Structural Engineering and Mechanics, *Vol. 75, No. 2 (2020) 157-174*
DOI: https://doi.org/10.12989/sem.2020.75.2.157

A simple analytical model for free vibration and buckling analysis of orthotropic rectangular plates

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(Received December 31, 2019, Revised February 10, 2020, Accepted February 12, 2020)

Abstract. In the present paper, a simple analytical model is developed based on a new refined parabolic shear deformation theory (RPSDT) for free vibration and buckling analysis of orthotropic rectangular plates with simply supported boundary conditions. The displacement field is simpler than those of other higher-order theories since it is modeled with only two unknowns and accounts for a parabolic distribution of the transverse shear stress through the plate thickness. The governing differential equations related to the present theory are obtained from the principle of virtual work, while the solution of the eigenvalue problem is achieved by assuming a Navier technique in the form of a double trigonometric series that satisfy the edge boundary conditions of the plate. Numerical results are presented and compared with previously published results for orthotropic rectangular plates in order to verify the precision of the proposed analytical model and to assess the impacts of several parameters such as the modulus ratio, the side-to-thickness ratio and the geometric ratio on natural frequencies and critical buckling loads. From these results, it can be concluded that the present computations are in excellent agreement with the other higher-order theories.

Keywords: orthotropic rectangular plates; free vibration; buckling; RPSDT; Navier technique

1. Introduction

Structural designers are constantly looking for new materials that are more efficient, lighter, more durable and less expensive. For this purpose, orthotropic rectangular plates have been developed and widely used over the last three decades in several applications such as mechanics, aeronautics, civil engineering, biomechanics and other industries because of their good orthotropic mechanical properties than those composed of conventional materials such as steel, wood and concrete. These structures are new assortments of anisotropic materials which having different mechanical properties according to their three orthogonal directions with an inhomogeneous and complex structural nature used to produce parts or lightweight bodies of high reliability and mechanical strength. Thus, the intensive applications of orthotropic rectangular plates in engineering require comprehensive set of information regarding the behaviour of anisotropic plates. Previous research results show that the transverse shear effects are more important for orthotropic plates than for isotropic ones (Wang and Huang 1991); moreover, it is also exposed to the combined loading during their service life. For this reason, numerous analytical and numerical methods have been carried out to solve the free vibration and buckling problems of orthotropic plates.

Therefore, it is very necessary to develop a suitable and simplified theory with a minimized number of variables and equilibrium equations to handle the orthotropic plates subjected to different kinds of in-plane loading that can cause buckling and to calculate the natural frequencies of these structures in the event of vibration.

The study of the free vibration and buckling of orthotropic plates is not a new topic. There are a variety of theoretical approaches on free vibration frequencies and critical buckling load of rectangular plates with a high diversity of boundary conditions. The most popular known are those of Leissa (1969, 1973, 1981), Laura *et al*. (1977), and Warburton and Edney (1984), The most important objective of Leissa's work is to present in one place reasonably and accurate analytical results for free vibration frequencies and buckling response of various combinations of classical boundary conditions for rectangular plates, while Laura and his colleagues have applied a variational formulation by using a very simple polynomial expression, which identically satisfies the boundary conditions for determining the fundamental frequency of rectangular plates with elastically restrained edges. Although a simple approach based on the Rayleigh-Ritz method has been used by Warburton and Edney (1984) to compute natural frequencies in transverse vibration of isotropic rectangular plates with elastically restrained edges, it should be noted from this investigation that the obtained results agree very close with those of Laura *et al*. (1977). Subsequently, the same approach was extended to orthotropic plates and considerable efforts have been expended by several investigators to find an accurate solution for the free vibration and buckling analysis of orthotropic plates. Among these we mention the results presented by

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http://www.techno-press.com/journals/sem&subpage=7
ISSN: 1225-4568 (Print), 1598-6217 (Online)

Dickinson and Di Blasio (1986) for a number of flexural vibration and buckling problems for orthotropic rectangular plates with various boundary conditions. Gorman (1993) employed the method of superposition to obtain the analytical solution for the free vibration of orthotropic rectangular plates, in which the eigenvalue matrix is developed in a manner similar to that described earlier for completely free isotropic plates. Sladek *et al*. (2006) developed a meshless local Petrov–Galerkin method (MLPG) to solve the static and dynamic problems of thick orthotropic plates resting on a Winkler elastic foundation. According to this computational method, the Laplace transformation technique is applied to all partial differential equations governing the elastodynamic bending of Reissner-Mindlin plates. The general mathematical expression based on the method of separation of variables was proposed by Xing and Liu (2009) to solve the exact solutions for the free vibrations of thin orthotropic rectangular plates with all combinations of simply supported and clamped boundary conditions. Ghugal and Pawar (2011) developed a refined hyperbolic shear deformation theory based on three variables to determine the natural frequencies and buckling loads of single layer isotropic and orthotropic plates. Thai and Kim (2011, 2012) applied the state space approach to the Levy-type solution based on a two variable refined plate theory for free vibration and bucking analysis of orthotropic plates with two opposite edges simply supported and the other two edges having arbitrary boundary conditions. Papkov and Banerjee (2015) have developed a new approach for free vibration and buckling analysis of rectangular orthotropic plates. The method suggested in their work enhances the superposition method significantly to reduce the boundary value problem to an infinite system of linear algebraic equations. A unified solution procedure was developed by Wang *et al*. (2016) using the first-order shear deformation theory to obtain some useful results for the free vibration analysis of moderately thick orthotropic rectangular plates with general boundary restraints, internal line supports and resting on elastic foundation. Furthermore, Abualnour *et al*. (2019) proposed a new four variable trigonometric refined plate theory by using the principle of virtual works for thermo-mechanical analysis of antisymmetric laminated reinforced composite plates.

In recent years, many studies and numerical investigations based on different shear deformation plate theories have been widely embraced by many researchers to examine the static and dynamic responses of orthotropic and multilayered composite structures (Panda and Katariya 2015, Nguyen *et al*. 2016, Sarangan and Singh 2016, Sahoo *et al*. 2016ab, Singh and Panda 2016, Katariya and Panda 2016, Hirwani *et al*. 2016, Swain *et al*. 2017, Adhikari and Singh 2017, Kolahchi *et al*. 2017, Sayyad and Ghugal 2017, Singh *et al*. 2017, Chikh *et al*. 2017, Joshan *et al*. 2017, Katariya *et al*. 2017a, Sahoo *et al*. 2017, Mehar *et al*. 2018, Karkon and Pajand 2018, Patni *et al*. 2018, Nor Hafizah *et al*. 2018, Benhenni *et al*. 2018, Sahoo *et al*. 2018, Chandra Mouli *et al*. 2018, Panda and Kolahchi 2018, Patle *et al*. 2018, Katariya *et al*. 2018ab, Das *et al*. 2018ab, Katariya and Panda 2019ab, Medani *et al*. 2019, Sahoo *et al*. 2019, Hirwani and Panda 2019, Ranjan *et al*. 2019, Zaoui *et al*. 2019, Batou *et al*. 2019, Chaabane *et al*. 2019, Boulefrakh *et al*. 2019, Boutaleb *et al*. 2019, Meksi *et al*. 2019, Boukhlif *et al*. 2019, Khiloun *et al*. 2019, Bourada *et al*. 2019, Zarga *et al*. 2019, Salah *et al*. 2019, Tounsi *et al*. 2020, Kaddari *et al*. 2020, Boussoula *et al*. 2020).

In this paper, a simple analytical model is presented based on a new refined higher-order shear deformation theory with two unknown variables recently developed by Fellah *et al*. (2019) for the buckling analysis of isotropic plates is successfully extended for the free vibration and buckling analysis of orthotropic rectangular plates with various loading conditions. The kinematics of the proposed theoretical model is defined by an undetermined integral component and uses the parabolic shape function of Reddy's theory to include the influences of the transverse shear stress through the plate thickness without requiring a shear correction factor. The governing equations and its boundary conditions are established by utilizing the principle of virtual works and solved via Navier-type analytical procedure. Numerical results of natural frequencies and critical buckling load for simply supported orthotropic plates are provided and compared with other shear deformation theories to confirm the validity and effectiveness of the present theory.

2. Theoretical formulation

2.1. Orthotropic plate under consideration

Consider an elastic orthotropic rectangular plate of the length a, width b and a constant thickness h in z-direction. The plate is simply supported on all four edges and subjected to various in-plane compressive loads N_x^0, N_y^0 and N_{xy}^0. The plate under consideration occupies the region $0 \le x \le a$, $0 \le y \le b$, $-h/2 \le z \le h/2$ in Cartesian coordinate system.

2.2. Kinematic and constitutive relations

The conventional higher-order shear deformation theory is usually based on five independent unknowns, but this number can be decreased by introducing some simplifying suppositions, in which it is assumed that, the axial displacements in the *x* and *y* directions, respectively, consist of bending and shear components, while the transverse displacement in the *z*-direction consists of bending component. The displacement field at any point (x, y, z) of the plate becomes in the following form.

$$\begin{aligned} u(x,y,z) &= -z\frac{\partial w_0}{\partial x} + f(z)\ \varphi(x,y) \\ v(x,y,z) &= -z\frac{\partial w_0}{\partial y} + f(z)\ \psi(x,y) \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{1}$$

where u, v and w denote the displacement components along the x, y and z coordinate directions, respectively, φ and ψ represents the rotations about the y and x axes, whereas $f(z)$ denote a shape function determining the distribution of the transverse shear strains and the stresses through the thickness of the plate. By employing that $\varphi = \int \theta(x,y)dx$ and $\psi = \int \theta(x,y)dy$, the new displacement field of the proposed refined parabolic shear deformation theory (RPSDT) can be expressed only with two unknowns in the most simplified form as (Fellah *et al.* 2019)

$$\begin{aligned} u(x,y,z) &= -z\frac{\partial w_0}{\partial x} + k_1 f(z)\int \theta(x,y)dx \\ v(x,y,z) &= -z\frac{\partial w_0}{\partial y} + k_2 f(z)\int \theta(x,y)dy \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{2}$$

where w_0 and θ are two unknowns displacement functions of middle surface of the orthotropic plate. The constants k_1 and k_2 depends on the geometry. In this paper, the present theory is obtained by putting

$$f(z) = z - \frac{4z^3}{3h^2} \tag{3}$$

Using the strain-displacement relationships from linear theory of elasticity, we can obtain the following strain field associated with Eq. (2)

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{Bmatrix} = z\begin{Bmatrix} \varepsilon_x^1 \\ \varepsilon_y^1 \\ \gamma_{xy}^1 \end{Bmatrix} + f(z)\begin{Bmatrix} \varepsilon_x^2 \\ \varepsilon_y^2 \\ \gamma_{xy}^2 \end{Bmatrix},$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \end{Bmatrix} = g(z)\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} \tag{4}$$

where

$$\begin{Bmatrix} \varepsilon_x^1 \\ \varepsilon_y^1 \\ \gamma_{xy}^1 \end{Bmatrix} = \begin{Bmatrix} -\frac{\partial^2 w_0}{\partial x^2} \\ -\frac{\partial^2 w_0}{\partial y^2} \\ -2\frac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}, \begin{Bmatrix} \varepsilon_x^2 \\ \varepsilon_y^2 \\ \gamma_{xy}^2 \end{Bmatrix} = \begin{Bmatrix} k_1\theta \\ k_2\theta \\ k_1\frac{\partial}{\partial y}\int \theta dx + k_2\frac{\partial}{\partial x}\int \theta dy \end{Bmatrix},$$

$$\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} k_2\int \theta dy \\ k_1\int \theta dx \end{Bmatrix} \tag{5}$$

and

$$g(z) = \frac{df(z)}{dz} \tag{6}$$

The integrals adopted in the previous relations shall be resolved by a Navier solution and can be determined by

$$\begin{aligned} &\frac{\partial}{\partial y}\int \theta dx = A'\frac{\partial^2\theta}{\partial x \partial y}, \quad \frac{\partial}{\partial x}\int \theta dy = B'\frac{\partial^2\theta}{\partial x \partial y}, \\ &\int \theta dx = A'\frac{\partial\theta}{\partial x}, \quad \int \theta dy = B'\frac{\partial\theta}{\partial y} \end{aligned} \tag{7}$$

where A' and B' are defined according to the type of solution employed, in this case via Navier. Thus, the parameters A' and B' are expressed by

$$A' = -\frac{1}{\alpha^2}, \quad B' = -\frac{1}{\beta^2}, \quad k_1 = \alpha^2, \quad k_2 = \beta^2 \tag{8}$$

where the parameters α and β are given as

$$\alpha = m\pi/a, \quad \beta = n\pi/b \tag{9}$$

The stress-strain relationships accounting for transversal shear deformation in the orthotropic plate coordinates, can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{10}$$

where Q_{ij} are the reduced stiffness coefficients as given below

$$\begin{aligned} &Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}, \; Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}}, \; Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}}, \\ &Q_{66} = G_{12}, \quad Q_{44} = G_{23}, \quad Q_{55} = G_{13} \end{aligned} \tag{11}$$

In which E_1 and E_2 are Young's moduli, G_{12}, G_{13} and G_{23} are shear moduli, and ν_{12} and ν_{21} are Poisson's ratios.

2.3 Governing equations

In the proposed RPSDT, the dynamic version of the principle of virtual work is used to obtain the governing equations and boundary conditions for the orthotropic rectangular plate under consideration. The principle can be stated in analytical form as (Akbas 2016 and 2017c, Ebrahimi and Barati 2017 ,2018 and 2019, Mirjavadi *et al.* 2019b, Eltaher and Mohamed 2020, Hamed *et al.* 2020, Barati and Shahverdi 2020)

$$\int_{t_1}^{t_2} (\delta U - \delta V + \delta K)dt = 0 \tag{12}$$

where symbol δ denotes the variational operator, t_1 is the initial time, t_2 is the final time, δU, δV and δK represents the variations of strain energy, work done by external forces and kinetic energy, respectively. Eq. (12) leads to the following form

$$\int_{t_1}^{t_2}\left\{\begin{array}{l}\int_{-h/2}^{h/2}\int_A\left(\begin{array}{l}\sigma_x\delta\varepsilon_x+\sigma_y\delta\varepsilon_y+\tau_{xy}\delta\gamma_{xy}\\+\tau_{yz}\delta\gamma_{yz}+\tau_{xz}\delta\gamma_{xz}\end{array}\right)dAdz\\-\int_A q(x,y)\delta w dA\\-\int_A\left(N_x^0\frac{\partial^2 w}{\partial x^2}+N_y^0\frac{\partial^2 w}{\partial y^2}+2N_{xy}^0\frac{\partial^2 w}{\partial x\partial y}\right)\delta w dA\\+\rho\int_{-h/2}^{h/2}\int_A\left(\ddot{u}\delta u+\ddot{v}\delta v+\ddot{w}\delta w\right)dAdz\end{array}\right\}dt=0 \tag{13}$$

where dot-superscript convention represents the second derivative with respect to the time variable t, A is the top surfaceof the plate, ρ is the mass density, q and (N_x^0, N_y^0, N_{xy}^0) are transverse and in-plane distributed loads, respectively. By substituting the terms for virtual strains given in Eq. (4) into Eq. (13) and integrating over the thickness direction, the principle of virtual work can be rewritten as

$$\int_{t_1}^{t_2}\left\{\int_A\left\{\begin{array}{l}M_x^b\delta\varepsilon_x^1+M_y^b\delta\varepsilon_y^1+M_{xy}^b\delta\gamma_{xy}^1\\+M_x^s\delta\varepsilon_x^2+M_y^s\delta\varepsilon_y^2+M_{xy}^s\delta\gamma_{xy}^2\\+S_{yz}^s\delta\gamma_{yz}^0+S_{xz}^s\delta\gamma_{xz}^0-q\delta w_0\\+N_x^0\frac{\partial w_0}{\partial x}\frac{\partial\delta w_0}{\partial x}+N_y^0\frac{\partial w_0}{\partial y}\frac{\partial\delta w_0}{\partial y}\\+2N_{xy}^0\frac{\partial w_0}{\partial x}\frac{\partial\delta w_0}{\partial y}+I_1\dot{w}_0\delta\dot{w}_0\\+I_2\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial\delta\dot{w}_0}{\partial x}+\frac{\partial\dot{w}_0}{\partial y}\frac{\partial\delta\dot{w}_0}{\partial y}\right)\\-I_3\left(\begin{array}{l}k_1A'\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial\delta\dot{\theta}}{\partial x}+\frac{\partial\dot{\theta}}{\partial x}\frac{\partial\delta\dot{w}_0}{\partial x}\right)\\+k_2B'\left(\frac{\partial\dot{w}_0}{\partial y}\frac{\partial\delta\dot{\theta}}{\partial y}+\frac{\partial\dot{\theta}}{\partial y}\frac{\partial\delta\dot{w}_0}{\partial y}\right)\end{array}\right)\\+I_4\left((k_1A')^2\frac{\partial\dot{\theta}}{\partial x}\frac{\partial\delta\dot{\theta}}{\partial x}+(k_2B')^2\frac{\partial\dot{\theta}}{\partial y}\frac{\partial\delta\dot{\theta}}{\partial y}\right)\end{array}\right\}dA\right\}dt=0 \tag{14}$$

where M^b, M^s and S^s are the stress resultants and $\mathrm{I_i}$ $(i=1,\ 2,\ 3,\ 4)$ are the inertia coefficients can be defined by the following integrations

$$\begin{Bmatrix} M_x^b, & M_y^b, & M_{xy}^b \\ M_x^s, & M_y^s, & M_{xy}^s \end{Bmatrix} = \int_{-h/2}^{h/2} \left(\sigma_x, \sigma_y, \tau_{xy}\right) \begin{Bmatrix} z \\ f(z) \end{Bmatrix} dz, \tag{15a}$$

$$\left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} \left(\tau_{xz}, \tau_{yz}\right) g(z)\, dz$$

$$\left(I_1, I_2, I_3, I_4\right) = \rho \int_{-h/2}^{h/2} \left(1, z^2, z\, f(z), f^2(z)\right) dz \tag{15b}$$

Substituting stress-strain relations from Eq. (10) into the Eq. (15), the stress resultants are obtained in terms of strains as following form

$$\begin{aligned} M_x^b &= D_{11}\varepsilon_x^1 + D_{12}\varepsilon_y^1 + F_{11}\varepsilon_x^2 + F_{12}\varepsilon_y^2, \\ M_y^b &= D_{12}\varepsilon_x^1 + D_{22}\varepsilon_y^1 + F_{12}\varepsilon_x^2 + F_{22}\varepsilon_y^2, \\ M_{xy}^b &= D_{66}\gamma_{xy}^1 + F_{66}\gamma_{xy}^2, \\ M_x^s &= F_{11}\varepsilon_x^1 + F_{12}\varepsilon_y^1 + H_{11}\varepsilon_x^2 + H_{12}\varepsilon_y^2, \\ M_y^s &= F_{12}\varepsilon_x^1 + F_{22}\varepsilon_y^1 + H_{12}\varepsilon_x^2 + H_{22}\varepsilon_y^2, \\ M_{xy}^s &= F_{66}\gamma_{xy}^1 + H_{66}\gamma_{xy}^2, \\ S_{yz}^s &= A_{44}^s\gamma_{yz}^0, \\ S_{xz}^s &= A_{55}^s\gamma_{xz}^0 \end{aligned} \tag{16}$$

where D_{ij}, F_{ij}, H_{ij} and A_{ij}^s are the plate stiffness coefficients given by

$$\left(D_{ij}, F_{ij}, H_{ij}\right) = \int_{-h/2}^{h/2} Q_{ij}\left(z^2, z\, f(z), f^2(z)\right) \mathrm{d}z, \quad i,j = 1,2,6 \tag{17a}$$

$$A_{ij}^s = \int_{-h/2}^{h/2} Q_{ij}\, g^2(z)\, dz, \quad i,j = 4,5 \tag{17b}$$

Substituting strain-displacement and stress-strain relations from Eq. (5) and (10) of the proposed theory into Eq. (14) and integrating by parts and collecting the coefficients of δw_0 and $\delta\theta$,the governing differential equations in terms of stress resultants are obtained as follows

$$\begin{aligned} \delta w_0: \quad & \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} + q + N_x^0 \frac{\partial^2 w_0}{\partial x^2} \\ & + N_y^0 \frac{\partial^2 w_0}{\partial y^2} + 2N_{xy}^0 \frac{\partial^2 w_0}{\partial x \partial y} = I_1 \ddot{w}_0 - I_2 \nabla \ddot{w}_0 \\ & + I_3\left(k_1 A' \frac{\partial^2 \ddot{\theta}}{\partial x^2} + k_2 B' \frac{\partial^2 \ddot{\theta}}{\partial y^2}\right) \end{aligned} \tag{18a}$$

$$\begin{aligned} \delta\theta: \quad & -k_1 M_x^s - k_2 M_y^s - \left(k_1 A' + k_2 B'\right) \frac{\partial^2 M_{xy}^s}{\partial x \partial y} \\ & + k_1 A' \frac{\partial S_{xz}^s}{\partial x} + k_2 B' \frac{\partial S_{yz}^s}{\partial y} = I_3\left(k_1 A' \frac{\partial^2 \ddot{w}_0}{\partial x^2} + k_2 B' \frac{\partial^2 \ddot{w}_0}{\partial y^2}\right) \\ & - I_4\left(\left(k_1 A'\right)^2 \frac{\partial^2 \theta}{\partial x^2} + \left(k_2 B'\right)^2 \frac{\partial^2 \theta}{\partial y^2}\right) \end{aligned} \tag{18b}$$

Next, by substituting the stress resultants from Eq. (16) into Eq. (18), the following differential equations of the proposed theory can be rewritten in terms of displacement variables as follow

$$\begin{aligned} \delta w_0: \quad & -D_{11}\frac{\partial^4 w_0}{\partial x^4} - 2\left(D_{12} + 2D_{66}\right)\frac{\partial^4 w_0}{\partial x^2 \partial y^2} - D_{22}\frac{\partial^4 w_0}{\partial y^4} \\ & + k_1 A' F_{11}\frac{\partial^4 \theta}{\partial x^4} + k_1 A'\left(F_{12} + 2F_{66}\right)\frac{\partial^4 \theta}{\partial x^2 \partial y^2} \\ & + k_2 B'\left(F_{12} + 2F_{66}\right)\frac{\partial^4 \theta}{\partial x^2 \partial y^2} + k_2 B' F_{22}\frac{\partial^4 \theta}{\partial y^4} + q + N_x^0 \frac{\partial^2 w_0}{\partial x^2} \\ & + N_y^0 \frac{\partial^2 w_0}{\partial y^2} + 2N_{xy}^0 \frac{\partial^2 w_0}{\partial x \partial y} = I_1 \ddot{w}_0 - I_2 \nabla \ddot{w}_0 \\ & + I_3\left(k_1 A' \frac{\partial^2 \ddot{\theta}}{\partial x^2} + k_2 B' \frac{\partial^2 \ddot{\theta}}{\partial y^2}\right) \end{aligned} \tag{19a}$$

$$\begin{aligned} \delta\theta: \quad & k_1 A' F_{11}\frac{\partial^4 w_0}{\partial x^4} + k_1 A'\left(F_{12} + 2F_{66}\right)\frac{\partial^4 w_0}{\partial x^2 \partial y^2} \\ & + k_2 B'\left(F_{12} + 2F_{66}\right)\frac{\partial^4 w_0}{\partial x^2 \partial y^2} + k_2 B' F_{22}\frac{\partial^4 w_0}{\partial y^4} \\ & - \left(k_1 A'\right)^2 H_{11}\frac{\partial^4 \theta}{\partial x^4} - 2k_1 A' k_2 B'\left(H_{12} + H_{66}\right)\frac{\partial^4 \theta}{\partial x^2 \partial y^2} \\ & - \left(\left(k_1 A'\right)^2 + \left(k_2 B'\right)^2\right) H_{66}\frac{\partial^4 \theta}{\partial x^2 \partial y^2} - \left(k_2 B'\right)^2 H_{22}\frac{\partial^4 \theta}{\partial y^4} \\ & + \left(k_2 B'\right)^2 As_{44}\frac{\partial^2 \theta}{\partial y^2} + \left(k_1 A'\right)^2 As_{55}\frac{\partial^2 \theta}{\partial x^2} = \\ & I_3\left(k_1 A' \frac{\partial^2 \ddot{w}_0}{\partial x^2} + k_2 B' \frac{\partial^2 \ddot{w}_0}{\partial y^2}\right) \\ & - I_4\left(\left(k_1 A'\right)^2 \frac{\partial^2 \ddot{\theta}}{\partial x^2} + \left(k_2 B'\right)^2 \frac{\partial^2 \ddot{\theta}}{\partial y^2}\right) \end{aligned} \tag{19b}$$

2.4 Analytical solution for simply supported orthotropic plates

According to the present theory, the free vibration and buckling analysis of a simply supported orthotropic plate is obtained by using Navier solution procedure. The following boundary conditions along the edges of the simply supported plate can be obtained as

$$w_0 = M_x^b = M_x^s = \theta = 0 \quad \text{On edges } (x = 0, a) \tag{20a}$$

$$w_0 = M_y^b = M_y^s = \theta = 0 \quad \text{On edges } (y = 0, b) \tag{20b}$$

For the analytical solution of Eq. (19), the Navier procedure is used under the specified boundary conditions. The displacement variables that satisfy the equations of boundary conditions given by Eqs. (20), are assumed as the following Fourier series

$$\begin{Bmatrix} w_0 \\ \theta \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} W_{mn}\sin(\alpha\, x)\sin(\beta\, y)e^{i\omega t} \\ \Phi_{mn}\sin(\alpha\, x)\sin(\beta\, y)e^{i\omega t} \end{Bmatrix} \tag{21}$$

where W_{mn} and Φ_{mn} are the unknown coefficients and ω is the eigenfrequency or buckling load factor related to the (m, n) eigenmode of the orthotropic plate. By substitution Eq. (21) into the governing equations (19), analytical solutions can be obtained from the following general equation

$$\{([K_{ij}] - N_0[N_{ij}]) - \omega^2[M_{ij}]\}\{\Delta\} = \{f_i\} \tag{22}$$

The Eq. (22) can elaborately be written as

$$\begin{Bmatrix} \left(\begin{bmatrix} K_{11} & K_{12} \\ K_{12} & K_{22} \end{bmatrix} - N_0\begin{bmatrix} N_{11} & 0 \\ 0 & 0 \end{bmatrix}\right) \\ -\omega^2\begin{bmatrix} M_{11} & M_{12} \\ M_{12} & M_{22} \end{bmatrix} \end{Bmatrix} \begin{Bmatrix} W_{mn} \\ \Phi_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ q_{mn} \end{Bmatrix} \tag{23}$$

where $\{\Delta\}$ is the vector of unknown coefficients, $\{f_i\}$ is the force vector as specified in Eq. (23) and $[K_{ij}]$, $[M_{ij}]$ and $[N_{ij}]$ are stiffness, inertia and geometric matrices, respectively. The elements of these matrices can be defined for orthotropic plates as follows

$$\begin{aligned}
&K_{11} = 2\alpha^2\beta^2(D_{12} + 2D_{66}) + \alpha^4 D_{11} + \beta^4 D_{22}, \\
&K_{12} = -k_1 A'\alpha^2(\alpha^2 F_{11} + \beta^2 F_{12}) \\
&-k_2 B'\beta^2(\alpha^2 F_{12} + \beta^2 F_{22}) - 2\alpha^2\beta^2(k_1 A' + k_2 B')F_{66}, \\
&K_{22} = k_1 A'\alpha^2(k_1 A'\alpha^2 H_{11} + k_2 B'\beta^2 H_{12}) \\
&+k_2 B'\beta^2(k_1 A'\alpha^2 H_{12} + k_2 B'\beta^2 H_{22}) \\
&+\alpha^2\beta^2(k_1 A' + k_2 B')(k_1 A' + k_2 B')H_{66} \\
&+(k_2 B')^2\beta^2 A_{44}^s + (k_1 A')^2\alpha^2 A_{55}^s, \\
&N_{11} = \gamma_1\alpha^2 + \gamma_2\beta^2, \\
&M_{11} = I_1 + I_2(\alpha^2 + \beta^2), \\
&M_{12} = -I_3(k_1 A'\alpha^2 + k_2 B'\beta^2), \\
&M_{22} = I_4(k_1^2 A'^2\alpha^2 + k_2^2 B'^2\beta^2)
\end{aligned} \tag{24}$$

Eq. (22) is a general form for bending, buckling and free vibration analysis of simply supported orthotropic rectangular plates subjected to in-plane and transverse loads. In case of free vibration problem the in-plane compressive and transverse loads $(N_0 = q_0 = 0)$ are set to zero, which leads to an eigenvalue equation described as follows

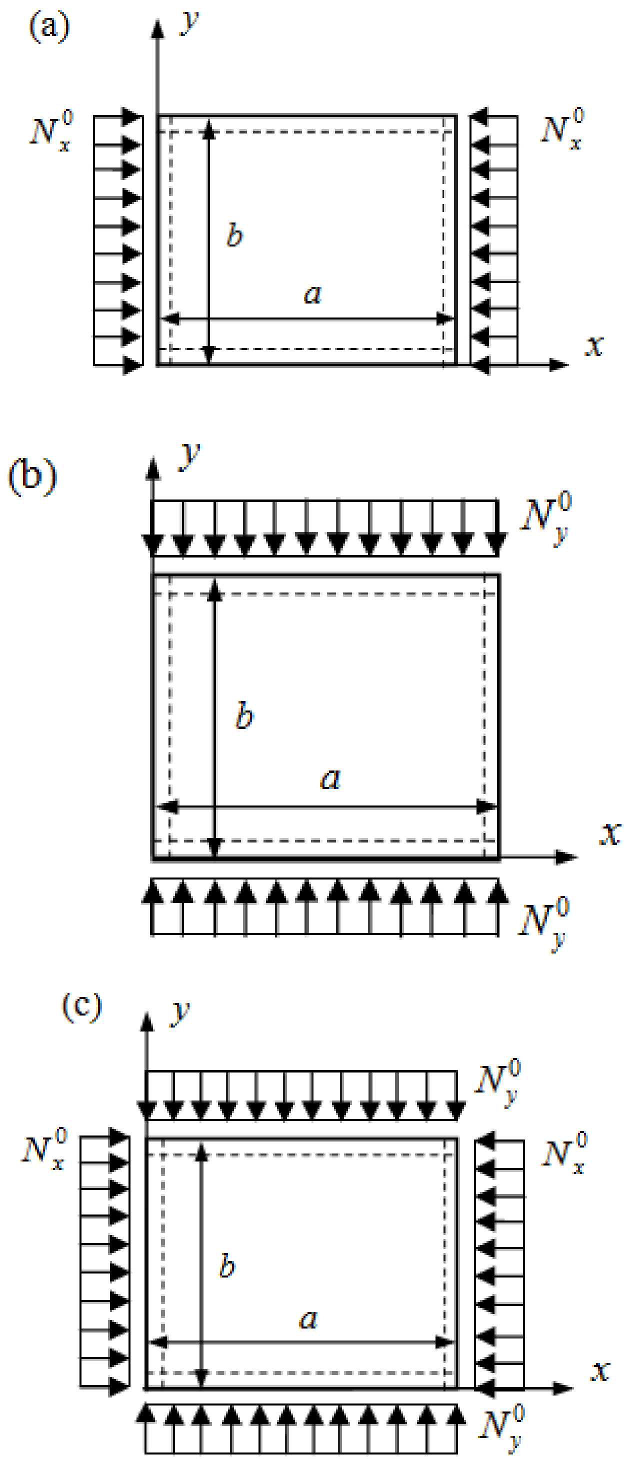

Fig. 1 The loading conditions of orthotropic rectangular plate: (a) uniaxial compression along x-axis, (b)uniaxial compression along y-axis and(c) biaxial compression.

Table 1 Displacement models of several plate theories

Model	Theory	Unknowns
CPT	Classical plate theory(Kirchhoff 1850)	3
FSDT	First-order shear deformation theory (Mindlin 1951)	5
HSDT	Higher-order shear deformation theory (Reddy 1984)	5
TSDT	Trigonometric shear deformation theory (Ghugaland Sayyad 2011)	4
ESDT	Exponential shear deformation theory (Sayyadand Ghugal 2014)	3
RPT	Two variablerefined plate theory (Thai and Kim 2012)	4
Present	Refined parabolic shear deformation theory	2

Table 2 Comparison of non-dimensional natural frequencies $\hat{\omega}$ of simply supported orthotropic square plate (*a/h*=10, material 1

Mode	Exact [a]	Present	Sayyad and Ghugal (2014)	Reddy (1984)	Ghugal and Sayyad (2011)	Shimpi and Patel (2006)	Mindlin (1951) [a]	CPT [a]
(1, 1)	0.0474	0.0477	0.0474	0.0474	0.0474	0.0477	0.0474	0.0497
(1, 2)	0.1033	0.1040	0.1033	0.1033	0.1031	0.1040	0.1032	0.1120
(1, 3)	0.1888	0.1898	0.1888	0.1888	0.1793	0.1898	0.1884	0.2154
(1, 4)	0.2969	0.2980	0.2969	0.2969	0.2932	0.2980	0.2959	0.3599
(2, 1)	0.1188	0.1198	0.1190	0.1189	0.1196	0.1198	0.1187	0.1354
(2, 2)	0.1694	0.1722	0.1697	0.1695	0.1696	0.1722	0.1692	0.1687
(2, 3)	0.2475	0.2520	0.2480	0.2477	0.2478	0.2520	0.2469	0.3029
(2, 4)	0.3476	0.3533	0.3482	0.3479	0.3468	0.3534	0.3463	0.4480
(3, 1)	0.2180	0.2197	0.2191	0.2184	0.2199	0.2197	0.2178	0.2779
(3, 2)	0.2624	0.2675	0.2637	0.2629	0.2671	0.2675	0.2619	0.3418
(3, 3)	0.3320	0.3407	0.3337	0.3326	0.3326	0.3407	0.3310	0.4470
(4, 1)	0.3319	0.3340	0.3351	0.3330	0.3346	0.3344	0.3311	0.4773
(4, 2)	0.3707	0.3774	0.3743	0.3720	0.3727	0.3774	0.3696	0.5415

[a] Results taken from reference of Srinivas and Rao (1970)

$$\left([K_{ij}]-\omega^2[M_{ij}]\right)\{\Delta\}=\{0\} \tag{25}$$

However the critical buckling loads N_{cr} as shown in Fig.1, can be obtained by setting the transverse load q_0 equal to zero and neglecting the inertia terms as follows

$$\left([K_{ij}]-N_0[N_{ij}]\right)\{\Delta\}=\{0\} \tag{26}$$

3. Numerical results and discussions

To ensure the precision and effectiveness of the present RPSDT in predicting the free vibration and buckling responses of simply supported orthotropic plates, various numerical examples are presented and compared with those obtained by the classical plate theory (CPT) of Kirchhoff (1850), first-order shear deformation theory (FSDT) of Mindlin (1951), higher-order shear deformation theories (HSDTs) and the exact elasticity theory available in the literature. The description of the different displacement models for the plate theories is presented in Table 1.

The following material properties are used to obtain the numerical results

Material 1 (Srinivas and Rao 1970)

$$\begin{aligned}&E_2/E_1=0.52500,\quad G_{12}/E_1=0.26293,\\&G_{13}/E_1=0.15991,\quad G_{23}/E_1=0.26681,\\&\nu_{12}=0.44046\end{aligned} \tag{27}$$

Material 2 (Reddy 1984)

$$\begin{aligned}&E_1/E_2=open,\quad G_{12}/E_2=G_{13}/E_2=0.5,\\&G_{23}/E_2=0.2,\quad \nu_{12}=0.25\end{aligned} \tag{28}$$

The following non-dimensional form is used while presenting numerical result of natural frequencies and critical buckling loads

$$\hat{\omega}=\omega h\sqrt{\frac{\rho}{Q_{11}}},\quad \bar{\omega}=\omega\frac{a^2}{h}\sqrt{\frac{\rho}{E_2}},\quad \bar{N}_{cr}=\frac{N_0a^2}{E_2h^3} \tag{29}$$

Table 3 Comparison of non-dimensional natural frequencies $\hat{\omega}$ of simply supported orthotropic rectangular plate ($a/h=10, b/a=\sqrt{2}$, material 1)

Mode	Present	Ghugal and Sayyad (2011)	Reddy (1984)	Shimpi and Patel (2006)	Mindlin (1951) [(b)]	CPT [(b)]
(1, 1)	0.0378	0.0376	0.0378	0.0378	0.0377	0.0390
(1, 2)	0.0670	0.0653	0.0676	0.0670	0.0669	0.0701
(1, 3)	0.1130	0.1066	0.1142	0.1130	0.1132	0.1210
(1, 4)	0.1733	0.1768	0.1750	0.1733	0.1739	0.1903
(2, 1)	0.1105	0.1104	0.1104	0.1105	0.1100	0.1225
(2, 2)	0.1377	0.1371	0.1377	0.1377	0.1362	0.1533
(2, 3)	0.1805	0.1728	0.1804	0.1805	0.1779	0.2032
(2, 4)	0.2366	0.2136	0.2366	0.2366	0.2333	0.2711
(3, 1)	0.2112	0.2114	0.2110	0.2112	0.2102	0.2575
(3, 2)	0.2360	0.2365	0.2352	0.2360	0.2329	0.2870
(3, 3)	0.2751	0.2701	0.2735	0.2751	0.2695	0.3352
(4, 1)	0.3264	0.3269	0.3262	0.3264	0.3246	0.4381
(4, 2)	0.3488	0.3500	0.3475	0.3488	0.3442	0.4661

[(b)] Results taken from reference of Ghugal andSayyad (2011)

3.1 Free vibration analysis of orthotropic plates

In this example, free vibration analysis of moderately thick (*a/h*=10) orthotropic square and rectangularplates is investigated using Eq. (25) in the absence of external load. The material properties of orthotropic plates utilized in the present study are given in Eq. (27).

The obtained resultsof non-dimensional natural frequencies for different modes of vibration are depicted in Tables 2 and 3, respectively. These results are subsequentlycompared with those presented by Reddy (1984), Shimpi and Patel (2006), Ghugal and Sayyad (2011), Sayyad and Ghugal (2014), the exact elasticity solution given by Srinivas *et al*. (1970), and with the corresponding values of FSDT and CPT. From the examination of Tables 2 and 3, it can be seen that the present refined parabolic shear deformation theory (RPSDT) shows the best accuracy and agree well with those reported by Shimpi and Patel (2006), exact elasticity solution and the previous studies based on the HSDTs. However, FSDT gives the lower values of frequencies as compared to those of HSDTs and exact results, whereas CPT gives the higher values for these frequencies for all modes of vibration.

Moreover, another example is extended from the previous one; the analytical model of the proposed theory is checked for the free vibration analysis of simply supported orthotropic rectangular plates made up of material 2. Numerical results of natural frequencies for first mode of vibration are illustrated in Table 4 for various values of modulus ratio $(E_1/E_2=3, 10, 20, 30, 40)$ and for different values of both side-to-thickness ratio $(a/h=5, 10, 20, 50)$ and geometric ratio $(a/b=0.5, 1, 2)$. Again the obtained results are compared with the solution reported by Thai and Kim (2012) using the two variable refined plate theory (RPT) and those computed using FSDT and CPT. This comparison displays clearly that the present results are in excellent agreement with those presented by Thai and Kim (2012) for all parameters.

The impacts of modulus ratio,side-to-thickness and geometric ratio on the natural frequencies of simply supported orthotropic rectangular plates made up of material 2 are shown from Figs. 2, 3 and 4, respectively. For all Figs., it can be proved that the present theory is more precise and efficient in predicting the natural frequencies of moderately thick orthotropic rectangular plates when compared to the analytical model provided by Thai and Kim (2012). Moreover, it is observed that the difference in the curves obtained using the present theory, RPT and FSDT becomes small when the modulus ratio decreases (see Fig. 2). The non-dimensional natural frequencies increase with the side-to-thickness ratio *a*/*h*, but the rate of increase in frequencies is negligible as the plate becomes thinner as shown in Fig. 3. Whereas the CPT overestimates the natural frequencies as compared to the results of other shear deformation theories due to the neglect of transverse shear strains and yields acceptable results only for thin orthotropic plates. It is also observed that the increase of the geometric ratio have a significant impact on the increase of the natural frequencies (see Fig. 4).

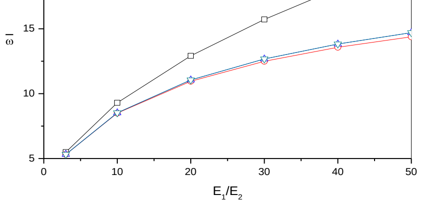

Fig. 2 The impact of modulus ratio on non-dimensional natural frequencies $\overline{\omega}$ of simply supported orthotropic rectangular plate with ($a/h=10, a/b=0.5$, material 2)

Table 4 Comparison of non-dimensional natural frequencies $\overline{\omega}$ of simply supported orthotropic rectangular plate, material 2

a/b	a/h	Theory	Model	E_1/E_2				
				3	10	20	30	40
0.5	5	Present	RPSDT	4.8399	6.9857	8.2401	8.8813	9.2832
		Thai and Kim (2012)	RPT	4.8399	6.9857	8.2401	8.8813	9.2832
		Mindlin(1951)	FSDT	4.8382	6.9158	8.0499	8.5941	8.9153
		Kirchhoff (1850)	CPT	5.4110	9.1558	12.7197	13.5648	13.5746
	10	Present	RPSDT	5.3202	8.5241	11.0551	12.6703	13.8239
		Thai and Kim (2012)	RPT	5.3202	8.5241	11.0551	12.6703	13.8239
		Mindlin(1951)	FSDT	5.3199	8.4965	10.9543	12.4918	13.5717
		Kirchhoff (1850)	CPT	5.4930	9.2945	12.9124	15.7186	18.0948
	20	Present	RPSDT	5.4685	9.1141	12.4009	14.7974	16.7105
		Thai and Kim (2012)	RPT	5.4685	9.1141	12.4009	14.7974	16.7105
		Mindlin(1951)	FSDT	5.4684	9.1061	12.3666	14.7293	16.6047
		Kirchhoff (1850)	CPT	5.5141	9.3301	12.9619	15.7789	18.1643
	50	Present	RPSDT	5.5126	9.3044	12.8804	15.6246	17.9239
		Thai and Kim (2012)	RPT	5.5126	9.3044	12.8804	15.6246	17.9239
		Mindlin(1951)	FSDT	5.5126	9.3030	12.8743	15.6120	17.9033
		Kirchhoff (1850)	CPT	5.5201	9.3402	12.9759	15.7960	18.1838
1	5	Present	RPSDT	6.1425	7.8304	9.0458	9.7339	10.1864
		Thai and Kim (2012)	RPT	6.1425	7.8304	9.0458	9.7339	10.1864
		Mindlin(1951)	FSDT	6.1305	7.7094	8.6452	9.1072	9.3832
		Kirchhoff (1850)	CPT	7.0877	10.1671	13.3976	15.9889	18.2155
	10	Present	RPSDT	6.9515	9.5628	11.9334	13.5598	14.7744
		Thai and Kim (2012)	RPT	6.9515	9.5628	11.9334	13.5598	14.7744
		Mindlin(1951)	FSDT	6.9468	9.5089	11.6839	13.0892	14.0902
		Kirchhoff (1850)	CPT	7.2577	10.4110	13.7190	16.3725	18.6524
	20	Present	RPSDT	7.2194	10.2349	13.2676	15.5845	17.4839
		Thai and Kim (2012)	RPT	7.2194	10.2349	13.2676	15.5845	17.4839
		Mindlin(1951)	FSDT	7.2180	10.2185	13.1790	15.3966	17.1822
		Kirchhoff (1850)	CPT	7.3021	10.4748	13.8030	16.4728	18.7666
	50	Present	RPSDT	7.3012	10.4530	13.7360	16.3474	18.5726
		Thai and Kim	RPT	7.3012	10.4530	13.7360	16.3474	18.5726
		Mindlin(1951)	FSDT	7.3009	10.4502	13.7201	16.3120	18.5132
		Kirchhoff (1850)	CPT	7.3147	10.4928	13.8268	16.5012	18.7990
2	5	Present	RPSDT	10.9975	11.6394	12.3588	12.9019	13.3277
		Thai and Kim (2012)	RPT	10.9975	11.6394	12.3588	12.9019	13.3277
		Mindlin(1951)	FSDT	10.8944	11.5883	12.0725	12.3295	12.4887
		Kirchhoff (1850)	CPT	14.3271	15.8800	17.9300	19.7753	21.4640
	10	Present	RPSDT	13.7909	14.9934	16.4739	17.7038	18.7467
		Thai and Kim (2012)	RPT	13.7909	14.9934	16.4739	17.7038	18.7467
		Mindlin(1951)	FSDT	13.7345	14.9802	16.2729	17.1922	17.8809
		Kirchhoff (1850)	CPT	15.1523	16.7946	18.9626	20.9142	22.7001
	20	Present	RPSDT	14.9772	16.5030	18.4742	20.2036	21.7468
		Thai and Kim (2012)	RPT	14.9772	16.5030	18.4742	20.2036	21.7468
		Mindlin(1951)	FSDT	14.9578	16.5002	18.3978	19.9828	21.3363
		Kirchhoff (1850)	CPT	15.3818	17.0490	19.2499	21.2310	23.0440
	50	Present	RPSDT	15.3796	17.0294	19.1992	21.1436	22.9151
		Thai and Kim (2012)	RPT	15.3796	17.0294	19.1992	21.1436	22.9151
		Mindlin(1951)	FSDT	15.3761	17.0290	19.1851	21.1009	22.8324
		Kirchhoff (1850)	CPT	15.4480	17.1223	19.3327	21.3223	23.1432

Table 5 Comparison of non-dimensional critical buckling load $\overline{N}_{cr}$ of simply supported orthotropic square plates under uniaxial compression ($\gamma_1 = -1,\ \gamma_2 = 0,\ m = n = 1,$ material 2).

a/h	Theory	Model	E_1 / E_2				
			3	10	20	30	40
5	Present	RPSDT	3.9587	6.3478	8.3967	9.6821	10.578
	Sayyad and Ghugal (2014)	ESDT	3.9650	6.3014	8.0946	9.2166	10.049
	Reddy (1984)	HSDT	3.9434	6.2072	7.8292	8.7422	9.3472
	Ghugal and Sayyad (2011)	TSDT	4.0572	6.3212	7.9324	8.8418	9.4502
	Kim *et al.* (2009)	RPT	—	6.3478	—	—	10.579
	Mindlin (1951)	FSDT	3.9386	6.1804	7.7450	8.5848	9.1084
	Kirchhoff (1850)	CPT	5.4248	11.163	19.383	27.606	35.830
10	Present	RPSDT	4.9637	9.3732	14.563	18.772	22.258
	Sayyad and Ghugal (2014)	ESDT	4.9612	9.2998	14.080	17.748	20.676
	Reddy (1984)	HSDT	4.9568	9.2772	14.001	17.577	20.386
	Ghugal and Sayyad (2011)	TSDT	5.0128	9.3646	14.116	17.711	20.534
	Kim *et al.* (2009)	RPT	—	9.3732	—	—	22.258
	Mindlin (1951)	FSDT	4.9562	9.2734	13.982	17.532	20.304
	Kirchhoff (1850)	CPT	5.4248	11.163	19.383	27.606	35.830
20	Present	RPSDT	5.3015	10.653	17.898	24.689	31.069
	Sayyad and Ghugal (2014)	ESDT	5.3004	10.625	17.681	24.146	30.094
	Reddy (1984)	HSDT	5.2994	10.621	17.664	24.108	30.025
	Ghugal and Sayyad (2011)	TSDT	5.3194	10.653	17.714	24.175	30.107
	Kim *et al.* (2009)	RPT	—	10.653	—	—	31.069
	Mindlin (1951)	FSDT	5.2994	10.620	17.662	24.102	30.014
	Kirchhoff (1850)	CPT	5.4248	11.163	19.383	27.606	35.830
50	Present	RPSDT	5.4046	11.078	19.129	27.094	34.972
	Sayyad and Ghugal (2014)	ESDT	5.4044	11.072	19.087	26.982	34.758
	Reddy (1984)	HSDT	5.4040	11.072	19.085	26.976	34.748
	Ghugal and Sayyad (2011)	TSDT	5.4116	11.081	19.098	26.993	34.769
	Kim *et al.* (2009)	RPT	—	11.078	—	—	34.972
	Mindlin (1951)	FSDT	5.4046	11.072	19.085	26.976	34.748
	Kirchhoff (1850)	CPT	5.4248	11.163	19.383	27.606	35.830
100	Present	RHSDT	5.4197	11.141	19.319	27.477	35.612
	Sayyad and Ghugal (2014)	ESDT	5.4196	11.400	19.308	27.447	35.554
	Reddy (1984)	HSDT	5.4192	11.139	19.307	27.466	35.553
	Ghugal and Sayyad (2011)	TSDT	5.4250	11.145	19.314	27.453	35.562
	Kim *et al.* (2009)	RPT	—	11.142	—	—	35.612
	Mindlin (1951)	FSDT	5.4206	11.142	19.309	27.448	35.554
	Kirchhoff (1850)	CPT	5.4248	11.163	19.383	27.606	35.830

3.2 Buckling analysis of orthotropic plates

In the case of static problem, the efficiency of present theory is investigated for buckling response of simply supported orthotropic square and rectangular plates subjected to in-plane distributed loads $(N_x^0 = \gamma_1 N_0, N_y^0 = \gamma_2 N_0, N_{xy}^0 = 0)$, as shown in Fig.1. The properties of orthotropic material used for this problem are given in Eq. (28). Three specific loading conditions are examined in this study, which are uniaxial compression along the *x*-axis, uniaxial compression along the *y*-axis and biaxial compression.

Since the exact elasticity solution is not available for the buckling analysis of orthotropic plates, the present results are compared and discussed with the corresponding results generated by using HSDT of Reddy (1984), RPT developed by Kim *et al.* (2009), TSDT reported by Ghugal and Sayyad

Table 6 Comparison of non-dimensional critical buckling load $\overline{N}_{cr}$ of simply supported orthotropic square plates under biaxial compression ($\gamma_1 = -1,\ \gamma_2 = -1,\ m = n = 1$, material 2).

a/h	Theory	Model	E_1/E_2				
			3	10	20	30	40
5	Present	RPSDT	1.9793	3.1739	4.1984	4.8410	5.2892
	Sayyad and Ghugal (2014)	ESDT	1.9825	3.1507	4.0473	4.6083	5.0246
	Reddy (1984)	HSDT	1.9717	3.1036	3.9146	4.3711	4.6736
	Ghugal and Sayyad (2011)	TSDT	2.0281	3.1606	3.9662	4.4209	4.7251
	Kim *et al.* (2009)	RPT	—	3.1739	—	—	5.2895
	Mindlin (1951)	FSDT	1.9693	3.0902	3.8725	4.2924	4.5542
	Kirchhoff (1850)	CPT	2.7124	5.5814	9.6917	13.8034	17.9154
10	Present	RPSDT	2.4818	4.6866	7.2816	9.3862	11.1290
	Sayyad and Ghugal (2014)	ESDT	2.4806	4.6499	7.0402	8.8741	10.3380
	Reddy (1984)	HSDT	2.4784	4.6386	7.0002	8.7885	10.1929
	Ghugal and Sayyad (2011)	TSDT	2.5064	4.6823	7.0582	8.8558	10.2674
	Kim *et al.* (2009)	RPT	—	4.6866	—	—	11.1290
	Mindlin (1951)	FSDT	2.4781	4.6367	6.9910	8.7662	10.1522
	Kirchhoff (1850)	CPT	2.7124	5.5814	9.6917	13.8034	17.9154
20	Present	RHSDT	2.6508	5.3267	8.9490	12.3448	15.5343
	Sayyad and Ghugal (2014)	ESDT	2.6502	5.3125	8.8405	12.0731	15.0470
	Reddy (1984)	HSDT	2.6497	5.3101	8.8320	12.0540	15.0127
	Ghugal and Sayyad (2011)	TSDT	2.6597	5.3266	8.8574	12.0875	15.0537
	Kim *et al.* (2009)	RPT	—	5.3267	—	—	15.5345
	Mindlin (1951)	FSDT	2.6497	5.3100	8.8311	12.0513	15.0070
	Kirchhoff (1850)	CPT	2.7124	5.5814	9.6917	13.8034	17.9154
50	Present	RHSDT	2.7023	5.5390	9.5646	12.3448	17.4859
	Sayyad and Ghugal (2014)	ESDT	2.7022	5.5364	9.5437	13.4911	17.3791
	Reddy (1984)	HSDT	2.7020	5.5360	9.5424	13.4884	17.3744
	Ghugal and Sayyad (2011)	TSDT	2.7058	5.5407	9.5490	13.4969	17.3849
	Kim *et al.* (2009)	RPT	—	5.5390	—	—	17.4860
	Mindlin (1951)	FSDT	2.7023	5.5362	9.5425	13.4885	17.3745
	Kirchhoff (1850)	CPT	2.7124	5.5814	9.6917	13.8034	17.9154
100	Present	RPSDT	2.7098	5.5707	9.6596	13.7383	17.8060
	Sayyad and Ghugal (2014)	ESDT	2.7098	5.5700	9.6542	13.7238	17.7779
	Reddy (1984)	HSDT	2.7096	5.5697	9.6533	13.7230	17.7767
	Ghugal and Sayyad (2011)	TSDT	2.7124	5.5727	9.6571	13.7269	17.7811
	Kim *et al.* (2009)	RPT	—	5.5710	—	—	17.8060
	Mindlin (1951)	FSDT	2.7103	5.5710	9.6544	13.7241	17.7772
	Kirchhoff (1850)	CPT	2.7124	5.5814	9.6917	13.8034	17.9154

(2011), ESDT presented by Sayyad and Ghugal (2014), FSDT and CPT.

The results of the non-dimensional critical buckling load of simply supported orthotropic square plates subjected to the uniaxial and biaxial loading conditions for various values of both side-to-thickness ratio a/h and modular ratio E_1/E_2 are presented in Tables 5 and 6.

It can be noticed that the numerical results obtained by using the present theory are in excellent agreement with those computed according to the different shear deformation plate theories, especially with those presented by Kim *et al.* (2009). It may be noted that the results of the critical buckling load increase significantly with the increase of the modular ratios. Besides,FSDT underestimates the value of critical buckling loads, whereas the CPT overestimates for all thickness ratio and modular ratio due to the negligence of the transverse shear deformation effect.

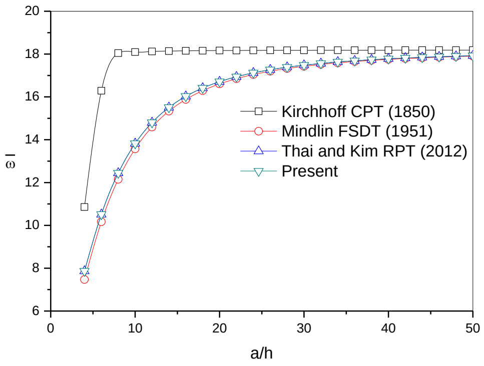

Fig. 3 The impact of side-to-thickness ratio on non-dimensional natural frequencies $\overline{\omega}$ of simply supported orthotropic rectangular plate with($E_1/E_2 = 40, a/b = 0.5$, material 2).

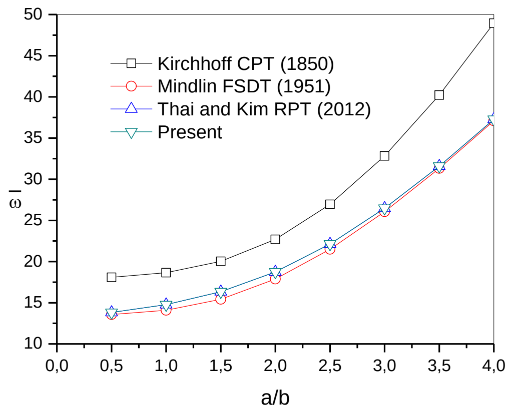

Fig. 4 The impact of geometric ratio on non-dimensional natural frequencies $\overline{\omega}$ of simply supported orthotropic rectangular plate with($a/h = 10, E_1/E_2 = 40$, material 2).

Tables 7–9 display the comparison of non-dimensional critical buckling load for simply supported thick orthotropic rectangular plates $(a/h = 5)$ for three modular ratio values ($E_1/E_2 = 10, 25, 40$) and various values of geometric ratio*b/a*. To confirm again the accuracy of this model, three loading conditions were considered in this analysis, uniaxial compression along *x*-axes, uniaxial compression along *y*-axes and biaxial compression, respectively.It can be seen that, the non-dimensional critical buckling load obtained by present refined theory (RPSDT) and Kim *et al.* (2009) is in excellent agreement for geometric ratio $(b/a = 1, 2)$ and with the analytical solutions provided by Sayyad and Ghugal (2014) for all geometric ratios. Besides, it should be noted that in the case of uniaxial compression along *x*-axes, the critical buckling load decreases with increasing value of geometric ratio, whereas it increases in the case of uniaxial compression along the *y*-axes.

Table 7 Comparison of non-dimensional critical buckling load $\overline{N}_{cr}$ of simply-supported orthotropic rectangular plates subjected to uniaxial compression along *x*-axes ($a/h = 5,\ \gamma_1 = -1,\ \gamma_2 = 0,\ m = n = 1$, material 2).

E_1/E_2	Theory	Model	*b/a* 1	1.5	2	2.5	3	3.5	4
10	Present	RPSDT	6.3478	5.3284	5.0109	4.8706	4.7961	4.7518	4.7232
	Sayyad and Ghugal (2014)	ESDT	6.3014	5.3026	5.0148	4.8939	4.8317	4.7953	4.7723
	Reddy (1984)	HSDT	6.2072	5.2245	4.9412	4.8223	4.7611	4.7253	4.7026
	Ghugal and Sayyad (2011)	TSDT	6.3212	5.2923	4.9940	4.8682	4.8033	4.7654	4.7412
	Kim *et al.* (2009)	RPT	6.3478	—	5.0109	—	—	—	—
	Mindlin (1951)	FSDT	6.1804	5.2025	4.9205	4.8021	4.7412	4.7056	4.6831
	Kirchhoff (1850)	CPT	11.163	9.3549	8.8428	8.6270	8.5154	8.4500	8.4083
25	Present	RPSDT	9.1039	7.9407	7.5409	7.3561	7.2557	7.1952	7.1559
	Sayyad and Ghugal (2014)	ESDT	8.7062	7.8373	7.6007	7.5047	7.4562	7.4281	7.4109
	Reddy (1984)	HSDT	8.3394	7.4929	7.2631	7.1701	7.1231	7.0961	7.0792
	Ghugal and Sayyad (2011)	TSDT	8.4398	7.5414	7.2929	7.1909	7.1391	7.1091	7.0905
	Kim *et al.* (2009)	RPT	9.1039	—	7.5409	—	—	—	—
	Mindlin (1951)	FSDT	8.2199	7.3805	7.1530	7.0610	7.0154	6.9883	6.9713
	Kirchhoff (1850)	CPT	23.495	21.690	21.179	20.964	20.854	20.783	20.744
40	Present	RPSDT	10.578	9.2342	8.7587	8.5368	8.4158	8.3426	8.2950
	Sayyad and Ghugal (2014)	ESDT	10.049	9.2310	9.0145	8.9282	8.8853	8.8608	8.8454
	Reddy (1984)	HSDT	9.3472	8.5541	8.3455	8.2628	8.2217	8.1983	8.1837
	Ghugal and Sayyad (2011)	TSDT	9.4502	8.6015	8.3719	8.2791	8.2324	8.2056	8.1888
	Kim *et al.* (2009)	RPT	10.579	—	8.7587	—	—	—	—
	Mindlin (1951)	FSDT	9.1084	8.3237	8.1178	8.0363	7.9958	7.9728	7.9585
	Kirchhoff (1850)	CPT	35.830	34.027	33.516	33.300	33.189	33.124	33.082

Table 8 Comparison of non-dimensional critical buckling load $\overline{N}_{cr}$ of simply supported orthotropic rectangular plates subjected to uniaxial compression along y-axes ($a/h=5,\ \gamma_1=0,\ \gamma_2=-1,\ m=n=1,$ material 2).

E_1/E_2	Theory	Model	b/a 1	1.5	2	2.5	3	3.5	4
10	Present	RPSDT	6.3478	11.989	20.044	30.441	43.165	58.210	75.572
	Sayyad and Ghugal (2014)	ESDT	6.3014	11.930	20.059	30.587	43.485	58.743	76.356
	Reddy (1984)	HSDT	6.2072	11.755	19.765	30.139	42.849	57.885	75.242
	Ghugal and Sayyad (2011)	TSDT	6.3212	11.907	19.975	30.426	43.229	58.375	75.859
	Kim *et al.*(2009)	RPT	6.3478	—	20.044	—	—	—	—
	Mindlin (1951)	FSDT	6.1804	11.705	19.682	30.013	42.670	57.644	74.929
	Kirchhoff (1850)	CPT	11.163	21.048	35.371	53.918	76.638	103.51	134.53
25	Present	RPSDT	9.1039	17.866	30.164	45.976	65.302	88.141	114.49
	Sayyad and Ghugal (2014)	ESDT	8.7062	17.634	30.403	46.904	67.107	90.999	118.57
	Reddy (1984)	HSDT	8.3394	16.859	29.052	44.813	64.110	86.931	113.27
	Ghugal and Sayyad (2011)	TSDT	8.4398	16.968	29.171	44.943	64.253	87.089	113.44
	Kim *et al.*(2009)	RPT	9.1039	—	30.164	—	—	—	—
	Mindlin (1951)	FSDT	8.2199	16.606	28.611	44.131	63.132	85.604	111.54
	Kirchhoff (1850)	CPT	23.495	48.803	84.716	131.02	187.66	254.63	331.92
40	Present	RPSDT	10.578	20.777	35.034	53.355	75.742	102.19	132.72
	Sayyad and Ghugal (2014)	ESDT	10.049	20.769	36.058	55.801	79.968	108.55	141.42
	Reddy (1984)	HSDT	9.3472	19.246	33.382	51.642	73.995	100.42	130.93
	Ghugal and Sayyad (2011)	TSDT	9.4502	19.353	33.487	51.744	74.092	100.52	131.02
	Kim *et al.*(2009)	RPT	10.579	—	35.034	—	—	—	—
	Mindlin (1951)	FSDT	9.1084	18.728	32.471	50.226	71.962	97.667	127.33
	Kirchhoff (1850)	CPT	35.830	76.560	134.06	208.12	298.69	405.76	529.31

Table 9 Comparison of non-dimensional critical buckling load $\overline{N}_{cr}$ of simply supported orthotropic rectangular plates subjected to biaxial compression ($a/h=5,\ \gamma_1=-1,\ \gamma_2=-1,\ m=n=1$, material 2).

E_1/E_2	Theory	Model	b/a 1	1.5	2	2.5	3	3.5	4
10	Present	RPSDT	3.1739	3.6889	4.0087	4.1988	4.3165	4.3932	4.4454
	Sayyad and Ghugal (2014)	ESDT	3.1507	3.6710	4.0118	4.2189	4.3485	4.4334	4.4915
	Reddy (1984)	HSDT	3.1036	3.6170	3.9530	4.1571	4.2849	4.3687	4.4260
	Ghugal and Sayyad (2011)	TSDT	3.1606	3.6639	3.9952	4.1967	4.3230	4.4057	4.4623
	Kim *et al.*(2009)	RPT	3.1739	—	4.0087	—	—	—	—
	Mindlin (1951)	FSDT	3.0902	3.6017	3.9364	4.1398	4.2671	4.3505	4.4076
	Kirchhoff (1850)	CPT	5.5814	6.4765	7.0743	7.4371	7.6638	7.8122	7.9137
25	Present	RPSDT	4.5519	5.4974	6.0327	6.3415	6.5302	6.6521	6.7349
	Sayyad and Ghugal (2014)	ESDT	4.3531	5.4258	6.0806	6.4696	6.7107	6.8678	6.9750
	Reddy (1984)	HSDT	4.1697	5.1874	5.8105	6.1811	6.4110	6.5609	6.6631
	Ghugal and Sayyad (2011)	TSDT	4.2199	5.2210	5.8343	6.1991	6.4253	6.5728	6.6734
	Kim *et al.*(2009)	RPT	4.5519	—	6.0327	—	—	—	—
	Mindlin (1951)	FSDT	4.1099	5.1096	5.7224	6.0870	6.3132	6.4607	6.5613
	Kirchhoff (1850)	CPT	11.747	15.016	16.943	18.072	18.767	19.217	19.524
40	Present	RPSDT	5.2892	6.3929	7.0069	7.3593	7.5742	7.7129	7.8070
	Sayyad and Ghugal (2014)	ESDT	5.0246	6.3907	7.2116	7.6967	7.9968	8.1920	8.3251
	Reddy (1984)	HSDT	4.6736	5.9221	6.6764	7.1231	7.3995	7.5796	7.7023
	Ghugal and Sayyad (2011)	TSDT	4.7251	5.9549	6.6975	7.1372	7.4092	7.5863	7.7071
	Kim *et al.*(2009)	RPT	5.2895	—	7.0069	—	—	—	—
	Mindlin (1951)	FSDT	4.5542	5.7626	6.4942	6.9278	7.1963	7.3711	7.4903
	Kirchhoff (1850)	CPT	17.915	23.557	26.813	28.707	29.870	30.623	31.136

4. Conclusions

In this article, a refined parabolic shear deformation theory (RPSDT) with only two unknowns is applied for free vibration and buckling analysis of simply supported orthotropic rectangular plates, in which the displacement field is modeled by an undetermined integral term in order to reduce the number of variables and the governing equations. The proposed theory accounts for a parabolic distribution of the transverse shear stress through the thickness direction and satisfies the edge boundary conditions of the plate, without needing a shear correction factor. The governing equations are obtained from the principle of virtual work and solved analytically by using Navier's solution procedure. Several parameters are considered in this study such as the modular ratio, the side-to-thickness ratio and the geometric ratio. The numerical results of natural frequencies and critical buckling load for simply supported orthotropic square and rectangular plates are obtained and compared with various theories available in the literature. Finally, it can be stated that the simple analytical model is not only more precise but also simple than the other shear deformation theories in predicting the free vibration and buckling responses of orthotropic rectangular plates. An improvement of the present formulation will be considered in the future work to consider other type of materials (Othman and Lotfy 2009, Abbas and Othman 2009, Kar and Panda 2015, Mehar *et al*. 2017, Akbas 2017ab, Katariya *et al*. 2017b, Faleh *et al*. 2018, Othman and Mahdy 2018, Panjehpour *et al*. 2018,Mehar *et al*. 2018 and 2019, Alimirzaei *et al*. 2019, Pandey *et al*. 2019, Mehar and Panda 2019, Ramteke *et al*. 2019, Kunche *et al*. 2019, Hussain *et al*. 2019, Balubaid *et al*. 2019, Rajabi and Mohammadimehr 2019, Karami *et al*. 2019ab, Mirjavadi *et al*. 2019a,Adda Bedia *et al*. 2019, Eltaher *et al*. 2019,Semmah *et al*. 2019, Draoui *et al*. 2019, Ebrahimi *et al*. 2019, Berghouti *et al*. 2019, Al-Osta 2019, Karami *et al*. 2019cd, Selmi 2019, Karami *et al*. 2019e and 2020, Eltaher and Wagih 2020, Khosravi *et al*. 2020, Hussain *et al*. 2020ab, Asghar *et al*. 2020, Bousahla *et al*. 2020).

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