h/2}\sigma_x\begin{Bmatrix}1\\ z\\ f(z)\end{Bmatrix}dz \tag{14a}$$

$$Q=\int_{-\frac{h}{2}}^{\frac{h}{2}}\tau_{xz}\left(g(z)-k_1A'f'(z)\right)dz \tag{14b}$$

$$N_z=\int_{-\frac{h}{2}}^{\frac{h}{2}}\sigma_z g'(z)dz \tag{14c}$$

Using Eq. (10) in Eq. (14), the stresses resultants, for the present FG beam, can be related to the displacements as:

$$\begin{Bmatrix} N_x \\ M_x^b \\ M_x^s \end{Bmatrix} = \begin{bmatrix} A & B & B^s \\ B & D & D^s \\ B^s & D^s & H^s \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ k_x^b \\ k_x^\theta \end{Bmatrix} - \begin{Bmatrix} N^T \\ M_x^{b^T} \\ M_x^{s^T} \end{Bmatrix} + \begin{Bmatrix} L \\ L^a \\ R \end{Bmatrix} \varepsilon_z^0 , \; Q = A^s \gamma , \quad (15)$$

$$N_z = L\varepsilon_x^0 + L^a k_x^b + R k_x^\theta + R^a \varepsilon_z^0 \quad (16)$$

where A , B , D , etc., are the beam stiffness, defined by

$$\left\{A, B, B^s, D, D^s, H^s\right\} = \int_{-h/2}^{h/2} Q_{11}.\left\{1, z, f(z), z^2, zf(z), f(z)^2\right\} dz \quad (17a)$$

$$\left\{L, L^a, R, R^a\right\} = \int_{-h/2}^{h/2} Q_{13}.\left\{g'(z), zg'(z), g'(z) f(z), g'(z)^2\right\} dz \quad (17b)$$

$$A_s = \int_{-h/2}^{h/2} Q_{66}.\left(g(z) - k_1 A' f'(z)\right)^2 dz \quad (17c)$$

The stress and moment resultants, N_x^T , $M_x^{b^T}$, $M_x^{s^T}$ due to thermal loading are defined by

$$\begin{Bmatrix} N_x^t \\ M_x^{bt} \\ M_x^{\theta t} \end{Bmatrix} = \int_{-h/2}^{h/2} \frac{E(z)}{(1-\nu)} \begin{Bmatrix} 1 \\ z \\ f(z) \end{Bmatrix} \alpha T dz \quad (18)$$

$$N_z^t = \int_{-h/2}^{h/2} \frac{E(z)}{(1-\nu)} g'(z) \alpha T dz$$

The potential energy of the foundation and distributed load is expressed as:

$$\delta K = \int_0^L \int_{-h/2}^{h/2} \left(k_w w - k_s \frac{d^2 w}{dx^2} \right) \delta\left(w_0 + g(z)\theta\right) - q\delta\left(w_0 + g(z)\theta\right) dxdz \quad (19)$$

Also, the variation of the kinetic energy can be expressed as

$$\delta T = -\int_0^L \int_{-h/2}^{h/2} \left[\left[\ddot{u}\delta u + \left(\ddot{w}_0 + g(z)\ddot{\theta}\right)\delta\left(w_0 + g(z)\theta\right)\right]\rho(z)\right] dxdz \quad (20)$$

$$\delta T = -\int_0^l \begin{bmatrix} I_0(\ddot{u}_0\delta u_0 + \ddot{w}_0\delta w_0) + I_1(-\ddot{u}_0 \frac{dw_0}{dx} - \frac{d\ddot{w}_0}{dx}\delta u_0) \\ +I_2(\frac{d\ddot{w}_0}{dx}\frac{d\delta\ddot{w}_0}{dx}) + J_0(\ddot{w}_0\delta\theta + \ddot{\theta}\delta w_0) \\ +J_1(K_1A'\ddot{u}_0\frac{d\delta\theta}{dx} + K_1A'\frac{d\ddot{\theta}}{dx}\delta u_0) \\ +J_2(-K_1A'\frac{d\ddot{w}_0}{dx}\frac{d\delta\theta}{dx} - K_1A'\frac{d\ddot{\theta}}{dx}\frac{d\delta w_0}{dx}) \\ +K_0\ddot{\theta}\delta\theta + K_2(K_1A')^2\frac{d\ddot{\theta}}{dx}\frac{d\delta\theta}{dx} \end{bmatrix} dx \quad (21)$$

Where dot-superscript convention indicates the differentiation with respect to the time variable t ;$\rho(z)$ is the mass density; and (I_0, I_1, J_0, J_1, I_2, J_2, K_0, K_2) are mass inertias defined as:

$$\left(I_0, I_1, I_2\right) = \int_{-h/2}^{h/2} \rho(z)\left(1, z, z^2\right) dz \quad (22a)$$

$$\left(J_0, J_1, J_2\right) = \int_{-h/2}^{h/2} \left(g(z), f(z), zf(z)\right) dz \quad (22b)$$

$$\left(K_0, K_2\right) = \int_{-h/2}^{h/2} \left(1, z, f, z^2, zf, f^2\right) dz \quad (22c)$$

Substituting the expressions for δU, δK and δT from Eqs. (13)-(19) and (21) into Eq. (11) and integrating by parts and collecting the coefficients of $\delta u_0, \delta w_0$ and $\delta\theta$, the following equations of motion of the beam are obtained

$$\delta u_0 : \frac{dN_x}{dx} = I_0\ddot{u}_0 - I_1\frac{d\ddot{w}_0}{dx} + J_1K_1A'\frac{d\ddot{\theta}}{dx}$$

$$\delta w_0 : \frac{d^2M_x^b}{dx^2} + q - k_w w_0 + k_s\frac{d^2w_0}{dx^2} = I_0\ddot{w}_0 + I_1\frac{d\ddot{u}_0}{dx} - I_2\frac{d^2\ddot{w}_0}{dx} + J_0\ddot{\theta} + J_2K_1A'\frac{d^2\ddot{\theta}}{dx}$$

$$\delta\theta : -K_1A'\frac{d^2M_x^s}{dx^2} - N_z + qg_0 - k_w w_0 g_0 + k_s\frac{d^2w_0}{dx^2}g_0 + \frac{dQ}{dx} = J_0\ddot{w}_0 - J_1K_1A'\frac{d\ddot{u}_0}{dx} + J_2K_1A'\frac{d^2\ddot{w}_0}{dx^2} + K_0\ddot{\theta} - K_2(k_1A')^2\frac{d^2\ddot{\theta}}{dx^2} \quad (23)$$

By substituting Eqs. (15)-(16) into Eq. (23), the equations of motion can be expressed in terms of displacements (u_0, w_0, θ) as

$$\delta u_0 : A\frac{d^2u_0}{dx^2} - B\frac{d^3w_0}{dx^3} + B^s(K_1A')\frac{d^3\theta}{dx^3} + L\frac{d\theta}{dx} = I_0\ddot{u}_0 - I_1\frac{d\ddot{w}_0}{dx} + J_1K_1A'\frac{d\ddot{\theta}}{dx} \quad (24a)$$

$$\delta w_0 : A\frac{d^3u_0}{dx^3} - D\frac{d^4w_0}{dx^4} + D^s(K_1A')\frac{d^4\theta}{dx^4} + L^a\frac{d^2\theta}{dx^2} - q - k_w w_0 + k_s\frac{d^2w_0}{dx^2} = I_0\ddot{w}_0 + I_1\frac{d\ddot{u}_0}{dx} - I_2\frac{d^2\ddot{w}_0}{dx} + J_0\ddot{\theta} + J_2K_1A'\frac{d^2\ddot{\theta}}{dx} \quad (24b)$$

$$\delta\theta : -K_1A'(B^s\frac{d^3u_0}{dx^3} - D^s\frac{d^4w_0}{dx^4} + H^s(K_1A')\frac{d^4\theta}{dx^4} + R\frac{d^2\theta}{dx^2}) - L\frac{du_0}{dx} + L^a\frac{d^2w_0}{dx^2} - RK_1A'\frac{d^2\theta}{dx^2} - R^a\theta + A^s\frac{d^2\theta}{dx^2} + qg_0 - k_w w_0 g_0 + k_s\frac{d^2w_0}{dx^2}g_0 = J_0\ddot{w}_0 - J_1K_1A'\frac{d\ddot{u}_0}{dx} + J_2K_1A'\frac{d^2\ddot{w}_0}{dx^2} + K_0\ddot{\theta} - K_2(k_1A')^2\frac{d^2\ddot{\theta}}{dx^2} \quad (24c)$$

The variation of temperature is assumed to occur in the thickness direction according to a power law form. The temperature field variation through the thickness is assumed to be (Zenkour and Sobhy (2013))

$$T(x,z,t) = \hat{t}(z)\,\hat{T}(x,t), \tag{25a}$$

where

$$\hat{t}(z) = T^{-} e^{\gamma\left(\frac{z}{h}+\frac{1}{2}\right)^{\eta}}, \quad \gamma = \ln\left(\frac{T^{+}}{T^{-}}\right), \quad 0 \le \eta \le \infty \tag{25b}$$

$$\hat{T}(x,z,t) = \bar{t}\,\sin(\lambda x)\, e^{i\omega t} \tag{25c}$$

where $\bar{t}$ is arbitrary parameter and $T^{+}$and T^{-} are the top and the bottom temperature.

In which η is the temperature exponent. Note that $\eta = 0$ represents a top surface temperature of the beam while $\eta = \infty$ represents a bottom surface temperature.

3. Analytical solution

In this paragraph, the Navier solution for simply supported beams will be used to solve the problem. The variables u_0, $w_0$and θcan be written by assuming the following variations:

$$\begin{pmatrix} u_0 \\ w_0 \\ \theta \end{pmatrix} = \sum_{m=1}^{\infty} \begin{pmatrix} U_m e^{i\omega t} \cos(\lambda x) \\ W_m e^{i\omega t} \sin(\lambda x) \\ X_m e^{i\omega t} \sin(\lambda x) \end{pmatrix} \tag{26}$$

where: U_m , W_m , X_m are arbitrary parameters to be determined,$\boldsymbol{\omega}$is the eigen frequency associated with m^{th} eigen mode, and $\frac{m\pi}{L}$. The transverse load q is also expanded in Fourier series as:

$$q(x) = \sum_{m=1}^{\infty} Q_m \sin\left(\frac{m\pi}{L}x\right) e^{i\omega t} \tag{27}$$

where Q_m is the load amplitude calculated from:

$$Q_m = \int_0^L q(x)\sin(\lambda x) \tag{28}$$

For the case of uniform distributed load, the coefficient $Q_m$is given as:

$$Q_m = \frac{4q_0}{m\pi} \qquad (m = 1,3,5,.....) \tag{29}$$

Substituting Eqs. (26)-(27) into Eq. (24), the analytical solutions can be obtained by:

- For the free vibration problem

$$\left([K] - \omega^2 [M]\right)\{\Delta\} = 0 \tag{30}$$

- For the case of the static problems

$$[K]\{\Delta\} = \{F\} \tag{31}$$

- For the case of the thermodynamic

$$\left([K] - \omega^2 [M]\right)\{\Delta\} = \{F\} \tag{32}$$

where $[M]$is the mass stiffness matrix, $[K]$is the stiffness matrix, $\{\Delta\}$ is the displacement of the nodal value, and $\{F\}$is the distributive force vector, and $\omega^2$is the eigen values of dynamic system (natural frequencies).

$$[K] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}; \quad [M] = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{bmatrix}; \tag{33a}$$

$$\{\Delta\} = \begin{Bmatrix} U_m \\ W_m \\ X_m \end{Bmatrix}; \quad \{F\} = \begin{Bmatrix} -A^t \bar{t} \\ B^t \bar{t} + q_0 \\ -k_1 A' C^t \bar{t} \lambda^2 + D^t \bar{t} + q_0 g_0 \end{Bmatrix} \tag{33b}$$

$$\begin{aligned} a_{11} &= A\lambda^2 \\ a_{12} &= -B\lambda^3 \\ a_{13} &= k_1 A' B^s \lambda^3 - L\lambda \\ a_{22} &= D\lambda^4 + k_s\lambda^2 + k_w \\ a_{23} &= -k_1 A' D^s \lambda^4 + k_s g_0 \lambda^2 + k_w g_0 + L^a \lambda^2 \\ a_{33} &= (k_1 A')^2 H^s \lambda^4 + A^s \lambda^2 - 2k_1 A' R\lambda^2 \\ &+ k_s g_0{}^2 \lambda^2 + k_w g_0{}^2 + R^a \end{aligned} \tag{33c}$$

where $g_0 = g(0)$

$$\begin{aligned} m_{11} &= I_0 \\ m_{12} &= -I_1\lambda \\ m_{13} &= k_1 A' J_1 \lambda \\ m_{22} &= I_0 + I_2\lambda^2 \\ m_{23} &= J_0 - k_1 A' J_2 \lambda^2 \\ m_{33} &= K_2 (k_1 A')^2 \lambda^2 + K_0 \end{aligned} \tag{33d}$$

4. Results and discussion

In this section, numerical computations of the simply supported FG beams resting on elastic foundation by the present method are suggested for investigation.

In absence of works relating to FG beams subjected to thermodynamic loading in literature, the results of this model have been validated for the static case in the presence of a mechanical load and in the dynamic case for free vibration.

Numerical calculations are performed for a mixture made of Aluminum and Alumina, where the top region is Alumina rich and the bottom region is Aluminum rich with the following material properties:

Table 1 Comparisons of the mid-span non-dimensional deflection ($\bar{w}$) of an isotropic homogeneous beam on elastic foundations using various beam theories

Foundation parameters		L/h=120			L/h=15			L/h=5		
K_w	K_s	Ying *et al.* (2008)	Ait Atmane *et al.* (2015) $\varepsilon_z \neq 0$	Present $\varepsilon_z \neq 0$	Ying *et al.* (2008)	Ait Atmane *et al.* (2015) $\varepsilon_z \neq 0$	Present $\varepsilon_z \neq 0$	Ying *et al.* (2008)	Ait Atmane *et al.* (2015) $\varepsilon_z \neq 0$	Present $\varepsilon_z \neq 0$
0	0	1.3023	1.3009	1.3021	1.3153	1.3022	1.3105	1.4202	1.3133	1.4091
	10	0.6448	0.6446	0.6448	0.6483	0.6532	0.6473	0.6745	0.7217	0.6730
	25	0.3661	0.3662	0.3661	0.3674	0.3796	0.3671	0.3767	0.4842	0.3763
10	0	1.1806	1.1794	1.1803	1.1913	1.1817	1.1875	1.2773	1.2003	1.2688
	10	0.6133	0.6131	0.6132	0.6165	0.6223	0.6156	0.6403	0.6945	0.6389
	25	0.3557	0.3557	0.3556	0.3568	0.3694	0.3566	0.3657	0.4755	0.3653
100	0	0.6401	0.6398	0.6400	0.6434	0.6486	0.6425	0.6685	0.7177	0.6670
	10	0.4256	0.4256	0.4255	0.4272	0.4380	0.4268	0.4388	0.5344	0.4383
	25	0.2829	0.2830	0.2828	0.2836	0.2981	0.2835	0.2894	0.4148	0.2891

- Metal (Aluminium, Al): $E_M = 70 \times 10^9$ N/m^2; $\nu = 0.3$; $\rho_M = 2702$ kg/m^3;

- Ceramic (Alumina, Al_2O_3): $E_C = 380 \times 10^9$; N/m^2; $\nu = 0.3$; $\rho_C = 3960$kg/m^3;

In all examples, the foundation parameters are presented in the non-dimensional form of $k_w = K_w L^4/EI$ and $k_p = K_p L^2/EI$.

For simplicity, the following non-dimensional parameters are used in the numerical examples:

$$\bar{w} = \frac{100 E_c I}{qL^4} w\left(\frac{L}{2}\right); \tilde{\omega} = \sqrt[4]{\frac{\rho_c A L^4 \omega^2}{EI}}; K_w = \frac{k_w L^4}{E_c I}; K_s = \frac{k_s L^2}{E_c I}.$$

$$\bar{\sigma}_x = -\frac{10^{-3} E_c I}{L^3} \sigma_x\left(\frac{L}{2}, \frac{h}{2}\right); \quad \bar{\tau}_{xz} = \frac{10^{-1} E_c I}{L^2} \tau_{xz}(0,0);$$

where $I = \frac{bh^3}{12}$ (beam stiffness)

4.1 Comparison studies

The accuracy and efficiency of the present method to characterize well the free vibration and the static bending behaviors is firstly demonstrated.

Tables 1-2 present respectively the comparisons of the non-dimensional natural frequency and the mid-span deflection obtained from the present computational model with other reported methods (Ying *et al.* (2008), Ait Atmane *et al.* (2017)). Results are presented for different values of foundation parameters, and thickness-to-length ratio.

In general, good agreements are observed between the present results and the exact two-dimensional theory of elasticity by Ying *et al.* (2008) and shear and normal deformation beam theory of Ait Atmane *et al.* (2017).

Table 1 Comparisons of the mid-span non-dimensional deflection ($\bar{w}$) of an isotropic homogeneous beam on elastic foundations using various beam theories.

Another comparison is presented in Table 3 where the

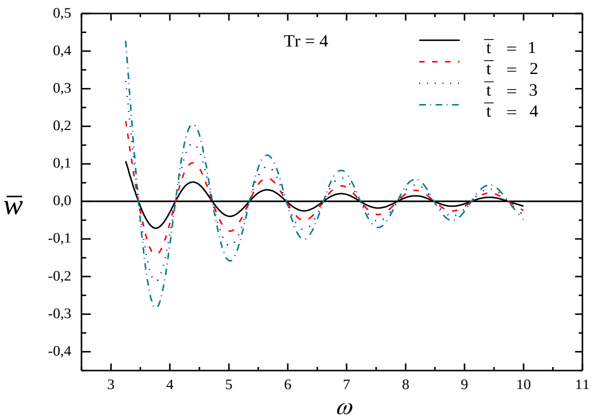

Fig. 1 Variation of deflection $\bar{w}$ versus the natural frequency ω ($L/h = 5, t = 5,\ \eta = 1,\ T^- = 25,\ k = 1,\ K_w = 100,\ K_p = 10$)

results of present method are compared with those of Li *et al.* (2010) and Ould Larbi *et al.* (2015).

The results are presented in terms of displacements and stress for different values of the power index and thickness-to-length ratio.

In this case too, an excellent agreement between results is observed. The difference is found to be relatively small while the thickness-to-length *a/h*=5 compared to the other two solutions.

This is due to the fact that the two solutions are 2D theories, whereas the present is a quasi 3D theory so it takes into account the stretching effect.

4.2 Parametric studies

In this section, a series of parametric studies are conducted to investigate the effects of beam parameters, elastic foundation and thermal loading on the thermodynamic behavior of simply supported FG beam resting on elastic foundation.

Table 2 Comparisons of the non-dimensional-fundamental frequency ($\tilde{\omega}$) parameter of an isotropic homogeneous beam on elastic foundations using various beam theories

Foundation parameters		L/h=120			L/h=15			L/h=5		
K_w	K_s/π^2	Ying *et al.* (2008)	Ait Atmane *et al.* (2017) $\varepsilon_z \neq 0$	Present $\varepsilon_z \neq 0$	Ying *et al.* (2008)	Ait Atmane *et al.* (2017) $\varepsilon_z \neq 0$	Present $\varepsilon_z \neq 0$	Ying *et al.* (2008)	Ait Atmane *et al.* (2017) $\varepsilon_z \neq 0$	Present $\varepsilon_z \neq 0$
0	0	3.1415	3.1421	3.1414	3.1323	3.1309	3.1329	3.0637	3.0484	3.0552
	1	3.7359	3.7363	3.7358	3.7278	3.7270	3.7278	3.6665	3.6598	3.6644
	2.5	4.2969	4.2972	4.2968	4.2889	4.2884	4.2887	4.2232	4.2249	4.2296
10^2	0	3.7482	3.7486	3.7482	3.7401	3.7393	3.7402	3.6788	3.6724	3.6769
	1	4.1436	4.1438	4.1438	4.1356	4.1350	4.1354	4.0720	4.0712	4.0758
	2.5	4.5823	4.5825	4.5822	4.5741	4.5738	4.5738	4.5028	4.5097	4.5153
10^4	0	10.0240	10.0240	10.0240	9.9958	10.0066	10.0059	7.3408	7.5525	7.7604
	1	10.0481	10.0481	10.0481	10.0197	10.0306	10.0299	7.3410	7.5525	7.7604
	2.5	10.0839	10.0839	10.0839	10.0552	10.0663	10.0656	7.3412	7.5525	7.7604

Table 3 Comparisons of *t*he non-dimensional $\bar{w}$,$\bar{\sigma}_x$,$\bar{\tau}_{xz}$of an FGM beam on using various beam theories

p	Theory	$\bar{w}$		$\bar{u}$		$\bar{\sigma}_x$		$\bar{\tau}_{xz}$	
		a/h=5	a/h=20	a/h=5	a/h=20	a/h=5	a/h=20	a/h=5	a/h=20
0	Li *et al.* (2010)	3.1653	2.8962	0.9402	0.2306	3.8018	15.0128	0.7321	0.7435
	Ould Larbi *et al.* (2015)	3.1653	2.8962	0.9406	0.2305	3.8018	15.0128	0.7330	0.7436
	Present	3.2986	2.9148	0.9293	0.2273	3.9572	15.5018	0.8006	0.7754
0.5	Li *et al.* (2010)	4.8285	4.4644	1.6603	0.4087	4.9922	19.7002	0.7493	0.7604
	Ould Larbi *et al.* (2015)	4.8285	4.4644	1.6608	0.4087	4.9476	19.6891	0.7501	0.7605
	Present	4.9851	4.4640	1.6089	0.3959	5.2021	20.3442	0.8427	0.8426
1	Li *et al.* (2010)	6.2594	5.8049	2.3045	0.5686	5.8834	23.2051	0.7321	0.7435
	Ould Larbi *et al.* (2015)	6.2594	5.8048	2.3052	0.5685	5.8066	23.1860	0.7329	0.7436
	Present	6.4055	5.7573	2.2123	0.5445	6.1287	23.9620	0.8006	0.7754
2	Li *et al.* (2010)	8.0676	7.4420	3.1134	0.7691	6.8823	27.0989	0.6696	0.6810
	Ould Larbi *et al.* (2015)	8.0676	7.4421	3.1146	0.7691	6.7591	27.0682	0.6704	0.6811
	Present	8.1941	7.3256	2.9753	0.7303	7.1607	27.9772	0.6711	0.5966
5	Li *et al.* (2010)	9.8280	8.8182	3.7089	0.9133	8.1104	31.8127	0.5896	0.6011
	Ould Larbi *et al.* (2015)	9.8280	8.8186	3.7128	0.9134	7.9252	31.7667	0.5903	0.6012
	Present	9.9849	8.6951	3.5761	0.8693	8.4172	32.8328	0.5067	0.4002
10	Li *et al.* (2010)	10.9381	9.6905	3.8860	0.9536	9.7119	38.1382	0.6456	0.6584
	Ould Larbi *et al.* (2015)	10.9381	9.6907	3.8898	0.9537	9.5285	38.0926	0.6465	0.6585
	Present	11.2077	9.6274	3.7888	0.9162	10.0728	39.3600	0.5473	0.4287

Fig. 1 displays the variation of deflection $\bar{w}$ versus the natural frequency ω for various values of the arbitrary parameter $\bar{t}$. From this figure, it can be seen that the displacements are strongly influenced in the low frequency zone, that is to say at the level of the first modes of vibration. This influence is clearly noted for cases of $\bar{t} > 1$. While for the case where $\bar{t} = 1$, the displacements are little disturbed. Then, there is a gradual attenuation of the displacements to approach the zero value that is to say the stability of the beam for the high modes.

Fig. 2 presents the variation of the axial stress $\bar{\sigma}_x$ versus the natural frequency ω for different values of the arbitrary parameter $\bar{t}$.

It can be seen from the figure that, for all cases of the values of the arbitrary parameter $\bar{t}$, the stresses have a sinusoidal shape whatever the mode of vibration. The increase in the values of this parameter leads to an increase in the stresses.

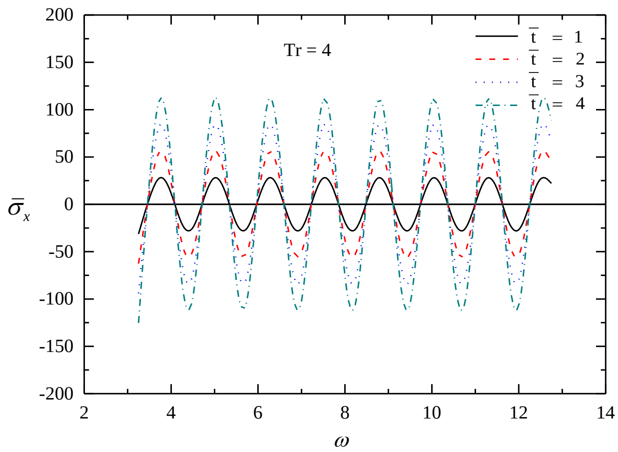

Fig. 2 Variation of the axial stress $\bar{\sigma}_x$ versus the natural frequency ω $(L/h = 5, t = 5,\ \eta = 1, T^{-} = 25,\ k = 1,\ K_w = 100,\ K_p = 10)$

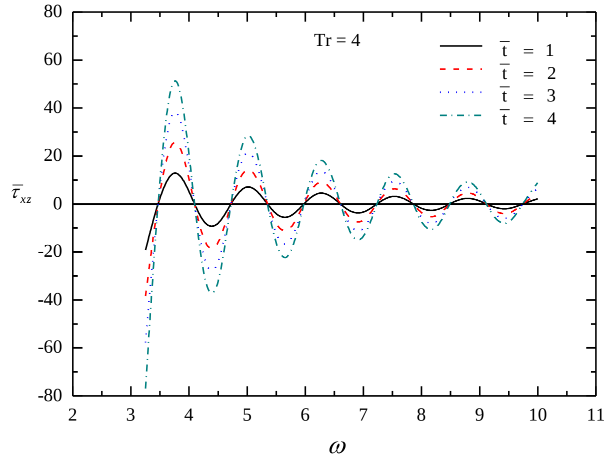

Fig. 3 Variation of the transverse shear stress $\bar{\tau}_{xz}$ versus the natural frequency ω $(L/h = 5, t = 5,\ \eta = 1, T^{-} = 25,\ k = 1,\ K_w = 100,\ K_p = 10)$

Fig. 3 depicts the variation of the transverse shear stress $\bar{\tau}_{xz}$versus the natural frequency. The same observation as Fig. 1 is noted, namely that the tangential stresses are strongly affected during the first modes of vibration. Then, as the natural frequency of the plate rise, the transverse shear stress decrease until canceled (vanished).

Figs. 4-6 show the variation of the deflection $\bar{w}$, the axial stress $\bar{\sigma}_x$ and the transverse shear stress $\bar{\tau}_{xz}$ with the time t respectively for different values of the temperature exponent η.

From these figures, the following remarks can be made:

- Increasing in the temperature exponent η values leads to a reduction of the deflection and stresses (axial and shear transverse),
- The shape of the displacement and stresses is sinusoidal regardless of the value of the temperature exponent η or time.
- Deflections and stresses reach their maxima at time $t = 0, \dfrac{\pi}{2}, \pi, \ldots$

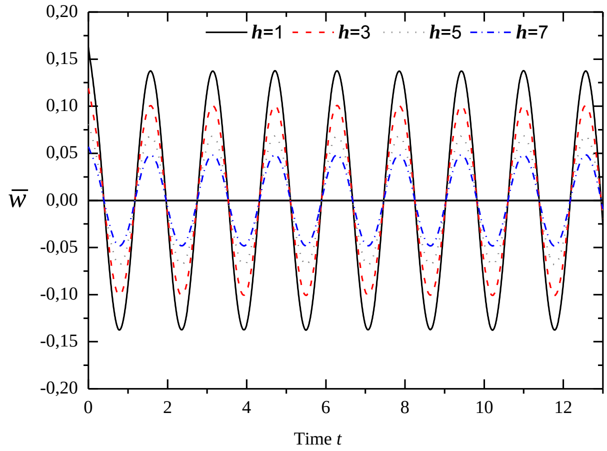

Fig. 4 Effect of time "t" on deflection " $\bar{w}$ "of FG beam for different values of the temperature exponent η with $(L/h = 5, \omega = 4,\ \bar{t} = 2,\ T^{-} = 25,\ T_r = 4,\ k = 1,\ K_w = 100,\ K_p = 10)$

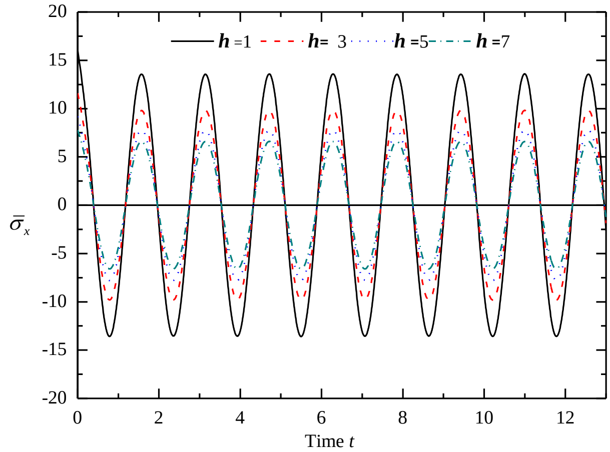

Fig. 5 Effect of time "t" on the longitudinal stress $\bar{\sigma}_x$ of FG beam for different values of the temperature exponent η with $(L/h = 10, \omega = 4,\ \bar{t} = 2,\ T^{-} = 25,\ T_r = 4,\ k = 1,\ K_w = 100,\ K_p = 10)$

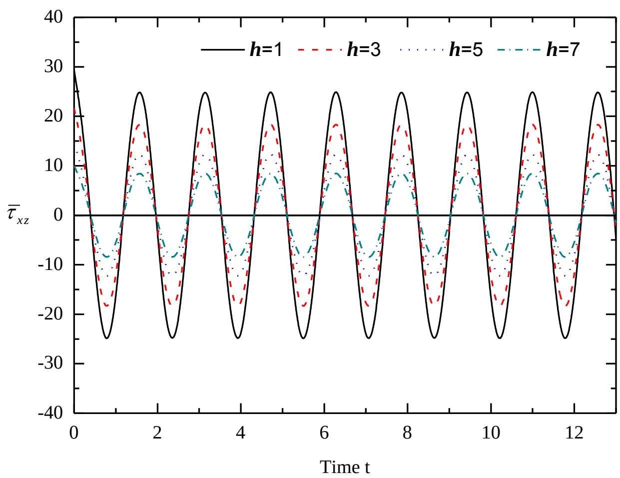

Fig. 6 Effect of time "t" on the transverse shear stress $\bar{\tau}_{xz}$ of FG beam for different values of the temperature exponent η with $(L/h = 5, \omega = 4,\ \bar{t} = 2,\ T^{-} = 25,\ T_r = 4,\ k = 1, K_w = 100, K_p = 10)$

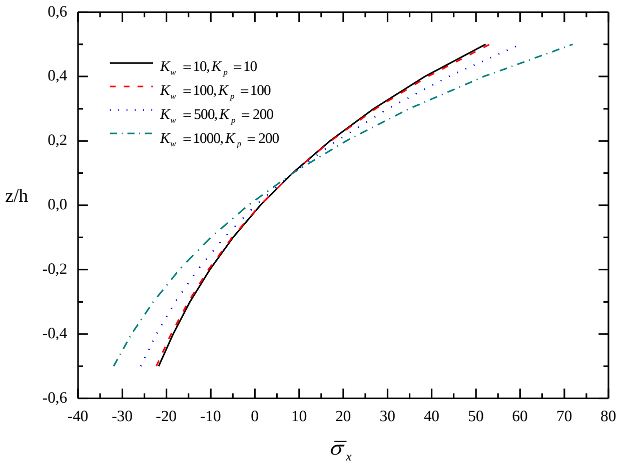

Fig. 7 Variation of the axial stress $\bar{\sigma}_x$ through the thickness of the FG beam resting on elastic foundations for different values of elastic foundations parameters $(L/h = 5, t = 3,\ \eta = 1, \omega = 4, \bar{t} = 2, T^- = 25,\ k = 1,\ T_r = 4)$

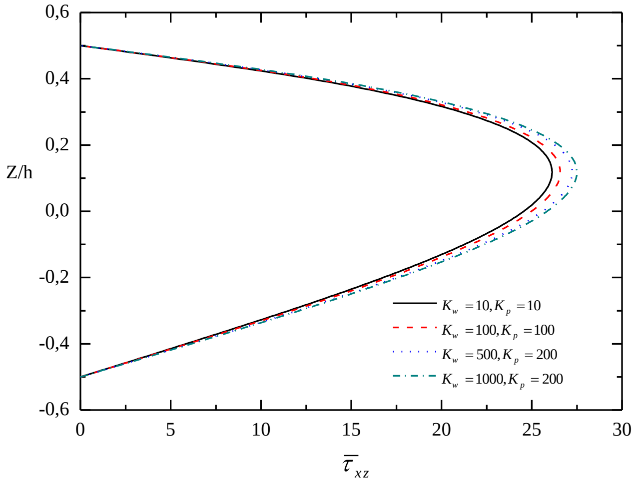

Fig. 8 Variation of the transverse shear stress $\bar{\tau}_{xz}$ through the thickness of the FG beam for different values of elastic foundations parameters $(L/h = 5, t = 3,\ \eta = 1, \omega = 4, \bar{t} = 2, T^- = 25,\ k = 1,\ T_r = 4)$

The effect of the elastic foundation parameters on the variation of the axial stresses across the thickness is plotted on Fig. 7.

According to this figure, it can be concluded that,

- Increasing in values of elastic foundation parameters leads to a increasing in values of the maximum axial stresses,
- The maximum compressive stresses occur at a point on the top surface and the maximum tensile stresses occur at a point on the bottom surface of the FG beam.

In the same way, the variation of the shear stresses through the thickness of the FG beam resting on elastic foundations for different values of parameter of these foundations is represented in Fig.8. It is observed that the increase in the parameters of the elastic foundation increases the transverse shear stresses.

5. Conclusion

In this paper, the thermodynamic response of FG beam resting on elastic foundation subjected to a harmonic temperature field across its thickness is analyzed by a refined quasi 3D theory. Influences of the temperature and elastic foundation parameters are investigated in great details. From the numerical examples it can be concluded that above mentioned parameters all affect the thermodynamic response of the FG beam especially in the first modes as it is the case of the variation of the displacement and the shear stress as a function of the arbitrary time parameter.

In addition, the present method is very efficient for the thermodynamic response analysis of FG beam resting on elastic foundation. This work can be extended in the future work for other type of materials (Arani and Kolahchi 2016, Daouadji 2017, Sharma *et al.* 2018abc, Ayat *et al.* 2018, Panjehpour *et al.* 2018, Narwariya *et al.* 2018, Mehar and Panda 2018abcd and 2019ab, Malikan 2018 and 2019, Othman and Fekry 2018, Hussain and Naeem 2019, Belbachir *et al.* 2019 and 2020, Pandey *et al.* 2019, Semmah *et al.* 2019, Hussain *et al.* 2019, Selmi 2019, Bensattalah *et al.* 2019, Abualnour *et al.* 2019, Alimirzaei *et al.* 2019, Karami *et al.* 2019bc, Adda Bedia *et al.* 2019, Timesli 2020, Asghar *et al.* 2020, Taj *et al.* 2020, Matouk *et al.* 2020, Al-Maliki *et al.* 2020, Khorasani *et al.* 2020, Bourada *et al.* 2020, Bousahla *et al.* 2020, Chikr *et al.* 2020, Bisen *et al.* 2020).

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References

Abualnour, M., Chikh, A., Hebali, H., Kaci, A., Tounsi, A., Bousahla, A.A., Tounsi, A. (2019), "Thermomechanical analysis of antisymmetric laminated reinforced composite plates using a new four variable trigonometric refined plate theory", *Comput. Concrete*, **24**(6), 489-498. https://doi.org/10.12989/cac.2019.24.6.489.

Adda Bedia, W., Houari, M.S.A., Bessaim, A., Bousahla, A.A., Tounsi, A., Saeed, T., Alhodaly, M.Sh. (2019), "A New Hyperbolic Two-Unknown Beam Model for Bending and Buckling Analysis of a Nonlocal Strain Gradient Nanobeams", *J. Nano Res.*, **57**, 175-191. https://doi.org/10.4028/www.scientific.net/JNanoR.57.175.

Addou, F.Y., Meradjah, M., Bousahla, A.A, Benachour, A., Bourada, F., Tounsi, A., Mahmoud, S.R. (2019), "Influences of porosity on dynamic response of FG plates resting on Winkler/Pasternak/Kerr foundation using quasi 3D HSDT", *Comput. Concrete*, **24**(4), 347-367. https://doi.org/10.12989/cac.2019.24.4.347.

Ahmed, R.A., Fenjan, R.M., Faleh, N.M. (2019), "Analyzing post-buckling behavior of continuously graded FG nanobeams with geometrical imperfections", *Geomech. Eng.*, **17**(2), 175-180. https://doi.org/10.12989/gae.2019.17.2.175.

Ait Atmane, H., Tounsi, A., Bernard, F. (2017), "Effect of thickness stretching and porosity on mechanical response of a functionally graded beams resting on elastic foundations", *Int. J. Mech. Mater. Des.,* **13**(1), 71–84. https://doi.org/10.1007/s10999-015-9318-x.

Al-Maliki, A.F.H., Ahmed, R.A., Moustafa, N.M., Faleh, N.M. (2020), "Finite element based modeling and thermal dynamic analysis of functionally graded graphene reinforced beams", *Adv. Comput. Design*, **5**(2), 177-193. https://doi.org/10.12989/acd.2020.5.2.177

Alimirzaei, S., Mohammadimehr, M., Tounsi, A. (2019), "Nonlinear analysis of viscoelastic micro-composite beam with geometrical imperfection using FEM: MSGT electro-magneto-elastic bending, buckling and vibration solutions", *Struct. Eng. Mech.*, **71**(5), 485-502. https://doi.org/10.12989/sem.2019.71.5.485.

Arani, A.J., and Kolahchi, R. (2016), "Buckling analysis of embedded concrete columns armed with carbon nanotubes", *Comput. Concrete*., **17**(5), 567-578.http://dx.doi.org/10.12989/cac.2016.17.5.567.

Asghar, S., Naeem, M.N., Hussain, M., Taj, M., Tounsi, A. (2020), "Prediction and assessment of nonlocal natural frequencies of DWCNTs: Vibration analysis", *Comput. Concrete*, **25**(2), 133-144. https://doi.org/10.12989/cac.2020.25.2.133.

Avcar, M. (2019), "Free vibration of imperfect sigmoid and power law functionally graded beams", *Steel Compos. Struct.*, **30**(6), 603-615. https://doi.org/10.12989/scs.2019.30.6.603.

Avcar, M., & Mohammed, W.K.M., (2018), "Free vibration of functionally graded beams resting on Winkler-Pasternak foundation", *Arab. J. Geosci.,* **11**(10), 232. https://doi.org/10.1007/s12517-018-3579-2

Ayat, H., Kellouche, Y., Ghrici, M., Boukhatem, B. (2018), "Compressive strength prediction of limestone filler concrete using artificial neural networks", *Adv. Comput. Design*, **3**(3), 289-302. https://doi.org/10.12989/acd.2018.3.3.289.

Bachir Bouiadjra, R., Mahmoudi, A., Benyoucef, S., Tounsi, A. and Bernard, F., (2018), "Analytical investigation of bending response of FGM plate using a new quasi 3D shear deformation theory: Effect of the micromechanical models", *Struct. Eng. Mech*, **66**(3), 317-328. https://doi.org/10.12989/sem.2018.66.3.317.

Bachiri, A., Bourada, M., Mahmoudi, A., Benyoucef, S., Tounsi, A., (2018), "Thermodynamic effect on the bending response of elastic foundation FG plate by using a novel four variable refined plate theory", *J. Thermal Stresses,* **41**(8), 1042–1062. https://doi.org/10.1080/01495739.2018.1452169.

Balubaid, M., Tounsi, A., Dakhel, B., Mahmoud, S.R. (2019), "Free vibration investigation of FG nanoscale plate using nonlocal two variables integral refined plate theory", *Comput. Concrete,* **24**(6), 579-586. https://doi.org/10.12989/cac.2019.24.6.579.

Barati, M.R. and Shahverdi, H. (2016), "A four-variable plate theory for thermal vibration of embedded FG nanoplates under non-uniform temperature distributions with different boundary conditions", *Struct. Eng. Mech.,* **60**(4), 707-727. https://doi.org/10.12989/sem.2016.60.4.707.

Batou, B., Nebab, M., Bennai, R., Ait Atmane, H., Tounsi, A., Bouremana, M. (2019), "Wave dispersion properties in imperfect sigmoid plates using various HSDTs", *Steel Composite Structures*, **33**(5), 699-716. https://doi.org/10.12989/scs.2019.33.5.699.

Behera, S., Kumari, P. (2018), "Free vibration of Levy-type rectangular laminated plates using efficient zig-zag theory", *Adv. Comput. Design*, **3**(3), 213-232. https://doi.org/10.12989/acd.2017.2.3.165.

Belbachir, N., Draich, K., Bousahla, A.A., Bourada, M., Tounsi, A., Mohammadimehr, M. (2019), "Bending analysis of anti-symmetric cross-ply laminated plates under nonlinear thermal and mechanical loadings", *Steel Compos. Struct.*, **33**(1), 81-92. https://doi.org/10.12989/scs.2019.33.1.081.

Belbachir, N., Bourada, M., Draiche, K., Tounsi, A., Bourada, F., Bousahla, A.A., Mahmoud, S.R. (2020), "Thermal flexural analysis of anti-symmetric cross-ply laminated plates using a four variable refined theory", *Smart Struct. Syst.*, **25**(4), 409-422. https://doi.org/10.12989/sss.2020.25.4.409

Bellal, M., Hebali, H., Heireche, H., Bousahla, A.A., Tounsi, A., Bourada, F., Mahmoud, S.R., Adda Bedia, E.A., Tounsi, A. (2020), "Buckling behavior of a single-layered graphene sheet resting on viscoelastic medium via nonlocal four-unknown integral model", *Steel Compos. Struct.,* **34**(5), 643-655. https://doi.org/10.12989/scs.2020.34.5.643.

Bensattalah, T., Zidour, M. and Daouadji, T.H. (2019), "A new nonlocal beam model for free vibration analysis of chiral single-walled carbon nanotubes", *Compos. Mater. Eng.*, **1**(1), 21-31. https://doi.org/10.12989/cme.2019.1.1.021

Berghouti, H., Adda Bedia, E.A., Benkhedda, A., Tounsi, A. (2019), "Vibration analysis of nonlocal porous nanobeams made of functionally graded material", *Adv. Nano Res.*, **7**(5), 351-364. https://doi.org/10.12989/anr.2019.7.5.351.

Bisen, H.B., Hirwani, C.K., Satankar, R.K., Panda, S.K., Mehar, K., Patel, B. (2020), "Numerical study of frequency and deflection responses of natural fiber (Luffa) reinforced polymer composite and experimental validation", *J. Natural Fiber,* **17**(4), 505-519. https://doi.org/10.1080/15440478.2018.1503129.

Boukhlif, Z., Bouremana, M., Bourada, F., Bousahla, A.A., Bourada, M., Tounsi, A., Al-Osta, M.A. (2019), "A simple quasi-3D HSDT for the dynamics analysis of FG thick plate on elastic foundation", *Steel Compos. Struct.,* **31**(5), 503-516. https://doi.org/10.12989/scs.2019.31.5.503.

Boulefrakh, L., Hebali, H., Chikh, A., Bousahla, A.A., Tounsi, A., Mahmoud, S.R. (2019), "The effect of parameters of visco-Pasternak foundation on the bending and vibration properties of a thick FG plate", *Geomech. Eng.*, **18**(2), 161-178. https://doi.org/10.12989/gae.2019.18.2.161.

Bourada, F., Bousahla, A.A., Bourada, M., Azzaz, A., Zinata, A., Tounsi, A. (2019), "Dynamic investigation of porous functionally graded beam using a sinusoidal shear deformation theory", *Wind Struct.*, **28**(1), 19-30. https://doi.org/10.12989/was.2019.28.1.019.

Bourada, F., Bousahla, A.A., Tounsi, A., Adda Bedia, E.A., Mahmoud, S.R., Benrahou, K.H., Tounsi, A. (2020), "Stability and dynamic analyses of SW-CNT reinforced concrete beam resting on elastic-foundation", *Comput. Concrete*, (Accepted).

Bousahla, A.A., Bourada, F., Mahmoud, S.R., Tounsi, A., Algarni, A., Adda Bedia, E.A., Tounsi, A. (2020), "Buckling and dynamic behavior of the simply supported CNT-RC beams using an integral-first shear deformation theory", *Comput. Concrete*, **25**(2), 155-166. https://doi.org/10.12989/cac.2020.25.2.155.

Boussoula, A., Boucham, B., Bourada, M., Bourada, F., Tounsi, A., Bousahla, A.A., Tounsi, A. (2020), "A simple nth-order shear deformation theory for thermomechanical bending analysis of different configurations of FG sandwich plates", *Smart Struct. Syst.*, **25**(2), 197-218. https://doi.org/10.12989/sss.2020.25.2.197.

Boutaleb, S., Benrahou, K.H., Bakora, A., Algarni, A., Bousahla, A.A., Tounsi, A., Mahmoud, S.R., Tounsi, A. (2019), "Dynamic Analysis of nanosize FG rectangular plates based on simple nonlocal quasi 3D HSDT", *Adv. Nano Res.*, **7**(3), 191-208. https://doi.org/10.12989/anr.2019.7.3.191.

Caliò, I., Greco, A., (2013), "Free vibrations of Timoshenko beam-columns on Pasternak foundations", *J. Vib. Control*, **19**, 686-696. https://doi.org/10.1177/1077546311433609.

Chaabane, L.A., Bourada, F., Sekkal, M., Zerouati, S., Zaoui, F.Z., Tounsi, A., Derras, A., Bousahla, A.A., Tounsi, A. (2019), "Analytical study of bending and free vibration responses of functionally graded beams resting on elastic foundation", *Struct. Eng. Mech.*, **71**(2), 185-196. https://doi.org/10.12989/sem.2019.71.2.185.

Chen, Y., Jin, G., Zhang, C., Ye, T., Xue, Y., (2018), "Thermal vibration of FGM beams with general boundary conditions using a higher-order shear deformation theory", *Compos. Part B. Eng.*, **153**, 376-386

Chikr, S.C., Kaci, A., Bousahla, A.A., Bourada, F., Tounsi, A., Adda Bedia, E.A., Mahmoud, S.R., Benrahou, K.H., Tounsi, A. (2020), "A novel four-unknown integral model for buckling response of FG sandwich plates resting on elastic foundations under various boundary conditions using Galerkin's approach", *Geomech. Eng.*, **21**(5), 471-487. https://doi.org/10.12989/gae.2020.21.5.471

Daouadji, T.H. (2017), "Analytical and numerical modeling of interfacial stresses in beams bonded with a thin plate", *Adv. Comput. Design*, **2**(1), 57-69. https://doi.org/10.12989/acd.2017.2.1.057.

Dash, S., Mehar, K., Sharma, N., Mahapatra, T.R., Panda, S.K. (2018), "Modal analysis of FG sandwich doubly curved shell structure", *Struct. Eng. Mech.*, **68**(6), 721-733. https://doi.org/10.12989/sem.2018.68.6.721.

Dash, S., Mehar, K., Sharma, N., Mahapatra, T.R., Panda, S.K. (2019), "Finite element solution of stress and flexural strength of functionally graded doubly curved sandwich shell panel", *Earthq. Struct.*, **16**(1), 55-67. https://doi.org/10.12989/eas.2019.16.1.055.

Draiche, K., Bousahla, A.A., Tounsi, A., Alwabli, A.S., Tounsi, A., Mahmoud, S.R. (2019), "Static analysis of laminated reinforced composite plates using a simple first-order shear deformation theory", *Comput. Concrete*, **24**(4), 369-378. https://doi.org/10.12989/cac.2019.24.4.369.

Draoui, A., Zidour, M., Tounsi, A., Adim, B. (2019), "Static and dynamic behavior of nanotubes-reinforced sandwich plates using (FSDT)", *J. Nano Res.*, **57**, 117-135. https://doi.org/10.4028/www.scientific.net/JNanoR.57.117.

Duy, H.T., Van, T.N., Noh, H.C., (2014), "Eigen analysis of functionally graded beams with variable cross-section resting on elastic supports and elastic foundation", *Struct. Eng. Mech.* **52**(5), 1033–1049. https://doi.org/10.12989/sem.2014.52.5.1033.

Faleh, N.M., Ahmed, R.A., Fenjan, R.M. (2018), "On vibrations of porous FG nanoshells*", J. Eng. Sci.,* **133**, 1-14.

Faleh, N.M., Fenjan, R. M., & Ahmed, R. A. (2020). "Forced Vibrations of Multi-phase Crystalline Porous Shells Based on Strain Gradient Elasticity and Pulse Load Effects", *J. Vib. Eng. Technol.*., 10.1007/s42417-020-00203-8.

Fenjan, R.M., Ahmed, R.A., Alasadi, A.A., Faleh, N.M. (2019b). "Nonlocal strain gradient thermal vibration analysis of double-coupled metal foam plate system with uniform and non-uniform porosities", *Coupled Syst Mech*., **8**(3),247–257. https://doi.org/10.12989/csm.2019.8.3.247.

Fenjan, RM, Ahmed RA, Faleh, N.M. (2019a), "Investigating dynamic stability of metal foam nanoplates under periodic in-plane loads via a three-unknown plate theory", *Adv. Aircr. Spacecr. Sci.*, **6**(4), 297-314. https://doi.org/10.12989/aas.2019.6.4.297.

Fu, Y., Wang, J., Mao, Y., (2012), "Nonlinear analysis of buckling, free vibration and dynamic stability for the piezoelectric functionally graded beams in thermal environment", *Appl. Math. Model*. **36**, 4324–4340. https://doi.org/10.1016/j.apm.2011.11.059.

Ghiasian, S.E., Kiani, Y., Eslami, M.R. (2013), "Dynamic buckling of suddenly heated or compressed FGM beams resting on nonlinear elastic foundation", *Compos. Struct.*, **106**, 225–234. https://doi.org/10.1016/j.compstruct.2013.06.001.

Ghiasian, S.E., Kiani, Y., Eslami, M.R. (2015), "Nonlinear thermal dynamic buckling of FGM beams", *European J. Mech.* , **54**, 232-242. https://doi.org/10.1016/j.euromechsol.2015.07.004.

Giunta, G., Crisafulli, D., Belouettar, S., Carrera, E. (2013), "A thermo-mechanical analysis of functionally graded beams via hierarchical modeling", *Compos. Struct.*, **95**(1), 676-90. https://doi.org/10.1016/j.compstruct.2012.08.013.

Gul, U., Aydogdu, M., Karacam, F. (2019), "Dynamics of a functionally graded Timoshenko beam considering new spectrums", *Compos. Struct.*, **207**, 273-291. https://doi.org/10.1016/j.compstruct.2018.09.021.

Hebbar, N., Bourada, M., Sekkal, M., Tounsi A., and. Mahmoud, S.R. (2018), "A novel four-unknown quasi-3D shear deformation theory for functionally graded plates", *Steel Compos. Struct.,* **27**(5), 599-611. https://doi.org/10.12989/scs.2018.27.5.599.

Hussain, M., Naeem, M.N. (2019), "Effects of ring supports on vibration of armchair and zigzag FGM rotating carbon nanotubes using Galerkin's method", *Compos. Part B. Eng.,* **163**, 548-561. https://doi.org/10.1016/j.compositesb.2018.12.144.

Hussain, M., Naeem, M.N., Tounsi, A., Taj, M. (2019), "Nonlocal effect on the vibration of armchair and zigzag SWCNTs with bending rigidity", *Adv. Nano Res.*, **7**(6), 431-442. https://doi.org/10.12989/anr.2019.7.6.431.

Kaddari, M., Kaci, A., Bousahla, A.A.,Tounsi, A., Bourada, F., Tounsi, A., Adda Bedia, E.A., Al-Osta, M.A. (2020), "A study on the structural behaviour of functionally graded porous plates on elastic foundation using a new quasi-3D model: Bending and Free vibration analysis", *Comput. Concrete*, **25**(1), 37-57. https://doi.org/10.12989/cac.2020.25.1.037.

Karami, B., Janghorban, M. and Tounsi, A. (2019b), "Galerkin's approach for buckling analysis of functionally graded anisotropic nanoplates/different boundary conditions", *Eng. Comput.*, **35**, 1297-1316. https://doi.org/10.1007/s00366-018-0664-9.

Karami, B., Janghorban, M., Tounsi, A. (2019a), "Wave propagation of functionally graded anisotropic nanoplates resting on Winkler-Pasternak foundation", *Struct. Eng. Mech.*, **70**(1), 55-66. https://doi.org/10.12989/sem.2019.70.1.055.

Karami, B., Janghorban, M., Tounsi, A. (2019c), "On pre-stressed functionally graded anisotropic nanoshell in magnetic field", J. Brazilian Soc. Mech. Sci. Eng., **41**, 495. https://doi.org/10.1007/s40430-019-1996-0.

Khorasani, M., Eyvazian, A., Karbon, M., Tounsi, A., Lampani, L., Sebaey, T.A. (2020), "Magneto-Electro-Elastic Vibration Analysis of Modified Couple Stress-Based Three-Layered Micro Rectangular Plates Exposed to Multi-Physical Fields Considering the Flexoelectricity Effects", *Smart Struct. Syst.*, (Accepted).

Lal, A.,Jagtap, K.R., Singh, B.N. (2017), "Thermo-mechanically induced finite element based nonlinear static response of elastically supported functionally graded plate with random system properties", *Adv. Comput. Design*, **2**(3), 165-194. https://doi.org/10.12989/acd.2017.2.3.165.

Malikan, M. (2018), "Buckling Analysis of a Micro Composite Plate with Nano Coating Based on the Modified Couple Stress Theory", *J. Appl. Comput. Mech.*, **4**(1), 1-15. https://doi.org/10.22055/jacm.2017.21820.1117.

Malikan, M. (2019), "On the Buckling Response of Axially Pressurized Nanotubes Based on a Novel Nonlocal Beam Theory", *J. Appl. Comput. Mech.*, **5**(1), 103-112. https://doi.org/10.22055/jacm.2018.25507.1274

Mantari, J.L., Yarasca, J. (2015), "A simple and accurate generalized shear deformation theory for beams", *Compo. struc.*, **134**, 593-601. https://doi.org/10.1016/j.compstruct.2015.08.073.

Matouk, H., Bousahla, A.A., Heireche, H., Bourada, F., Adda Bedia, E. A., Tounsi, A., Mahmoud, S.R., Tounsi, A., Benrahou, K.H. (2020), "Investigation on hygro-thermal vibration of P-FG and symmetric S-FG nanobeam using integral Timoshenko beam theory", *Adv. Nano Res.*, **8**.

Medani, M., Benahmed, A., Zidour, M., Heireche, H., Tounsi, A., Bousahla, A.A., Tounsi, A., Mahmoud, S.R. (2019), "Static and dynamic behavior of (FG-CNT) reinforced porous sandwich plate using energy principle", *Steel Compos. Struct.,* **32**(5), 595-610. https://doi.org/10.12989/scs.2019.32.5.595.

Mehar, K. and Panda, S. (2018c), "Thermoelastic flexural analysis of FG-CNT doubly curved shell panel", *Aircraft Eng. Aerospace Technol.*, **90**(1), 11-23. https://doi.org/10.1108/AEAT-11-2015-0237.

Mehar, K. and Panda, S. (2018d), "Nonlinear finite element solutions of thermoelastic flexural strength and stress values of temperature dependent graded CNT-reinforced sandwich shallow shell structure", *Struct. Eng. Mech.*, **67**(6), 565-578. https://doi.org/10.12989/sem.2018.67.6.565.

Mehar, K., Mahapatra, T.R., Panda, S.K., Katariya, P.V., Tompe, U.K. (2018c), "Finite-element solution to nonlocal elasticity and scale effect on frequency behavior of shear deformable nanoplate structure", *J. Eng. Mech.*, **144**(9), 04018094. https://doi.org/10.1061/(ASCE)EM.1943-7889.0001519.

Mehar, K., Mishra, P.K., Panda, S.K. (2019b), "Numerical investigation of thermal frequency responses of graded hybrid smart nanocomposite (CNT-SMA-Epoxy) structure", *Mech. Adv. Mater. Struct.*, (In press). https://doi.org/10.1080/15376494.2020.1725193.

Mehar, K., Panda, S.K. (2018a), "Elastic bending and stress analysis of carbon nanotube-reinforced composite plate: Experimental, numerical, and simulation", *Adv. Polym. Technol.*, **37**(6), 1643-1657. https://doi.org/10.1002/adv.21821.

Mehar, K., Panda, S.K. (2018b), "Thermal free vibration behavior of FG-CNT reinforced sandwich curved panel using finite element method", *Polymer Compos.*, **39**(8), 2751-2764. https://doi.org/10.1002/pc.24266.

Mehar, K., Panda, S.K. (2019a), "Theoretical deflection analysis of multi-walled carbon nanotube reinforced sandwich panel and experimental verification", *Compos. Part B. Eng.*, **167**, 317-328. https://doi.org/10.1016/j.compositesb.2018.12.058.

Mehar, K., Panda, S.K. (2019b), "Multiscale modeling approach for thermal buckling analysis of nanocomposite curved structure", *Adv. Nano Res.*, **7**(3), 181-190. https://doi.org/10.12989/anr.2019.7.3.181.

Mehar, K., Panda, S.K. (2020), "Nonlinear deformation and stress responses of a graded carbon nanotube sandwich plate structure under thermoelastic loading", *Acta Mech*, **231**, 1105–1123. https://doi.org/10.1007/s00707-019-02579-5.

Mehar, K., Panda, S.K., Devarajan, Y., Choubey, G. (2019a), "Numerical buckling analysis of graded CNT-reinforced composite sandwich shell structure under thermal loading", *Compos. Struct.*, **240**, 112064. https://doi.org/10.1016/j.compstruct.2019.03.002.

Mehar, K., Panda, S.K., Mahapatra, T.R. (2018a), "Thermoelastic deflection responses of CNT reinforced sandwich shell structure using finite element method", *Scientia Iranica*, **25**(5), 2722-27370. https://doi.org/ 10.24200/sci.2017.4525.

Mehar, K., Panda, S.K., Mahapatra, T.R. (2018d), "Nonlinear frequency responses of functionally graded carbon nanotube-reinforced sandwich curved panel under uniform temperature field", *J. Appl. Mech.*, **10**(3), 1850028. https://doi.org/10.1142/S175882511850028X.

Mehar, K., Panda, S.K., Mahapatra, T.R. (2019c), "Large deformation bending responses of nanotube-reinforced polymer composite panel structure: Numerical and experimental analyses", *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering*, **233**(5), 1695-1704. https://doi.org/10.1177/0954410018761192.

Mehar, K., Panda, S.K., Patle, B.K. (2018b), "Stress, deflection, and frequency analysis of CNT reinforced graded sandwich plate under uniform and linear thermal environment: A finite element approach", *Polymer Compos.*, **39**(10), 3792-3809. https://doi.org/10.1002/pc.24409.

Mehar, K., Panda, S.K., Sharma, N. (2020), "Numerical investigation and experimental verification of thermal frequency of carbon nanotube-reinforced sandwich structure", *Eng. Struct.*, **211**, 110444. https://doi.org/10.1016/j.engstruct.2020.110444.

Mekerbi, M., Benyoucef, S., Mahmoudi, A., Tounsi, A., Bousahla., A.A., Mahmoud, S.R. (2019), "Thermodynamic behavior of functionally graded sandwich plates resting on different elastic foundation and with various boundary conditions", *J. Sandwich Struct. Mater.*, (In press). https://doi.org/10.1177/1099636219851281.

Narwariya, M.,Choudhury, A., Sharma, A.K. (2018), "Harmonic analysis of moderately thick symmetric cross-ply laminated composite plate using FEM", *Adv. Comput. Design*, **3**(2), 113-132. https://doi.org/10.12989/acd.2018.3.2.113.

Nguyen, T.K. and Nguyen B. D., (2015), "A new higher-order shear deformation theory for static, buckling and free vibration analysis of functionally graded sandwich beams", *J. Sandwich Struct. Mater.*, **17**(6), 613-631. https://doi.org/10.1177/1099636215589237.

Othman, M. and Fekry, M. (2018), "Effect of rotation and gravity on generalized thermo-viscoelastic medium with voids", *Multidiscipline Modeling Mater. Struct.*, **14**(2), 322-338. 10.1108/MMMS-08-2017-0082.

Pandey, H.K., Hirwani, C.K., Sharma, N., Katariya, P.V., Panda, S.K. (2019), "Effect of nano glass cenosphere filler on hybrid composite eigenfrequency responses - An FEM approach and experimental verification", *Adv. Nano Res.*, **7**(6), 419-429. https://doi.org/10.12989/anr.2019.7.6.419.

Panjehpour, M., Eric Woo Kee Loh, Deepak TJ. (2018), "Structural Insulated Panels: State-of-the-Art", *Trends in Civil Eng. Architecture*, **3**(1) 336-340. 10.32474/TCEIA.2018.03.000151.

Pradhan, K. K. and Chakraverty, S. (2013), "Free vibration of Euler and Timoshenko functionally graded beams by Rayleigh–Ritz method", *Compos. Part B Eng*, **51**, 175-184. https://doi.org/10.1016/j.compositesb.2013.02.027.

Rahmani, M.C., Kaci, A., Bousahla, A.A., Bourada, F., Tounsi, A., Adda Bedia, E.A., Mahmoud, S.R., Benrahou, K.H., Tounsi, A. (2020), "Influence of boundary conditions on the bending and free vibration behavior of FGM sandwich plates using a four-unknown refined integral plate theory", *Comput. Concrete*, **25**(3), 225-244. https://doi.org/10.12989/cac.2020.25.3.225.

Ramteke, P.M., Panda, S.K., Sharma, N. (2019), "Effect of grading pattern and porosity on the eigen characteristics of porous functionally graded structure", *Steel Compos. Struct.,* **33**(6), 865-875. https://doi.org/10.12989/scs.2019.33.6.865.

Ranjan, S., Khan, R., Dash, S., Sharma, N., Mahapatra, T.R., Panda, S.K. (2019), "Thermo-elastic free vibration analysis of functionally graded flat panel with temperature gradient along thickness", *IOP Conference Series: Materials Science and Engineering*, **577**(1), 012123. https://doi.org/10.1088/1757-899X/577/1/012123.

Refrafi, S., Bousahla, A.A., Bouhadra, A., Menasria, A., Bourada, F., Tounsi, A., Adda Bedia, E.A., Mahmoud, S.R., Benrahou, K.H., Tounsi, A. (2020), "Effects of hygro-thermo-mechanical conditions on the buckling of FG sandwich plates resting on elastic foundations", *Comput. Concrete*, **25**(4).

Rezaiee-Pajand, M., Masoodi, A.R.,Mokhtari, M. (2018), "Static analysis of functionally graded non-prismatic sandwich beams", *Adv. Comput. Design*, **3**(2), 165-190. https://doi.org/10.12989/acd.2018...165.

Sahla, F., Saidi, H., Draiche, K., Bousahla, A.A., Bourada, F., Tounsi, A. (2019), "Free vibration analysis of angle-ply laminated composite and soft core sandwich plates", *Steel Compos. Struct.,* **33**(5), 663-679.

https://doi.org/10.12989/scs.2019.33.5.663.

Sahu, P., Sharma, N., Panda, S.K. (2020a), "Numerical prediction and experimental validation of free vibration responses of hybrid composite (Glass/Carbon/Kevlar) curved panel structure", *Compos. Struct.* **241**, 112073. https://doi.org/10.1016/j.compstruct.2020.112073.

Sahu, P., Sharma, N., Panda, S.K. (2020b), "Numerical eigen-frequencies of hybrid fiber composite curved shell panels", *Materials Today: Proceedings*, (In press). https://doi.org/10.1016/j.matpr.2020.02.473.

Salah, F., Boucham, B., Bourada, F., Benzair, A., Bousahla, A.A., Tounsi, A. (2019), "Investigation of thermal buckling properties of ceramic-metal FGM sandwich plates using 2D integral plate model", *Steel Compos. Struct.,* **33**(6), 805-822. https://doi.org/10.12989/scs.2019.33.6.805.

Selmi, A. (2019), "Effectiveness of SWNT in reducing the crack effect on the dynamic behavior of aluminium alloy", *Adv. Nano Res.*, **7**(5), 365-377. https://doi.org/10.12989/anr.2019.7.5.365.

Semmah, A., Heireche, H., Bousahla, A.A., Tounsi, A. (2019), "Thermal buckling analysis of SWBNNT on Winkler foundation by non local FSDT", *Adv. Nano Res.*, **7**(2), 89-98. https://doi.org/10.12989/anr.2019.7.2.089.

Sharma, N., Mahapatra, T.R., Panda, S.K. (2018a), "Analysis of vibro-acoustic response of un-baffled laminated composite conical shell panel with varying thickness", *IOP Conference Series: Mater. Sci. Eng.*, **377**(1), 012139. https://doi.org/10.1088/1757-899X/377/1/012139.

Sharma, N., Mahapatra, T.R., Panda, S.K., Mazumdar, A. (2018b), "Acoustic radiation characteristics of un-baffled laminated composite conical shell panels", *Materials Today: Proceedings*, **5**(11), 24387-24396. https://doi.org/10.1016/j.matpr.2018.10.234.

Sharma, N., Mahapatra, T.R., Panda, S.K., Mehar, K. (2018c), "Evaluation of vibroacoustic responses of laminated composite sandwich structure using higher-order finite-boundary element model", *Steel Compos. Struct.,* **28**(5), 629-639. https://doi.org/10.12989/scs.2018.28.5.629.

Şimşek, M., (2010), "Fundamental frequency analysis of functionally graded beams by using different higher-order beam theories", *Nucl. Eng. Des.*, **240**(4), 697-705. https://doi.org/10.1016/j.nucengdes.2009.12.013.

Singh, V.K., Hirwani, C.K., Panda, S.K., Mahapatra, T.R., Mehar, K. (2019), "Numerical and experimental nonlinear dynamic response reduction of smart composite curved structure using collocation and non-collocation configuration", *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Sciences,* **233**(5), 1601-1619. https://doi.org/10.1177/0954406218774362.

Taj, M., Majeed, A., Hussain, M., Naeem, M.N., Safeer, M., Ahmad, M., Khan, H.U., Tounsi, A. (2020), "Non-local orthotropic elastic shell model for vibration analysis of protein microtubules", *Comput. Concrete*, **25**(3), 245-253. https://doi.org/10.12989/cac.2020.25.3.245.

Timesli, A. (2020), "An efficient approach for prediction of the nonlocal critical buckling load of double-walled carbon nanotubes using the nonlocal Donnell shell theory", *SN Appl. Sci.*, **2**, 407. https://doi.org/10.1007/s42452-020-2182-9.

Tlidji, Y., Zidour, M., Draiche, K., Safa, A., Bourada, M., Tounsi, A., Bousahla, A.A., Mahmoud, S.R. (2019), "Vibration analysis of different material distributions of functionally graded microbeam", *Struct. Eng. Mech.*, **69**(6), 637-649. https://doi.org/10.12989/sem.2019.69.6.637.

Tounsi, A., Al-Dulaijan, S.U., Al-Osta, M.A., Chikh, A., Al-Zahrani, M.M., Sharif, A., Tounsi, A. (2020), "A four variable trigonometric integral plate theory for hygro-thermo-mechanical bending analysis of AFG ceramic-metal plates resting on a two-parameter elastic foundation", *Steel Compos. Struct.,* **34**(4), 511-524. https://doi.org/10.12989/scs.2020.34.4.511.

Ying, J., Lu, C.F., Chen, W.Q., (2008), "Two-dimensional elasticity solutions for functionally graded beams resting on elastic foundations", *Compos. Struct.*, **84**, 209–219. 10.1016/j.compstruct.2007.07.004.

Zaoui, F.Z., Ouinas, D., Tounsi, A. (2019), "New 2D and quasi-3D shear deformation theories for free vibration of functionally graded plates on elastic foundations", *Compos. Part B*, **159**, 231-247. https://doi.org/10.1016/j.compositesb.2018.09.051.

Zarga, D., Tounsi, A., Bousahla, A.A., Bourada, F., Mahmoud, S.R. (2019), "Thermomechanical bending study for functionally graded sandwich plates using a simple quasi-3D shear deformation theory", *Steel Compos. Struct.,* **32**(3), 389-410. https://doi.org/10.12989/scs.2019.32.3.389.

Zenkour, A. M., Sobhy, M., (2013), "Dynamic bending response of thermoelastic functionally graded plates resting on elastic foundations", *Aerosp. Sci. Technol.*, **29**(1), 7-17. https://doi.org/10.1016/j.ast.2013.01.003.

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