Structural Engineering and Mechanics, *Vol. 76, No. 1 (2020) 17-26*
DOI: https://doi.org/10.12989/sem.2020.76.1.017

Effect of length scale parameters on transversely isotropic thermoelastic medium using new modified couple stress theory

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(Received April 30, 2019, Revised May 4, 2020, Accepted May 18, 2020)

Abstract. The objective of this paper is to study the deformation in transversely isotropic thermoelastic solid using new modified couple stress theory subjected to ramp-type thermal source and without energy dissipation. This theory contains three material length scale parameters which can determine the size effects. The couple stress constitutive relationships are introduced for transversely isotropic thermoelastic solid, in which the curvature (rotation gradient) tensor is asymmetric and the couple stress moment tensor is symmetric. Laplace and Fourier transform technique is applied to obtain the solutions of the governing equations. The displacement components, stress components, temperature change and couple stress are obtained in the transformed domain. A numerical inversion technique has been used to obtain the solutions in the physical domain. The effects of length scale parameters are depicted graphically on the resulted quantities. Numerical results show that the proposed model can capture the scale effects of microstructures.

Keywords: new modified couple stress theory; length scale parameters; transversely isotropic; ramp type heat; Laplace and Fourier transform

1. Introduction

Transverse isotropy is important because many artificial and natural materials behave effectively as transversely isotropic elastic materials. A large number of joints in advanced electronic devices are carried out with the application of these materials. Couple stress theory is an extension to continuum theory that includes the effects of couple stresses, in addition to the classical direct and shear forces per unit area. First mathematical model to examine the materials with couple stresses was presented by Cosserat and Cosserat (1909). In this theory, both curvature tensor and the couple stress moment tensor are asymmetric and every particle is assumed to be capable of both linear displacement and rotation during the deformation of the material. Because of the failure of establishing the constitutive relationships, this theory was not given importance by researchers. However, Tiwari (1971) determined the effect of couple stress on deflection produced in a semi-infinite elastic body because of impulsive twist over surface using Cosserat equations.

Mindlin and Tierstein (1962) were first to formulate the complete boundary value problem of couple stress theory. Koiter (1964) introduced the constitutive relationships for couple stress theory, involving length scale parameters to predict the size effects. This version of theory is known as Mindlin-Tierstein-Koiter Couple stress theory. This theory suffers from some inconsistencies, such as the indeterminacy of the couple-stress tensor, inconsistent boundary conditions and the consideration of the redundant body couple distribution. Employing the balance law for moments of couple besides the balance laws for forces and moment of forces a modified couple stress theory (M-CST) with one length scale parameter was offered by Yang *et al.* (2002). Application of this equilibrium equation leads to a symmetric couple-stress tensor. In M-CST

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right),$$

$$\widehat{\chi_{ij}} = \frac{1}{2}\left(\omega_{i,j} + \omega_{j,i}\right),$$

$$\omega_i = \frac{1}{2} e_{ijk} u_{k,j},\ \sigma_{ij} = \lambda \varepsilon_{kk}\delta_{ij} + 2G\varepsilon_{ij}$$

$$\widehat{m_{ij}} = 2l^2 G \widehat{\chi_{ij}}$$

Here u_i is the displacement vector, m_{ij} is couple stress moment tensor, λ and G are lame's constant, l is material length scale, σ_{ij} is stress tensor, ϵ_{ij} is strain tensor, χ_{ij} is curvature tensor, ω_i are rotation components. Tsiatas and Yiotis (2010) proposed a modified couple stress model for the static study of orthotropic micro-plates with various shapes, aspect and Poisson's ratios subjected to various boundary conditions and on the basis of principle of minimum potential energy. Ke *et al.* (2012) studied the nonlinear free vibration problem of a functionally graded micro-beam according to the modified couple stress theory. Najafi (2012) investigated the quality factor of thermo-elastic damping in an electro-statically deflected micro-beam resonator using Hamilton principles based on modified couple stress theory and hyperbolic heat conduction model. Free vibration analysis of a three

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http://www.techno-press.com/journals/sem&subpage=7
ISSN: 1225-4568 (Print), 1598-6217 (Online)

dimensional cylindrical micro-beam on the basis of modified couple stress theory was done by Wang *et al.* (2013). Roque *et al.* (2013) examined the bending of simply supported micro isotropic plates using modified couple stress theory and a meshless method. The added scale parameter produces an effect according to the size of the plate, the effect getting to be smaller as the plate size increases. Shaat *et al.* (2014) studied the bending analysis of nano-sized Kirchhoff plates using modified couple-stress theory including surface energy and microstructure effects. Kumar *et al.* (2015) studied the impact of Hall current and rotation in thermoelastic diffusive media caused by ramp type loading on the basis of modified couple stress theory using Laplace and Fourier Transform techniques. Khorshidi and Shariati (2015) gave an exact solution on the basis of modified couple stress theory for the analysis of postbuckling conduct of shear deformable micro-/nanoscale carbon beams. Shafiei *et al.* (2016a) did the nonlinear vibration study of axially nonuniform FG microbeams using modified couple stress theory and on the basis of Euler–Bernoulli beam theory and Von-Kármán's strain. Atanasov *et al.* (2017) examined the thermal effect on the free vibration and buckling of the Euler-Bernoulli double microbeam system based on the modified couple stress theory using Bernoulli–Fourier method. Togun and Bağdatlı. (2017) presented the linear free vibration of a simply-supported by using modified couple stress theory and Hamilton's principle and analyzed the effects of the length scale parameter and the Poisson's ratio on natural frequency showing that the natural frequency is decreased as the dimensionless scale parameter is magnified. Vibrational frequency of a tapered microbeam resonator was examined via a generalized thermoelastic theory in connection with modified couple stress theory by Zenkour (2018). Despite of this several researchers worked on different theory of thermoelasticity and similar concept as Marin (1997,2007), Marin and Baleanu (2016), Marin and Stan (2013), Marin (1998,2009,2010) and Lata and Kaur (2019a,2019b,2019c), Khorshidi (2018), Li *et al.* (2019), Zhang and Li(2020),Guo *et al.*(2016,2018), Reddy *et al.*(2016), El-Karamany and Ezzat(2011), Ezzat and Ewad(2010), Ezzat and Abd-Elaal(1997,1997a)..

M-CST cannot describe the pure bending of plate properly as no couple stresses and no size-effects are predicted for pure bending of plate. So, Hadjesfandiari and Dargush (2011) gave consistent couple stress theory (C-CST) with the skew-symmetric couple-stresses. Here,

$$\chi_{ij} = \frac{1}{2}\left(\omega_{i,j} - \omega_{j,i}\right)$$

$$\sigma_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + 2G\varepsilon_{ij} + 2Gl^2 e_{ijk}\nabla^2\omega_k$$

$$m_{ij} = 4l^2 G\chi_{ij}$$

Hadjesfandiari *et al.* (2018) developed size-dependent Timoshenko beam model using C-CST. Laminated composite materials are anisotropic and are usually used in engineering. Modified couple stress theory could not be applied to anisotropic materials. So, Chen and Li (2014) introduced the new modified couple stress theory (NM-CST) for anisotropic materials containing three length scale parameters. For NM- couple stress theory,

$$\chi_{ij} = \omega_{i,j}$$

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl}$$

$$m_{ij} = l_i^2 G_i\chi_{ij} + l_j^2 G_j\chi_{ji}$$

$c_{ijkl}$are elastic parameters, l_i and l_j are material length scale parameters. Chen *et al.* (2014) studied the scale effects of composite laminated plates using new modified couple stress theory by finite element method.

In the present investigation, our objective is to study the deformation in transversely isotropic thermoelastic solid using new modified couple stress theory without energy dissipation. The solid is employed to ramp-type heating. The couple stress constitutive relationships are introduced for transversely isotropic thermoelasticity, in which the curvature (rotation gradient) tensor is asymmetric and the couple stress moment tensor is symmetric. Laplace and Fourier transform technique is applied to obtain the solutions of the governing equations. The displacement components, stress components, temperature change and couple stress are obtained in the transformed domain and are presented graphically for different values of displacement. The effects of length scale parameters on resulting quantities are also depicted graphically.

2. Basic equations

Following Chen and Li (2014), Kumar and Devi (2015) and Kumar *et al.*(2015c), the field equations transversely isotropic thermoelastic solid using new modified couple stress theory in the absence of body forces, body couple and without energy dissipation are given by

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} + \frac{1}{2}e_{ijk}m_{lk,l} - \beta_{ij}T \tag{1}$$

$$c_{ijkl}\varepsilon_{kl,j} + \frac{1}{2}e_{ijk}m_{lk,lj} - \beta_{ij}T_{,j} = \rho\ddot{u}_i \tag{2}$$

$$K_{ij}T_{,ij} - \rho C_E\ddot{T} = \beta_{ij}T_0\ddot{\varepsilon}_{ij} \tag{3}$$

where

$$\beta_{ij} = c_{ijkl}\alpha_{ij} \tag{4}$$

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{5}$$

$$m_{ij} = l_i^2 G_i\chi_{ij} + l_j^2 G_j\chi_{ji} \tag{6}$$

$$\chi_{ij} = \omega_{i,j} \tag{7}$$

$$\omega_i = \frac{1}{2}e_{ijk}u_{k,j} \tag{8}$$

Here $u = (u_1, u_2, u_3)$ is the components of displacement vector, $c_{ijkl}(c_{ijkl} = c_{ijlk} = c_{jikl} = c_{jilk})$ are elastic parameters, σ_{ij} are the components of stress tensor, ε_{ij} are the components of strain tensor,e_{ijk} is alternate tensor, m_{ij} are the components of couple-stress, α_{ij} are the coefficients of linear thermal expansion, β_{ij} is thermal tensor, T is the temperature change, $l_i(\mathrm{i}=1,2,3)$ are material length scale parameters , χ_{ij} is curvature, ω_i is the rotational vector, ρ is the density, K_{ij} isthe thermal conductivity, c_E is the specific heat at constant strain, T_0 is the reference temperature assumed to be such that $T/T_0 \ll 1$, G_i are the elasticity constants and $\beta_1=(c_{11}+c_{12})\alpha_1+c_{13}\alpha_3$, $\beta_3=2c_{13}\alpha_1+c_{33}\alpha_3$.

3. Formulation and solution of the problem

We consider a two dimensional homogeneous transversely isotropic thermoelastic solid using new modified couple stress theory initially at uniform temperature T_0 occupying the region of a half space $x_3\geq 0$ .A rectangular coordinate system (x_1,x_2,x_3) having origin on the surface $x_3=0$ has been taken. All the field quantities depend on (x_1,x_3,t). We have used appropriate transformation using Slaughter (2002), on the set of equation (1) - (3) to derive the equations for transversely isotropic thermoelastic solid under consideration.

Equation of motion in u_1-u_3 plane are given by

$$c_{11}u_{1,11}+\left(c_{44}-\frac{1}{4}l_2^2G_2\nabla^2\right)u_{1,33}+\left(c_{13}+c_{44}+\frac{1}{4}l_2^2G_2\nabla^2\right)u_{3,13}-\beta_1\frac{\partial T}{\partial x_1}=\rho\ddot{u}_1 \tag{9}$$

$$c_{33}u_{3,33}+\left(c_{44}+c_{13}+\frac{1}{4}l_2^2G_2\nabla^2\right)u_{1,31}+\left(c_{44}+\frac{1}{4}l_2^2G_2\nabla^2\right)u_{3,11}-\beta_3\frac{\partial T}{\partial x_3}=\rho\ddot{u}_3 \tag{10}$$

Equation of heat conduction without energy dissipation is given by

$$K_1\frac{\partial^2 T}{\partial x_1^2}+K_3\frac{\partial^2 T}{\partial x_3^2}-\rho c_E\frac{\partial^2 T}{\partial t^2}=T_0\frac{\partial}{\partial t}\left(\beta_1\frac{\partial u_1}{\partial x_1}+\beta_3\frac{\partial u_3}{\partial x_3}\right) \tag{11}$$

where

$$\beta_1=(c_{11}+c_{12})\alpha_1+c_{13}\alpha_3\ ,\qquad \beta_3=2c_{13}\alpha_1+c_{33}\alpha_3$$

where $\nabla^2=\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_3^2}\right)$ .And comma in the subscript denotes the derivative w.r.t. displacement component written after comma. In the above equation we use contracting subscript notation $(1\to 11, 2\to 22, 3\to 33, 4\to 23, 5\to 31, 6\to 12)$ to relate c_{ijkl} to c_{mn}.

And the constitutive relationships are

$$\sigma_{33}=c_{13}\frac{\partial u_1}{\partial x_1}+c_{33}\frac{\partial u_3}{\partial x_3}-\beta_3 T \tag{12}$$

$$\sigma_{31}=c_{44}\left(\frac{\partial u_1}{\partial x_3}+\frac{\partial u_3}{\partial x_1}\right)+\frac{1}{4}\left((l_1^2G_1-l_2^2G_2)\left(-\frac{\partial^3 u_1}{\partial x_3\partial x_1^2}+\frac{\partial^3 u_3}{\partial x_1^3}\right)+(l_3^2G_3-l_2^2G_2)\left(-\frac{\partial^3 u_1}{\partial x_3^3}+\frac{\partial^3 u_3}{\partial x_1^3}\right)\right) \tag{13}$$

$$m_{32}=\frac{1}{2}(l_2^2G_2-l_3^2G_3)\left(\frac{\partial^2 u_1}{\partial x_3^2}-\frac{\partial^2 u_3}{\partial x_1\partial x_3}\right) \tag{14}$$

To facilitate the solution following dimensionless quantities are used

$$x_1'=\frac{x_1}{L}, x_3'=\frac{x_3}{L}, u_1'=\frac{\rho c_1^2}{L\beta_1T_0}u_1\ , u_3'=\frac{\rho c_1^2}{L\beta_1T_0}u_3, T'=\frac{T}{T_0}, t'=\frac{c_1}{L}t, \sigma_{11}'=\frac{\sigma_{11}}{\beta_1T_0}, \sigma_{33}'=\frac{\sigma_{33}}{\beta_1T_0}, m_{32}'=\frac{m_{32}}{L\beta_1T_0} \tag{15}$$

where $c_1^2=\frac{c_{11}}{\rho}$ and L is constant of dimension of length.

Using the dimensionless quantities defined by (15) into equations (9) - (14) and suppressing the primes, we obtain

$$\frac{\partial^2 u_1}{\partial x_1^2}+\left(\delta_1-\frac{1}{4L^2c_{11}}l_2^2G_2\left(\frac{\partial^2}{\partial x_1'^2}+\frac{\partial^2}{\partial x_3'^2}\right)\right)\frac{\partial^2 u_1}{\partial x_3^2}+\left(\delta_2+\frac{1}{4L^2c_{11}}l_2^2G_2\left(\frac{\partial^2}{\partial x_1'^2}+\frac{\partial^2}{\partial x_3'^2}\right)\right)\frac{\partial^2 u_3}{\partial x_1\partial x_3}-\frac{\partial T}{\partial x_1}=\frac{\partial^2 u_1}{\partial t^2} \tag{16}$$

$$\delta_4\frac{\partial^2 u_3}{\partial x_3^2}+\left(\delta_2+\frac{1}{4L^2c_{11}}l_2^2G_2\left(\frac{\partial^2}{\partial x_1'^2}+\frac{\partial^2}{\partial x_3'^2}\right)\right)\frac{\partial^2 u_1}{\partial x_1\partial x_3}+\left(\delta_1-\frac{1}{4L^2c_{11}}l_2^2G_2\left(\frac{\partial^2}{\partial x_1'^2}+\frac{\partial^2}{\partial x_3'^2}\right)\right)\frac{\partial^2 u_3}{\partial x_1^2}-p_5\frac{\partial T}{\partial x_3}=\frac{\partial^2 u_3}{\partial t^2} \tag{17}$$

$$c_1\frac{\partial^2 T}{\partial x_1^2}+p_3c_1\frac{\partial^2 T}{\partial x_3^2}=\zeta_1L\frac{\partial^2 u_1}{\partial t\partial x_1}+\zeta_2L\frac{\partial^2 u_3}{\partial t\partial x_3}+\zeta_3c_1\frac{\partial^2 T}{\partial t^2} \tag{18}$$

$$\sigma_{33}=\frac{c_{13}}{\rho c_1^2}\frac{\partial u_1}{\partial x_1}+\frac{c_{33}}{\rho c_1^2}\frac{\partial u_3}{\partial x_3}-p_5T \tag{19}$$

$$\sigma_{31}=\frac{c_{44}}{\rho c_1^2}\left(\frac{\partial u_1}{\partial x_3}+\frac{\partial u_3}{\partial x_1}\right)+\frac{1}{4\rho c_1^2L^2}\left((l_1^2G_1-l_2^2G_2)\left(-\frac{\partial^3 u_1}{\partial x_3\partial x_1^2}+\frac{\partial^3 u_3}{\partial x_1^3}\right)+(l_3^2G_3-l_2^2G_2)\left(-\frac{\partial^3 u_1}{\partial x_1\partial x_3^2}+\frac{\partial^3 u_3}{\partial x_1^2\partial x_3}\right)\right) \tag{20}$$

$$m_{32} = \frac{1}{2}\frac{\beta_1 T_0}{L^2 \rho c_1^2}(l_2^2 G_2 - l_3^2 G_3)(\frac{\partial^2 u_1}{\partial x_3^2} - \frac{\partial^2 u_3}{\partial x_1 \partial x_3}) \quad (21)$$

where

$\delta_1 = \frac{c_{44}}{c_{11}}$ $\delta_2 = \frac{c_{13}+c_{44}}{c_{11}}$ $\delta_4 = \frac{c_{33}}{c_{11}}$

$p_5 = \frac{\beta_3}{\beta_1}$ $p_3 = \frac{K_3}{K_1}$ $\zeta_1 = \frac{T_0\beta_1^2}{K_1\rho}$ $\zeta_2 = \frac{T_0\beta_1\beta_3}{K_1\rho}$

The initial and regularity conditions are given by

$$\begin{gathered} u_1(x_1, x_3, 0) = 0 = \dot{u_1}(x_1, x_3, 0) \\ u_3(x_1, x_3, 0) = 0 = \dot{u_3}(x_1, x_3, 0) \\ T(x_1, x_3, 0) = 0 = \dot{T}(x_1, x_3, 0) \\ for\ x_3 \geq 0, -\infty < x_1 < \infty \\ u_1(x_1, x_3, t) = u_3(x_1, x_3, t) = T(x_1, x_3, 0) = 0 \\ for\ t > 0\ when\ x_3 \to \infty \end{gathered} \quad (22)$$

Applying Laplace and Fourier transformation defined by

$$\overline{f}(x_1, x_3, s) = \int_0^\infty f(x_1, x_3, t) e^{-st} dt \quad (23)$$

$$\hat{f}(\xi, x_3, s) = \int_{-\infty}^{\infty} \overline{f}(x_1, x_3, s) e^{i\xi x_1} dx_1 \quad (24)$$

to the equation (16)-(21), we obtain system of three homogeneous equations from equations (16)-(18). These resulting equations have non trivial solution if the determinant of the coefficient $(\hat{u}_1, \hat{u}_3, \hat{T})$ vanishes, which yields the following characteristic equation

$$P\frac{d^8}{d_{x_3}^8} + Q\frac{d^6}{d_{x_3}^6} + R\frac{d^4}{d_{x_3}^4} + S\frac{d^2}{d_{x_3}^2} + T)(\hat{u}_1, \hat{u}_3, T) = 0 \quad (25)$$

where

$$P = p_3 c_1 \delta_4 \frac{l_2^2 G_2}{4L^2 c_{11}},$$

$$Q = -(\zeta_2 sLp_5 \frac{l_2^2 G_2}{4L^2 c_{11}} + p_3 c_1 \left(\begin{gathered} \left(\delta_1 + \frac{l_2^2 G_2}{4L^2 c_{11}}\xi^2\right)\delta_4 \\ + \frac{-l_2^2 G_2}{4L^2 c_{11}}(-s^2) + 2\xi^2(\delta_1 + \delta_2)\frac{l_2^2 G_2}{4L^2 c_{11}} \end{gathered} \right) + \delta_4 \frac{l_2^2 G_2}{4L^2 c_{11}}(c_1\xi^2 + \zeta_3 c_1 s^2))$$

$$\begin{aligned} R = -\xi^2\zeta_1 sL \frac{p_5}{4L^2 c_{11}} &- p_3 c_1 \Bigg(-\delta_4(\xi^2 + s^2) - \xi^2\left(\delta_1^2 - \delta_2^2 + 2(\delta_1 + \delta_2)\frac{l_2^2 G_2}{4L^2 c_{11}}\xi^2\right) \\ &- \xi^2(\xi^2 + s^2)\frac{l_2^2 G_2}{4L^2 c_{11}} + \left(\delta_1 + \frac{l_2^2 G_2}{4L^2 c_{11}}\xi^2\right)(-s^2)\Bigg) \\ &+ (c_1\xi^2 + \zeta_3 c_1 s^2)\left(\left(\left(\delta_1 + \frac{l_2^2 G_2}{4L^2 c_{11}}\xi^2\right)\delta_4 + \frac{l_2^2 G_2}{4L^2 c_{11}}(s^2) + 2\xi^2(\delta_1 + \delta_2)\frac{l_2^2 G_2}{4L^2 c_{11}}\right)\right) \\ &+ \zeta_2 sL\left(p_5\left(\delta_1 + \frac{l_2^2 G_2}{4L^2 c_{11}}\xi^2\right) - \xi^2\frac{l_2^2 G_2}{4L^2 c_{11}}\right) \end{aligned}$$

$$\begin{aligned} S = \xi^2\zeta_1 sL(p_5(\delta_2 - \frac{l_2^2 G_2}{4L^2 c_{11}}\xi^2) &+ \left(\delta_4 + \frac{l_2^2 G_2}{4L^2 c_{11}}\xi^2\right)) - \zeta_2 sL(p_5(\xi^2 + s^2) - \xi^2(\delta_2 - \frac{l_2^2 G_2}{4L^2 c_{11}}\xi^2)) \\ &+ (c_1\xi^2 + \zeta_3 c_1 s^2)\Bigg(-\delta_4(\xi^2 + s^2) - \xi^2(\xi^2 + s^2)\frac{l_2^2 G_2}{4L^2 c_{11}} + \left(\delta_1 + \frac{l_2^2 G_2}{4L^2 c_{11}}\xi^2\right)(-s^2) \\ &- \xi^2\left(\delta_1^2 - \delta_2^2 + 2(\delta_1 + \delta_2)\frac{l_2^2 G_2}{4L^2 c_{11}}\xi^2\right)\Bigg) \\ &- p_3 c_1((\xi^2 + s^2)s^2 + (\xi^2 + s^2)\xi^2\left(\delta_1 + \frac{l_2^2 G_2}{4L^2 c_{11}}\xi^2\right)) \end{aligned}$$

$$T = (c_1\xi^2 + \zeta_3 Ls)\left((\xi^2 + s^2)s^2 + (\xi^2 + s^2)\xi^2\left(\delta_1 + \frac{l_2^2 G_2}{4L^2 c_{11}}\xi^2\right)\right) + \xi^2\zeta_1 sL\left(s^2 + \xi^2\left(\delta_1 + \frac{l_2^2 G_2}{4L^2 c_{11}}\xi^2\right)\right)$$

The roots of the equation (25) are $\pm\lambda_i$ ($i = 1, 2, 3, 4$), using the radiation condition that $\widehat{u_1}, \widehat{u_3}, \hat{T} \to 0$ as $x_3 \to \infty$ the solution of equation (24) may be written as

$$\widehat{u_1} = A_1 e^{-\lambda_1 x_3} + A_2 e^{-\lambda_2 x_3} + A_3 e^{-\lambda_3 x_3} + A_4 e^{-\lambda_4 x_3} \quad (26)$$

$$\widehat{u_3} = d_1 A_1 e^{-\lambda_1 x_3} + d_2 A_2 e^{-\lambda_2 x_3} + d_3 A_3 e^{-\lambda_3 x_3} + d_4 A_4 e^{-\lambda_4 x_3} \quad (27)$$

$$\hat{T} = g_1 A_1 e^{-\lambda_1 x_3} + g_2 A_2 e^{-\lambda_2 x_3} + g_3 A_3 e^{-\lambda_3 x_3} + g_4 A_4 e^{-\lambda_4 x_3} \quad (28)$$

where

$$d_i=\frac{-\lambda_i^6 p_3 c_1\,A+\lambda_i^4(-p_3c_1\ \epsilon_1+c_1(\xi^2+\zeta_3 s^2)A)+\lambda_i^2\big((\xi^2+s^2)p_3c_1+c_1\ \epsilon_1\ (\xi^2+\zeta_3 s^2)\big)-c_1\,(\xi^2+s^2)(\xi^2+\zeta_3 s^2)-\xi^2\zeta_1 sL}{-\lambda_i^4 p_3c_1\,(\delta_4-A\xi^2)+\lambda_i^2(p_3c_1\,(s^2+\xi^2\ \epsilon_1)+c_1\,(\xi^2+\zeta_3s^2)(\delta_4-A\xi^2)+\zeta_2 sLp_5))-c_1\,(s^2+\xi^2\ \epsilon_1\ (\xi^2+\zeta_3s^2))}$$

$$g_i=\frac{\begin{array}{c}\lambda_i^6\big(\xi^2A^2+A(\delta_4-A\xi^2)\big)+\lambda_i^4(\epsilon_1(\delta_4-A\xi^2)+A(s^2+\xi^2\epsilon_1)-2\xi^2(\delta_2+A\xi^2)A)+\lambda_i^2(-(\xi^2+s^2)(\delta_4-A\xi^2)\\+\epsilon_1(s^2+\xi^2\epsilon_1)+\xi^2(\delta_2+A\xi^2)^2)+(\xi^2+s^2)(s^2+\xi^2\epsilon_1)\end{array}}{-\lambda_i^4p_3c_1\,(\delta_4-A\xi^2)+\lambda_i^2((s^2+\xi^2\epsilon_1)p_3c_1+c_1(\xi^2+\zeta_3s^2)(\delta_4-A\xi^2)+\zeta_2sLp_5))-c_1(s^2+\xi^2\epsilon_1(\xi^2+\zeta_3s^2))},$$

$$\mathrm{A}=-\frac{l_2^2G_2}{4L^2c_{11}},\qquad \epsilon_1=(\delta_1-A\xi^2)$$

4. Boundary conditions

For boundary conditions we suppose that the boundary plane$x_3=0$ is subjected to 1. Vanishing of normal stress

$$\sigma_{33}=0 \tag{29}$$

2. Vanishing of tangential stress

$$\sigma_{31}=0 \tag{30}$$

3. Vanishing of tangential couple stress

$$m_{32}=0 \tag{31}$$

4. Condition of temperature change

$$T(x,t)=G(t)\delta(x) \tag{32}$$

where $\delta(x)$Dirac delta function and $G(t)$ is a function defined as

$$G(t)=\begin{cases}0 & t\le 0\\ T_1\dfrac{t}{t_0} & 0<t\le t_0\\ T_1 & t>t_0\end{cases} \tag{33}$$

where t_0 indicates the length of time to rise the heat and T_1 is a constant, this means that the boundary of the half-space, which is initially at rest and has a fixed temperature t_0, is suddenly raised to a temperature equal to the function $G(t)\delta(x)$ and is maintained at this temperature afterwards.

Applying the Laplace and Fourier transforms to both sides of (33), we obtain

$$\tilde{\bar{T}}(\xi,s)=\tilde{G}(s)$$

$$\text{where }\ \tilde{G}(s)=T_1\frac{(1-e^{-st_0})}{t_0s^2}$$

Substituting the values of $\widehat{u_1}$,$\widehat{u_3}$,$\hat{T}$ from equations(26)-(28) in the boundary conditions(29)-(33) and with the aid of (1), (5)-(8),(23)-(24),(19)-(21), we obtain the components of displacement, normal stress, tangential stress, tangential couple stress and temperature change as

$$\widehat{u_1}=\frac{\widetilde{G(s)}}{\Delta}\ (B_{41}e^{-\lambda_1x_3}+B_{42}e^{-\lambda_2x_3}+B_{43}e^{-\lambda_3x_3}+B_{44}e^{-\lambda_4x_3} \tag{34}$$

$$\widehat{u_3}=\frac{\widetilde{G(s)}}{\Delta}(d_1B_{41}e^{-\lambda_1x_3}+d_2B_{42}e^{-\lambda_2x_3}+d_3B_{43}e^{-\lambda_3x_3}+d_4B_{44}e^{-\lambda_4x_3}) \tag{35}$$

$$\hat{T}=\frac{\widetilde{G(s)}}{\Delta}(g_1B_{41}e^{-\lambda_1x_3}+g_2B_{42}e^{-\lambda_2x_3}+g_3B_{43}e^{-\lambda_3x_3}+g_4B_{44}e^{-\lambda_4x_3}) \tag{36}$$

$$\begin{aligned}\widehat{\sigma_{33}}=\frac{\widetilde{G(s)}}{\Delta}(&B_{41}\left(\frac{c_{13}}{\rho c_1^2}\mathrm{i}\xi-d_1\lambda_1\frac{c_{33}}{\rho c_1^2}-p_5g_1\right)e^{-\lambda_1x_3}\\&+B_{42}\left(\frac{c_{13}}{\rho c_1^2}\mathrm{i}\xi-d_2\lambda_2\frac{c_{33}}{\rho c_1^2}-p_5g_2\right)e^{-\lambda_2x_3}\\&+B_{43}\left(\frac{c_{13}}{\rho c_1^2}\mathrm{i}\xi-d_3\lambda_3\frac{c_{33}}{\rho c_1^2}-p_5g_3\right)e^{-\lambda_3x_3}\\&+B_{44}\left(\frac{c_{13}}{\rho c_1^2}\mathrm{i}\xi-d_4\lambda_4\frac{c_{33}}{\rho c_1^2}-p_5g_4\right)e^{-\lambda_4x_3})\end{aligned} \tag{37}$$

$$\begin{aligned}\widehat{\sigma_{31}}&=\frac{\widetilde{G(s)}}{\rho c_1^2\Delta}(B_{41}(-\lambda_1c_{44}+\mathrm{i}\xi d_1\\&+\frac{1}{4L^2}\big((l_1^2G_1-l_2^2G_2)(-\xi^2\lambda_1-\mathrm{i}\xi^3d_1)+(l_3^2G_3\\&-l_2^2G_2)(\lambda_1^3+\mathrm{i}\xi\lambda_1^2d_1)\big))e^{-\lambda_1x_3}+B_{42}(-\lambda_2c_{44}+\mathrm{i}\xi d_2\\&+\frac{1}{4L^2}\big((l_1^2G_1-l_2^2G_2)(-\xi^2\lambda_2-\mathrm{i}\xi^3d_2)+(l_3^2G_3\\&-l_2^2G_2)(\lambda_2^3+\mathrm{i}\xi\lambda_2^2d_2)\big))e^{-\lambda_2x_3}+B_{43}(-\lambda_3c_{44}+\mathrm{i}\xi d_3\\&+\frac{1}{4L^2}\begin{pmatrix}(l_1^2G_1-l_2^2G_2)(-\xi^2\lambda_3-\mathrm{i}\xi^3d_3)+\\(l_3^2G_3-l_2^2G_2)(\lambda_3^3+\mathrm{i}\xi\lambda_3^2d_3)\end{pmatrix})e^{-\lambda_3x_3}\\&+B_{44}(-\lambda_4c_{44}+\mathrm{i}\xi d_4\\&+\frac{1}{4L^2}\big((l_1^2G_1-l_2^2G_2)(-\xi^2\lambda_4-\mathrm{i}\xi^3d_4)+(l_3^2G_3\\&-l_2^2G_2)(\lambda_4^3+\mathrm{i}\xi\lambda_4^2d_4)\big))e^{-\lambda_4x_3})\end{aligned} \tag{38}$$

$$\begin{aligned}\widehat{m_{32}}=\frac{1}{2}\frac{\widetilde{G(s)}}{\rho c_1^2L^2\Delta}(l_2^2G_2&\\-l_3^2G_3)(&\,B_{41}e^{-\lambda_1x_3}(\lambda_1^2+\mathrm{i}\xi\lambda_1d_1)\\&+B_{42}e^{-\lambda_2x_3}(\lambda_2^2+\mathrm{i}\xi\lambda_2d_2)\\&+B_{43}e^{-\lambda_3x_3}(\lambda_3^2+\mathrm{i}\xi\lambda_3d_3)\\&+B_{44}e^{-\lambda_4x_3}(\lambda_4^2+\mathrm{i}\xi\lambda_4d_4))\end{aligned} \tag{39}$$

where

$$B_{41} = -A_{12}(A_{23}A_{34} - A_{33}A_{24}) + A_{13}(A_{22}A_{34} - A_{32}A_{24}) - A_{14}(A_{22}A_{33} - A_{32}A_{23})$$

$$B_{42} = A_{11}(A_{23}A_{34} - A_{33}A_{24}) - A_{13}(A_{21}A_{34} - A_{31}A_{24}) + A_{14}(A_{21}A_{33} - A_{31}A_{23})$$

$$B_{43} = -A_{11}(A_{22}A_{34} - A_{33}A_{24}) + A_{12}(A_{21}A_{34} - A_{31}A_{24}) - A_{14}(A_{21}A_{32} - A_{31}A_{22})$$

$$B_{44} = A_{11}(A_{22}A_{33} - A_{23}A_{32}) - A_{12}(A_{21}A_{33} - A_{31}A_{23}) + A_{13}(A_{21}A_{32} - A_{31}A_{22})$$

$$A_{11} = \frac{\text{ı}\xi c_{13}}{\rho c_1^2} - \frac{c_{33}\lambda_1 d_1}{\rho c_1^2} - p_5 g_1$$

$$A_{12} = \frac{\text{ı}\xi c_{13}}{\rho c_1^2} - \frac{c_{33}\lambda_2 d_2}{\rho c_1^2} - p_5 g_2$$

$$A_{13} = \frac{\text{ı}\xi c_{13}}{\rho c_1^2} - \frac{c_{33}\lambda_3 d_3}{\rho c_1^2} - p_5 g_3$$

$$A_{14} = \frac{\text{ı}\xi c_{13}}{\rho c_1^2} - \frac{c_{33}\lambda_4 d_4}{\rho c_1^2} - p_5 g_4$$

$$A_{21} = c_{44}(-\lambda_1 \frac{1}{\rho c_1^2} + \text{ı}\xi d_1 \frac{1}{\rho c_1^2}) + \frac{1}{4\rho c_1^2 L^2}((l_1^2 G_1 - l_2^2 G_2)(-\xi^2 \lambda_1 - \text{ı}\xi^3 d_1) + (l_3^2 G_3 - l_2^2 G_2)(-\lambda_1^3 + \text{ı}\xi \lambda_1^2 d_1))$$

$$A_{22} = c_{44}(-\lambda_2 \frac{1}{\rho c_1^2} + \text{ı}\xi d_2 \frac{1}{\rho c_1^2}) + \frac{1}{4\rho c_1^2 L^2}((l_1^2 G_1 - l_2^2 G_2)(-\xi^2 \lambda_2 - \text{ı}\xi^3 d_2) + (l_3^2 G_3 - l_2^2 G_2)(-\lambda_2^3 + \text{ı}\xi \lambda_2^2 d_2))$$

$$A_{23} = c_{44}(-\lambda_3 \frac{1}{\rho c_1^2} + \text{ı}\xi d_3 \frac{1}{\rho c_1^2}) + \frac{1}{4\rho c_1^2 L^2}((l_1^2 G_1 - l_2^2 G_2)(-\xi^2 \lambda_3 - \text{ı}\xi^3 d_3) + (l_3^2 G_3 - l_2^2 G_2)(-\lambda_3^3 + \text{ı}\xi \lambda_3^2 d_3))$$

$$A_{24} = c_{44}(-\lambda_4 \frac{1}{\rho c_1^2} + \text{ı}\xi d_4 \frac{1}{\rho c_1^2}) + \frac{1}{4\rho c_1^2 L^2}((l_1^2 G_1 - l_2^2 G_2)(-\xi^2 \lambda_4 - \text{ı}\xi^3 d_4) + (l_3^2 G_3 - l_2^2 G_2)(-\lambda_4^3 + \text{ı}\xi \lambda_4^2 d_4))$$

$$A_{31} = \frac{1}{2\rho c_1^2 L^2}(l_2^2 G_2 - l_3^2 G_3)(\lambda_1^2 + \text{ı}\xi \lambda_1 d_1)$$

$$A_{32} = \frac{1}{2\rho c_1^2 L^2}(l_2^2 G_2 - l_3^2 G_3)(\lambda_2^2 + \text{ı}\xi \lambda_2 d_2)$$

$$A_{33} = \frac{1}{2\rho c_1^2 L^2}(l_2^2 G_2 - l_3^2 G_3)(\lambda_3^2 + \text{ı}\xi \lambda_3 d_3)$$

$$A_{34} = \frac{1}{2\rho c_1^2 L^2}(l_2^2 G_2 - l_3^2 G_3)(\lambda_4^2 + \text{ı}\xi \lambda_4 d_4)$$

$$A_{41} = g_1 e^{-\lambda_1 x_3}$$

$$A_{42} = g_2 e^{-\lambda_2 x_3}$$

$$A_{43} = g_3 e^{-\lambda_3 x_3}$$

$$A_{44} = g_4 e^{-\lambda_4 x_3}$$

$$\Delta = \Delta_1 - \Delta_2 + \Delta_3 - \Delta_4$$

$$\Delta_1 = A_{11}A_{22}(A_{33}A_{44} - A_{43}A_{34}) - A_{11}A_{23}(A_{32}A_{44} - A_{42}A_{34}) + A_{11}A_{24}(A_{32}A_{43} - A_{42}A_{33})$$

$$\Delta_2 = A_{12}A_{21}(A_{33}A_{44} - A_{43}A_{34}) - A_{12}A_{23}(A_{31}A_{44} - A_{41}A_{34}) + A_{24}A_{12}(A_{31}A_{43} - A_{41}A_{33})$$

$$\Delta_3 = A_{13}A_{21}(A_{32}A_{44} - A_{42}A_{34}) - A_{22}A_{13}(A_{31}A_{44} - A_{41}A_{34}) + A_{13}A_{24}(A_{31}A_{42} - A_{41}A_{32})$$

$$\Delta_4 = A_{14}A_{21}(A_{32}A_{43} - A_{42}A_{33}) - A_{22}A_{14}(A_{31}A_{43} - A_{41}A_{33}) + A_{14}A_{23}(A_{31}A_{42} - A_{41}A_{32})$$

and $A_i = \frac{1}{\Delta} B_{4i} \widetilde{G(s)}$

5. Inversion of the transformations:

To obtain the solution of the problem in physical domain, we must invert the transforms in Eqs. (34)-(39). Here the displacement components, normal and tangential stresses and temperature change, couple stress are functions of x_3, the parameters of Laplace and Fourier transforms s and ξ respectively and hence are of the form $f(\xi, x_3, s)$. To obtain the function $f(x, x_3, t)$ in the physical domain, we first invert the Fourier transform using

$$\bar{f}(x, x_3, t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-\text{ı}\xi x_1} \hat{f}(\xi, x_3, s)\, d\xi = \frac{1}{2\pi}\int_{-\infty}^{\infty} |cos\,(\xi\, x) f_e - isin(\xi\, x) f_0| d\xi\,. \tag{40}$$

where f_e and f_0 are respectively the odd and even parts of $\hat{f}(\xi, x_3, s)$. Thus the expression (40) gives the Laplace transform $\bar{f}(\xi, x_3, s)$. of the function $f(x, x_3, t)$. Following Honig and Hirdes (1984), the Laplace transform function $\bar{f}(\xi, x_3, s)$ can be inverted to $f(x, x_3, t)$.

The last step is to calculate the integral in Eq. (40). The method for evaluating this integral is described in Press *et al*. (1986). It involves the use of Romberg's integration with adaptive step size. This also uses the results from successive

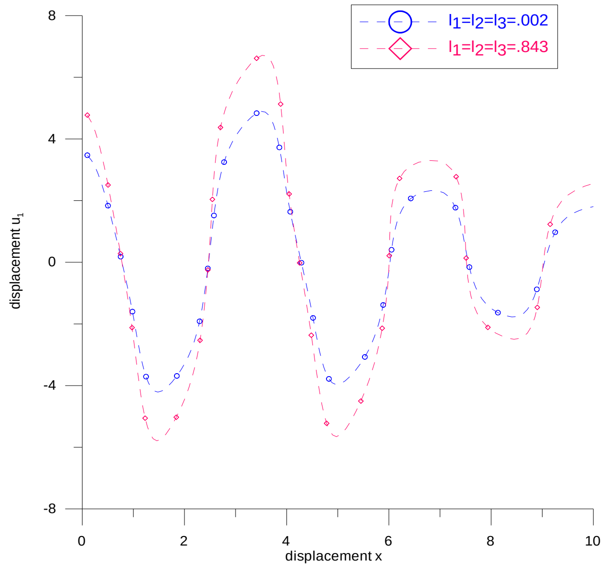

Fig. 1 variation of displacement component u_1 with the displacement x

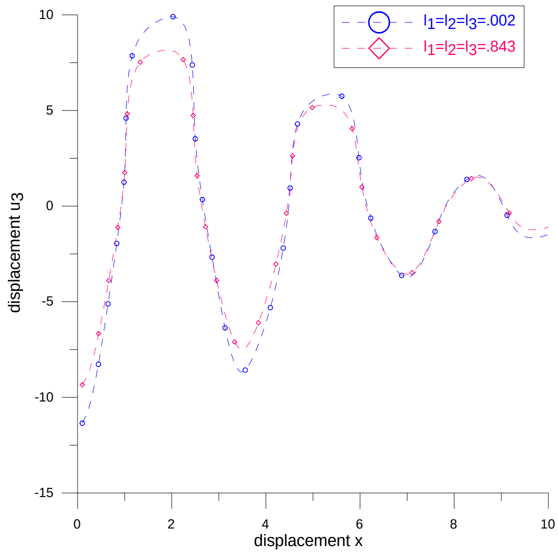

Fig. 2 variation of displacement component u_3 with the displacement x

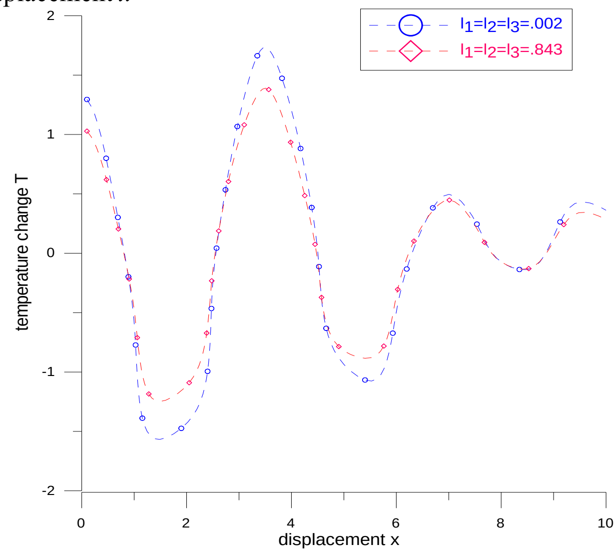

Fig. 3 variation of temperature T with the displacement x

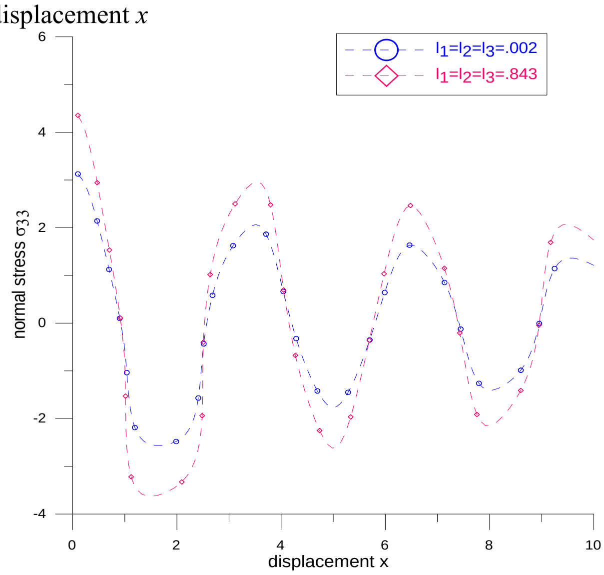

Fig. 4 variation of normal stress σ_{33} with the displacement x

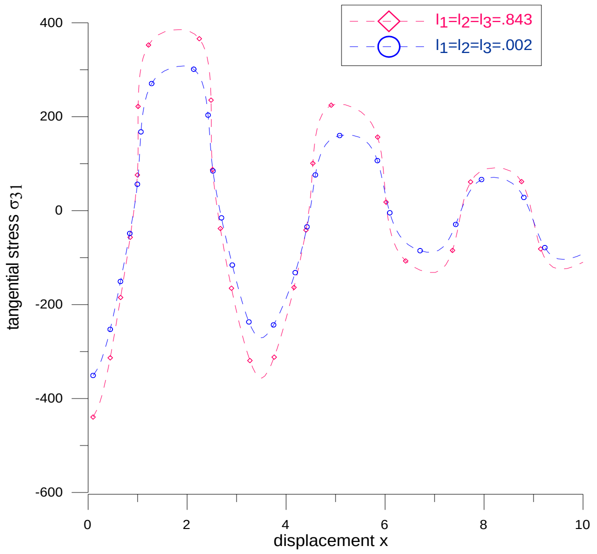

Fig. 5 variation of tangential stress σ_{31} with the displacement x

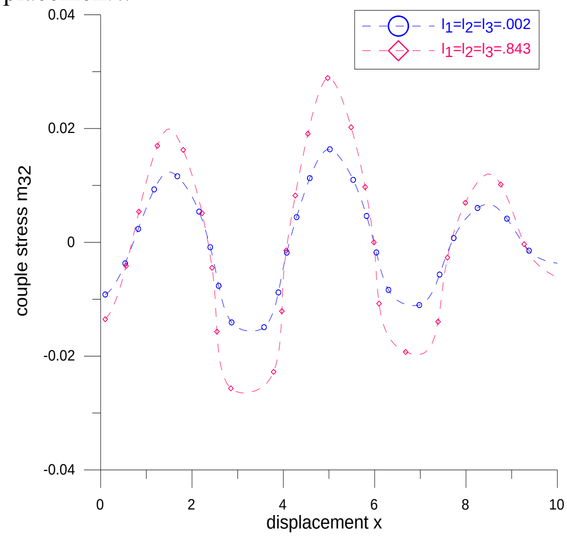

Fig. 6 variation of couple stress m_{32} with the displacement x

refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

6. Numerical results and discussions:

For numerical computations, we take the copper material which is transversely isotropic

$$c_{11} = 18.78 \times 10^{10}\ Kgm^{-1}s^{-2},$$

$$c_{12} = 8.76 \times 10^{10}\ Kgm^{-1}s^{-2}\ ,$$

$$c_{13} = 8.0 \times 10^{10}\ Kgm^{-1}s^{-2} c_{33} = 17.2 \times 10^{10}\ Kgm^{-1}s^{-2},$$

$$c_{44} = 5.06 \times 10^{10}\ Kgm^{-1}s^{-2} \quad C_E = 0.6331 \times 10^{3} JKg^{-1}K^{-1} \alpha_1 = 2.98 \times 10^{-5}K^{-1},$$

$$\alpha_3 = 2.4 \times 10^{-5}K^{-1}, \rho = 8.954 \times 10^{3}Kgm^{-3},$$
$$K_1 = 0.433 \times 10^{3}Wm^{-1}K^{-1},$$

$$K_3 = 0.450 \times 10^{3}Wm^{-1}K^{-1},$$

$$T_0 = 293K, G_1 = 0.1\ ,$$

$$G_2 = 0.2\ ,\quad G_3 = 0.3,\ L = 1, t_0 = .02s$$

Components of displacement, stress, temperature change and couple stress are computed numerically. Software Grapher 3 has been used to determine and compare the values of normal stress, tangential stress, couple stress, temperature change and components of displacement for transversely isotropic thermoelastic solid with distance x for two different values of length scale parameters graphically. In Figs. 1-6, dotted line with centre symbol ($- - \lozenge - -$) corresponds to $l_1 = l_2 = l_3 = .843$ and dotted line with centre symbol ($- -\ o\ - -$) corresponds to $l_1 = l_2 = l_3 = .002$.

NM-CST for $l_1 = l_2 = l_3 = .843$ and $l_1 = l_2 = l_3 = .002$

Figs 1-6 shows the variation of displacement component u_1, displacement component u_3, temperature change T, normal stress σ_{33}, tangential stress σ_{31}, couple stress m_{32} with the displacement x resp. for both the cases ($l_1 = l_2 = l_3 = .843$ and $l_1 = l_2 = l_3 = .002$). Displacement has oscillatory effect on all the physical components mentioned above. However not much change in the behaviors of curves has been noticed when compared for both the cases except at the amplitude of the curves.

In fig. 1 value of displacement u_1 sharply decreases for $0 \le x \le 1.5$,then sharply increases for $1.5 \le x \le 3.5$, decreases for $3.5 \le x \le 5$, then increases for $5 \le x \le 7$,again decreases for $7 \le x \le 8.5$ and increases for $8.5 \le x \le 10$ for both cases and value of u_1 lies in the range (-6,6). Maximum amplitude is in the range $2.5 \le x \le 4.5$.

In fig. 2 value of displacement u_3 increases for $0 \le x \le 2$, then decreases for $2 \le x \le 3.5$, increases for $3.5 \le x \le 5$, then decreases for $5 \le x \le 7$,again increases for $7 \le x \le 8.5$ and decreases for $8.5 \le x \le 10$ and value of u_3 lies in the range (-15,10). The amplitude is maximum near the origin and goes on decreasing as x increases for $0 \le x \le 10$. Also, the difference between the amplitudes of the curves reduces as we move along the displacement axes away from the origin.

In fig. 3 value of temperature change T decreases for $0 \le x \le 1.5$, $3.5 \le x \le 5.5$, $7 \le x \le 8.5$ and increases rapidly for $1.5 \le x \le 3.5, 5.5 \le x \le 7$ and $8.5 \le x \le 9.5$ and value of T lies in the range (-2, 2). Maximum and sharp peak amplitude is seen in the range $2.5 \le x \le 4.5$.

In fig. 4 value of normal stress σ_{33} decreases for $0 \le x \le 1.5$, then increases for $1.5 \le x \le 3.5$, decreases for $3.5 \le x \le 5.$, then increases for $5 \le x \le 6.5$,again decreases for $6.5 \le x \le 8$ and increases for $8 \le x \le 9.5$ and value of σ_{33} lies in the range (-4,6).Amplitudes of the curves are maximum at the origin, decreases gradually and slowly as x increases from 0 to 10.

In fig.5 value of normal stress σ_{31} increases for $0 \le x \le 2, 3.5 \le x \le 5, 7 \le x \le 8.5$ and decreases for $2 \le x \le 3.5$, decreases for $5 \le x \le 7$,$8.5 \le x \le 10$ and value of σ_{31} lies in the range (-400,400).Amplitude are maximum in the range $0 \le x \le 3$ and decrease afterwards. A sharp peak of amplitude can be seen in the range $2.5 \le x \le 4.5$.

In fig. 6 value of normal stress m_{32} increases for $0 \le x \le 1.5$, $3 \le x \le 5.2$, $6.7 \le x \le 8.5$ and decreases for $1.5 \le x \le 3, 5.2 \le x \le 6.7$,$8.5 \le x \le 10$ and value of m_{32} lies in the range (-.004,.004).But a noticeable difference between the amplitudes of both the curves drawn for both the cases for NM-CST is observed. Amplitude of the curve for $l_1 = l_2 = l_3 = .843$ is greater as compared to curve for $l_1 = l_2 = l_3 = .241$. Peaks of amplitudes are sharper for upper cycles than the amplitude of down cycles. Amplitudes of oscillations are maximum in the range $4 \le x \le 6$.

It is clear to see from the figs. that amplitudes of the curves sketched for displacement component u_1, stress components and couple stress magnifies reduces as the parameters $l_1 = l_2 = l_3$ decreases and amplitude of curves drawn for variation of displacement component u_3, and temperature change T with the displacement reduces as the material parameters $l_1 = l_2 = l_3$ decreases. But, variation of couple stress m_{32} with displacement x depicts clear difference among the amplitudes of the curves. Amplitude of the curve decreases sharply as length scale parameters decreases.

7. Conclusions

New modified couple stress theory for transverse isotropic thermoelastic solid is presented in this paper. Size effects are considered *using* length scale parameters. Analysis of stresses, temperature change and displacement components due to thermal and mechanical change in transversely isotropic material is a significant problem in solid mechanics. The interactions of a transversely isotropic thermoelastic material in the new modified couple stress theory have been investigated using Laplace transform and

Fourier transform technique. A numerical inversion technique has been used to recover the solutions in the physical domain. The expressions for components of stress, components of displacement, temperature change and couple stress have been derived successfully and shown graphically in the presence of material length parameter. The resulting quantities depicted graphically are observed to be very sensitive towards length scale parameters. All the analysis has been done by taking $l_1 = l_2 = l_3 = l(say)$.Figures show that the length scale parameters have appreciable effects on the numerical values of the physical quantities obtained after computational process. As length scale parameters are varied, amplitude of the curve as sketched above also changes. Amplitudes of the curves sketched for displacement componentu_1, stress components and couple stress magnifies reduces as the parameters l are decreased and amplitude of curves drawn for variation of displacement componentu_3, and temperature change T with the displacement reduces as the material parameters l are increased.The results obtained in the study should be beneficial for people working in medical science, thermomechanical, engineering, accelerometers, sensors, resonators and also in future work.

Acknowledgment

The corresponding author Harpreet Kaur duly acknowledges the Junior Research Fellowship (JRF) received from University Grants Commission (UGC), Delhi India for pursuing her PhD under the sanctioned no. 19/6/2016/(i) EU-V.

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