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Anti-sparse representation for structural model updating using l_∞ norm regularization

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Abstract. Finite element (FE) model based structural damage detection (SDD) methods play vital roles in effectively locating and quantifying structural damages. Among these methods, structural model updating should be conducted before SDD to obtain benchmark models of real structures. However, the characteristics of updating parameters are not reasonably considered in existing studies. Inspired by the l_∞ norm regularization, a novel anti-sparse representation method is proposed for structural model updating in this study. Based on sensitivity analysis, both frequencies and mode shapes are used to define an objective function at first. Then, by adding l_∞ norm penalty, an optimization problem is established for structural model updating. As a result, the optimization problem can be solved by the fast iterative shrinkage thresholding algorithm (FISTA). Moreover, comparative studies with classical regularization strategy, i.e. the l_2 norm regularization method, are conducted as well. To intuitively illustrate the effectiveness of the proposed method, a 2-DOF spring-mass model is taken as an example in numerical simulations. The updating results show that the proposed method has a good robustness to measurement noises. Finally, to further verify the applicability of the proposed method, a six-storey aluminum alloy frame is designed and fabricated in laboratory. The added mass on each storey is taken as updating parameter. The updating results provide a good agreement with the true values, which indicates that the proposed method can effectively update the model parameters with a high accuracy.

Keywords: structural model updating; l_∞ norm regularization; anti-sparse representation; sensitivity analysis

1. Introduction

The safety of in-service engineering structures has attracted increasing attentions in recent years. Structural damage detection (SDD), a technology to timely detect potential damages, can effectively help to ensure the safety of structures and achieve long-term structural health monitoring (SHM) (Chen and Yu 2017, Lakshmi 2019).

In the existing methods, the finite element (FE) model based SDD methods have advantages to accurately locate and quantify structural damages (Ha *et al*. 2017, Vahidi *et al*. 2019). An accurate FE model is necessary for SDD because it is taken as a benchmark model for real structures. If the FE model cannot be used to describe damage characteristics of real structures, it will fail to effectively locate and quantify structural damages by using these methods. That is to say, reasonable model updating results are the premise for effective SDD. Thus, the differences of characteristics between the FE model and the real structure should be carefully reduced at first (Behmanesh *et al*. 2018, Zhang *et al*. 2019).

For model updating, structural parameters of FE model are changed during the updating process (Arora 2014). Dependent variables of structural parameters should be reasonably selected because they will greatly affect the updating process and results. It is common to select the dependent variables which are sensitive to structural parameters and insensitive to measurement noises (Goksu *et al*. 2017). Meanwhile, with the help of sensitivity analysis, sensitivity-based parameter updating method can effectively achieve the process of model updating (Link 1999, Batou 2019). Commonly, sensitivity analysis relates the changes in dependent variables to the changes in structural parameters. Due to this important ability, sensitivity analysis has been widely investigated for model updating in existing studies. For example, based on the sensitivity of frequencies, Cao *et al*. (2019) proposed a multistage model updating method for establishing an effective model of stitched sandwich composite. Based on frequency response function and sensitivity analysis, enhanced decomposed transfer function (EDTF) was introduced to mitigate the influence of incomplete measurement. Comparative studies with dynamic expansion of measured mode shapes proved the superiority of the proposed method for model updating (Pedram and Esfandiari 2019). Moreover, Esfandiari *et al*. (2018) accurately detected stiffness parameters by using the sensitivity of power spectral density (PSD) functions. Identified results of the truss model illustrated that the sensitivity based model updating method has great ability for parameter estimation under the influence of noise.

On the other hand, the model updating problem is a well-known ill-posed problem, because the structural parameters should be indirectly updated from the output information of structure. It is effective to improve the ill-

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posedness of this problem by constraining the ranges of corresponding parameters.

Penalties in regularization methods can constrain solution in a limited range (Wang and Yang, 2012), so regularization methods, such as the l_q ($0 < q \leq 1$), l_2 and l_∞ norm regularization methods, perform well in solving ill-posed problems. However, because the characteristics of penalties are usually different, these regularization methods have different application scopes.

Among these three regularization methods, the l_q ($0 < q \leq 1$) norm penalty has the strongest constrained function for updating parameters (Xu *et al*. 2012, Fan *et al*. 2018, Ding *et al*. 2019), which can be used to obtain sparse solutions. It is obvious that the l_q norm regularization methods are inappropriate strategies for solving the model updating problem because the updating parameters are usually not sparse.

Contrary to the l_q norm penalty, the l_2 norm penalty does not restrict the number of nonzero elements, but limits the sum of squares in solution. Therefore, dense solutions are obtained from the l_2 norm regularization. This regularization method has been widely studied for model updating problem (Hua *et al*. 2011, Zhang and Guo 2016).

For example, regularization parameter controls the balance between regularization errors and perturbation errors. Based on the l_2 norm regularization, the effect of regularization parameter for model updating results has been investigated in the existing studies (Mares *et al*. 2002, Hua *et al*. 2009). Moreover, based on incomplete modal data, Chen and Maung (2014) employed the l_2 norm regularization incorporating the L-curve criterion method for updating model parameters.

However, the l_2 norm regularization method is not reasonable for all situations (Deo and Walker 1997). Magnitudes of individual elements are not controlled by the l_2 norm penalty, so it is easily misestimated in model updating problem due to the influence of noises.

The l_∞ norm penalty is conductive to obtain the anti-sparse representation of solution and reduce errors in elements of solution. Previously, the l_∞ norm regularization method has been used in the minimum effort problem for minimizing the amplitude of the control input required (Cadzow 1971). Then, Fuchs (2011) proposed anti-sparse representations based on the l_∞ norm to represent a vector on a redundant basis. Because of the ability to spread energy equally, the l_∞ norm regularization method has numerous applications (Studer *et al*. 2012). A binarization scheme based on l_∞ penalty for high dimensional vectors was proposed and performed well in the approximate nearest neighbor search (Jégou *et al*. 2012). In wireless communication and control system, the l_∞ norm regularization method was introduced to reduce peak-to-average power ratio (PAPR) by Shen and Gu (2015). Moreover, for promoting anti-sparsity, the l_∞ norm has been applied to define probability distribution, which is called as democratic distribution (Elvira *et al*. 2016).

For the model updating problem, magnitudes of updating parameters are usually of the same order. Based on this characteristic, the l_∞ norm regularization method is considered for model updating in this study.

For the model updating problem, this paper focuses on the situation that magnitudes of updating parameters are of the same order, and a novel model updating method is proposed. Rather than the l_2 norm regularization, the l_∞ norm regularization is introduced for its ability to obtain an anti-sparse representation of solution. Based on sensitivity analysis, the l_∞ norm penalty is added to define an objective function. Then, a fast iterative shrinkage thresholding algorithm (FISTA) is adopted to solve the objective function for updating results. Numerical simulations on a 2-DOF spring-mass model and experimental verifications on a six-storey aluminum alloy frame fabricated in laboratory are conducted to investigate the performances of the proposed method. Meanwhile, comparative studies with the l_2 norm regularization method are also studied in these examples.

2. Theoretical background

2.1 Sensitivity analysis

The FE model should be updated to approximate the real structure. For obtaining a reasonable FE model, both physical and geometric parameters are updated in the model updating problem (Friswell *et al*. 2001, Khademi-Zahedi and Alimouri 2019). For these parameters, the following relationship between the FE model and the real structure is given at first:

$$p_j^{RE} = p_j^{FE}\left(1-\Delta\alpha_j\right) \tag{1}$$

where, p_j^{RE} (j = 1, 2, …, n) and p_j^{FE} (j = 1, 2, …, n) represent the jth parameters of the real structure and the FE model, respectively. α_j (j = 1, 2, …, n) is the design variable of the jth parameter. n is the total element number of updated parameters.

Frequencies and mode shapes are important modal parameters for model updating. Herein, these two modal parameters are used to update FE model. Based on sensitivity analysis, the following expression can be obtained:

$$\begin{bmatrix}\mathbf{S}_1 & \mathbf{S}_2 & \cdots & \mathbf{S}_n\end{bmatrix}\boldsymbol{\Delta\alpha} = \mathbf{S}\boldsymbol{\Delta\alpha} \approx \mathbf{\Delta f} = \begin{Bmatrix}\mathbf{\Delta f}_\upsilon \\ \mathbf{\Delta f}_\varphi\end{Bmatrix} \tag{2}$$

where, $\mathbf{\Delta f}_\upsilon$ and $\mathbf{\Delta f}_\varphi$ represent differences in frequencies and mode shapes between the real structure and corresponding FE model, respectively. $\boldsymbol{\Delta\alpha}$ is the difference of design variables. $\mathbf{S}$ is the sensitivity matrix. $\mathbf{S}_j$ (j = 1, 2, …, n) is the sensitivity matrix obtained by the forward finite difference approach (Li *et al*. 2015):

$$\mathbf{S}_j = \frac{\mathbf{f}(\gamma_j + \Delta\gamma_j) - \mathbf{f}(\gamma_j)}{\Delta\gamma_j} \tag{3}$$

where, $\mathbf{f}(\gamma_j + \Delta\gamma_j)$ represents the vector of modal parameters when the design variable γ_j changes into $\gamma_j + \Delta\gamma_j$. The value of $\Delta\gamma_j$ is important for accurate model updating. In this study, the step of $\Delta\gamma_j$ is selected as 0.0001.

The least squares (LS) method can be used to solve $\boldsymbol{\Delta\alpha}$ from Eq. (2) by minimizing sum of the squared errors between $\mathbf{S}\boldsymbol{\Delta\alpha}$ and $\boldsymbol{\Delta}\mathbf{f}$:

$$J_{\text{LS}}(\boldsymbol{\Delta\alpha}) = \arg\min_{\boldsymbol{\Delta\alpha}} \frac{1}{2}\left\|\mathbf{S}\boldsymbol{\Delta\alpha} - \boldsymbol{\Delta}\mathbf{f}\right\|_2^2 \tag{4}$$

However, model updating problem is ill-posed, and the modal parameters are inevitably polluted by noise. As a result, stable solutions cannot be obtained by the LS method (Hansen *et al*. 2007).

Regularization methods have good performance in improving the ill-posedness of model updating problem by constraining solution ranges of updating parameters.

It should be noted that, the constraint effects to solution ranges by using different regularization methods are different. For example, the l_q ($0 < q \leq 1$) norm regularization methods are suitable for solving the problems with sparse properties (Hou *et al*. 2018). On the contrary, the l_∞ norm regularization method can obtain dense solutions.

For the model updating problem, the updating parameters are not sparse in most cases. The elements in $\boldsymbol{\Delta\alpha}$ usually have same order of magnitudes. By considering this feature, the l_∞ norm regularization method is introduced into model updating problem to improve the ill-posedness. As a result, an anti-sparse representation of $\boldsymbol{\Delta\alpha}$ can be obtained for the model updating result.

2.2 L∞ norm regularization method

A system of linear equations can be simply written as:

$$\mathbf{A}\mathbf{x} = \mathbf{b} \tag{5}$$

where, $\mathbf{A} \in R^{m\times n}$ is a matrix. both $\mathbf{b} \in R^m$ and $\mathbf{x} \in R^n$ are vectors. m and n are the number of row and column in matrix $\mathbf{A}$, respectively.

Multiple solutions of vector $\mathbf{x}$ can be obtained from Eq. (5) when $\mathbf{A}$ is a full rank matrix. To find an anti-sparse representation of vector $\mathbf{x}$ and improve the ill-posedness of the problem, the l_∞ norm regularization is added into Eq. (5), which can be described as follows (Fuchs 2011):

$$J(\mathbf{x}) = \arg\min_{\mathbf{x}\in\mathbf{R}^n} \left\{\frac{1}{2}\left\|\mathbf{A}\mathbf{x} - \mathbf{b}\right\|_2^2 + \lambda\left\|\mathbf{x}\right\|_\infty\right\} \tag{6}$$

where, $\left\|\mathbf{x}\right\|_\infty = \max\left(|x_1|,\ |x_2|,\ \cdots,\ |x_n|\right)$ represents the l_∞ norm of $\mathbf{x}$. λ ($\lambda > 0$) is the regularization parameter.

As explaining by Elvira *et al*. (2017), the l_∞ norm regularization can obtain anti-sparsity solution, because the l_∞ norm penalty spreads the energy of $\mathbf{b}$ equally over elements in the vector $\mathbf{x}$ with respect to $\mathbf{A}$.

However, the norm penalty $\|\boldsymbol{x}\|_\infty$ is not a strictly convex function. To obtain a unique and stable solution from Eq. (6), the given matrix $\mathbf{A}$ must be restricted. Mangasarian and Recht (2011) derived necessary and sufficient conditions for an integer and unique solution by transforming the problem to some linear equations. It showed that the dimension of matrix $\mathbf{A}$ is a critical factor for successful solution. The ratio of row to column in $\mathbf{A}$ is stated that:

$$\mathrm{P} = \frac{m}{n} > \frac{1}{2} \tag{7}$$

where, P is an increasing function. When n is fixed, m should be larger than half of n, so the most integer programming problems can be successfully resolved.

For the model updating problem, the mathematical expression in Eq. (2) is same as Eq. (5). Meanwhile, by considering the characteristics of model updating problem, the l_∞ norm regularization method can be introduced to spread the information for all elements in design variables equally. As a result, the vector $\boldsymbol{\Delta\alpha}$ with the same order of magnitudes can be obtained.

With the help of Eq. (6), the model updating problem based on sensitivity analysis and the l_∞ norm regularization is described as follows:

$$J_\infty(\boldsymbol{\Delta\alpha}) = \arg\min_{\boldsymbol{\Delta\alpha}\in\mathbf{R}^n} \left\{\frac{1}{2}\left\|\mathbf{S}\boldsymbol{\Delta\alpha} - \boldsymbol{\Delta}\mathbf{f}\right\|_2^2 + \lambda\left\|\boldsymbol{\Delta\alpha}\right\|_\infty\right\} \tag{8}$$

Eq. (8) can be solved by the FISTA (Toh and Yun 2010). The l_∞ norm regularization method offers a robust representation of solution by sharing information evenly, so errors in elements of vector $\boldsymbol{\Delta\alpha}$ can be reduced well.

2.3 L2 norm regularization method

As a special case of Tikhonov regularization method, the l_2 norm regularization method has been widely applied in many fields (Pan *et al*. 2017). The constrained function of l_2 norm penalty is weaker than the l_q ($0 < q \leq 1$) norm penalties, and the l_2 norm regularization method can obtain dense solutions in many cases.

For the model updating problem, the l_2 regularization method has been used in the existing studies. To compare the updating results by respectively using the l_2 norm and the l_∞ norm regularization methods, the model updating problem based on sensitivity analysis method and the l_2 norm regularization method is expressed as follows:

$$J_2(\boldsymbol{\Delta\alpha}) = \arg\min_{\boldsymbol{\Delta\alpha}\in\mathbf{R}^n} \left\{\frac{1}{2}\left\|\mathbf{S}\boldsymbol{\Delta\alpha} - \boldsymbol{\Delta}\mathbf{f}\right\|_2^2 + \lambda\left\|\boldsymbol{\Delta\alpha}\right\|_2^2\right\} \tag{9}$$

The solution of vector $\boldsymbol{\Delta\alpha}$ solved from Eq. (9) can be given as (Zhang and Xu 2016):

$$\boldsymbol{\Delta\hat{\alpha}} = \left(\mathbf{S}\mathbf{S}^T + \lambda\mathbf{I}\right)^{-1}\mathbf{S}^T\boldsymbol{\Delta}\mathbf{f} \tag{10}$$

where, $\mathbf{I}$ is the identity matrix.

According to Eqs. (6) and (9), it should be pointed out that the regularization parameter λ greatly affects the updating results by controlling the balance between regularization error and perturbation error. To obtain reasonable solutions, the values of regularization parameter in different regularization methods are generally different. Thus, to effectively compare the l_2 norm regularization with the l_∞ norm regularization, the regularization parameters in different regularization methods are selected when same values of an indicator θ are obtained. The indicator is defined as follows:

$$\theta = \left\|\mathbf{f}^{RE} - \mathbf{f}^{FE}\right\|_2 \tag{11}$$

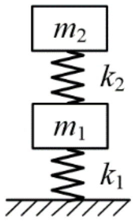


Fig. 1 Spring-mass model

where, $\mathbf{f}^{RE}$ is the rearranged vector of modal parameters for the real structure. $\mathbf{f}^{FE}$ is the rearranged vector of modal parameters for FE model updated by the l_2 norm regularization method or the l_∞ norm regularization method.

3. Numerical simulations

As shown in Fig. 1, a 2-DOF spring-mass model is used to intuitively analyze constrained functions of different norm penalties. The mass and stiffness of each DOF are 100 kg and 150 kN/m, respectively.

Different modal parameters polluted by noise are respectively simulated as follows:

$$\upsilon_i = \upsilon_i^{\,c} + \varepsilon_\upsilon R_i \left(\frac{\|\boldsymbol{\upsilon}_s - \boldsymbol{\upsilon}_e\|_2}{k}\right) \tag{12}$$

$$\varphi_{ij} = \varphi_{ij}^{\,c} + \varepsilon_\varphi R_{ij} \left(\frac{\|\boldsymbol{\varphi}_s - \boldsymbol{\varphi}_e\|_2}{lN}\right) \tag{13}$$

where, υ_i is the ith noisy frequency, φ_{ij} is the jth element of the ith noisy mode shape. Superscript 'c' represents corresponding calculated modal parameters. $\boldsymbol{\upsilon}$ and $\boldsymbol{\varphi}$ are vectors of frequencies and rearranged mode shapes, respectively. Subscripts 's' and 'e' represent modal parameters of the real structure and the initial FE model, respectively. ε_υ and ε_φ are noise level for frequencies and mode shapes, respectively. R_i and R_{ij} are random values drawn from the standard normal distribution.

A scenario in Table 1 is used to illustrate constrained functions of the l_∞ norm and l_2 norm penalties.

As shown in Fig. 2, the identified result obtained by the l_2 norm regularization method is given, and the sketch map about the regularization error $\|\Delta\alpha\|_2^2$ and the perturbation error $\|S\Delta\alpha - \Delta f\|_2^2$ is also given. Similarly, the corresponding identified result and the sketch map obtained by the l_∞ norm regularization are shown in Fig. 3.

From Figs. 2 and 3, it is obvious that the updating result obtained by the LS method is far from the actual value due to the influence of noises. Meanwhile, the regularization methods can improve the ill-posedness of the model updating problem, and the updating results are better than that obtained from the LS method.

From Fig. 2, it can be seen that both the perturbation error and the regularization error for the l_2 norm regularization are circles, and these two errors are tangent to the result which closes to the result obtained from the LS method. The function of l_2 norm penalty minimizes the sum of squares of $\boldsymbol{\Delta\alpha}$, but it cannot control amplitude of each element in $\boldsymbol{\Delta\alpha}$. As a result, the updating result in the first DOF is larger than the true value, and the result in the second DOF is less than the true value.

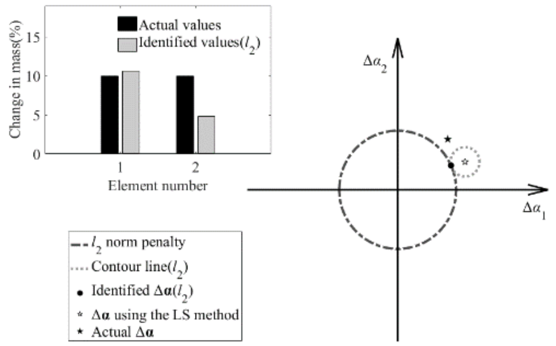


Fig. 2 Sketch map and identified results for spring-mass model in scenario 1 by adding l_2 norm penalty

Table 1 Simulated scenario for spring-mass model

Scenario no.	Changes in mass (%) @ DOFs	Values of m and l	Noise level	θ	Regularization methods	λ
1	10 @ D1, D2	$m = l = 2$	15%	0.0755	l_2	3.882
					l_∞	0.100

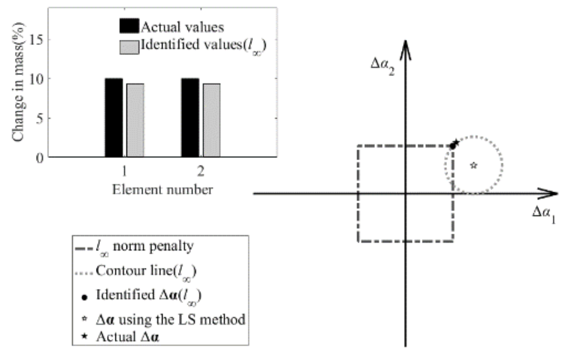


Fig. 3 Sketch map and identified results for spring-mass model in scenario 1 by adding l_∞ norm penalty

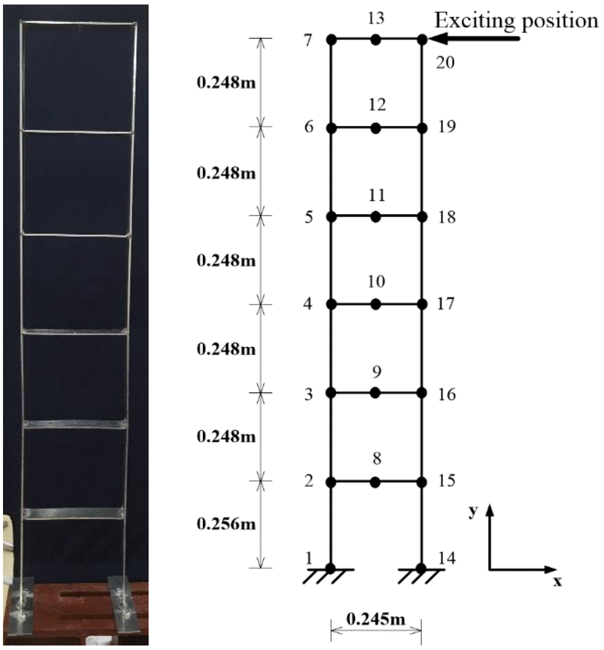
The maximum values of elements in vector $\boldsymbol{\Delta\alpha}$ are constrained by adding the l_∞ norm penalty. Compared the updating result in Fig. 3 with the true values, it is clear that the l_∞ norm regularization is more reasonable than the l_2 norm regularization in this scenario. Moreover, the elements of $\boldsymbol{\Delta\alpha}$ are equal. It indicates that the l_∞ norm regularization is suitable for scenarios when the elements of $\boldsymbol{\Delta\alpha}$ have the same order of magnitudes.

4. Experimental verifications

4.1 Experimental setup

As shown in Fig. 4(a), to verify the effectiveness of the proposed method, a six-storey aluminum alloy frame is designed and fabricated in laboratory.

The corresponding experimental setup is shown in Fig. 4(b). The total height and width of the frame is 1.496 m and



(a) Frame (b) Experimental setup

Fig. 4 Six-storey aluminum alloy frame

Table 2 Initial parameters of FE model for six-storey aluminum alloy frame

Elastic modulus (N/m^2)	Density (kg/m^3)	Width of column (m)	Width of beam (m)	Rotational stiffness (N·m/rad)
7×10^{10}	2700	0.06	0.055	100

0.245 m, respectively. The thickness of both columns and beams are 0.004 m. Six accelerometers are placed at nodes 15-20. The frame is excited by a vibration exciter placed at node 20.

An excitation stinger is used to provide excitation connection between the vibration exciter and the frame. It should be pointed out that the stinger may affect dynamic characteristics of structure by introducing rotational stiffness into structure (Avitabile 2010). Thus, in this study, adding rotational stiffness is considered at node 20.

After performing experimental modal analysis, the first four frequencies and mode shapes are obtained and used to updating the FE model. Initial parameters of the frame are given in Table 2.

4.2 Benchmark model updating

As mentioned above, to obtain the benchmark model of frame, both physical and geometric parameters should be updated. Thus, by considering machining errors of frame, parameters of elastic modulus, density, width of the column, width of the beam and rotational stiffness are selected as

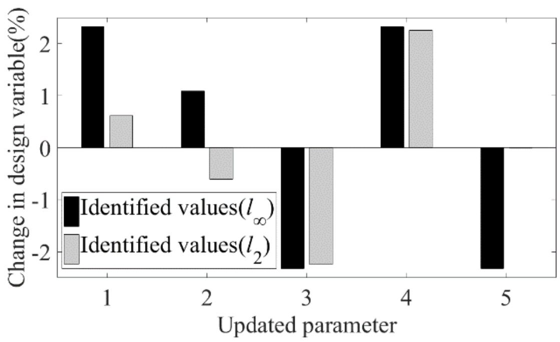


Fig. 5 Updated results of structural parameters by usin g different regularization methods

updated parameters, and the updating results are obtained based on the l_2 norm and l_∞ norm regularization methods, respectively.

In Fig. 5, updated parameters 1-5 represent elastic modulus, density, width of the column, width of the beam and rotational stiffness, respectively. All parameters are simultaneously updated by the l_∞ norm regularization method, and the changes in design variables are in the same order of magnitudes. Different from the l_∞ norm regularization method, the rotational stiffness is hardly updated by the l_2 norm regularization method, which shows the differences between these two methods.

According to the updated values in design variables, the corresponding updating results can be seen in Table 3. Widths of column and beam updated by the two regularization methods have little difference. However, the updated parameters are close to the initial values and consistent with the actual values. Elastic modulus and density updated by the l_∞ norm regularization is smaller than that updated by the l_2 norm regularization. Moreover, the values of added rotational stiffness at node 20 are approximate by these two methods.

By comparing the calculated frequencies with the measured frequencies, the percentage errors of frequencies are given in Table 4. The percentage errors of the first two frequencies obtained by the l_∞ norm regularization are smaller than that obtained by the l_2 norm regularization. On the contrary, the percentage errors of the other two frequencies obtained by the l_∞ norm regularization are larger than that obtained by the l_2 norm regularization.

The updated FE models respectively obtained by the two methods are taken as the benchmark models, which will be used to verify the effectiveness of the model updating methods in the next section.

4.3 Verification on effectiveness of proposed method

To verify the effectiveness of the proposed method, six known masses are added to the frame, as shown in Fig. 6.

Table 3 Updated parameters of FE models obtained by different regularization methods

Regularization methods	Elastic modulus (N/m^2)	Density (kg/m^3)	Width of column (m)	Width of beam (m)	Rotational stiffness (N·m/rad)	λ	θ
l_2	6.9570×10^{10}	2716.4	0.0613	0.0538	100.0077	9.200	2.8329
l_∞	6.8376×10^{10}	2670.7	0.0614	0.0537	102.3202	0.015	

Table 4 Percentage errors of frequencies for updated FE models

Mode	Measured frequencies (Hz)	Percentage error (%)	
		l_2	l_∞
1	6.8423	0.5551	0.5092
2	20.9105	0.5573	0.5358
3	37.9170	0.0952	0.1143
4	57.8745	0.0438	0.0549

Table 5 Actual masses of aluminum alloy blocks in each storey

Storey number	Actual mass (kg)
1	0.6832
2	0.6622
3	0.6586
4	0.6670
5	0.6726
6	0.6646

Table 6 Experimental scenarios for six-storey aluminum alloy frame with added masses

Scenarios no.	Scenarios description	θ	Regularization methods	λ
1	Added masses in 1-2 storeys	3.0837	l_2	33.082
			l_∞	0.295
2	Added masses in 1-3 storeys	3.0609	l_2	24.961
			l_∞	0.252
3	Added masses in 1-4 storeys	3.0917	l_2	5.891
			l_∞	0.434
4	Added masses in 1-5 storeys	3.1088	l_2	18.373
			l_∞	0.445
5	Added masses in 1-6 storeys	3.5585	l_2	24.177
			l_∞	0.808

The actual masses of these aluminum alloy blocks are given in Table 5. By adding different numbers of masses, different scenarios can be obtained.

Herein, five scenarios are introduced as given in Table 6. As a result, the updated results obtained by different regularization methods are given in Figs. 7-11.

In different regularization methods, the values of regularization parameter λ are different. The indicator θ in Table 6 are calculated by Eq. (11) for reasonably selecting regularization parameter. To control identification accuracy of all scenarios, the difference in values of θ is small. As the numbers of masses adding, the values of regularization parameter change, but accurate results still are obtained.

In Figs. 7-11, the added masses identified by the regularization methods are compared with the true values. It can be seen that the identified accuracy is different by adding different norm penalties into the objective function in Eq. (4).

Fig. 6 Adding aluminum alloy blocks on frame

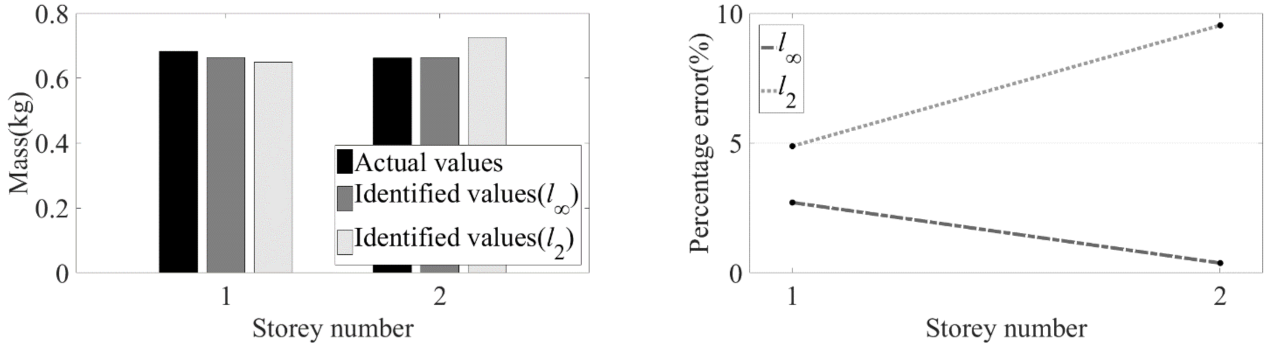
By using the l_2 norm regularization, percentage errors of identified masses are more than 5% except the storey 1 in Fig. 7, the storey 3 in Fig. 9, the storey 4 in Fig. 10, the storeys 3 and 6 in Fig. 11. For the given scenarios, some elements in identified masses are overvalued and other elements are undervalued. Amplitude of each element cannot be controlled by the l_2 norm regularization, so the identified accuracy of all elements will be affected when an element is inaccurately evaluated. Moreover, when dimensions of identified masses increase, the updated results will be more inaccurate.

On the other hand, percentage errors of identified masses obtained from the l_∞ norm regularization is always less than 5% in the given scenarios. Identified mass in storey 1 is overvalued in all scenarios, but the identified results of other storeys are not affected. For different scenarios, the values of same identified masses are similar. The identified masses are spread to each storey equally, so the percentage errors are small.

It indicates that the proposed method based on the l_∞ norm regularization can obtain variables of the same order of magnitudes accurately and withstand errors in information with high stability.

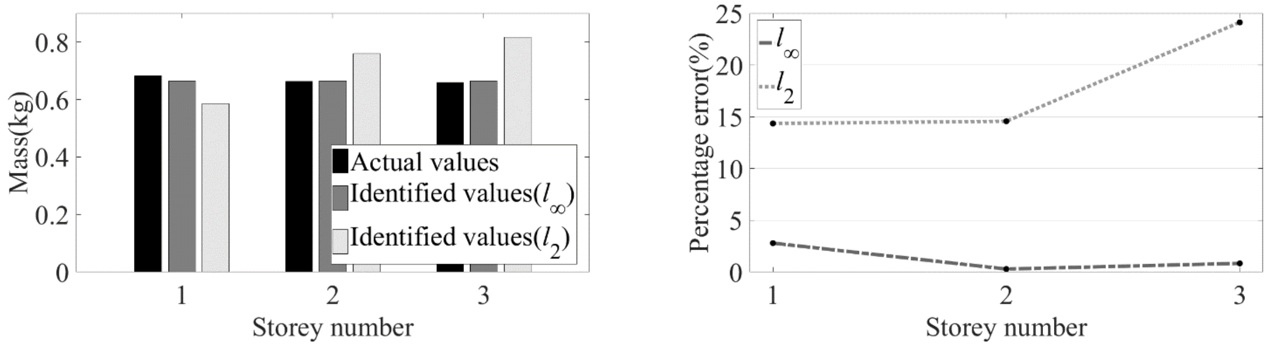
5. Conclusions

In this paper, an anti-sparse representation method of model updating is proposed based on the l_∞ norm regularization. According to the characteristics of structural model updating problem, the sensitivity of modal parameters and the l_∞ norm regularization are introduced to define an objective function. Then, a fast iterative shrinkage thresholding algorithm (FISTA) is adopted to solve the objective function. As a result, an anti-sparse representation of solution is obtained by the proposed method. Numerical simulations on a 2-DOF spring-mass model have been carried out to investigate the performance of the proposed method under the influence of noises. Finally, to further verify the effectiveness of the proposed method, experimental verifications on a six-storey aluminum alloy frame are conducted. Meanwhile, comparative studies with the l_2 norm regularization are conducted as well. Based on



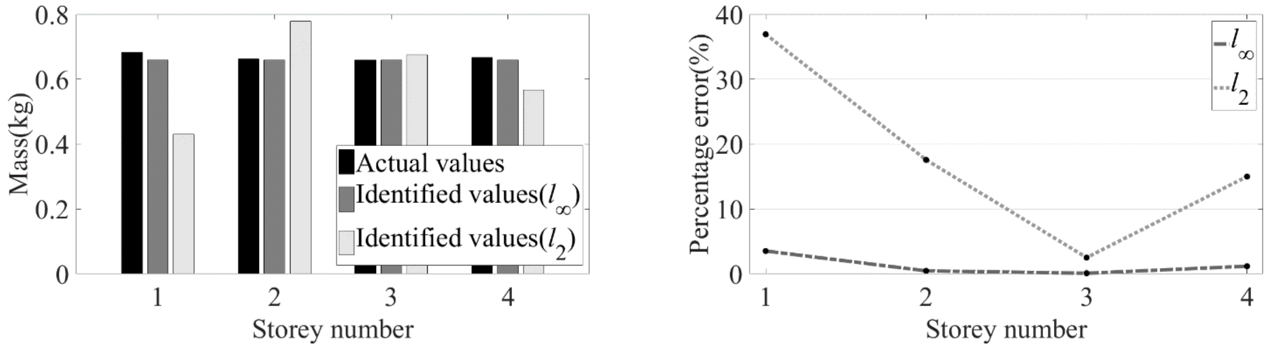
(a) Identified mass in each storey (b) Percentage errors of identified mass in each storey

Fig. 7 Updated results of FE model with masses in scenario 1 by using different regularization methods



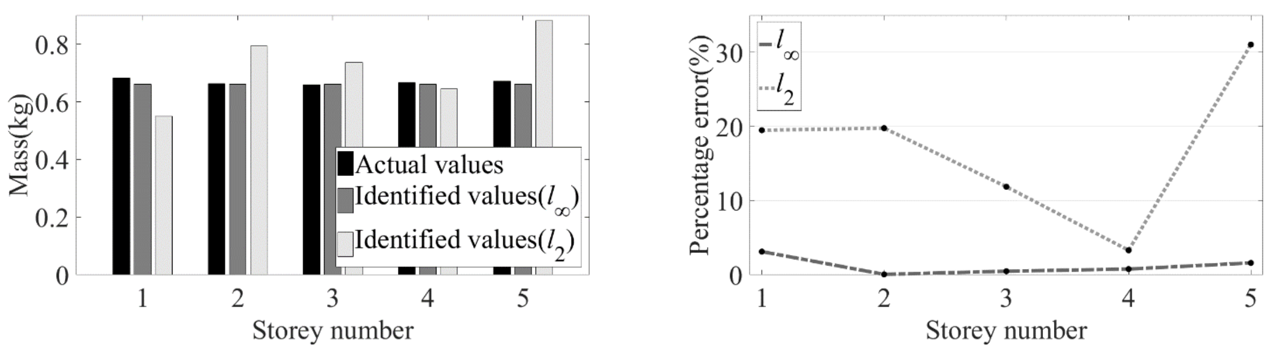
(a) Identified mass in each storey (b) Percentage errors of identified mass in each storey

Fig. 8 Updated results of FE model with masses in scenario 2 by using different regularization methods



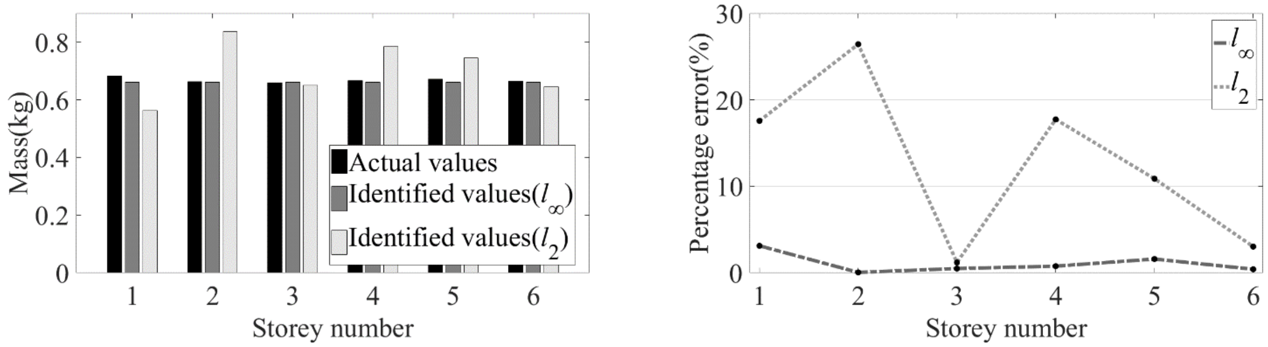
(a) Identified mass in each storey (b) Percentage errors of identified mass in each storey

Fig. 9 Updated results of FE model with masses in scenario 3 by using different regularization methods



(a) Identified mass in each storey (b) Percentage errors of identified mass in each storey

Fig. 10 Updated results of FE model with masses in scenario 4 by using different regularization methods



(a) Identified mass in each storey (b) Percentage errors of identified mass in each storey

Fig. 11 Updated results of FE model with masses in scenario 5 by using different regularization methods

the updated results of numerical simulations and experimental studies, some conclusions can be made as follows:

- The proposed method has good performance in updating variables of the same order of magnitudes because the l_∞ norm regularization has great ability to spread information equally.
- With the help of the l_∞ norm penalty, errors in elements of updated parameters can be reduced well in the proposed method, so the updated results have high accuracy.
- Compared with the l_2 norm regularization, the updated results obtained by the proposed method are more reasonable because it has great ability to describe the characteristics of the given model updating problem.
- The finite element (FE) model is needed to effectively reflect the changing rule of real structures, so FE model updated by the l_∞ norm regularization will be used in structural damage detection (SDD) for investigating reasonability of the proposed method.

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