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Dynamic response uncertainty analysis of vehicle-track coupling system with fuzzy variables

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Abstract. Dynamic analysis of a vehicle-track coupling system is important to structural design, damage detection and condition assessment of the structural system. Deterministic analysis of the vehicle-track coupling system has been extensively studied in the past, however, the structural parameters of the coupling system have uncertainties in engineering practices. It is essential to treat the parameters of the vehicle-track coupling system with consideration of uncertainties. In this paper, a method for predicting the bounds of the vehicle-track coupling system responses with uncertain parameters is presented. The uncertain system parameters are modeled as fuzzy variables instead of conventional random variables with known probability distributions. Then, the dynamic response functions of the coupling system are transformed into a component function based on the high dimensional representation approximation. The Lagrange interpolation method is used to approximate the component function. Finally, the bounds of the system's dynamic responses can be predicted by using Monte Carlo method for the interpolation polynomials of the Lagrange interpolation function. A numerical example is introduced to illustrate the ability of the proposed method to predict the bounds of the system's dynamic responses, and the results are compared with the direct Monte Carlo method. The results show that the proposed method is effective and efficient to predict the bounds of the system's dynamic responses with fuzzy variables.

Keywords: vehicle-track coupling system; dynamic response; fuzzy variables; high dimensional representation approach; Monte Carlo method

1. Introduction

The subway system has been fast developed for public transportation. The dynamic analysis of the subway vehicle-track coupling system is very important to ensure the serviceability of the track and the safety of the train vehicles. The dynamic analysis of the vehicle-track coupling system has been studied by many researchers. Among most of the previous studies, the parameters of the system are assumed to be deterministic (Chen 2018, Chen *et al*. 2017). The variation in parameters is often taken into consideration by assuming a series of values within certain ranges in advance. Then the deterministic analysis is conducted according to all these parameters with different values (Deng and Cai 2010, Liu *et al*. 2013). However, in practice, both the vehicle and the track are subjected to many types of uncertainties, which are unavoidable and difficult to forecast. In order to predict the system's dynamic responses accurately, non-determinism has to be taken into account.

Non-determinism approach is typically adopted in whole process of the design. There are different sources of uncertainties, such as parametric non-determinism, non-determinism in the mathematical modeling and numerical error (Farkas *et al.* 2010). Normally, there are three main methods to deal with these uncertainties, namely probabilistic theory (Wang *et al.* 2014, Tannert and Haukaas 2013), interval analysis (Wang and Matthies 2019, Zou *et al*. 2016) and fuzzy set (Chong and Zhi 2014, Doan *et al.* 2019). The probabilistic theory has been widely used in the analysis of many engineering problems (Chen and Xiao 2015, Obrien *et al*. 2010). It normally treats the uncertain parameters as random variables whose probability distributions are known in advance. Sufficient statistical data are needed to build reliable probability distributions of these variables. Therefore, the probabilistic theory is limited to certain circumstances, such as cases when valid data are inaccurate or sparse and the uncertain parameters are not random naturally (Ma *et al*. 2014).

Interval analysis and fuzzy approaches are becoming increasing popular to solve non-probabilistic uncertain problems. A fuzzy variable is actually a generalization of an interval variable (Adhikari *et al*. 2011). When the interval method is used to model an uncertain variable, the bounds of the variable are determined. The fuzzy approaches develop this concept by producing a membership function. A fuzzy finite element analysis aims to obtain the bounds of certain output responses by giving the membership of data in the set of input variables. It is actually an uncertainty propagation problem, which in principle can be solved

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based on the so called direct Monte Carlo method. In real cases, the direct Monte Carlo method usually requires a great amount of computing time, thus, the most important task is to reduce the computational cost. As the fuzzy variables are a generalization of interval variables, the methods used in interval analysis such as classical interval arithmetic (Moore 1966), affine analysis (Manson 2005, Degrauwe *et al*. 2010) and vertex theorems (Qui *et al*. 2005) can be adopted. Many methods (Hinke *et al*. 2009, De *et al*. 2008, De *et al*. 2009) have been developed for fuzzy uncertainty propagation. Among these methods, the high dimensional model representation (HDMR) method (Alis and Rabitz 2001, Li *et al*. 2001, Sobol 2003) has been extensively studied. The HDMR method (Chowdhury 2008) is developed to express the complex input-output relationship in terms of hierarchical correlated function expansions. The uncertain analysis of the approximated function can then be well estimated by a Monte Carlo method. For the finite element analysis, the approximated function are quantitatively reduced, and computational cost can be reduced severely without compromising accuracy.

In this paper, the parameters of the vehicle-track coupling system are defined as fuzzy variables to consider the uncertainty of the railway system. A new approach based on the HDMR method is proposed to calculate the bounds of the system dynamic response. Firstly, the dynamic responses of the vehicle-track coupling system with fuzzy variables are expressed by a component function based on the HDMR method. Then the component function is transformed into Lagrange interpolation function. Finally, the bounds of the system's dynamic responses are estimated by the dynamic responses of several sample points. This method predicts the bounds of the system's dynamic responses by using the Monte Carlo method on the interpolation polynomials of Lagrange interpolation function. This avoids directly the expensive Monte Carlo process on the motion equation of the vehicle-track coupling system, which greatly shortens the calculation time and improves the efficiency. A numerical example of the vehicle-track coupling system is analyzed to verify the proposed method. The calculated results are compared with those calculated by the direct Monte Carlo method, which shows that the proposed method is effective and efficient to predict the bounds of the system's dynamic responses with fuzzy variables.

2. Dynamic response analysis with uncertainty

For the vehicle-track coupling system, e.g. vehicle system or track system, there is more or less uncertainty, thus, it is particularly important to analyze its uncertainty. Assuming that the vehicle-track coupling system has uncertain parameters $\boldsymbol{z} = [z_1, z_2, \ldots, z_{Ncal}]^T$, where *Ncal* represents the total number of uncertain parameters, which are defined as L-R fuzzy variables (Li *et al*. 2020). According to the motion equation of the vehicle-track coupling system (Ye *et al*. 2019), the motion equation of the vehicle-track coupling system with fuzzy variables can be expressed as

$$\begin{cases} \mathbf{M}_a(z)\ddot{\mathbf{x}}_a(z) + \mathbf{C}_a(z)\dot{\mathbf{x}}_a(z) + \mathbf{K}_a(z)\mathbf{x}_a(z) = \mathbf{D}_{a1}\mathbf{f}_g(z) - \mathbf{D}_{a2}\mathbf{f}_{wr}(z) \\ \mathbf{M}_b(z)\ddot{\mathbf{x}}_b(z) + \mathbf{C}_b(z)\dot{\mathbf{x}}_b(z) + \mathbf{K}_b(z)\mathbf{x}_b(z) = \mathbf{D}_b\mathbf{N}_b\mathbf{f}_{wr}(z) \\ \mathbf{f}_{wr}(z) = K_w\left(\mathbf{D}_w\mathbf{x}_a(z) - \mathbf{D}_r\mathbf{N}_b{}^{\mathrm{T}}\mathbf{x}_b(z) - \mathbf{X}_{irr}\right)^{3/2} \end{cases} \tag{1}$$

where $\boldsymbol{M}_a(z)$, $\boldsymbol{C}_a(z)$, and $\boldsymbol{K}_a(z)$ are the mass, damping and stiffness matrices of the vehicle system with uncertainties, respectively; $\ddot{\boldsymbol{x}}_a(z)$, $\dot{\boldsymbol{x}}_a(z)$, and $\boldsymbol{x}_a(z)$ are the acceleration, velocity and displacement response vectors of the vehicle system with uncertainties, respectively; $\boldsymbol{f}_g(z)$ is the gravity vector with uncertainty; $\boldsymbol{f}_{wr}(z)$ is the vehicle-track interaction force vector with uncertainty; $\mathbf{D}_{a1}$ and $\mathbf{D}_{a2}$ represent the indicator matrices of the position of the gravitational load and the normal load of the wheel and rail of the vehicle model, respectively, and have values of 0 or 1; $\boldsymbol{M}_b(z)$, $\boldsymbol{C}_b(z)$, and $\boldsymbol{K}_b(z)$ are the mass, damping and stiffness matrices of the track system with uncertainties, respectively; $\ddot{\boldsymbol{x}}_b(z)$, $\dot{\boldsymbol{x}}_b(z)$, and $\boldsymbol{x}_b(z)$ are the acceleration, velocity and displacement response vectors of the track system with uncertainties, respectively; $\mathbf{D}_b$ is an indicator matrix of the external load position of the track model, and has values of 0 or 1; $\mathbf{N}_b$ is a track integral shape function composed of a set of shape functions of all units of the track system; $\mathbf{D}_w$ and $\mathbf{D}_r$ represent the indicator matrices for the transformation of the local displacements to the global displacements; K_w is a contact constant; $\mathbf{X}_{irr}$ is the track irregularity; and the superscript T represents the transpose of the matrix. When the system parameters are known, the Eq. (1) can be solved by the numerical method (Ye *et al*. 2019).

According to the concept of L-R fuzzy variables (Li *et al*. 2020), the uncertainty interval of fuzzy variables of the vehicle-track coupling system can be obtained. The bounds of the system's dynamic responses are predicted by the direct Monte Carlo method. However, as the motion equation of the vehicle-track coupling system is very complex, it takes long time to solve the motion equation of the system. Therefore, it is extremely inefficient by using the Monte Carlo method directly to predict the bounds of the system's dynamic responses. In this paper, the HDMR method (Li 2001) is introduced to express the system's dynamic responses with fuzzy variables by a component function as

$$\begin{aligned} \begin{Bmatrix} \mathbf{y}_r(z) \\ \mathbf{y}_v(z) \end{Bmatrix} &= \begin{Bmatrix} \mathbf{y}_r(z) \\ \mathbf{y}_v(z) \end{Bmatrix}_0 + \sum_{q=1}^{Ncal} \begin{Bmatrix} \mathbf{y}_r(z_q) \\ \mathbf{y}_v(z_q) \end{Bmatrix} + \sum_{1 \le q_1 < q_2 \le Ncal} \begin{Bmatrix} \mathbf{y}_r(z_{q_1}, z_{q_2}) \\ \mathbf{y}_v(z_{q_1}, z_{q_2}) \end{Bmatrix} \\ &+ \cdots + \sum_{1 \le q_1 < \ldots < q_j \le Ncal} \begin{Bmatrix} \mathbf{y}_r(z_{q_1}, z_{q_2}, \ldots, z_{q_j}) \\ \mathbf{y}_v(z_{q_1}, z_{q_2}, \ldots, z_{q_j}) \end{Bmatrix} \\ &+ \cdots + \begin{Bmatrix} \mathbf{y}_r(z_{q_1}, z_{q_2}, \ldots, z_{q_{Ncal}}) \\ \mathbf{y}_v(z_{q_1}, z_{q_2}, \ldots, z_{q_{Ncal}}) \end{Bmatrix} \end{aligned} \tag{2}$$

where $\{\boldsymbol{y}_r(z), \boldsymbol{y}_v(z)\}_0{}^T$ are the average values of the system's dynamic responses, which are constant terms; $\{\boldsymbol{y}_r(z_q), \boldsymbol{y}_v(z_q)\}^T$ represent the system dynamic response calculated by only considering the fuzziness of z_q in the vehicle-track coupling system; $\{\boldsymbol{y}_r(z_{q_1}, z_{q_2}), \boldsymbol{y}_v(z_{q_1}, z_{q_2})\}^T$

represent the system dynamic response with the fuzziness of z_{q1} and z_{q2} taken into account; Similarly, the more fuzzy variables are then included in the formula, and the response equation represents the system's dynamic response calculated taking all their fuzziness into account; The last term $\{\boldsymbol{y}_r(z_{q_1}, z_{q_2}, \ldots, z_{q_{Ncal}}), \boldsymbol{y}_v(z_{q_1}, z_{q_2}, \ldots, z_{q_{Ncal}})\}^T$ represents the system's dynamic responses considering the fuzziness of all parameters of the system. By defining $\bar{z} = \{\bar{z}_1, \bar{z}_2, \ldots, \bar{z}_{Ncal}\}^T$ as the reference point, the terms in Eq. (2) can be given as

$$\begin{Bmatrix} \mathbf{y}_r(z) \\ \mathbf{y}_v(z) \end{Bmatrix}_0 = \begin{Bmatrix} \mathbf{y}_r(\bar{z}) \\ \mathbf{y}_v(\bar{z}) \end{Bmatrix} \tag{3}$$

$$\begin{Bmatrix} \mathbf{y}_r(z) \\ \mathbf{y}_v(z) \end{Bmatrix}_q (z_q) = \begin{Bmatrix} \mathbf{y}_r(z) \\ \mathbf{y}_v(z) \end{Bmatrix} (z_q; \bar{z}^q) - \begin{Bmatrix} \mathbf{y}_r(z) \\ \mathbf{y}_v(z) \end{Bmatrix}_0 \tag{4}$$

$$\begin{aligned} \begin{Bmatrix} \mathbf{y}_r(z) \\ \mathbf{y}_v(z) \end{Bmatrix}_{q_1 q_2} (z_{q_1}, z_{q_2}) &= \begin{Bmatrix} \mathbf{y}_r(z) \\ \mathbf{y}_v(z) \end{Bmatrix} (z_{q_1}, z_{q_2}; \bar{z}^{q_1 q_2}) \\ &- \begin{Bmatrix} \mathbf{y}_r(z) \\ \mathbf{y}_v(z) \end{Bmatrix}_{q_1} (z_{q_1}) - \begin{Bmatrix} \mathbf{y}_r(z) \\ \mathbf{y}_v(z) \end{Bmatrix}_{q_2} (z_{q_2}) - \begin{Bmatrix} \mathbf{y}_r(z) \\ \mathbf{y}_v(z) \end{Bmatrix}_0 \end{aligned} \tag{5}$$

Substituting Eqs. (3)-(5) into Eq. (2), the system's dynamic responses can be approximately expressed as

$$\begin{aligned} \begin{Bmatrix} \tilde{\boldsymbol{y}}_r(z) \\ \tilde{\boldsymbol{y}}_v(z) \end{Bmatrix} &= \sum_{1 \le q_1 < \cdots < q_\tau \le Ncal} \begin{Bmatrix} \boldsymbol{y}_r(z_{q_1}, \ldots z_{q_\tau}; \bar{z}^{q_1, \ldots q_\tau}) \\ \boldsymbol{y}_v(z_{q_1}, \ldots z_{q_\tau}; \bar{z}^{q_1, \ldots q_\tau}) \end{Bmatrix} \\ &- (Ncal - \tau) \sum_{1 \le q_1 < \cdots < q_{\tau-1} \le Ncal} \begin{Bmatrix} \boldsymbol{y}_r(z_{q_1}, \ldots z_{q_{\tau-1}}; \bar{z}^{q_1, \ldots q_{\tau-1}}) \\ \boldsymbol{y}_v(z_{q_1}, \ldots z_{q_{\tau-1}}; \bar{z}^{q_1, \ldots q_{\tau-1}}) \end{Bmatrix} \\ &+ \frac{(Ncal - \tau + 1)!}{2!\,(Ncal - \tau - 1)!} \sum_{1 \le q_1 < \cdots < q_{\tau-2} \le Ncal} \begin{Bmatrix} \boldsymbol{y}_r(z_{q_1}, \ldots z_{q_{\tau-2}}; \bar{z}^{q_1, \ldots q_{\tau-2}}) \\ \boldsymbol{y}_v(z_{q_1}, \ldots z_{q_{\tau-2}}; \bar{z}^{q_1, \ldots q_{\tau-2}}) \end{Bmatrix} \\ &- \ldots \mp \frac{(Ncal - 2)!}{(\tau - 1)!\,(Ncal - \tau - 1)!} \sum_{1 \le q \le Ncal} \begin{Bmatrix} \boldsymbol{y}_r(z_q; \bar{z}^q) \\ \boldsymbol{y}_v(z_q; \bar{z}^q) \end{Bmatrix} \\ &\pm \frac{(Ncal - 1)!}{\tau!\,(Ncal - \tau - 1)!} \begin{Bmatrix} \boldsymbol{y}_r(\bar{z}) \\ \boldsymbol{y}_v(\bar{z}) \end{Bmatrix} \end{aligned} \tag{6}$$

where $\{\tilde{\boldsymbol{y}}_r(z), \tilde{\boldsymbol{y}}_v(z)\}^T$ is the dynamic responses of the system; $\{\boldsymbol{y}_r(\bar{z}), \boldsymbol{y}_v(\bar{z})\}^T$ is the calculated system's dynamic responses when all the fuzzy variables are taken as reference values, and is a constant term; $\{\boldsymbol{y}_r(z_q; \bar{z}^q), \boldsymbol{y}_v(z_q; \bar{z}^q)\}^T$ is the calculated system's dynamic responses when z_q is taken as a fuzzy variable while the others are taken as their reference values, namely $z_p \equiv \bar{z}_p$, $p \neq q$; $\{\boldsymbol{y}_r(z_{q_1} z_{q_2}; \bar{z}^{q_1 q_2}), \boldsymbol{y}_v(z_{q_1} z_{q_2}; \bar{z}^{q_1 q_2})\}^T$ is the calculated system's dynamic responses when z_{q1} and z_{q2} are taken as fuzzy variables while the others are taken as their reference values, namely $z_p \equiv \bar{z}_p$, $p \neq q_1, q_2$; and τ is the order of the HDMR method, and satisfies $1 \le \tau \le (Ncal - 1)$. When τ is odd, the last term in Eq. (6) is preceded by "-"; when τ is an even number, the last term in Eq. (6) is preceded by "+". According to the HDMR method (Sobol 2003), only a few order polynomials of Eq. (6) are needed to achieve the accuracy of approximate substitution. In this paper, only the first order model is used to approximate the system's dynamic responses. The interaction between fuzzy variables and the influence of multiple fuzzy variables on the bounds of system's dynamic responses are not considered. Equation (6) can be simplified as

$$\begin{Bmatrix} \tilde{\mathbf{y}}_r(z) \\ \tilde{\mathbf{y}}_v(z) \end{Bmatrix} = \sum_{1 \le q \le Ncal} \begin{Bmatrix} \mathbf{y}_r(z_q; \bar{z}^q) \\ \mathbf{y}_v(z_q; \bar{z}^q) \end{Bmatrix} - \begin{Bmatrix} \mathbf{y}_r(\bar{z}) \\ \mathbf{y}_v(\bar{z}) \end{Bmatrix} \tag{7}$$

From Eq. (7), when k sample points are assumed for each fuzzy variable, in order to obtain the bounds of the system's dynamic responses by using the direct Monte Carlo method, the number of parameter combinations is $\sum_{i=0}^{1} [Ncal!/i!\,(Ncal - i)\ !](k-1)^i$. The computation process is time consuming. To improve the calculation efficiency, Equation (7) is approximately simplified based on the Lagrange polynomial method. The bounds of the system's dynamic responses is approximately simulated by the responses at a small number of sample points. Equation (7) can be rewritten as

$$\begin{Bmatrix} \tilde{\mathbf{y}}_r(z) \\ \tilde{\mathbf{y}}_v(z) \end{Bmatrix} = \sum_{q=1}^{Ncal} \sum_{j=1}^{s} \Phi_j(z_q) \begin{Bmatrix} \mathbf{y}_r(z_q^j; \bar{z}^q) \\ \mathbf{y}_v(z_q^j; \bar{z}^q) \end{Bmatrix} - 2 \begin{Bmatrix} \mathbf{y}_r(\bar{z}) \\ \mathbf{y}_v(\bar{z}) \end{Bmatrix} \tag{8}$$

where s is the odd number of the dynamic response functions used in the Lagrange interpolation function (Ma *et al.* 2014); and z_q^j represents the jth sample point of fuzzy variable z_q. After obtaining the uncertainty interval $[z_{qL}, z_{qR}]$ of fuzzy variable z_q, a group of regularly spaced sample points are obtained according to

$$z_q^1 = z_{qL},\ z_q^2 = \bar{z}_q - (s-3)(\bar{z}_q - z_{qL})/(s-1),$$
$$z_q^3 = \bar{z}_q - (s-5)(\bar{z}_q - z_{qL})/(s-1), \ldots,$$
$$z_q^{(s+1)/2} = \bar{z}_q, \ldots,$$
$$z_q^{s-2} = \bar{z}_q + (s-5)(z_{qR} - \bar{z}_q)/(s-1),$$
$$z_q^{s-1} = \bar{z}_q + (s-3)(z_{qR} - \bar{z}_q)/(s-1) \text{ and } z_q^s = z_{qR}.$$

Here, $\bar{z}$, z_{qL} and z_{qR} represent the mean, minimum and maximum values of the fuzzy parameters, respectively, where the number of parameter combinations reduces to $[(s-1)\times Ncal+1]$. $\boldsymbol{\Phi}_j(z_q)$ is the interpolation function. The moving least-squares interpolation functions (Balu and Rao 2012) are used here, expressed as

$$\boldsymbol{\Phi}_j(z_q) = \frac{(z_q - z_q^1)\ldots(z_q - z_q^{j-1})(z_q - z_q^{j+1})\ldots(z_q - z_q^s)}{(z_q^j - z_q^1)\ldots(z_q^j - z_q^{j-1})(z_q^j - z_q^{j+1})\ldots(z_q^j - z_q^s)} \quad j = 1, 2, \ldots, s \tag{9}$$

Through the above approximation process, the dynamic responses of the vehicle-track coupling system are approximated as a component function based on the HDMR method. Then the component function is expressed as a set of interpolation polynomials of the Lagrange interpolation

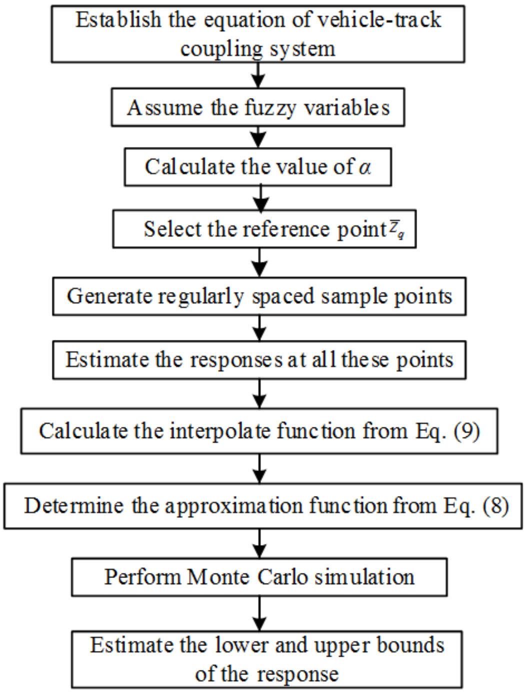


Fig. 1 Flowchart of the process for estimating the bounds of the system's dynamic responses

function. The bounds of the system's dynamic responses can be predicted by using the Monte Carlo method on the interpolation polynomials of the Lagrange interpolation function, avoiding adopting the Monte Carlo method on the motion equation of the vehicle-track coupling system. This transformation greatly reduces the amount of calculation and improves the calculation efficiency. The detailed procedure for calculating the bounds of the system's dynamic responses with fuzzy variables can be described as follows:

Step 1: Construct the equation of vehicle-track coupling system. Calculate the motion equation of the vehicle-track coupling system with fuzzy variables from Eq. (1).

Step 2: Calculate the optimum left and right spreading coefficients α and β. Confirm the uncertainty interval [z_{qL}, z_{qR}] of fuzzy variable at membership degree ζ, which takes as a small value as possible. Generate a group of regularly spaced sample points according to

$$z_q^1 = z_{qL},\ z_q^2 = \bar{z}_q - (s-3)(\bar{z}_q - z_{qL})/(s-1),$$
$$z_q^3 = \bar{z}_q - (s-5)(\bar{z}_q - z_{qL})/(s-1), \ldots,$$
$$z_q^{(s+1)/2} = \bar{z}_q, \ldots,$$
$$z_q^{s-2} = \bar{z}_q + (s-5)(z_{qR} - \bar{z}_q)/(s-1),$$
$$z_q^{s-1} = \bar{z}_q + (s-3)(z_{qR} - \bar{z}_q)/(s-1) \ \text{and} \ z_q^s = z_{qR}.$$

Estimate the system's dynamic responses at all these sample points from Eq. (1).

Step 3: Express the system's dynamic responses as a component function based on the HDMR method from Eq. (7). Transform the component function to a set of interpolation polynomials of the Lagrange interpolation function. Calculate the interpolation function $\boldsymbol{\Phi}_j(z_q)$ from Eq. (9).

Step 4: Calculate the bounds of the system's dynamic responses from Eq. (8). In this process, the number of the motion equations of the vehicle-track coupling system needed to be solved is [(s-1)×$Ncal$+1].

The flowchart of the prediction process of the system's dynamic responses is shown in Fig. 1.

3. Numerical example

A vehicle-track coupling system with a car model of 10 degree of freedoms and a discrete point-supported track model is analyzed to verify the applicability and efficiency of the proposed method. The models of the track and the vehicle are shown in Fig. 2-3.

The calculated length of the track is 98.1 m. The track is divided into 180 units with the unit distance of 0.545 m according to the location of the sleeper. The beam has 181 nodes and 540 degree of freedoms in total.

The parameters of the track and the vehicle are shown in Table 1-2. In this example, the vehicle parameters are defined as symmetric normal fuzzy variables distribution without considering the influence of the track parameters and track irregularity parameters on the bounds of system's dynamic responses. It is assumed that the fuzzy variables are independent of each other. The prediction results of the bounds of the vehicle acceleration dynamic responses are utilized to verify the precision and efficiency of the proposed method.

According to the properties of the membership function of the fuzzy variables, the membership function is determined by the spreading coefficient and the nominal value of the fuzzy variables. Different spreading coefficients produce different shapes of membership function, which affects the bounds of the fuzzy variables. In theory, the degree of a fuzzy variable changes with the requirement of the actual consideration of different projects, which is not available in advance. If the bounds of the fuzzy variables are too wide, the predicted results will be too conservative to lose the significance. Therefore, the fuzzy variables are simulated as the normal distribution based on the Monte Carlo method to find the optimum spreading coefficient initially. For an L-R fuzzy variable with normal distribution, the membership degree is always greater than 0. The lower membership degree can be taken to make a horizontal cut in order to obtain the bounds of the fuzzy variable. The fuzzy variable can be described as an uncertainty set, which can be simulated by probabilistic

Table 1 Parameters of the track

Parameter	Unit	Value	Parameter	Unit	Value
E	N/m^2	2.06×10^{11}	l	m	0.545
I	m^4	3.217×10^{-5}	ρ_b	kg/m^3	1.8×10^3
k_b	N/m	1.1×10^8	c_p	N/(m/s)	7.5×10^4
m_r	kg/m	60.64	k_p	N/m	1×10^8
m_s	kg	125.5	c_b	N/(m/s)	5.8×10^4

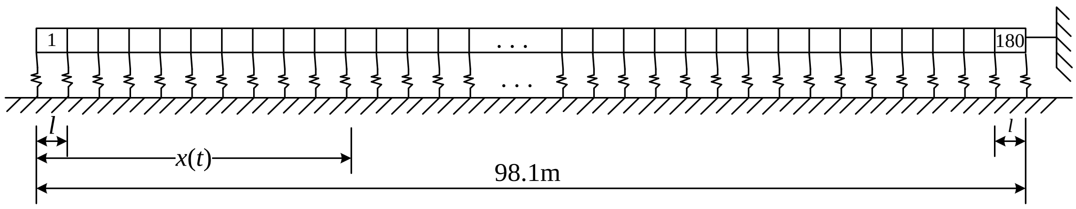


Fig. 2 Division of rail unit

Table 2 Parameters of the vehicle

Parameter	Unit	Value	Parameter	Unit	Value	Parameter	Unit	Value
M_c	kg	18600	J_e	kg·m^2	3120	K_{s2}	N/m	1.7×10^6
M_e	kg	3200	L_1	m	1.25	C_{s1}	N/(m/s)	5×10^5
M_w	kg	1400	L_2	m	9	C_{s2}	N/(m/s)	1.9×10^5
J_e	kg·m^2	2.3×10^6	K_{s1}	N/m	1.8×10^6	R	m	0.4575

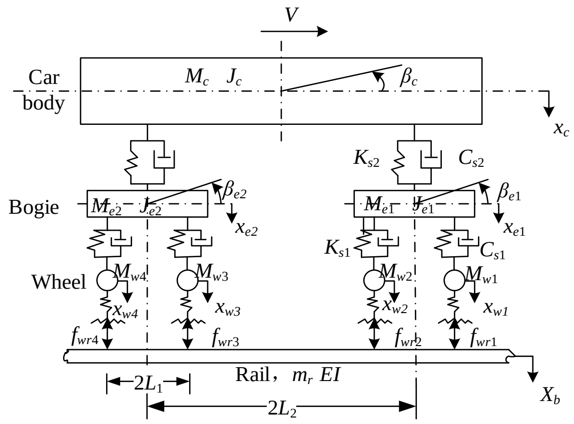


Fig. 3 Vehicle model with 10 degree of freedoms

Table 3 Bounds of the input fuzzy variables of vehicles

Parameter	Unit	$z_q^{\#}$	α	u_L	u_R
M_c	kg	18600	3720	14880	22320
M_e	kg	3200	127.17	2560	3840
M_w	kg	1400	111.27	840	1960
J_c	kg·m^2	2.3×10^6	9.14×10^4	1.84×10^6	2.76×10^6
J_e	kg·m^2	3120	247.98	1872	4368
K_{s1}	N/m	1.8×10^6	7.15×10^4	1.44×10^6	2.16×10^6
K_{s2}	N/m	1.7×10^6	1.35×10^5	1.02×10^6	2.38×10^6
C_{s1}	N·s/m	5×10^5	3.97×10^4	3×10^5	7×10^5
C_{s2}	N·s/m	1.9×10^5	1.51×10^4	1.14×10^5	2.66×10^5
L_1	m	1.25	0.0497	1	1.5
L_2	m	9	0.5193	6	11
R	m	0.4575	0.0182	0.366	0.549

approaches, such as Monte Carlo method. Then the influence of fuzzy variables in the vehicle-track coupling system on the bounds of the system's dynamic responses is investigated. The fuzzy variable z_q is expressed as the standard form $(z_q^{\#}, \alpha, \alpha)_{LR}$ of the L-R fuzzy variable. The full range of the fuzzy variables at a special spreading coefficient can be obtained with a sufficiently small membership ζ. By assuming $\zeta=1\times10^{-11}$, the calculated nominal values, spreading coefficient and fuzzy bounds of each L-R vehicle fuzzy variable are given in Table 3.

The influence of the number s of the system's dynamic responses on the predicted bounds is investigated according to Eq. (8). A set of regularly spaced sample points can be calculated with different s values. The total number of sample points is calculated by $(s\text{-}1)\times12+1$. The system dynamic response at these sample points can be estimated according to the Eq. (1), which can be used to calculate the Lagrange interpolation function Eq. (9). The bounds of the system's dynamic responses then can be calculated based on the Monte Carlo method. To verify the accuracy and efficiency of the proposed method, the results are compared with those calculated by using the direct Monte Carlo method in Eq. (1). The number of the Monte Carlo simulations is taken as 1000 in this example.

Figure 4 shows the time history curve of the bounds of the vehicle acceleration response based on the proposed method and the direct Monte Carlo method. The vehicle acceleration response with deterministic parameters is also shown in Fig. 4. As shown in Fig. 4, the time history curves of the vehicle acceleration responses to the left and right bounds obtained by the proposed method are basically consistent with those calculated by the directly Monte Carlo method.

Table 4 gives the bounds of the vehicle acceleration responses and relative errors calculated by the proposed method and the direct Monte Carlo method. The relative errors are calculated by $\left|\frac{\ddot{y}_v|\mathrm{PM}-\ddot{y}_v|\mathrm{MCM}}{\ddot{y}_v|\mathrm{MCM}}\right|\times 100\%$, where $\ddot{\boldsymbol{y}}_v|\mathrm{PM}$ is the vehicle acceleration response calculated by the proposed method; and $\ddot{\boldsymbol{y}}_v|\mathrm{MCM}$ is the vehicle acceleration response calculated by the direct Monte Carlo method. In Table 4, $\ddot{\boldsymbol{x}}_{cL}$ and $\ddot{\boldsymbol{x}}_{cR}$ represent the left and right bounds of the vehicle acceleration responses; and the unit of vehicle acceleration response is m/s^2. The unit also apply to the tables given below.

From Table 4, it can be seen that the maximum relative error is 5.258% in these three cases of s value, which shows the accuracy of the proposed method. At the same time, with the increase of the number s of the responses applied in the Lagrange interpolation function, the maximum values

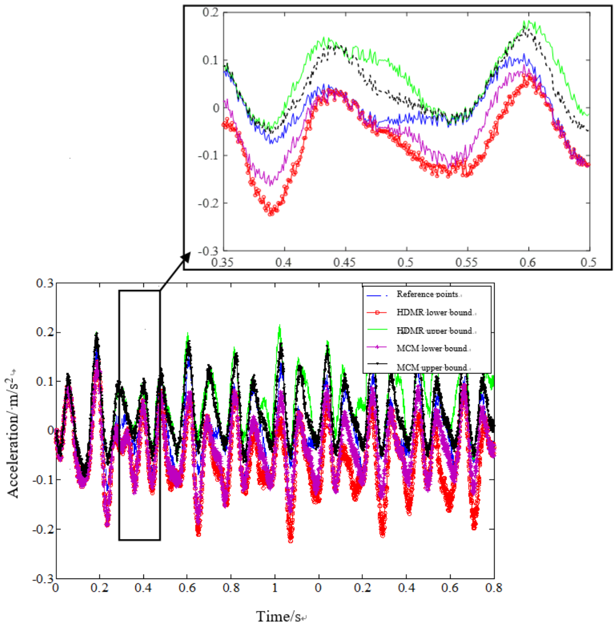


Fig. 4 The upper and lower bounds of vehicle acceleration responses calculated by the proposed method and the direct Monte Carlo method (s=5)

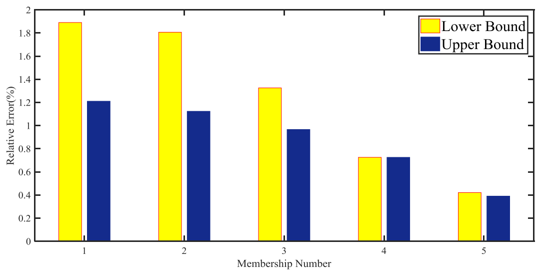


Fig. 5 Relative errors between the two methods in calculating the bounds of system responses

Table 4 Comparison of response bounds and relative errors with different s

	s=5		s=7		s=9	
	$\ddot{\mathbf{x}}_{cL}$	$\ddot{\mathbf{x}}_{cR}$	$\ddot{\mathbf{x}}_{cL}$	$\ddot{\mathbf{x}}_{cR}$	$\ddot{\mathbf{x}}_{cL}$	$\ddot{\mathbf{x}}_{cR}$
PD*	-0.2308	0.2622	-0.2253	0.2519	-0.2191	0.2475
DMCM*	-0.2213	0.2491	-0.2213	0.2491	-0.2213	0.2491
Relative error	4.292%	5.258%	1.807%	1.124%	0.994%	0.642%

*PD: Proposed method; DMCM: Direct Monte Carlo method

of relative errors calculated by both two methods discussed above are gradually reduced.

Table 5 gives the calculation times for the above two methods with different values of s. As shown in Table 5, when s=5, 7, 9, it takes 109.45 s, 256.12 s and 472.91 s, respectively, to calculate the bounds of the system's dynamic responses for the proposed method. In the meantime, it takes 18723.19 s for the direct Monte Carlo method, which is 39.59 times long of the proposed method. Table 5 shows that the maximum relative errors of the bounds are reduced to 0.994% and 0.642% under the

Table 5 Comparison of the computation cost

	Proposed method			DMCM*
	s=5	s=7	s=9	
Number	49	73	97	1000
Computing cost	109.45s	256.12s	472.91s	18723.19s

*DMCM: Direct Monte Carlo method

Table 6 Comparison of response bounds and relative errors with different ζ

		Proposed method	DMCM*	Relative error (%)
ζ=1×10^{-12}	$\ddot{\mathbf{x}}_{cL}$	-0.241	-0.237	1.891
	$\ddot{\mathbf{x}}_{cR}$	0.254	0.251	1.211
ζ=1×10^{-11}	$\ddot{\mathbf{x}}_{cL}$	-0.225	-0.221	1.807
	$\ddot{\mathbf{x}}_{cR}$	0.252	0.249	1.124
ζ=1×10^{-10}	$\ddot{\mathbf{x}}_{cL}$	-0.206	-0.203	1.326
	$\ddot{\mathbf{x}}_{cR}$	0.225	0.223	0.967
ζ=1×10^{-9}	$\ddot{\mathbf{x}}_{cL}$	-0.194	-0.193	0.726
	$\ddot{\mathbf{x}}_{cR}$	0.219	0.217	0.759
ζ=1×10^{-8}	$\ddot{\mathbf{x}}_{cL}$	-0.190	-0.189	0.421
	$\ddot{\mathbf{x}}_{cR}$	0.194	0.193	0.391

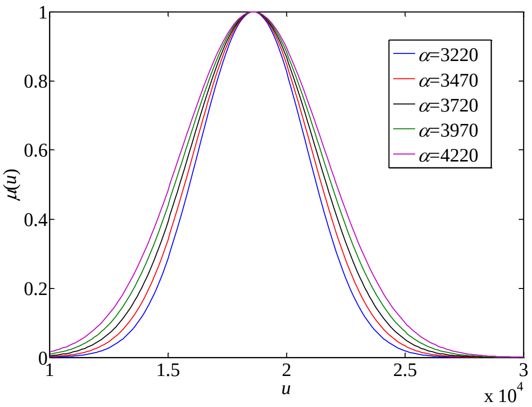
*DMCM: Direct Monte Carlo method

condition of s=9, which shows the accuracy and efficiency of the proposed method in calculating the bounds of the system's dynamic responses.

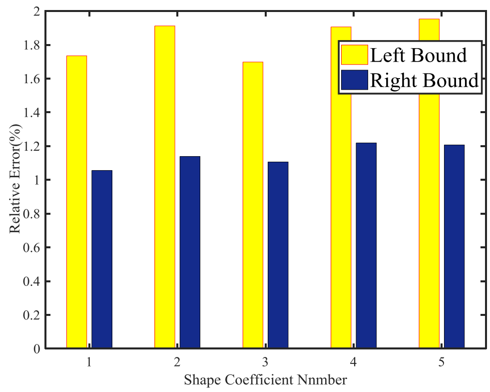
3.1 The effect of the membership degree on the bounds of dynamic responses

When the optimum spreading coefficients and membership function are determined, the bounds of fuzzy variables are determined by the membership degree. The influences of different membership degrees, e.g. ζ=1×10^{-12}, 1×10^{-11}, 1×10^{-10}, 1×10^{-9} and 1×10^{-8}, on the prediction of the response bounds are discussed here. The number s of dynamic responses applied in the Lagrange interpolation function is taken as 7. Figure 5 shows the maximum relative errors of the left and right bounds of the acceleration responses obtained from the proposed method and the direct Monte Carlo method under different values of ζ. As shown in Fig. 5, the relative errors of the bounds calculated by these two methods decreases gradually with the increase of the membership degree ζ.

Table 6 gives the bounds and relative errors calculated by above two methods with different ζ. As shown in Table 6, with the increase of membership degree ζ, the values of the left bounds calculated by the two methods are larger, while the right bounds are smaller. This means that the bounds of the calculated vehicle acceleration responses become narrower, which are closer to the responses



(a) Membership function



(b) Relative errors

Fig. 6 Membership functions and relative errors with different spreading coefficients

calculated by the deterministic method based on the nominal values of the parameters. This is because as the membership degree ζ increases, the bounds of fuzzy variables become narrower and closer to the nominal value of the parameters. At the same time, with the increase of membership degree ζ, the relative errors of the left and right bounds tend to decrease. However, when ζ is reduced from 1×10^{-11} to 1×10^{-12}, the effect on the calculation accuracy is very small. This is because fuzzy variables are defined as symmetric normal fuzzy variables. When ζ is small enough, most of the values near the nominal values are included already. The reduction of ζ value will only increase the excessive fuzzy variables, which has little influence on the calculation results.

From Tables 4 and 6, the proposed method can improve the accuracy by increasing the number of dynamic responses s applied in the Lagrange interpolation function or by increasing the value of membership degree ζ. However, the increase of s will lead to the increase of computing time, while the increase of the membership degree ζ will reduce the fuzziness. Therefore, it is necessary to select reasonable values of s and ζ to achieve the required accuracy and fuzziness based on the actual engineering situation.

3.2 Influence of the spreading coefficient on the bounds of dynamic responses

The vehicle parameters are assumed as fuzzy variables with known probability distributions in this paper. Firstly, the optimum spreading coefficient is obtained based on the Monte Carlo method. Then the influence of the number s of dynamic responses used in Lagrange interpolation function and the membership degree ζ on the bounds is studied. However, the probability distribution of many parameters in engineering practices cannot be known in advance, thus the optimal spreading coefficient cannot be estimated initially. The influence of the spreading coefficient α on the bounds of dynamic responses is now investigated. As the number of fuzzy variables is not considered, only the influence of vehicle parameter M_c with different spreading coefficients on the bounds are analyzed. In this paper, spreading

Table 7 Comparison of response bounds and relative errors with different α

		Proposed method	DMCM*	Relative error (%)
α=3220	$\ddot{\mathbf{x}}_{cL}$	-0.176	-0.173	1.735
	$\ddot{\mathbf{x}}_{cR}$	0.195	0.193	1.056
α=3470	$\ddot{\mathbf{x}}_{cL}$	-0.189	-0.185	1.912
	$\ddot{\mathbf{x}}_{cR}$	0.206	0.204	1.139
α=3720	$\ddot{\mathbf{x}}_{cL}$	-0.194	-0.191	1.698
	$\ddot{\mathbf{x}}_{cR}$	0.221	0.219	1.106
α=3970	$\ddot{\mathbf{x}}_{cL}$	-0.232	-0.228	1.906
	$\ddot{\mathbf{x}}_{cR}$	0.252	0.229	1.219
α=4220	$\ddot{\mathbf{x}}_{cL}$	-0.253	-0.248	1.953
	$\ddot{\mathbf{x}}_{cR}$	0.255	0.242	1.207

*DMCM: Direct Monte Carlo method

coefficient α considered are near the optimum spreading coefficients, i.e. α=3220, 3470, 3720, 3970 and 4220. The membership degree is taken as $\zeta=1\times10^{-11}$. The number of dynamic response applied in the Lagrange interpolation function is taken as s=7.

Figure 6 shows the membership functions of the fuzzy variable M_c with different spreading coefficients and the relative errors of the bounds calculated by the two methods. From results in Fig. 6, the bounds of fuzzy variable under the same membership degree increase as the increase of the expansion coefficient α. The relative errors of the bounds of the vehicle acceleration responses calculated by the two methods have no obvious regularity.

Table 7 shows the bounds of the vehicle acceleration responses and relative errors calculated by the two methods under different spreading coefficients. From Table 7, when the spreading coefficients are taken as α=3220, 3470, 3720, 3970 and 4220, respectively, the left bounds calculated by the proposed method are between -0.176 m/s^2 and -0.253 m/s^2, while the right bounds ranges from 0.195 m/s^2 to

0.255 m/s^2. The maximum relative errors of the left bounds are between 1.735% and 1.953%, and the maximum relative errors of the right bounds are within the range from 1.056% to 1.207%. It can be seen that with the increase of the spreading coefficient α, the bounds calculated by the two methods also increase. However, the relative error magnitude is less than 2%. The results show that the bounds of the system's dynamic responses can be simulated accurately under different spreading coefficients, indicating that the proposed method is not affected by the shape coefficient.

4. Conclusions

In order to solve the problem of low efficiency of calculating the bounds of the dynamic responses of the vehicle-track coupling system with fuzzy variables, a method based on high dimensional model representation method is proposed in this paper. The dynamic responses of the vehicle-track coupling system with fuzzy variables are expressed by a component function. The Lagrange interpolation method is used to approximate the component function. The bounds of system responses are approximately predicted based on the responses of several sample points, avoiding the expensive Monte Carlo process for the whole system responses.

A numerical study is adopted to demonstrate the accuracy and efficiency of the proposed method. The influence of the membership degree and spreading coefficient on the results of the predicted responses bounds is investigated in this study. From the numerical results, the following conclusions can be made. (1) The proposed method can predict the bounds of the system responses with high accuracy and high efficiency; (2) The accuracy can be improved by increasing the number of responses involved and the membership degree. The increase of the number of responses will lead to the increase of computing time, while the increase of the membership degree will reduce the fuzziness; (3) The bounds of the system's dynamic responses from the proposed method and the direct Monte Carlo method increase with the increase of the spreading coefficient. The relative error magnitude is typically less than 2%. The results show that the bounds of the system's dynamic responses can be predicted accurately under different spreading coefficients, which indicate that the proposed method is not affected by the shape coefficient.

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