Structural Engineering and Mechanics, *Vol. 75, No. 6 (2020) 713-722*
DOI: https://doi.org/10.12989/sem.2020.75.6.713

Damped dynamic responses of a layered functionally graded thick beam under a pulse load

Saeed A. Asiri*[1], Şeref D. Akbaş[2a] and Mohamed A. Eltaher*[1,3]

[1]*Mechanical Engineering Department, Faculty of Engineering, King Abdulaziz University, P.O. Box 80204, Jeddah, Saudi Arabia*
[2]*Deparment of Civil Engineering, Bursa Technical University, Bursa, Turkey*
[3]*Department of Mechanical Design & Production, Faculty of Engineering, Zagazig University,P.O. Box 44519, Zagazig, Egypt*

(Received March 3, 2020, Revised April 5, 2020, Accepted April 14, 2020)

Abstract. This article aims to illustrate the damped dynamic responses of layered functionally graded (FG) thick 2D beam under dynamic pulse sinusoidal load by using finite element method, for the first time. To investigate the response of thick beam accurately, two-dimensional plane stress problem is assumed to describe the constitutive behavior of thick beam structure. The material is distributed gradually through the thickness of each layer by generalized power law function. The Kelvin–Voigt viscoelastic constitutive model is exploited to include the material internal damping effect. The governing equations are obtained by using Lagrange's equations and solved by using finite element method with twelve –node 2D plane element. The dynamic equation of motion is solved numerically by Newmark implicit time integration procedure. Numerical studies are presented to illustrate stacking sequence and material gradation index on the displacement-time response of cantilever beam structure. It is found that, the number of waves increases by increasing the graduation distribution parameter. The presented mathematical model is useful in analysis and design of nuclear, marine, vehicle and aerospace structures those manufactured from functionally graded materials (FGM).

Keywords: dynamic analysis; thick beam; pulse load; layered FGM; viscoelastic; finite element method

1. Introduction

During last decades, functionally graded materials (Trough space-plane project in Japan in 1984, new class of composite material known as functionally graded material (FGM) was used for the first time as a thermal barrier, Alshorbagy (2011). FGMs are formed by mixing two or more different material constituent phases with a volume fraction that varies gradually with respect to spatial coordinates, thus combining the desired material properties to obtain a superior performance, Moleiro *et al*. (2020). FGMs are utilized extensively in many disciplines, such as aerospace technology, automobiles, electronics, optics, chemistry, biomedical, nuclear and mechanical engineering, Nowadays, FGMs are employed in micro/nanoelectro-mechanical system (MEMS/NEMS) and atomic force microscopes (AFMs) to achieve high sensitivity and desired performance, Eltaher *et al* (2014).

Recently, lots of attention is given to study the free and forced vibrational and dynamical behaviors of FGM beam structures. Kiani *et al*. (2013) and Khalili *et al*. (2013) studied low velocity impact response of thick FGM beams with general boundary conditions in thermal field. Sedighi *et al*. (2014) exploited strain gradient elasticity theory to study size-dependent dynamic pull-in instability of vibrating MEMS structure. Sedighi *et al*. (2015a) presented non-linear dynamic instability of a double-sided nano-bridge considering centrifugal force and rarefied gas flow. Sedighi *et al*. (2015b&2016) studied dynamic instability of FG nano-bridges considering Casimir attraction and electric filed actuation. Seguini and Nedjar (2017) presented the nonlinear structures response of deep beam resting on linear and nonlinear random soil. Akbas (2017) illustrated the dynamic free and forced responses of cracked and uncracked FGM damped microbeam by using modified couple stress. Eltaher *et al*. (2018) presented the applicability of FGM in the natural gas transmission pipeline subjected to internal high fluctuated pressure and unsteady temperature. Soliman *et al*. (2018) presented nonlinear transient analysis of FG pipe subjected to internal pressure and unsteady temperature by using finite element method. Song *et al*. (2018) studied dynamic responses of aerothermoelastic FG CNT reinforced composite panels in supersonic airflow subjected to low-velocity impact. Wu *et al*. (2018) exploited A finite element method to study free and forced vibration of FG porous beam type structures. She *et al*. (2018) predicted wave propagation behaviors of FG porous nanobeams based on higher-order shear deformation beam theory in conjunction with the non-local strain gradient theory. Emam *et al*. (2018) investigated the postbuckling and free vibration response of geometrically imperfect multilayer nanobeams subjected to a pre-stress compressive load.

Hamed *et al*. (2019) presented the effects of porosity

*Corresponding author, Associate Professor
E-mail: sasiri@kau.edu.sa; meltaher@kau.edu.sa
[a] Professor
E-mail: serefda@yahoo.com
[b] Professor
E-mail: mmeltaher@zu.edu.eg

http://www.techno-press.com/journals/sem&subpage=7
ISSN: 1225-4568 (Print), 1598-6217 (Online)

models on static behavior of size dependent FG nanobeam by using nonlocal elasticity. Bedia *et al*. (2019) developed new hyperbolic two-unknown beam model to study bending and buckling of a nonlocal strain gradient nanobeams. Ebrahimi *et al*. (2019) studied frequency response of curved magneto-electro-viscoelastic FG nanobeams resting on viscoelastic foundation. She *et al*. (2019) studied snap-buckling behaviors of FG porous curved nanobeams resting on three parameters elastic foundations. Esmaeili and Beni (2019) examined the buckling and vibration behaviors of FG flexoelectric nanobeam. Tlidji *et al*. (2019) developed a quasi-3D beam theory to study free vibration of FG microbeams by using modified coupled stress theory. Berghouti *et al*. (2019) investigated the vibration behavior of nonlocal porous FG nanobeams takes into the effect of shear deformation. Javani *et al*. (2019) obtained natural frequencies of circular deep FG arches based on the unconstrained higher-order shear deformation theory considering the depth change, complete effects of shear deformation, and rotary inertia. Hussain *et al*. (2019), Semmah *et al*. (2019) and Asghar *et al*. (2020) studied buckling and vibration characteristics of chiral and zigzag double-walled carbon nanotubes by nonlocal elasticity model. Khan *et al*. (2020) exploited refined third-order beam theory to study free and forced vibration of FG thick beams using finite element model. Li *et al*. (2020) studied the free vibration of the variable thickness beam made of FGMs and submerged in fluid by using differential quadrature method.

For FG plate structures, Balubaid *et al*. (2019) and Boutaleb *et al*. (2019) studied free vibration of FG nanoscale plate using nonlocal two variables integral refined plate theory and quasi 3D HSDT. Draoui *et al*. (2019) investigated static and dynamic behavior of nanotubes-reinforced sandwich plates using first order shear deformation. Belbachir *et al*. (2019) and Draiche *et al*. (2019) investigated bending of composite laminated plates under nonlinear thermal and mechanical loadings. Chikr *et al*. (2019) proposed a new higher-order shear and normal deformation theory for the buckling analysis of new type of FGM sandwich plates. Karami *et al*. (2019b&c) explored wave propagation of FG anisotropic nanoplates resting on Winkler-Pasternak foundation. Karami *et al*. (2019d) studied resonance behavior of FG polymer composite nanoplates reinforced with graphene nanoplatelets. Karami *et al*. (2019e) predicted elastic bulk wave characteristics of doubly curved FG nanoshell by using nonlocal strain gradient theory. Addou *et al*. (2019) and Kaddari *et al*. (2020) presented the effect of porosity on static and dynamic behaviors of FG and sandwich plate modeled by 3D HSDT. Boukhlif *et al*. (2019) and Boulefrakh *et al*. (2019) exploited 3D HSDT to study dynamic behavior of thick plate with and without viscoelastic foundation. Abualnour *et al*. (2019), Zarga *et al*. (2019), Tounsi *et al* (2020), and Boussoula *et al*. (2020) presented a simple four-variable trigonometric integral shear deformation model to study static and dynamic behavior of FG plates rested on elastic foundation and subjected to a nonlinear hygro-thermo-mechanical load. Bellal *et al*. (2020) presented buckling behavior of a single-layered graphene sheet resting on viscoelastic medium via nonlocal four-unknown integral model.

The modelling and analysis of layered FGM have gained lots of researcher's attention over the last years. Apetre *et al*. (2006) investigated low-velocity impact response of sandwich beams with FG core by combining static contact problem and dynamic of sandwich panel obtained via a simple nonlinear spring-mass model. Avila (2007) studied experimentally the failure mode of sandwich beams with functionally graded core. Etemadi *et al*. (2009) analyzed low velocity impact behavior of sandwich beams with FG core using 3D finite element method. Gardner *et al*. (2012) illustrated the performance of FG sandwich composite beams under shock wave loading. Yang *et al*. (2014) considered mesh free boundary-domain integral equation method to studied free vibration of FG sandwich beams. Nguyen *et al*. (2016) obtained an analytical solution using a quasi-3D beam theory for buckling and free vibration of (FG) sandwich beams having various boundary conditions. Şimşek and Al-Shujairi (2017) examined static, free and forced vibration of FG sandwich beams under double moving harmonic loads travelling with constant velocities using Timoshenko beam theory (TBT). Ebrahimi and Farazmandnia (2018) studied thermo-mechanical vibration of sandwich beams with a stiff core and face sheets made of FG-CNTRC by using TBT. Fan *et al*. (2018) presented an investigation on the low-velocity impact response of FG graphene reinforced composite laminated beam rested on viscoelastic foundation and subjected to a transverse impact load. Sucharda and Konecny (2018) recommended to model reinforced-concrete beam tests by 3D non-linear model analysis. Belabed *et al*. (2018) presented new 3-unknown hyperbolic shear deformation theory for analysis of vibration of FG sandwich plate.

Akbas (2018&2019) analyzed forced vibration of sandwich deep beams made of sandwich FGM including porosity effects. Lai *et al*. (2019) studied experimentally and numerically the penetration of three-layer FG cementitious composite subjected to multiple projectile impacts. Dash *et al*. (2019) derived FE solutions of deflection and corresponding in-plane stress of the FG sandwich shell via a higher-order polynomial shear deformation kinematics. Medani *et al*. (2019) investigated static and dynamic behaviors of FG-CNT reinforced porous sandwich plate by using first order shear deformation theory. Sahla *et al*. (2019) presented a simple four-variable trigonometric shear deformation model with undetermined integral terms to consider the influences of transverse shear deformation on the dynamic analysis of anti-symmetric laminated composite and soft-core sandwich plates. Moleiro *et al*. (2020) addressed the modelling and analysis of multilayered plates with embedded FGM layer(s) under hygro-thermo-mechanical loadings. Daikh *et al*. (2020) examined the influence of thickness ratio, the inhomogeneity parameter and the thermal loading kinds on the thermal buckling response of various types of FG sandwich beams. Huang and Ouyang (2020) derived exact analytical solutions for bending behavior of 2D FGM beam based on the TBT. Hamed *et al*. (2020) investigated buckling and stability analysis of sandwich beams rested on

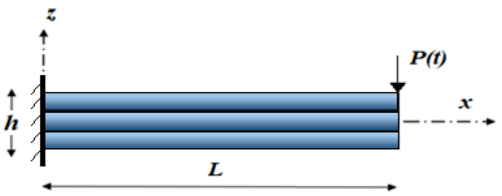

Fig. 1. A cantilever thick beam with three functionally graded layers under a pulse load

elastic foundation and subjected to varying axial loads by using differential quadrature method. Eltaher and Akbas (2020) studied dynamic response of 2D FG thick under a dynamic load by using finite element method.

According to previous studies and authors' knowledge the damped dynamic response of 2D layered FG thick beams under pulse load has not been studied previously. So, this article tends to fill this gap. The article is structured as follows: - Section 2 presents the mathematical formulation (including the constitutive equations and equation of motion), numerical formulations (included element matrices and force vectors), and numerical Newmark implicit time integration procedure. Section 3 is devoted to validation and parametric studies to present effects gradation parameter and stacking sequences on the time response of cantilever layered FG thick beam. Main remarks and conclusion points are highlighted and summarized in Section 4.

2. Problem formulation

2.1 Mathematical formulation

A cantilever thick beam with three functionally graded layers under a pulse load P(t) at free end is shown in Fig. 1. The beam has a length of L through the axial direction x-axis, and thickness h through the transverse z-axis. The functionally graded layers are located as symmetry according to mid-plane axis. The height of each layer is equal. Each layer has a graduation material property (i.e.; Young modulus E, Poisson's ratio ν, and density ρ) through transverse direction, those can be described by a power-law distribution as

$$E(z) = (E_T - E_B)\left[\frac{z}{h} + \frac{1}{2}\right]^n + E_B \tag{1.a}$$

$$\nu(z) = (\nu_T - \nu_B)\left[\frac{z}{h} + \frac{1}{2}\right]^n + \nu_B \tag{1.b}$$

$$\rho(z) = (\rho_T - \rho_B)\left[\frac{z}{h} + \frac{1}{2}\right]^n + \rho_B \tag{1.c}$$

which n is the positive power exponent parameter and subscript T and B being the top and bottom properties of the layers.

Based on the continuum mechanics, the thick beam is described by plane stress problem. Therefore, the kinematic strain-displacement relations are described by

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{zz} \\ \gamma_{xz} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial z} & \frac{\partial}{\partial x} \end{bmatrix} \begin{Bmatrix} u \\ w \end{Bmatrix} \tag{2}$$

in which u, w are the displacements in x and z directions, respectively. ε_{xx} and ε_{zz} are the normal in-plane strains, and γ_{xz} is the shear in-plane strain. The constitutive stress-strain equation with the Kelvin–Voigt viscoelastic constitutive model, in a case of FG layer, can be represented as

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{zz} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11}(z) & Q_{12}(z) & 0 \\ Q_{12}(z) & Q_{22}(z) & 0 \\ 0 & 0 & Q_{33}(z) \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{zz} \\ \gamma_{xz} \end{Bmatrix} + \begin{bmatrix} \eta\, Q_{11}(z) & \eta Q_{12}(z) & 0 \\ \eta\, Q_{12}(z) & \eta\, Q_{22}(z) & 0 \\ 0 & 0 & \eta\, Q_{33}(z) \end{bmatrix} \begin{Bmatrix} \frac{\partial \varepsilon_{xx}}{\partial t} \\ \frac{\partial \varepsilon_{zz}}{\partial t} \\ \frac{\partial \gamma_{xz}}{\partial t} \end{Bmatrix} \tag{3.a}$$

$$\{\sigma\} = [C]\{\varepsilon\} + [D]\{\dot{\varepsilon}\} \tag{3.b}$$

where coefficients of the stiffness can be portrayed as functions of elasticity and Poisson's ratio as following:

$$C_{11}(z) = C_{22}(z) = \frac{E(z)}{1-[\nu(z)]^2} \quad , \quad C_{33}(z) = \frac{E(z)}{2\,[1+\nu(z)]} \quad ; \; C_{12}(z) = \frac{\nu(z)E(z)}{1-[\nu(z)]^2} \tag{4}$$

and η, presented in Eq. (3), denotes the damping ratio that can be evaluated by

$$\eta = \frac{c}{E} \tag{5}$$

where, c is the damping coefficient. The Strain energy (U_i), the kinetic energy (T), the dissipation function (R) and the potential energy of the external loads (U_e) are presented as follows;

$$U_i = \frac{1}{2}\{d\}^T\left(\int_V [B]^T\,[C][B]\right)\{d\}dV \tag{6}$$

$$T = \frac{1}{2}\{\dot{d}\}^T\left(\int_V \rho(z)\right)\{\dot{d}\}dV \tag{7}$$

$$R = \frac{1}{2}\{\dot{d}\}^T\left(\int_V [B]^T\,[D][B]\right)\{\dot{d}\}dV \tag{8}$$

$$U_e = -P(t)v(x_p, t) \tag{9}$$

in which $\{\dot{d}\}$ is the velocity vector, x_p is the coordinate of the applied load. The Lagrangian functional of the problem is presented as follows

$$I = T - (U_i + U_e) \tag{10}$$

In the current analysis, the dynamic response of damped FG multilayered 2D thick beam has been studied. The governing equations for this system can't be analytically. Thus, numerical solution, such as, Bubnov-Galerkin and Energy Balance Method (Sedighi *et al*. (2012)), asymptotic approximate approaches (Sedighi *et al*. (2013)), homotopy perturbation method (Sedighi *et al*. (2014)), Galerkin's approach (Karami *et al*. (2019a)), differential Quadrature Method (Eltaher and Mohamed (2020) and Eltaher *et al*. (2020b)), and finite element method (FEM) (Akbas (2019), Alimirzaei *et al*. (2019) and Eltaher *et al*. (2020a)) can be used. Since, the FEM is more accurate and simpler in analysis of complex structures. So, the proposed model is solved by using finite element method with Twelve-node 2D-plane element model, as illustrated in Fig. 2

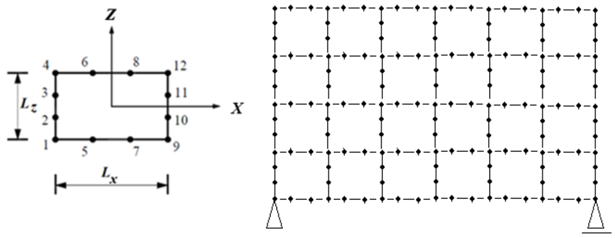

Fig. 2 Twelve –node 2D plane element and finite element model

where L_x and L_y are element lengths in X and Z directions respectively. The displacement vector ($\{d\}$) for Twelve-node plane element is expressed as:

$$\{d\} = [\emptyset]\{d_n\} \tag{11.a}$$

$$[\emptyset] = [\emptyset_1\ \emptyset_2 \ldots . \emptyset_{12}] \tag{11.b}$$

where $\{d_n\}$ indicates the node displacement vector.

$$\{d_n\} = \begin{Bmatrix} u_1 \\ u_2 \\ . \\ . \\ u_{12} \\ w_1 \\ w_2 \\ . \\ . \\ w_{12} \end{Bmatrix} \tag{8}$$

where $\{d_n\}$ is the node displacement vector and its components are u_i and v_i are the displacement components for i node. The displacement of any generic element can be represented by its nodal values and corresponding functions as,

$$u = (u_1\emptyset_1 + u_2\emptyset_2 + u_3\emptyset_3 + u_4\emptyset_4 + u_5\emptyset_5 + u_6\emptyset_6 + u_7\emptyset_7 + u_8\emptyset_8 + u_9\emptyset_9 + u_{10}\emptyset_{10} + u_{11}\emptyset_{11} + u_{12}\emptyset_{12}) \tag{13.a}$$

$$w = (w_1\emptyset_1 + w_2\emptyset_2 + w_3\emptyset_3 + w_4\emptyset_4 + w_5\emptyset_5 + w_6\emptyset_6 + w_7\emptyset_7 + w_8\emptyset_8 + w_9\emptyset_9 + w_{10}\emptyset_{10} + w_{11}\emptyset_{11} + w_{12}\emptyset_{12}) \tag{13.b}$$

where $\emptyset_i$ is the nonlinear interpolation shape functions, which can be represented as follows;

$$\begin{aligned}
\emptyset_1 &= \tfrac{1}{32}\left(1-\tfrac{2X}{L_x}\right)\left(1-\tfrac{2Z}{L_z}\right)\left(-10+9\left(\tfrac{4X^2}{{L_x}^2}+\tfrac{4Z^2}{{L_z}^2}\right)\right),\\
\emptyset_2 &= \tfrac{9}{32}\left(1-\tfrac{2X}{L_x}\right)\left(1-\tfrac{4Z^2}{{L_z}^2}\right)\left(1-\tfrac{6Z}{L_z}\right)\\
\emptyset_3 &= \tfrac{9}{32}\left(1-\tfrac{2X}{L_x}\right)\left(1-\tfrac{4Z^2}{{L_z}^2}\right)\left(1+\tfrac{6Z}{L_z}\right),\\
\emptyset_4 &= \tfrac{1}{32}\left(1-\tfrac{2X}{L_x}\right)\left(1+\tfrac{2Z}{L_z}\right)\left(-10+9\left(\tfrac{4X^2}{{L_x}^2}+\tfrac{4Z^2}{{L_z}^2}\right)\right)\\
\emptyset_5 &= \tfrac{9}{32}\left(1-\tfrac{2Z}{L_z}\right)\left(1-\tfrac{4X^2}{{L_x}^2}\right)\left(1-\tfrac{6X}{L_x}\right),\\
\emptyset_6 &= \frac{9}{32}\left(1+\frac{2Z}{L_z}\right)\left(1-\frac{4X^2}{{L_x}^2}\right)\left(1-\frac{6X}{L_x}\right)\\
\emptyset_7 &= \tfrac{9}{32}\left(1-\tfrac{2Z}{L_z}\right)\left(1-\tfrac{4X^2}{{L_x}^2}\right)\left(1+\tfrac{6X}{L_x}\right),\\
\emptyset_8 &= \frac{9}{32}\left(1+\frac{2Z}{L_z}\right)\left(1-\frac{4X^2}{{L_x}^2}\right)\left(1+\frac{6X}{L_x}\right)\\
\emptyset_9 &= \tfrac{1}{32}\left(1+\tfrac{2X}{L_x}\right)\left(1-\tfrac{2Z}{L_z}\right)\left(-10+9\left(\tfrac{4X^2}{{L_x}^2}+\tfrac{4Z^2}{{L_z}^2}\right)\right),\\
\emptyset_{10} &= \tfrac{9}{32}\left(1+\tfrac{2X}{L_x}\right)\left(1-\tfrac{4Z^2}{{L_z}^2}\right)\left(1-\tfrac{6Z}{L_z}\right)\\
\emptyset_{11} &= \tfrac{9}{32}\left(1+\tfrac{2X}{L_x}\right)\left(1-\tfrac{4Z^2}{{L_z}^2}\right)\left(1+\tfrac{6Z}{L_z}\right),\\
\emptyset_{12} &= \tfrac{1}{32}\left(1+\tfrac{2X}{L_x}\right)\left(1+\tfrac{2Z}{L_z}\right)\left(-10+9\left(\tfrac{4X^2}{{L_x}^2}+\tfrac{4Z^2}{{L_z}^2}\right)\right)
\end{aligned} \tag{17}$$

Substituting equations (13), into the energy Eqs. (6-9) and Lagrange's equations, and then Lagrange's equations are presented as following:

$$\begin{gathered}
\frac{\partial \mathrm{I}}{\partial \mathrm{q}_\mathrm{k}^{(\mathrm{e})}} - \frac{\partial}{\partial \mathrm{t}}\frac{\partial \mathrm{I}}{\partial \dot{\mathrm{q}}_\mathrm{k}^{(\mathrm{e})}} + \mathrm{Q}_{\mathrm{D}_\mathrm{k}} = 0,\\
\mathrm{Q}_{\mathrm{D}_\mathrm{k}} = -\frac{\partial \mathrm{R}}{\partial \dot{\mathrm{q}}_\mathrm{k}^{(\mathrm{e})}}, k = 1,2,3, \ldots 12
\end{gathered} \tag{15}$$

where $\mathrm{Q}_{\mathrm{D}_\mathrm{k}}$ is generalized damping load. After using the Lagrange procedure, the dynamic equilibrium equation is writeen as follows:

$$[K]\{d_n\} + [C]\{\dot{d_n}\} + [M]\{\ddot{d_n}\} = \{F\} \tag{16}$$

where $[K]$, $[C]$, $[M]$, $\{F\}$, $\{d_n\}$, $\{\dot{d_n}\}$ and $\{\ddot{d_n}\}$ are the stiffness matrix, damping matrix, mass matrix, load vector, displacement vector, velocity vector and acceleration vector respectively. The expansions of finite element matrices are represented as

$$[K] = b\int_A [H]^T[Q][H]dA \tag{17.a}$$

$$[C] = b\int_A \eta[H]^T[Q][H]dA \tag{17.b}$$

$$[M] = b\int_A \rho(z)[\emptyset]^T[\emptyset]dA \tag{17.c}$$

$$\{F\} = \int_\Gamma \{\delta d_n\}^T[\emptyset]^T P(t)d\Gamma \tag{17.d}$$

where

$$[H] = \begin{bmatrix} \frac{\partial}{\partial X} & 0 \\ 0 & \frac{\partial}{\partial Y} \\ \frac{\partial}{\partial Y} & \frac{\partial}{\partial X} \end{bmatrix}[\emptyset] \tag{18}$$

The pulse point load $P(t)$ is assumed to be sinusoidal form in time domain as following.

$$P(t) = P_0 sin(\Omega\, t) \qquad 0 \le t \ll t0 \quad \text{Sin Pulse} \tag{19}$$

where, P_0 is the amplitude of the dynamic load, t_0 is the time duration of the load and Ω is the frequency of the dynamic load. The governing equation of motions Eq. (16), is solved numerically by using implicit Newmark average acceleration $(\alpha = 0.5\,, \beta = 0.25)$ method in the time domain. By this procedure, the dynamic problem is transferred to system of static problem in each step as following:-

$$[\bar{K}(t,X)]\{d_n\}_{j+1} = \{\bar{F}(t)\} \tag{20}$$

in which

$$[\bar{K}(t,X)] = [K] + \frac{1}{\beta\Delta t^2}[M] + \frac{\alpha}{\beta\Delta t}[C] \tag{21.a}$$

$$\{\bar{F}(t)\} = \{F(t)\}_{j+1} + B_1\{d_n\}_j + B_2\{\dot{d}_n\}_j + B_3\{\ddot{d}_n\}_j \tag{21.b}$$

and constant coefficients can be evaluated by

$$B_1 = \frac{1}{\beta\Delta t^2}[M] + \frac{\alpha}{\beta\Delta t}[C]\,,$$
$$B_2 = \frac{1}{\beta\Delta t}[M] + [C]\left(\frac{\alpha}{\beta} - 1\right), \tag{22}$$
$$B_3 = [M]\left(\frac{1}{2\beta} - 1\right) + [C]\left(\frac{\alpha}{2\beta} - 1\right)$$

After evaluating $\{d_n\}_{j+1}$ at a time $t_{j+1} = t_j + \Delta t$, the acceleration and velocity vectors can be evaluated by

$$\{\ddot{d}_n\}_{j+1} = \frac{1}{\beta\Delta t^2}\left(\{d_n\}_{j+1} - \{d_n\}_j\right) - \frac{[M]}{\beta\Delta t}\{\dot{d}_n\}_j - \left(\frac{\alpha}{2\beta} - 1\right)\{\ddot{d}_n\}_j \tag{23.a}$$

$$\{\dot{d}_n\}_{j+1} = \{\dot{d}_n\}_j + \Delta t\,(1-\alpha)\{\ddot{d}_n\}_j + \Delta t\,\alpha\,\{\ddot{d}_n\}_{j+1} \tag{23.b}$$

3. Numerical results

In the numerical study, effects of graduation parameter, geometrical and stacking sequence of layers on the time response of thick multilayer FG beams under pulse load with damped effects. The materials of functionally graded layers are considered as Aluminium (Al; E=70 GPa, ν=0.3, ρ=2702 kg/m^3) and Zirconia (E=151GPa, ν=0.3, ρ=3000 kg/m^3). The bottom surface of the FG layer is Zirconia, the top surface material of the FG layer is Aluminium. The dimensions of the FG thick beam are considered as follows: b= 0.15 m, h = 0.15 m and the length of beam varied according to aspect ratio L/h=4, 5, 7 in the numerical process. The height of each layer is equal. It is noted that the following parameters are used in the numerical results: P_0=1000 kN, Ω=2 rad/s, $t_0 = 0.05\,s$, $\eta = 0.0001$. The Gaussian integration rule with five integration points is used for calculation of the integration. In the numerical process, the finite element number is taken as 30 in both X and Z directions.

In the numerical results, three different stacking sequences of layers are considered. The stacking sequences of layers used are ***stacking sequence 1:*** FGM-Homogeneous Aluminum -FGM, ***stacking sequence 2:*** Homogeneous Zirconia - FGM-Homogeneous Aluminum layers and ***stacking sequence 3:*** FGM - FGM - FGM layers as shown figure 3.

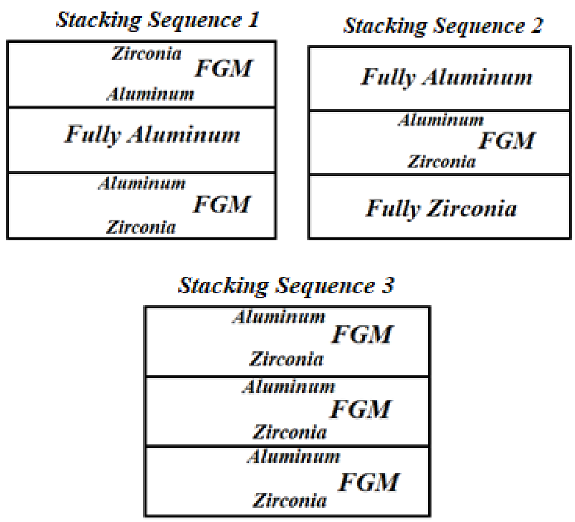

Fig. 3 Stacking sequence distributions through the beam thickness

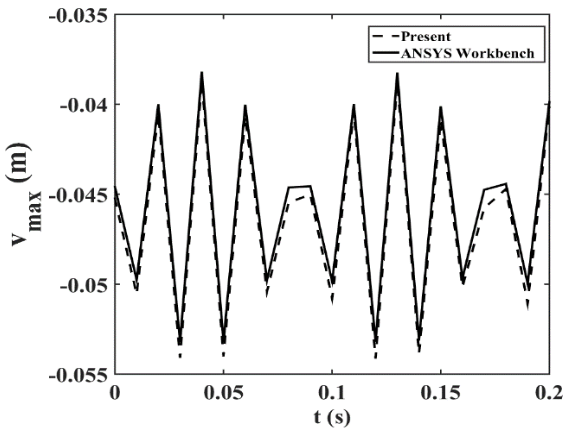

Fig. 4 Comparison study: Time responses of the Homogeneous Zirconia/ Homogeneous Aluminum/ Homogeneous Zirconia beam under pulse load without damping effect for L/h=7, P_0=1000 kN, Ω=2 rad/s, $t_0 = 0.01\,sn$

In order to validate proposed model, a comparison study is performed. In the validation study, the dynamic vertical displacements at free end of a Homogeneous Zirconia/ Homogeneous Aluminum/ Homogeneous Zirconia beam are obtained and compared with ANSYS Workbench 14 program under pulse load without damping effect for L/h=7, P_0=1000 kN, Ω=5 rad/s, $t_0 = 0.01\,s$, as shown in figure 4. It is noticed from figure 4, that results of this study are approximately identical with results of ANSYS Workbench 14.

In figures 5, 6 and 7, the time response of the free end of thick cantilever FGM layered thick beam are presented with different stacking sequences for aspect ratio L/h=4, L/h=5 and L/h=7, respectively. It is seen from these figures, after removing the load, the displacement decays in progress of time naturally because of damping and approach to a steady state value without any oscillations, for all stacking sequences.

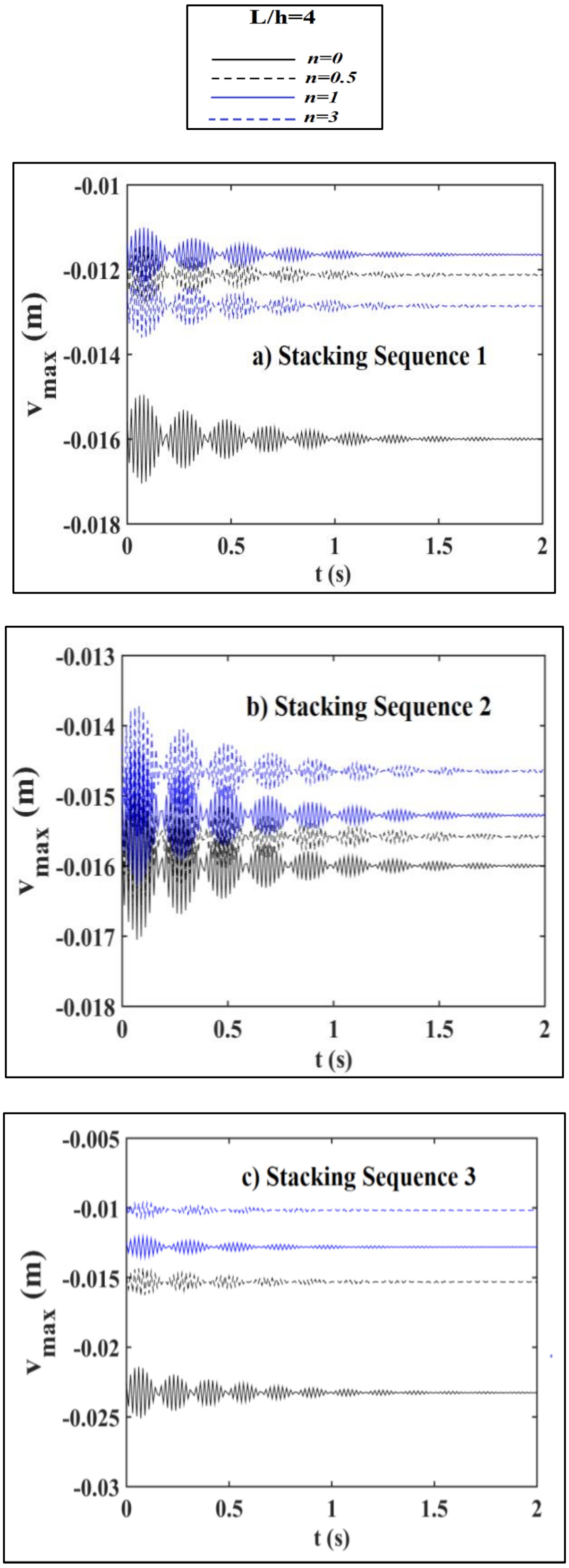

Fig. 5 Time responses of thick FGM layered beam for L/h=4 with different n parameters for a) *stacking sequence 1*, a) *stacking sequence 2* and c) *stacking sequence 3*.

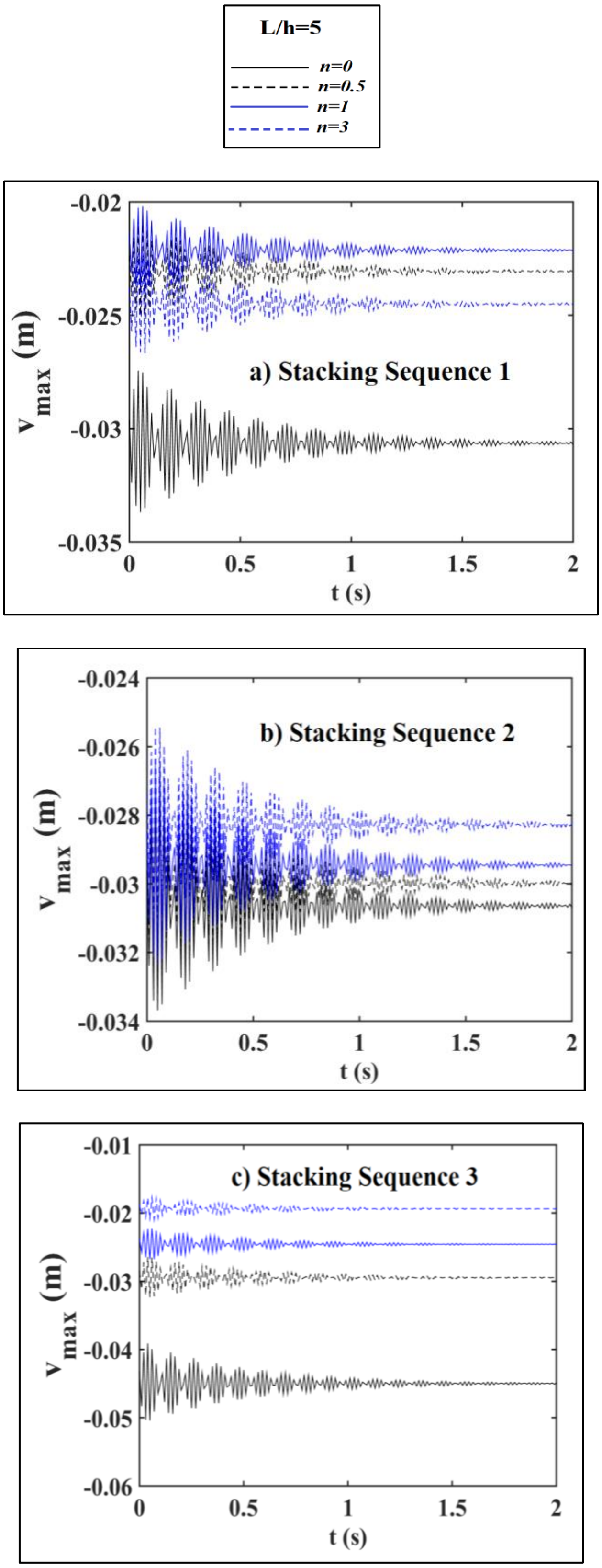

Fig. 6 Time responses of thick FGM layered beam for L/h=5 with different n parameters for a) *stacking sequence 1*, a) *stacking sequence 2* and c) *stacking sequence 3*.

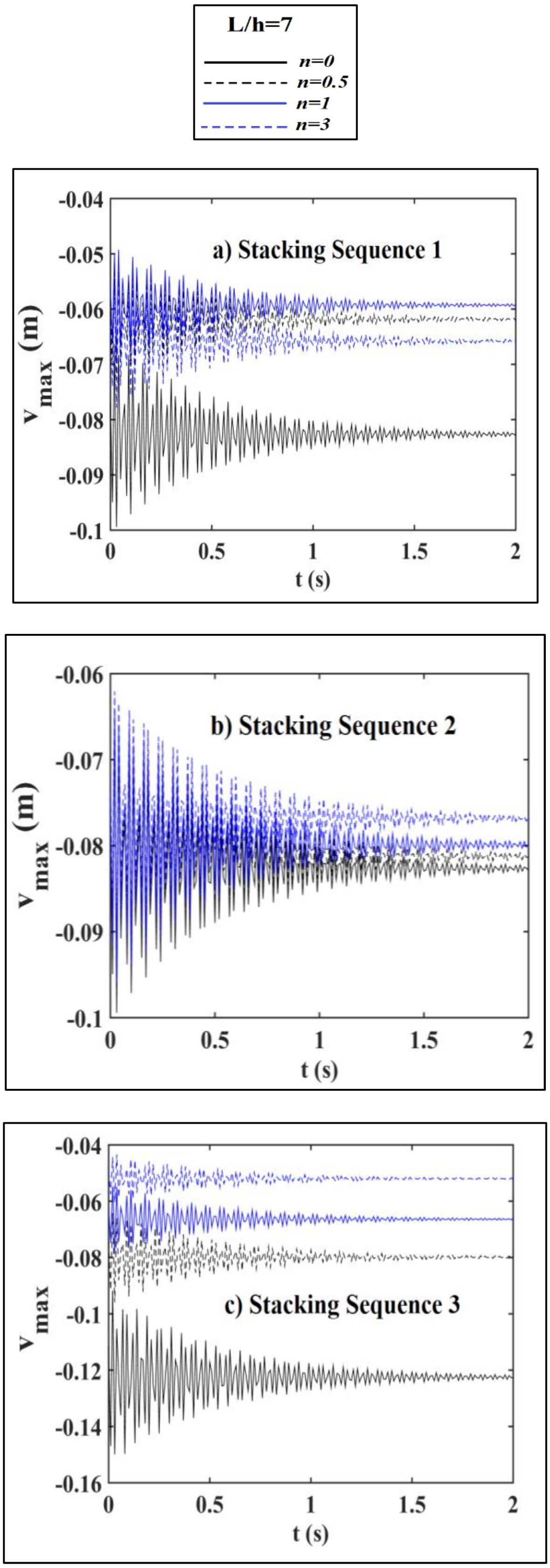

Fig. 7 Time responses of thick FGM layered beam for L/h=7 with different n parameters for a) *stacking sequence 1*, a) *stacking sequence 2* and c) *stacking sequence 3*.

The gradation material parameter(n) has a significant influence on the dynamic response of beam and its amplitude. As shown in Fig. 5, by increasing the gradation parameter (n), the amplitude of displacement waves decreased for all stacking sequences. This observation due to increasing the content of Zirconia by increasing the gradation parameter (n), that leads to increase the stiffness of structure and hence decreasing the deflection. It is noted also, the number of waves increases by increasing the graduation distribution parameter (n).

By comparing Figs. (5-7) for each other, it is observed that the increasing the aspect ratio (L/h) tends to decrease time period of oscillation and hence increase the number of the waves. In a case of hand, by fixing all conditions and changing the stacking sequences, it is observed that, the number of waves change according to the stacking sequences significantly.

The number of waves in the *stacking sequence 2* is bigger than the number of waves in other the stacking sequences*, considerably.*In designing process, if the beam designed based on the rigidity, the stacking sequence 1 is the best case. If the beam designed based on dynamic behavior, the stacking sequence 3 is preferred rather than others.

4. Conclusions

In the framework of 2D plane stress problem, the dynamic response of multilayered functionally graded thick beam with Kelvin–Voigt viscoelastic damped is investigated. Numerical finite element with twelve –node 2D plane element and Newmark implicit constant average acceleration procedure are exploited to solve the equation of motion. Materials are gradated through the thickness by power law. The beam thickness composed from three different layers with different gradation. So, three different stacking sequences are studied. The proposed model is validated with ANSYS and an excellent agreement was obtained. The most findings of the current analysis can be summarized as:

- Because of damping, the displacement decays in progress of time naturally and approach to a steady state value without any oscillations.
- By increasing the gradation parameter (n), the amplitude of displacement waves decreased for all stacking sequences.
- The number of waves increases by increasing the graduation distribution parameter (n).
- The increasing the aspect ratio (L/h) tends to decrease time period of oscillation and hence increase the number of the waves.
- The presented mathematical model is useful in analysis and design of nuclear, marine, vehicle and aerospace structures those manufactured from functionally graded materials (FGM).

Acknowledgments

This project was funded by the Deanship of Scientific Research (DSR), King Abdulaziz University, Jeddah, under grant No. (G-083-135-1441). The authors, therefore, gratefully acknowledge DSR technical and financial support.

References

Abualnour, M., Chikh, A., Hebali, H., Kaci, A., Tounsi, A., Bousahla, A. A. and Tounsi, A. (2019), "Thermomechanical analysis of antisymmetric laminated reinforced composite plates using a new four variable trigonometric refined plate theory", *Comput. Concrete*, **24**(6), 489-498. https://doi.org/10.12989/cac.2019.24.6.489

Addou, F. Y., Meradjah, M., Bousahla, A.A., Benachour, A., Bourada, F., Tounsi, A. and Mahmoud, S.R. (2019), "Influences of porosity on dynamic response of FG plates resting on Winkler/Pasternak/Kerr foundation using quasi 3D HSDT", *Comput. Concrete*, **24**(4), 347-367. https://doi.org/10.12989/cac.2019.24.4.347

Akbaş, Ş. D. (2017a), "Forced vibration analysis of functionally graded nanobeams", *J. Appl. Mech.*, **9**(07), 1750100. https://doi.org/10.1142/S1758825117501009

Akbaş, Ş. D. (2018), "Forced vibration analysis of functionally graded porous deep beams", *Compos. Struct.*, **186**, 293-302. https://doi.org/10.1016/j.compstruct.2017.12.013

Akbaş, Ş. D. (2019), "Forced vibration analysis of functionally graded sandwich deep beams", *Coupled Syst. Mech.*, **8**(3), 259-271. https://doi.org/10.12989/csm.2019.8.3.259

Alimirzaei, S., Mohammadimehr, M. and Tounsi, A. (2019), "Nonlinear analysis of viscoelastic micro-composite beam with geometrical imperfection using FEM: MSGT electro-magneto-elastic bending, buckling and vibration solutions", *Struct. Eng. Mech.*, **71**(5), 485-502. https://doi.org/10.12989/sem.2019.71.5.485.

Alshorbagy, A. E., Eltaher, M. A. and Mahmoud, F. F. (2011), "Free vibration characteristics of a functionally graded beam by finite element method", *Appl. Math. Modell.*, **35**(1), 412-425. https://doi.org/10.1016/j.apm.2010.07.006

Apetre, N. A., Sankar, B. V. and Ambur, D. R. (2006), "Low-velocity impact response of sandwich beams with functionally graded core", *J. Solids Struct.*, **43**(9), 2479-2496. https://doi.org/10.1016/j.ijsolstr.2005.06.003

Asghar, S., Naeem, M. N., Hussain, M., Taj, M. and Tounsi, A. (2020), "Prediction and assessment of nonlocal natural frequencies of DWCNTs: Vibration analysis", *Comput. Concrete*, **25**(2), 133. https://doi.org/10.12989/cac.2020.25.2.133

Avila, A. F. (2007), "Failure mode investigation of sandwich beams with functionally graded core", *Compos. Struct.*, **81**(3), 323-330. https://doi.org/10.1016/j.compstruct.2006.08.030

Balubaid, M., Tounsi, A., Dakhel, B. and Mahmoud, S. R. (2019), "Free vibration investigation of FG nanoscale plate using nonlocal two variables integral refined plate theory", *Comput. Concrete*, **24**(6), 579-586. https://doi.org/10.12989/cac.2019.24.6.579

Bedia, W. A., Houari, M. S. A., Bessaim, A., Bousahla, A. A., Tounsi, A., Saeed, T. and Alhodaly, M. S. (2019), "A new hyperbolic two-unknown beam model for bending and buckling analysis of a nonlocal strain gradient nanobeams", *J. Nano Res.*, **57**, 175-191. https://doi.org/10.4028/www.scientific.net/JNanoR.57.175

Belabed, Z., Bousahla, A. A., Houari, M. S. A., Tounsi, A. and Mahmoud, S. R. (2018), "A new 3-unknown hyperbolic shear deformation theory for vibration of functionally graded sandwich plate", *Earthq. Struct.*, **14**(2), 103-115. https://doi.org/10.12989/eas.2018.14.2.103

Belbachir, N., Draich, K., Bousahla, A. A., Bourada, M., Tounsi, A. and Mohammadimehr, M. (2019), "Bending analysis of anti-symmetric cross-ply laminated plates under nonlinear thermal and mechanical loadings", *Steel Compos. Struct.*, **33**(1), 81-92. https://doi.org/10.12989/scs.2019.33.1.081

Bellal, M., Hebali, H., Heireche, H., Bousahla, A. A., Tounsi, A., Bourada, F., ... & Tounsi, A, (2020), "Buckling behavior of a single-layered graphene sheet resting on viscoelastic medium via nonlocal four-unknown integral model", *Steel Compos. Struct.*, **34**(5), 643. https://doi.org/10.12989/scs.2020.34.5.643

Berghouti, H., Adda Bedia, E. A., Benkhedda, A. and Tounsi, A. (2019), "Vibration analysis of nonlocal porous nanobeams made of functionally graded material", *Adv. Nano Res.,* **7**(5), 351-364. https://doi.org/10.12989/anr.2019.7.5.351

Boukhlif, Z., Bouremana, M., Bourada, F., Bousahla, A. A., Bourada, M., Tounsi, A. and Al-Osta, M. A. (2019), "A simple quasi-3D HSDT for the dynamics analysis of FG thick plate on elastic foundation", *Steel Compos. Struct.*, **31**(5), 503-516. https://doi.org/10.12989/scs.2019.31.5.503

Boulefrakh, L., Hebali, H., Chikh, A., Bousahla, A. A., Tounsi, A. and Mahmoud, S. R. (2019), "The effect of parameters of visco-Pasternak foundation on the bending and vibration properties of a thick FG plate", *Geomech. Eng.*, **18**(2), 161-178. https://doi.org/10.12989/gae.2019.18.2.161

Boutaleb, S., Benrahou, K. H., Bakora, A., Algarni, A., Bousahla, A. A., Tounsi, A. and Mahmoud, S. R. (2019), "Dynamic analysis of nanosize FG rectangular plates based on simple nonlocal quasi 3D HSDT", *Adv. Nano Res.*, **7**(3), 191. https://doi.org/10.12989/anr.2019.7.3.191

Boussoula, A., Boucham, B., Bourada, M., Bourada, F., Tounsi, A., Bousahla, A. A. and Tounsi, A. (2020), "A simple nth-order shear deformation theory for thermomechanical bending analysis of different configurations of FG sandwich plates", *Smart Struct. Syst.*, **25**(2), 197. https://doi.org/10.12989/sss.2020.25.2.197

Chikr, S. C., Kaci, A., Yeghnem, R. and Tounsi, A. (2019), "A new higher-order shear and normal deformation theory for the buckling analysis of new type of FGM sandwich plates", *Struct. Eng. Mech.*, **72**(5), 653- 673. https://doi.org/10.12989/sem.2019.72.5.653

Daikh, A. A., Guerroudj, M., El Adjrami, M. and Megueni, A. (2020), "Thermal Buckling of Functionally Graded Sandwich Beams", *Adv. Mater. Res.*, **1156**, 43-59. Trans Tech Publications Ltd. https://doi.org/10.4028/www.scientific.net/AMR.1156.43.

Draiche, K., Bousahla, A. A., Tounsi, A., Alwabli, A. S., Tounsi, A. and Mahmoud, S. R. (2019), "Static analysis of laminated reinforced composite plates using a simple first-order shear deformation theory", *Comput. Concrete*, **24**(4), 369-378. https://doi.org/10.12989/cac.2019.24.4.369

Draoui, A., Zidour, M., Tounsi, A. and Adim, B. (2019), "Static and dynamic behavior of nanotubes-reinforced sandwich plates using (FSDT)", *J. Nano Res.*, **57**, 117-135. https://doi.org/10.4028/www.scientific.net/JNanoR.57.117.

Dash, S., Mehar, K., Sharma, N., Mahapatra, T. R. and Panda, S. K. (2019), "Finite element solution of stress and flexural strength of functionally graded doubly curved sandwich shell panel", *Earthq. Struct.*, **16**(1), 55-67. https://doi.org/10.12989/eas.2019.16.1.055.

Ebrahimi, F. and Farazmandnia, N. (2018), "Vibration analysis of functionally graded carbon nanotube-reinforced composite sandwich beams in thermal environment", *Adv. Aircraft Spacecraft Sci.*, **5**(1), 107. https://doi.org/10.12989/aas.2018.5.1.107.

Ebrahimi, F., Fardshad, R. E. and Mahesh, V. (2019), "Frequency response analysis of curved embedded magneto-electro-viscoelastic functionally graded nanobeams", *Adv. Nano Res.*, **7**(6), 391. https://doi.org/10.12989/anr.2019.7.6.391.

Eltaher, M. A., A. A. Abdelrahman, A. Al-Nabawy, M. Khater, and A. Mansour, (2014), "Vibration of nonlinear graduation of nano-Timoshenko beam considering the neutral axis position", *Appl. Math. Comput.*, **235**, 512-529. https://doi.org/10.1016/j.amc.2014.03.028

Eltaher, M. A., Attia, M. A., Soliman, A. E. and Alshorbagy, A. E.

(2018), "Analysis of crack occurs under unsteady pressure and temperature in a natural gas facility by applying FGM", *Struct. Eng. Mech.*, **66**(1), 97-111. https://doi.org/10.12989/sem.2018.66.1.097

Eltaher, M.A. and Akbas, S.D (2020), "Transient Response of 2D Functionally Graded Beam Structure", *Struct. Eng. Mech.*, **75**, https://doi.org/10.12989/sem.2020.75.3.357.

Eltaher, M. A., Omar, F. A., Abdraboh, A. M., Abdalla, W. S. and Alshorbagy, A.E. (2020a), "Mechanical behaviors of piezoelectric nonlocal nanobeam with cutouts", *Smart Struct. Syst.*, **25**(2), 219. https://doi.org/10.12989/sss.2020.25.2.219

Eltaher, M. A., Mohamed, S. A. and Melaibari, A. (2020b), "Static stability of a unified composite beams under varying axial loads", *Thin-Wall. Struct.*, **147**, 106488. https://doi.org/10.1016/j.tws.2019.106488.

Eltaher, M. A. and Mohamed, S. A. (2020), "Buckling and stability analysis of sandwich beams subjected to varying axial loads", *Steel Compos. Struct.*, **34**(2), 241. https://doi.org/10.12989/scs.2020.34.2.241.

Emam, S., Eltaher, M., Khater, M. and Abdalla, W. (2018), "Postbuckling and Free Vibration of Multilayer Imperfect Nanobeams under a Pre-Stress Load", *Appl. Sci.*, **8**(11), 2238. https://doi.org/10.3390/app8112238.

Etemadi, E., Khatibi, A. A. and Takaffoli, M. (2009), "3D finite element simulation of sandwich panels with a functionally graded core subjected to low velocity impact", *Compos. Struct.*, **89**(1), 28-34. https://doi.org/10.1016/j.compstruct.2008.06.013

Esmaeili, M. and Tadi Beni, Y. (2019), "Vibration and Buckling Analysis of Functionally Graded Flexoelectric Smart Beam", *J. Appl. Comput. Mech.*, **5**(5), https://doi/900-917. 10.22055/JACM.2019.27857.1439

Fan, Y., Xiang, Y., Shen, H. S. and Wang, H. (2018), "Low-velocity impact response of FG-GRC laminated beams resting on visco-elastic foundations", *J. Mech. Sci.*, **141**, 117-126. https://doi.org/10.1016/j.ijmecsci.2018.04.007.

Gardner, N., Wang, E. and Shukla, A. (2012), "Performance of functionally graded sandwich composite beams under shock wave loading", *Compos. Struct.*, **94**(5), 1755-1770. https://doi.org/10.1016/j.compstruct.2011.12.006

Hamed, M. A., Sadoun, A. M. and Eltaher, M. A. (2019), "Effects of porosity models on static behavior of size dependent functionally graded beam", *Struct. Eng. Mech.*, **71**(1), 89-98. https://doi.org/10.12989/sem.2019.71.1.089

Hamed M.A., Mohamed, S.A., Eltaher, M.A., (2020), "Buckling Analysis of Sandwich Beam Rested on Elastic Foundation and Subjected to Varying Axial In-Plane Loads", *Steel Compos. Struct.*, **34**(1), 75-89. https://doi.org/10.12989/scs.2020.34.1.075.

Huang, Y. and Ouyang, Z. Y. (2020), "Exact solution for bending analysis of two-directional functionally graded Timoshenko beams", *ARch. Appl. Mech.*, 1-19. https://doi.org/10.1007/s00419-019-01655-5

Hussain, M., Naeem, M. N., Tounsi, A. and Taj, M. (2019), "Nonlocal effect on the vibration of armchair and zigzag SWCNTs with bending rigidity", *Adv. Nano Res.*, **7**(6), 431. https://doi.org/10.12989/anr.2019.7.6.431

Javani, M., Kiani, Y. and Eslami, M. R. (2019), "Free vibration of arbitrary thick FGM deep arches using unconstrained higher-order shear deformation theory", *Thin-Walled Struct.*, **136**, 258-266. https://doi.org/10.1016/j.tws.2018.12.020

Kaddari, M., Kaci, A., Bousahla, A. A., Tounsi, A., Bourada, F., Bedia, E. A. and Al-Osta, M. A. (2020), "A study on the structural behaviour of functionally graded porous plates on elastic foundation using a new quasi-3D model: bending and free vibration analysis", *Comput. Concrete*, **25**(1), 37. https://doi.org/10.12989/cac.2020.25.1.037

Karami, B., Janghorban, M. and Tounsi, A. (2019a), "Galerkin's approach for buckling analysis of functionally graded anisotropic nanoplates/different boundary conditions", *Eng. Comput.*, **35**(4), 1297-1316. https://doi.org/10.1007/s00366-018-0664-9

Karami, B., Janghorban, M. and Tounsi, A. (2019b), "Wave propagation of functionally graded anisotropic nanoplates resting on Winkler-Pasternak foundation", *Struct. Eng. Mech.*, **70**(1), 55-66. https://doi.org/10.12989/sem.2019.70.1.055

Karami, B., Janghorban, M. and Tounsi, A. (2019c), "On exact wave propagation analysis of triclinic material using three-dimensional bi-Helmholtz gradient plate model", *Struct. Eng. Mech.*, **69**(5), 487-497. https://doi.org/10.12989/sem.2019.69.5.487

Karami, B., Shahsavari, D., Janghorban, M. and Tounsi, A. (2019d), "Resonance behavior of functionally graded polymer composite nanoplates reinforced with graphene nanoplatelets", *J. Mech. Sci.*, **156**, 94-105. https://doi.org/10.1016/j.ijmecsci.2019.03.036

Karami, B., Janghorban, M. and Tounsi, A. (2019e), "On pre-stressed functionally graded anisotropic nanoshell in magnetic field", *J. Brazilian Soc. Mech. Sci. Eng.*, **41**(11), 495. https://doi.org/10.1007/s40430-019-1996-0

Khalili, S. M. R., Malekzadeh, K. and Gorgabad, A. V. (2013), "Low velocity transverse impact response of functionally graded plates with temperature dependent properties", *Compos. Struct.*, **96**, 64-74. https://doi.org/10.1016/j.compstruct.2012.07.035

Khan, M. A., Yasin, M. Y., Beg, M. S. and Khan, A. H. (2020), "Free and Forced Vibration Analysis of Functionally Graded Beams Using Finite Element Model Based on Refined Third-Order Theory", *Emerging Trends in Mechanical Engineering,* Springer, Singapore.

Kiani, Y., Sadighi, M., Salami, S. J. and Eslami, M.R. (2013), "Low velocity impact response of thick FGM beams with general boundary conditions in thermal field", *Compos. Struct.*, **104**, 293-303. https://doi.org/10.1016/j.compstruct.2013.05.002.

Lai, J., Yang, H., Wang, H., Zheng, X. and Wang, Q. (2019), "Penetration experiments and simulation of three-layer functionally graded cementitious composite subjected to multiple projectile impacts", *Construct. Building Mater.*, **196**, 499-511. https://doi.org/10.1016/j.conbuildmat.2018.11.154.

Medani, M., Benahmed, A., Zidour, M., Heireche, H., Tounsi, A., Bousahla, A.A. and Mahmoud, S.R. (2019), "Static and dynamic behavior of (FG-CNT) reinforced porous sandwich plate using energy principle", *Steel Compos. Struct.*, **32**(5), 595-610. https://doi.org/10.12989/scs.2019.32.5.595.

Li, H. C., Ke, L. L., Yang, J., Kitipornchai, S. and Wang, Y. S. (2020), "Free vibration of variable thickness FGM beam submerged in fluid", *Compos. Struct.*, **233**, 111582. https://doi.org/10.1016/j.compstruct.2019.111582

Moleiro, F., Carrera, E., Ferreira, A. J. M. and Reddy, J. N. (2020), "Hygro-thermo-mechanical modelling and analysis of multilayered plates with embedded functionally graded material layers", *Compos. Struct.*, **233**, 111442. https://doi.org/10.1016/j.compstruct.2019.111442

Nguyen, T. K., Vo, T. P., Nguyen, B. D. and Lee, J. (2016), "An analytical solution for buckling and vibration analysis of functionally graded sandwich beams using a quasi-3D shear deformation theory", *Compos. Struct.*, **156**, 238-252. https://doi.org/10.1016/j.compstruct.2015.11.074

Sahla, M., Saidi, H., Draiche, K., Bousahla, A. A., Bourada, F. and Tounsi, A. (2019), "Free vibration analysis of angle-ply laminated composite and soft core sandwich plates", *Steel Compos. Struct.*, **33**(5), 663. https://doi.org/10.12989/scs.2019.33.5.663

Sedighi, H. M., Shirazi, K. H. and Noghrehabadi, A. (2012), "Application of recent powerful analytical approaches on the non-linear vibration of cantilever beams", *J. Nonlinear Sci. Numerical Simulation*, **13**(7-8), 487-494. https://doi.org/10.1515/ijnsns-2012-0030

Sedighi, H. M., Shirazi, K. H. and Attarzadeh, M. A. (2013), "A study on the quintic nonlinear beam vibrations using asymptotic approximate approaches", *Acta Astronautica*, **91**, 245-250. https://doi.org/10.1016/j.actaastro.2013.06.018.

Sedighi, H. M. and Daneshmand, F. (2014), "Nonlinear transversely vibrating beams by the homotopy perturbation method with an auxiliary term", *J. Appl. Comput. Mech.*, **1**(1), 1-9. https://doi.org/10.22055/JACM.2014.10545.

Sedighi, H. M. (2014), "Size-dependent dynamic pull-in instability of vibrating electrically actuated microbeams based on the strain gradient elasticity theory", *Acta Astronautica*, **95**, 111-123. https://doi.org/10.1016/j.actaastro.2013.10.020.

Sedighi, H. M., Koochi, A., Daneshmand, F. and Abadyan, M. (2015a), "Non-linear dynamic instability of a double-sided nano-bridge considering centrifugal force and rarefied gas flow", *J. Non-Linear Mech.*, **77**, 96-106. https://doi.org/10.1016/j.ijnonlinmec.2015.08.002.

Sedighi, H. M., Daneshmand, F. and Abadyan, M. (2015b), "Dynamic instability analysis of electrostatic functionally graded doubly-clamped nano-actuators", *Compos. Struct.*, **124**, 55-64. https://doi.org/10.1016/j.compstruct.2015.01.004.

Sedighi, H. M., Daneshmand, F. and Abadyan, M. (2016), "Modeling the effects of material properties on the pull-in instability of nonlocal functionally graded nano-actuators", *ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik*, **96**(3), 385-400. https://doi.org/10.1002/zamm.201400160.

Seguini, M. and Nedjar, D. (2017), "Nonlinear Analysis of Deep Beam Resting on Linear and Nonlinear Random Soil", *Arabian J. Sci. Eng.*, **42**(9), 3875-3893. https://doi.org/10.1007/s13369-017-2449-7

Semmah, A., Heireche, H., Bousahla, A. A. and Tounsi, A. (2019), "Thermal buckling analysis of SWBNNT on Winkler foundation by non local FSDT", *Adv. Nano Res.*, **7**(2), 89. https://doi.org/10.12989/anr.2019.7.2.089.

She, G. L., Yan, K. M., Zhang, Y. L., Liu, H. B. and Ren, Y. R. (2018), "Wave propagation of functionally graded porous nanobeams based on non-local strain gradient theory", *European Phys. J. Plus*, **133**(9), 368. https://doi.org/10.1140/epjp/i2018-12196-5.

She, G. L., Ren, Y. R. and Yan, K. M. (2019), "On snap-buckling of porous FG curved nanobeams", *Acta Astronautica*, 161, 475-484. https://doi.org/10.1016/j.actaastro.2019.04.010

Şimşek, M. and Al-Shujairi, M. (2017), "Static, free and forced vibration of functionally graded (FG) sandwich beams excited by two successive moving harmonic loads", *Compos. Part B Eng.*, **108**, 18-34. https://doi.org/10.1016/j.compositesb.2016.09.098.

Soliman, A. E., Eltaher, M. A., Attia, M. A. and Alshorbagy, A. E. (2018), "Nonlinear transient analysis of FG pipe subjected to internal pressure and unsteady temperature in a natural gas facility", *Struct. Eng. Mech.*, **66**(1), 85-96. https://doi.org/10.12989/sem.2018.66.1.085

Song, Z. G., He, X. and Liew, K. M. (2018), "Dynamic responses of aerothermoelastic functionally graded CNT reinforced composite panels in supersonic airflow subjected to low-velocity impact", *Compos. Part B Eng.*, **149**, 99-109. https://doi.org/10.1016/j.compositesb.2018.05.026

Sucharda, O. and Konecny, P. (2018), Recommendation for the modelling of 3D non-linear analysis of RC beam tests", *Comput. Concrete*, **21**(1), 11-20. https://doi.org/10.12989/cac.2018.21.1.011.

Tlidji, Y., Zidour, M., Draiche, K., Safa, A., Bourada, M., Tounsi, A. and Mahmoud, S. R. (2019), "Vibration analysis of different material distributions of functionally graded microbeam", *Struct. Eng. Mech.*, **69**(6), 637-649. https://doi.org/10.12989/sem.2019.69.6.637

Tounsi,A., Al-Dulaijan, S.U., Al-Osta, M.A., Chikh, A., Al-Zahrani, M.M., Sharif, A. and Tounsi, A., (2020), "A four variable trigonometric integral plate theory for hygro-thermo-mechanical bending analysis of AFG ceramic-metal plates resting on a two-parameter elastic foundation", *Steel Compos. Struct.*, **34**(4), 511-524 https://doi.org/10.12989/scs.2020.34.4.511.

Wu, D., Liu, A., Huang, Y., Huang, Y., Pi, Y. and Gao, W. (2018), "Dynamic analysis of functionally graded porous structures through finite element analysis", *Eng. Struct.,* **165**, 287-301. https://doi.org/10.1016/j.engstruct.2018.03.023

Yang, Y., Lam, C. C., Kou, K. P. and Iu, V. P. (2014), "Free vibration analysis of the functionally graded sandwich beams by a meshfree boundary-domain integral equation method", *Compos. Struct.*, **117**, 32-39. https://doi.org/10.1016/j.compstruct.2014.06.016.

Zarga, D., Tounsi, A., Bousahla, A. A., Bourada, F. and Mahmoud, S. R. (2019), "Thermomechanical bending study for functionally graded sandwich plates using a simple quasi-3D shear deformation theory", *Steel Compos. Struct.*, **32**(3), 389-410. https://doi.org/10.12989/scs.2019.32.3.389

CC