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# The effect of accidental eccentricities on the inelastic torsional response of buildings

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**Abstract.** This paper investigates the influence of spatial varations of accidental mass eccentricities on the torsional response of inelastic multistorey reinforced concrete buildings. It complements recent studies on the elastic response of structural buildings and extends the investigation into the inelastic range, with the aim of providing guidelines for minimising the torsional response of structural buildings. Four spatial mass eccentricity configurations of common nine story buildings, along with their reversed mass eccentricities subjected to the Erzincan-1992 and Kobe-1995 ground motions were investigated, and the results are discussed in the context of the structural response of the no eccentricity models. It is demonstrated that when the initial linear response is practically translational, it is maintained into the inelastic phase of deformation as long as the strength assignment of the lateral resisting bents is based on a planar static analysis where the applied lateral loads simulate the first mode of vibration of the uncoupled structure.

**Keywords:** earthquake engineering; mass eccentricity; torsion; inelastic numerical modeling

## 1. Introduction

Earthquake induced torsion in structural buildings may arise as a result of (a) stiffness and/or mass eccentricities, (b) uncertainties in the stiffness of structural elements (owing to fabrication methods, ambiguities about the material properties and possible variability of element dimensions, etc) and uneven mass distribution (c) other reasons that are not explicitly accounted for in the design of the structure (stiffness of non-structural elements such as brick infill walls, non-symmetric yielding of the load resisting elements, etc) and d) differential ground motion arising during seismic wave passage and/or by ground motion incoherency. Stiffness and/or mass eccentricities affect the natural torsion and its effect is reflected in the coupling of the translational and rotational motions, which may be easily quantified. Uncertainties in the mass distribution and the stiffness of structural elements are of a probabilistic nature and their effects are usually referred to as accidental torsion. In particular, discrepancies between the mass, stiffness and strength distribution assumed in the analysis and their real distribution at the time of a ground excitation are accounted for by shifting the center of mass of each floor to a distance equal to $\pm\beta$b, where b is the floor dimension of the building normal to ground motion and $\beta$ is a coefficient specified by the national codes. This procedure has become an accepted practice in structural applications, but it is not an apparent assumption regarding the rotational components of ground excitations. The phase shift in the arrival of seismic waves at various locations on the ground surface which results in differential ground motion and the corresponding torsional ground component is less easily quantified.

Due to the inherent ambiguities in quantifying the rotational ground excitations and the associated challenge in introducing these effects into everyday structural design practices, a number of researchers have proposed equivalent accidental mass eccentricities to account for the effects of the torsional ground motion on the structure, in the same manner the other sources of accidental torsion (cases b and c) are accounted for. For example, De la Llera and Chopra (1994b), who used relative translational accelerations, at different locations at the base of the structure, to evaluate the rotational ground acceleration, concluded that the computed values of accidental eccentricity, $\beta$b, are much smaller than the proposed design code values of 0.05b-0.1b except for systems with long plan dimension (say b>50m). Shakib and Tohidi (2002) adopted a random procedure in the evaluation of the effects of the rotational component of earthquakes on the accidental eccentricity of symmetric and asymmetric single storey systems and concluded that in torsionally stiff buildings the coefficient $\beta$ is generally less than 0.05 for almost all values of the translational period of the system, but for short periods and flexible systems the proposed design codes values of $\beta$=0.05 are not sufficient in terms of design requirements. Falamarz-Sheikhabadi and Ghafory-Ashtiany (2012), worked on linear single story systems and concluded that the value of $\beta$=0.05 is mostly a conservative approximation for accidental eccentricity reflecting only the effect of the torsional component in asymmetric buildings. However, in another paper of Falamarz-Sheikhabadi (2014) it is demonstrated that the value of $\beta$=0.05 is not a conservative approximation,

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particularly in symmetric, torsionally stiff, multistory buildings. In a recent study on single storey linear and nonlinear systems reported by Basu *et al.* (2014) an alternative definition of the accidental eccentricity was proposed. This new definition applies to a torsional ground motion, which is the product of a translational ground motion and a factor that is a function of the proposed accidental eccentricity. It is interesting that in most design codes the coefficient $\beta$ is proposed to be in the range of 0.05-0.10, accounting for all the possible sources of accidental torsion.

The results of the analytical and simplified methodologies to account for the effects of accidental eccentricity on the torsional response of building structures reported in the literature generally suggest that accidental mass eccentricities may have a significant effect on the elastic torsional response of building structures (De La Llera & Chopra, 1994a, 1994c, 1995; De-La-Colina *et al.* 2013). Although the effect of accidental mass eccentricity on the elastic torsional response of structural buildings is relatively well researched, a limited number of studies have been reported on the effects of accidental mass eccentricities on the inelastic torsional response of buildings. Recent numerical analysis has shown, that the effect of accidental eccentricity may be less significant for the inelastic torsional design of building structures (Stathopoulos and Anagnostopoulos, 2005, 2010; Anagnostopoulos *et al.* 2015). Numerical modeling results on the effects of accidental eccentricity on the inelastic torsional response of building structures showed that the differences in ductility demand between structural buildings designed for different accidental eccentricities were found to be marginal for symmetric buildings and negligible for eccentric buildings (Stathopoulos and Anagnostopoulos, 2010). If damage indices instead of ductility factors were used to assess the inelastic response of symmetric as well as eccentric concrete frames, the effect of accidental eccentricity on the torsional response of structural buildings was insignificant, as it did not generally lead to lower damage indices compared to those for designs with zero accidental eccentricities (Stathopoulos and Anagnostopoulos, 2003). In an attempt to quantify the effect of earthquake induced torsion on mass-irregular building structures Stathi *et al*. (2015) proposed a ratio of torsion index (ROT). The accuracy of ROT was verified in non-linear analyses on plan irregular single and multistory buildings and it was demonstrated that reducing the ROT leads to a reduction of the torsion induced shear forces.

In recent years a number of investigations have been carried out, which demonstrate the seismic vulnerability of buildings due to unexpected torsional displacements and a significant amount of qualitative reviews have been published on this issue (Chandler *et al*. (1996), Rutenberg (1998), De Stefano and Pintucchi (2008), Anagnostopoulos *et al*. (2013)). From the point of view of the practicing engineer the problem of mitigating the torsional effects during a strong ground motion is a challenging and still open issue. Myslimaj and Tso (2002, 2004) proposed methodologies for minimising the torsional effect on single storey systems, and Aziminejad *et al*. (2008) and Aziminejad and Moghadam (2009) on multistorey systems suggesting a proper configuration of the centres of mass, strength and stiffness at different limit states with promising results. This approach, which is recently extended to include the soil-structure interaction (e.g. Shakib and Atefatdoost (2014)), is based on the findings of Paulay's studies (Paulay, 1998, 2001) that the stiffness of a structural element is strength dependent. However, the code provisions for the design of new buildings recommend that the element stiffnesses are strength independent (e.g. in EC8-2004, clause 4.3.1, the stiffness of concrete elements may be taken equal to 50% of the corresponding stiffness of the uncracked section). This provision implies that an optimum (minimum) torsional response may be obtained with a suitable arrangement of the lateral load resisting elements. For example, in the case of single storey systems, when the center of mass (CM) and the center of stiffness (CS) of the floor slab are coicident, any inertia force applied at the CM causes only a translation of the slab. In multistorey buildings, when the CM of all the floor slabs are located on the same vertical line, a minimum torsional response may be attained when this line coincides with the optimum torsion axis (OTA), as defined by Georgoussis (Georgoussis, 2010, 2014, 2015, 2016, 2018). This axis can be either determined using the approximate method of the continuous medium, which defines the OTA as the vertical axis passing through the center of rigidity (m-CR) of an equivalent (modal) single story system (Georgoussis, 2010, 2014, 2015, 2016), or alternatively (Georgoussis 2018), using the discrete element approach (stiffness method). In practice, where the arrangement of most of the lateral load resisting elements is determined by architectural or functional considerations, the structural engineer needs to relocate only one of the resisting element (key element) in order to obtain a structural configuration, where the OTA coincides with the mass axis. The issue of designing structural buildings to sustain minimum torsion by considering height wise accidental mass eccentricities has been investigated in a recent authors' paper (Georgoussis and Mamou, 2018; Georgoussis *et al.* 2019). The systems analyzed were common multistorey building with lateral load resisting systems comprising frames, shear walls and coupled wall bents and it was demonstrated that in the linear phase of deformation, for any height wise mass eccentricity variation, a structural configuration of minimum torsional response may be obtained by a suitable relocation of the key element. It was also demonstrated that by reversing the spatial distribution of floor masses, with respect to their nominal locations (mass axis passing through the centroids of floor slabs), the required relocation of the key element is shifted to a symmetrical position with respect to its nominal location (determined when all floor masses are assumed to lie on the same vertical axis). It has been shown that small shifts of the key element from its optimum location result in rather large torsional distortions, but these effects gradually become less significant as the key element moves further away from its optimum location.

The aim of this paper was to investigate the significance of mass eccentricity effects on the response of building structures when they are pushed beyond their elastic limits

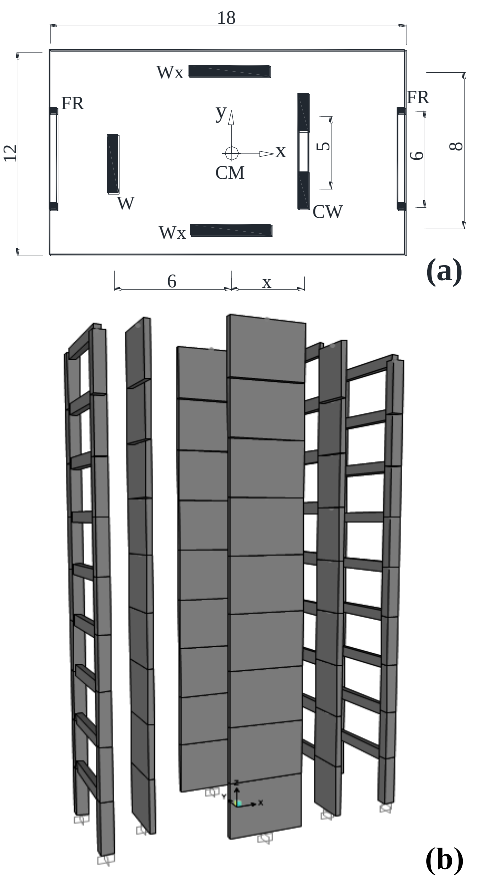


Fig. 1 Plan configuration of example model structure (all dimensions in meters); (b) perspective view (SAP2000 model)

during a strong ground motion. It complements the aforementioned paper and extends the research to the inelastic behavior of buildings structures. The main objective of this paper is to provide guidelines for designing structural buildings to sustain minimum torsion, as it is the main concern of practicing engineers. A parametric study is presented on 9-story common building types having a mixed-type lateral load resisting system (frames, walls, coupled wall bents) and representative heightwise variations of accidental eccentricities. Their response is investigated under the Erzincan-1992 and Kobe-1995 ground motions.

## 2. Methodology

### *2.1 Case study*

The plan view of the nine storey mono-symmetric reinforced concrete buildings investigated in this research is illustrated in Fig.1(a). The investigated building structure is a uniform over the height concrete building (Fig. 1(b)), analyzed using the SAP2000 numerical modeling software. All floor nodes were constrained by the diaphragmatic action of the slab, and the building was designed to resist a horizontal force, $V_d$, equal to 20% of its total weight, $W_{tot}$. The story height was taken equal to 3.5m and the total mass per floor was $m$=154 kNs$^2$/m uniformly distributed over the floor slab (assuming a total gravity load density of 7 kN/m$^2$). The radius of gyration about the center of mass (CM) was $r$=6.245m, the concrete Young's Modulus was equal to 40GPa and the flexural stiffness of all elements was equal to 50% of the corresponding stiffness of the uncracked section. It has a typical wall-frame dual system along the y-direction and a wall system in the x-direction. The lateral resistance along the x-axis is provided by a pair of 35/450cm flexural shear walls (Wx), located symmetrically to the axis of symmetry at ±4m distances as shown in Fig. 1(a) and the lateral resistance along the y-direction was provided by a 35/350cm flexural shear wall (W), a coupled wall bent (CW), composed of two 35/250cm walls set at 5m distance and connected by 30/80cm lintel beams at the floor levels and finally by two moment resisting frames (FR), comprising two 70/70cm columns set apart at 6m distance and connected by 40/70cm beams. The two FR frames are located at the edges of the floor plan and the flexural wall W is located on the left of the CM at a 6m distance (Fig. 1(a)). All the aforementioned lateral load resisting elements were assumed to have only in-plane stiffness. In order to assess the optimum location of the coupled wall bent (CW), assumed to be the key element, for which the torsional response of the structure was minimized when it was subjected to a translational ground motion along the y-direction, the CW bent was shifted to all possible locations between -7.5m to +7.5m, along the x-axis. As the objective of this paper was to investigate the response of inelastic systems, a strength assignment for all the lateral load resisting bents was required. The strength distribution of the various bents was based on static analysis of the symmetrical counterpart building. That is, when all the floors of the assumed model are restrained against any rotation. This assumption is associated with the fact that minimizing the torsional response of a structural building requires a strength assignment compatible with a practically translational response. It is worth mentioning here that when the coupled wall bent CW 'moves' to coordinates higher than +2m, both the criteria of EC8-2004 (Clause 4.2.3.2, Eqs. (4.1a), (4.1b)) for in plan regularity are satisfied and therefore a planar static analysis is permitted. The non-linear properties of the building are based on the capacity design assumption (strong column-weak beam model), which suggests that the potential locations of plastic hinges are at the end sections of beams and at the bases of columns and walls. A static linear analysis under an external lateral load with floor forces having the shape of the 'inverted triangle' (EC8-2004) and assuming a base shear equal to $V_d$=0.2$W_{tot}$=2721.6 kN, was performed. The storey shear distribution (shown as percentages of $V_d$) for each of the frames FR is shown in Fig. 2(a) and it is noticeable that these forces vary slightly and that the shear forces at the midpoints of the coupling beams (which are also the points of contraflexure) are not significantly different across the second to eighth floor (Fig. 2(b)).

For this reason, the maximum shear force, shown in the fourth floor (equal to 167 kN), was used to evaluate the bending (yield) capacity of all beams, at the faces of the columns. This capacity is equal to 167*(3-0.35)=442.5kNm. The gravity load distribution in the beams, was taken as 35kN/m, which means that the corresponding end moments at the column faces were less than 70 kNm. Such bending moments are less than 20% of the assumed bending capacity and within the code limits of permissible redistribution.

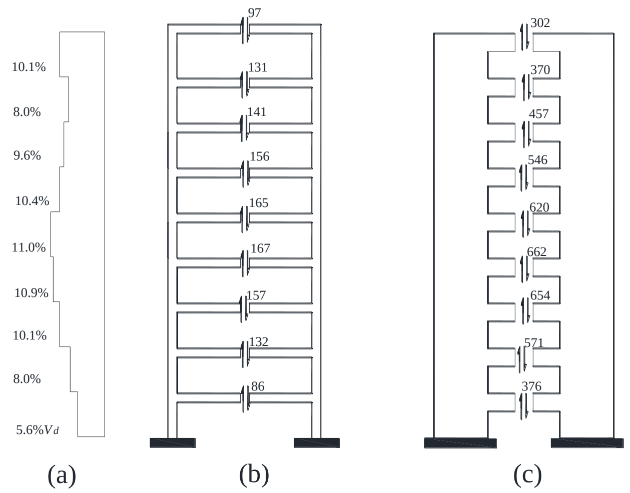


Fig. 2 Distribution of storey shears sustained by each FR frame of the exampled structure designed to withstand an xternal loading equal to $V_d$; (b) shear forces in the connecting beams of FR (in kN) and; (c) CW bent.

The aforementioned static analysis provides also an estimate of the required bending (yield) capacity at the column bases, which was found to be 277 kNm (neglecting the effect of the axial load). Taking into account the shear forces in the connecting beams of the CW bent (Fig. 2(c)), which show a small variation from the second to the seventh floor, their bending capacity was taken equal to 827.5(=662*1.25)kNm across all levels. The required bending capacity at the wall bases of the CW bent, as derived from the aforementioned static analysis, was 4719 kNm, while the corresponding bending capacities of the shear walls W and Wx, were 11895 and 30027 kNm respectively (the effect of axial loading on these vertical members was not taken into account). At the locations of plastic hinges the moment-rotation relationship was assumed to be bilinear, with a post-yielding stiffness ratio equal to 1% and the maximum plastic rotation capacity was $\theta_p$=0.015rads at the ultimate bending moment.

The elevation of the four mass eccentricity cases investigated in this research are shown in Fig. 3. The first three cases are representative of the uncertainties in the distribution of mass and/or stiffness, while the fourth case (Fig. 3(d)) may be regarded as an equivalent result of a ground rotational excitation, as all the floor mass eccentricities have the same algebraic sign. For each case, the corresponding building model is labelled as *MassEc*(+) and its response is compared with (i) the response of the model (labelled *MassEc*(-)) in which all mass eccentricities are reversed and (ii) the model (labelled *NoEc*) in which all the centres of floor masses are aligned along the vertical line passing through the centroids of the floors. The latter model may be regarded as the reference model as it is the same for all cases. In all the investigated models, it was assumed that the storey masses were lumped and equal to $m$ with a polar moment of inertia equal to $mr^2$(De la Llera and Chopra 1994a).

### 2.2 Numerical modeling

The numerical modeling was performed with the structural analysis program SAP2000-V16 using the 3D frame template and assuming end (length) offsets in the coupling beams of the CW bent. All the joints on each floor were constrained by the diaphragmatic action of the slab. The numerical analysis was performed for the Kobe 1995 (component KJM000) and Erzincan 1992 (EW component) ground excitations (Fig. 4), acting along the y-direction and scaled to PGA=0.5g. These time history data were obtained from the peer ground motion

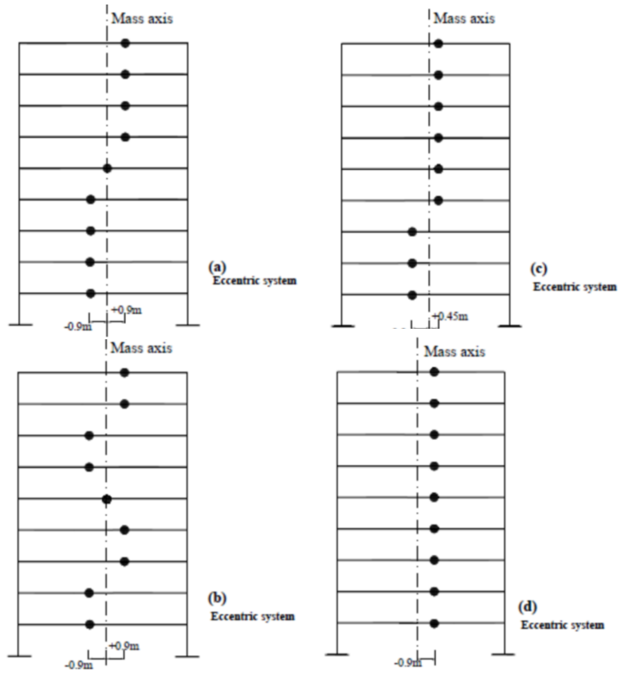


Fig. 3 Elevation of the investigated mass eccentricity cases (a), (b), (c) and (d)

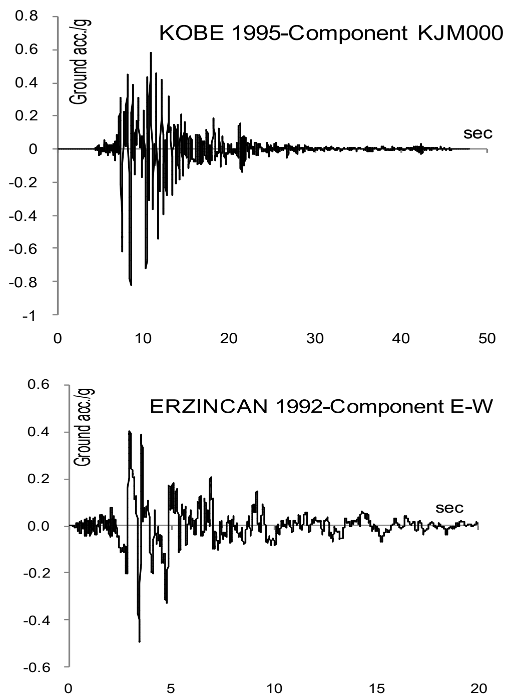


Fig. 4 Ground excitation for the Kobe 1995 (component KJM000) and Erzincan 1992 (EW component) as obtained from the peer ground motion database recorded by the Pacific Earthquake Engineering Research Center.

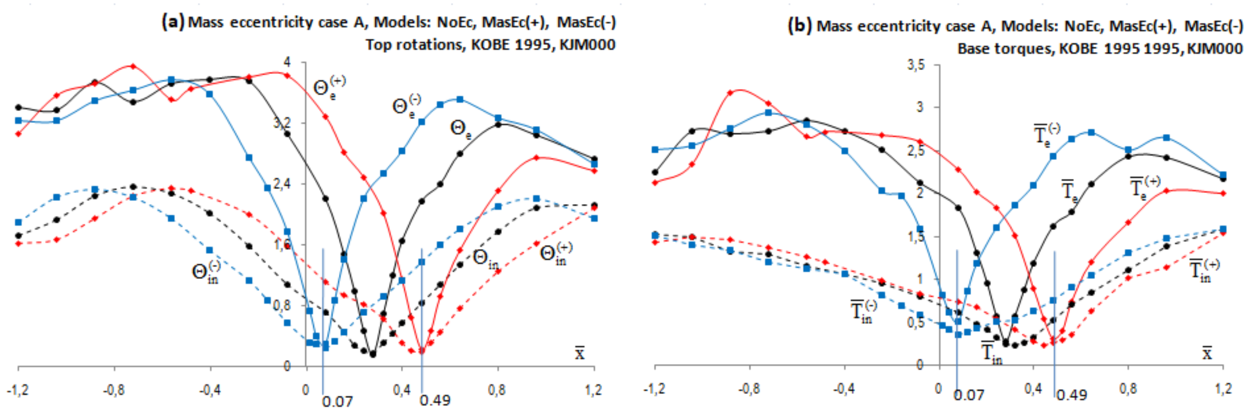


Fig. 5.1 Mass eccentricity case A: Top rotations Θ (x10$^{-2}$ rads), and normalized base torques $\overline{\mathrm{T}}$, of *NoEc* (black lines), *MassEc(+)* (red lines) and *MassEc(-)* (blue lines) models responding as elastic (subscript 'e') and inelastic (subscript 'in') systems under the Kobe 1995 (KJM000) ground excitation.

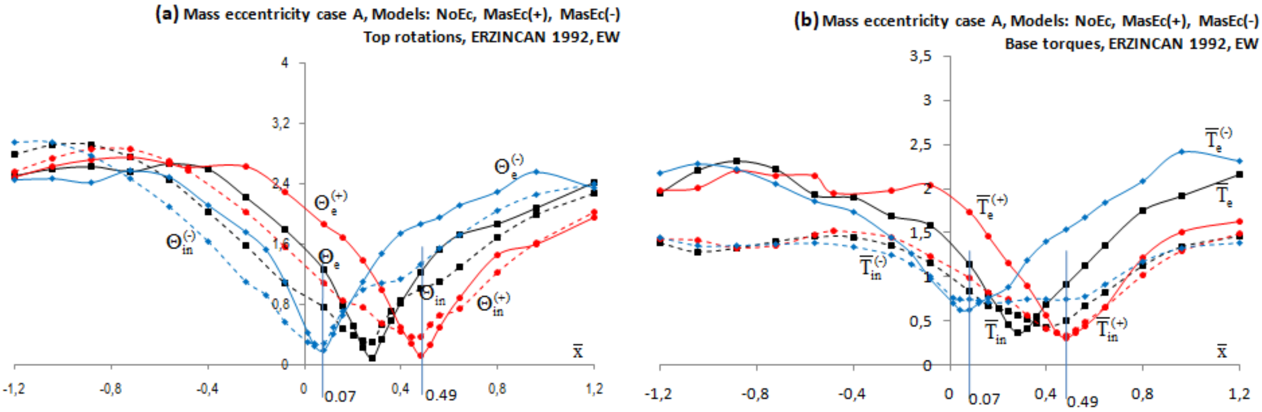


Fig. 5.2 Mass eccentricity case A: Top rotations Θ (x10$^{-2}$ rads), and normalized base torques $\overline{\mathrm{T}}$, of *NoEc* (black lines), *MassEc(+)* (red lines) and *MassEc(-)* (blue lines) models responding as elastic (subscript 'e') and inelastic (subscript 'in') systems under the Erzincan 1992 (EW) ground excitation

database recorded by the Pacific Earthquake Engineering Research Center. Based on the results reported by Stathopoulos & Anagnostopoulos (2003) that ground motions with different characteristics cause similar inelastic response, the two ground motion history data investigated were deemed sufficient to study the influence of accidental eccentricity on the torsional response of the investigated buildings. The non-linear analysis involved a direct integration history analysis using the Wilson time integration method with the theta parameter set equal to 1.4 and the damping matrix was assumed to be stiffness and mass proportional (the damping ratio was taken equal to 5% for the first and third coupled periods of vibration for each specific location of the coupled wall bent CW). Appendix A shows a typical variation of the first and third periods of vibration for the mass eccentricity case (b) (for both models MassEc(+) and MassEc(-)) in relation to the assumed normalized locations, $\overline{x}$ (=x/$r$), of the coupled wall bent. As stated above, prior to the application of the assumed ground motion, the effect of a gravity loading equal to 35 kN/m acting on the beams of the FR and CW bents was first considered. It was also assumed that the rest of the gravity loading is sustained by columns (not shown in Fig. 1(a)), which do not contribute to the lateral resistance of the system.

## 3. Discussion of results

Figures 5.1 and 5.2 present the response of models *MassEc*(+) (in red lines) and MassEc(-) (in blue lines) for the eccentricity case shown in Fig. 3(a), together with the response of the reference model NoEc (in black lines). The values along the x-axis in Fig. 5.1 and 5.2 represent the various locations of the coupled wall bent in a normalized form ($\overline{x} = x/r$). The diagrams of Fig. 5.1 present the rotations at the top floor, Θ, and the normalized base torques, $\overline{\mathrm{T}} = T/rV_d$, sustained under the unidirectional ground motion of Kobe 1995 (component KJM000) and similar are the diagrams of Fig. 5.2 obtained under the Erzincan 1992 (EW component) ground excitation. The elastic torsional response of the structure is denoted by the subscript 'e' (in solid lines), while the inelastic response is denoted by the subscript 'in' (in dotted lines). A similar logic applies to the results presented in Figs. 6.1-2, 7.1-2 and 8.1-2, which show the response of models, *MassEc*(+), *MassEc*(-) and *NoEc* for the eccentricity cases of Figs. 3(b) to (d). The results presented in the aforementioned figures show that the elastic torsional response curves (Θ and $\overline{\mathrm{T}}$) of all the mass eccentricities models considered, are of a minimum value

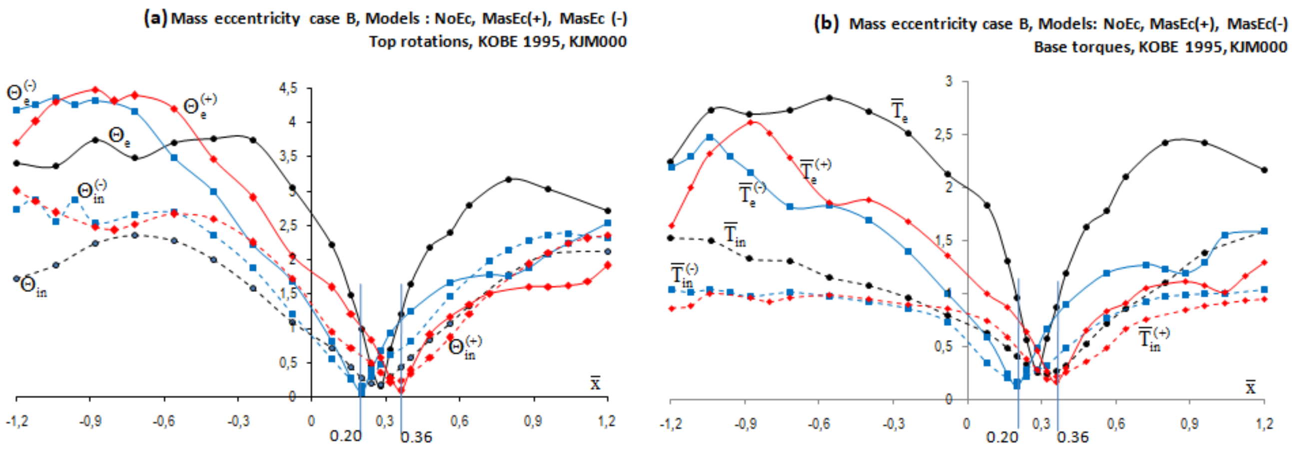


Fig. 6.1 Mass eccentricity case B: Top rotations Θ (x10$^{-2}$ rads), and normalized base torques $\overline{T}$, of *NoEc* (black lines), *MassEc(+)* (red lines) and *MassEc(-)* (blue lines) models responding as elastic (subscript 'e') and inelastic (subscript ' in') systems under the Kobe 1995 (KJM000) ground excitation.

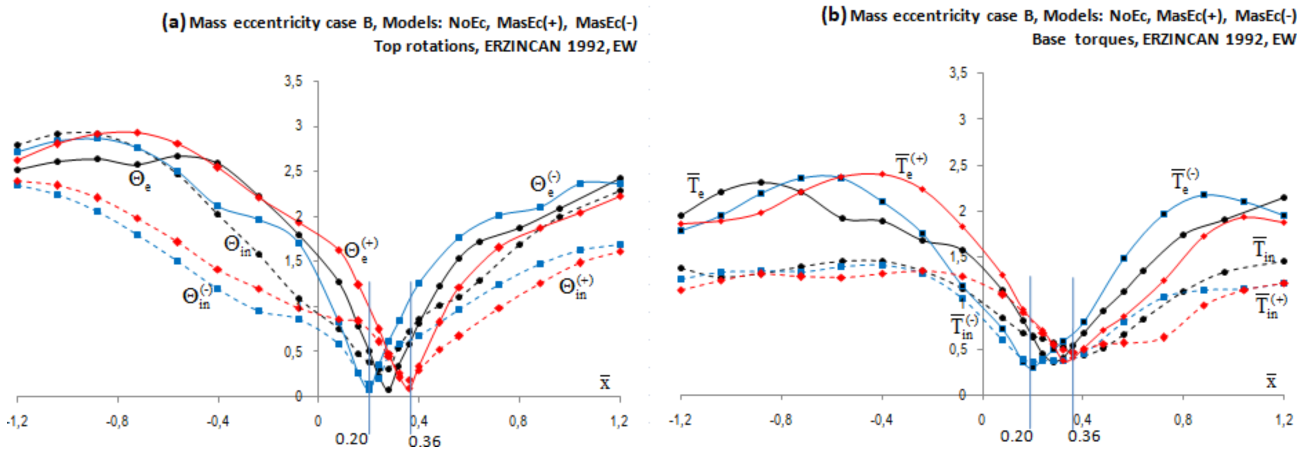


Fig. 6.2 Mass eccentricity case B: Top rotations Θ (x10$^{-2}$ rads), and normalized base torques $\overline{T}$, of *NoEc* (black lines), *MassEc(+)* (red lines) and *MassEc(-)* (blue lines) models responding as elastic (subscript 'e') and inelastic (subscript ' in') systems under the Erzincan 1992 (EW) ground excitation

when the coupled wall bent CW is positioned at a specific (optimum) location, as indicated by their inverted peak. These locations are almost coincident with the locations predicted by the analytical solution presented by Georgoussis and Mamou (2018), which are shown in Table 1. In this Table the optimum locations of the CW are denoted as $\overline{x}(+)$ or $\overline{x}(-)$ for the model *MassEc*(+) (or *MassEc*(-)) for each of the eccentricity cases of Figs. 3(a) to (d) and the corresponding values are also shown, by separate vertical lines, in Figs. 5 to 8. As demonstrated in this paper, the predicted optimum locations of the CW, for the eccentricity cases of Figs. 3(a) to (d), are pointing to almost symmetrical locations, with respect to the optimum location of the CW bent (for example $\overline{x}(0)$, when no accidental mass eccentricities are accounted for (this nominal coordinate of the CW is depicted by the inverted peak of the black lines of Model *NoEc*). It is evident that the elastic response of the structure is generally sensitive to small shifts of the coupled wall bent (from its optimum location), but with larger shifts, this response become smoother and eventually even flattens out.

The elastic torsional response illustrated in Figs. 5-8 is in agreement with results presented by Chandler & Hutchinson (1986), who reported that the torsional response of the investigated building structure increased rapidly with increasing mass eccentricity, and that the increase in the torsional response was more significant at small eccentricities. The inelastic torisonal response of the investigated mass eccentricities configurations, was generally smoother than the elastic response, with the inelastic torsional response curves exhibiting a more extended range of possible locations of the couple wall bent for which the torsional response of the structure was minimised. The results presented in Figs. 5-8 suggest that the inelastic torsional response of the structure was generally less sensitive to spatial variations of the coupled wall, confirming reports by Stathopoulos & Anagnostopoulos (2010) that the effects of accidental eccentricity on the inelastic torsional response of building structures may be insignificant. The results of Figs. 5-7 suggest that the variation of the inelastic response in terms of base torques, in the range of normalized coordinates of the coupled wall bent between $\overline{x}(+)$ to $\overline{x}(-)$ was insignificant and practically any location of the CW within this interval may potentially be an optimum location of the coupled wall bent. The $\overline{x}(+)$ to $\overline{x}(-)$ interval is rather extended for the eccentricity case of Fig. 3(d), which

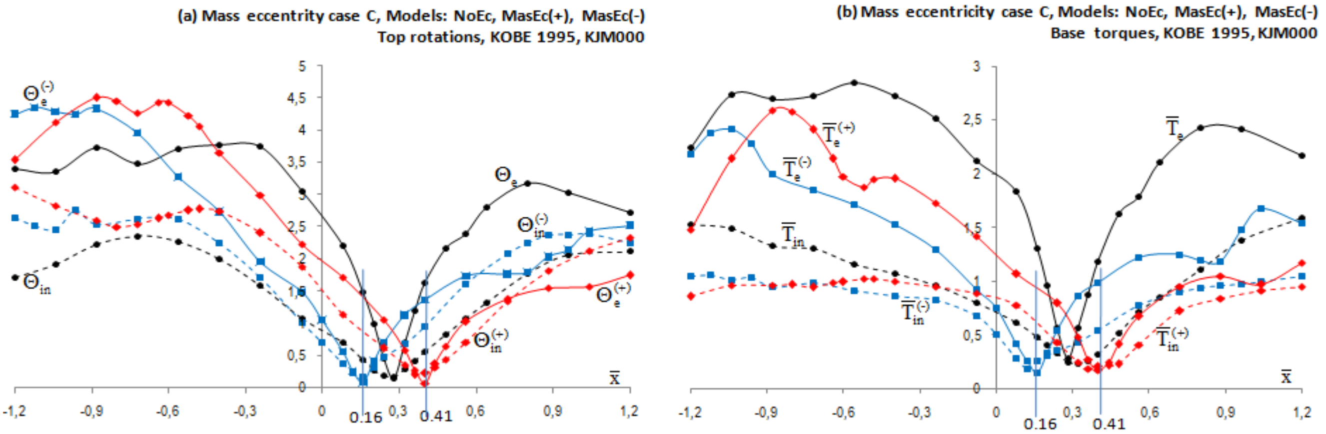


Fig. 7.1 Mass eccentricity case C: Top rotations Θ ($x10^{-2}$ rads), and normalized base torques $\overline{T}$, of *NoEc* (black lines), *MassEc(+)* (red lines) and *MassEc(-)* (blue lines) models responding as elastic (subscript 'e') and inelastic (subscript ' in') systems under the Kobe 1995 (KJM000) ground excitation.

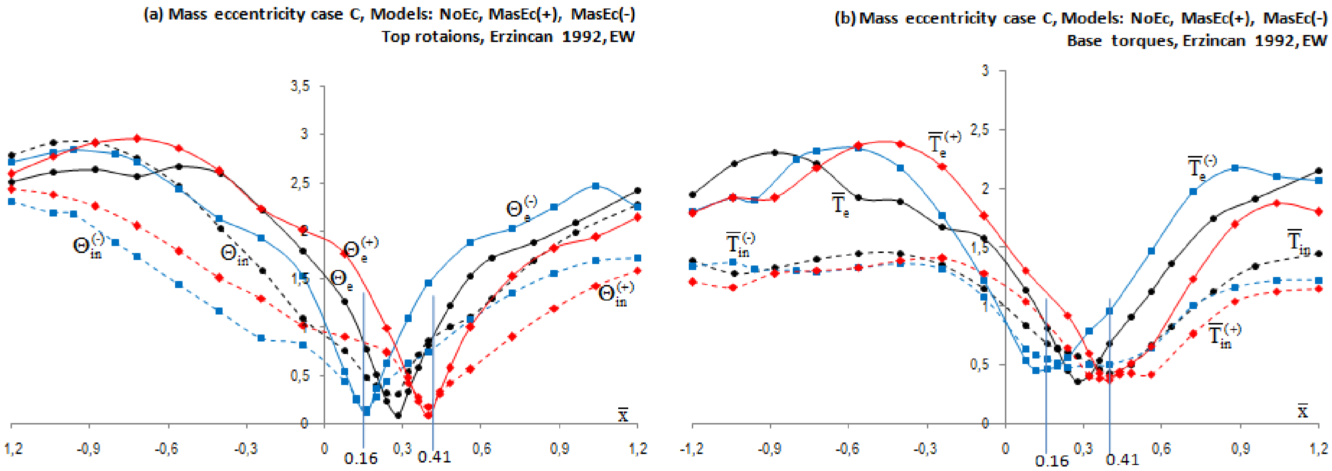


Fig. 7.2 Mass eccentricity case C: Top rotations Θ ($x10^{-2}$ rads), and normalized base torques $\overline{T}$, of *NoEc* (black line s), *MassEc(+)* (red lines) and *MassEc(-)* (blue lines) models responding as elastic (subscript 'e') and inelastic (subscrip t 'in') systems under the Erzincan 1992 (EW) ground excitation.

simulates a rotational ground excitation. It extends from 0.02 to 0.55 and suggests that the effects of the rotational component of a ground excitation are the most demanding, in terms of shifting the location of the key element in order to achieve a practically translational response of the building structure. In any case, at the locations of the coupled wall bent CW for which the initial elastic torsional response of the structure was minimized, the post elastic behavior of the structure maintained this response, which may be interpreted as follows: when the elastic behavior is practically translational, the effective seismic forces acting on a medium or low height structure are basically proportional to the first translational mode of vibration. Therefore, a strength assignment obtained from a planar static analysis under a set of lateral loads simulating the aforementioned mode of vibration, represents a system in which all potential plastic hinges at the critical sections are formed at approximately the same time. As a result the system is further pushed into the inelastic region in a translational mode. In other words, the almost concurrent yielding of the most stressed potential plastic hinges of all the bents in the direction of the ground motion, maintains the translational response, attained at the end of the elastic phase, into the inelastic phase.

This response is in agreement with the observations of Lucchini *et al.* (2009) in single story buildings where it is concluded: their nonlinear response depends on how the building enters the nonlinear range, which in turn depends on its elastic properties (i.e. the stiffness and mass distributions), and on the capacities of its resisting elements (i.e. the strength distribution). This response is also in agreement with the observation by Myslimaj and Tso (2002) and Tso and Myslimaj (2003), who reported that the torsional reponse increases when one structural element yields, while the other element is still in the elastic range, or when one element unloads, while the other element remains in the yield plateau. This implies that during the response, the elements would yield at approximately the same time, having similar yield durations and that they would unload at a similar time.

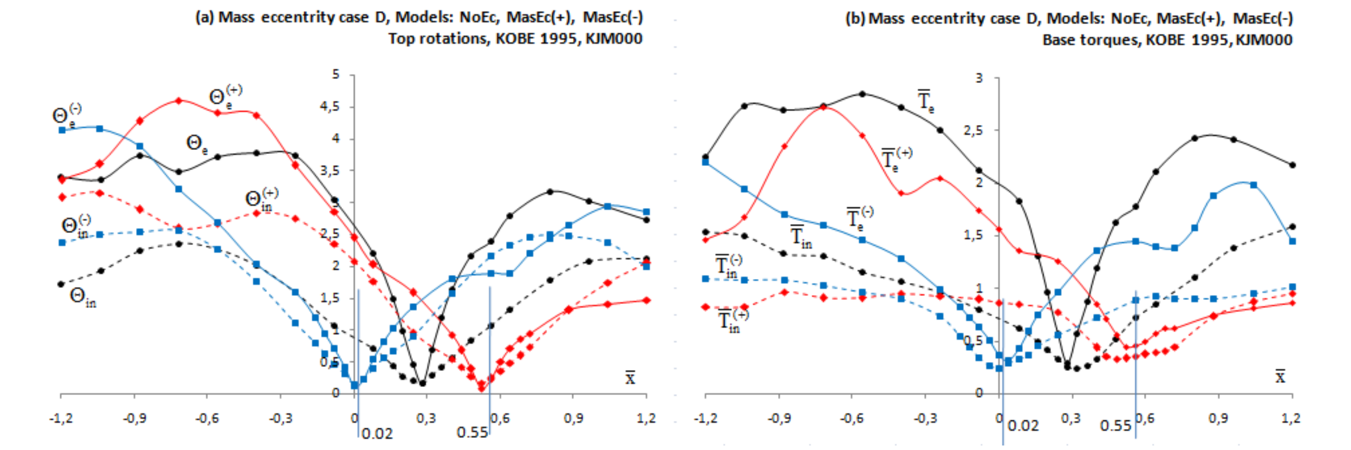


Fig. 8.1 Mass eccentricity case D: Top rotations Θ (x10$^{-2}$ rads), and normalized base torques $\overline{T}$, of *NoEc* (black lines), *MassEc(+)* (red lines) and *MassEc(-)* (blue lines) models responding as elastic (subscript 'e') and inelastic (subscript 'in') systems under the Kobe 1995 (KJM000) ground excitation.

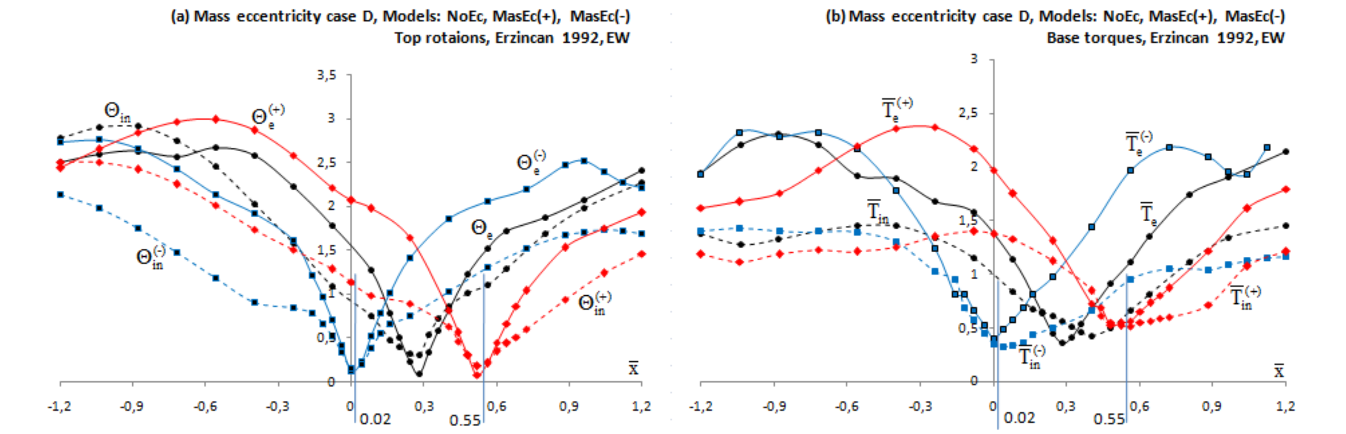


Fig. 8.2 Mass eccentricity case D: Top rotations Θ (x10$^{-2}$ rads), and normalized base torques $\overline{T}$, of *NoEc* (black lines), *MassEc(+)* (red lines) and *MassEc(-)* (blue lines) models responding as elastic (subscript 'e') and inelastic (subscript 'in') systems under the Erzincan 1992 (EW) ground excitation

## 4. Conclusions

This paper investigates the effect of spatial variations of mass eccentricities on the torsional response of medium height multi-storey asymmetric buildings subjected to two mono-directional ground excitations. Based on the numerical modelling results for the specific lateral resisting system investigated comprising frames, shear walls and coupled walls the main conclusions are presented bellow. Further analysis would be required to investigate whether the conclusions drawn for the specific lateral load resisting system can also be verified for bi-directional ground excitations and for different lateral load resisting systems.

- The elastic torsional response of the investigated mass eccentricity cases, indicate an optimum location of the key element (CW bent) for which the torsional response is minimised. It is shown that the elastic response of the structure is generally sensitive to small shifts of the CW bent, from the location of where the torsional response is minimised. With larger shifts of the key element, the torsional response curves become smoother and eventually even flatten out.
- The location of the inverted peaks, where the elastic torsional response is minimised, is predicted with sufficient accuracy by the analytical solution proposed by Georgoussis and Mamou (2018).
- The inelastic torsional response was generally smoother than the elastic response, with the inelastic results indicating an extended range of possible locations of the couple wall bent, for which insignificant or small variations in the torsional response of the structure occurred.
- Reversing the accidental eccentricities, shifted the elastic torsional response curves to approximately symmetric locations with respect to the no eccentricity reference torsional response curves. A similar observation can be made for the overall 'relocation' of the smoother response curves of the inelastic models.
- The variation of the inelastic response in terms of base torques, in the range of normalized coordinates of the key element from $\overline{x}(+)$, which is its optimum location

Table 1 Predicted optimum normalized locations of the key element (CW bent) in models *MassEc(+)* and *MassEc(-)* for the mass eccentricity cases shown in Figs. 3(a)-(d)

| Mass eccentricity case | Models | Optimum locations of CW |
|---|---|---|
| Fig.3(a) | *MassEc(+)* | $\overline{x}(+) = 0.49$ |
| Reversed case | *MassEc(-)* | $\overline{x}(-) = 0.07$ |
| Fig.3(b) | *MassEc(+)* | $\overline{x}(+) = 0.36$ |
| Reversed case | *MassEc(-)* | $\overline{x}(-) = 0.20$ |
| Fig.3(c) | *MassEc(+)* | $\overline{x}(+) = 0.41$ |
| Reversed case | *MassEc(-)* | $\overline{x}(-) = 0.16$ |
| Fig.3(d) | *MassEc(+)* | $\overline{x}(+) = 0.55$ |
| Reversed case | *MassEc(-)* | $\overline{x}(-) = 0.02$ |

under a given mass eccentricity distribution, to $\overline{x}(-)$, which shows its corresponding optimum location for the reversed eccentricity distribution, was insignificant and practically any location of the key element within this interval may potentially be an optimum location of the coupled wall bent.

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## APPENDIX A

Table 1A. First and third periods of vibration for the mass eccentricity case (b) shown in Fig. 3, for both models MassEc(+) and (MassEc(-), for the various normalised locations, $\bar{x}$, of the coupled wall bent.

| Mass Eccentricity Case (b) | | | | |
|---|---|---|---|---|
| | Model MassEc(+) | | Model MassEc(-) | |
| $\bar{x}$ | T1(s) | T3(s) | T1(s) | T3(s) |
| -1,201 | 1,373 | 0,374 | 1,325 | 0,348 |
| -1,041 | 1,344 | 0,365 | 1,295 | 0,340 |
| -0,881 | 1,309 | 0,355 | 1,260 | 0,330 |
| -0,721 | 1,270 | 0,343 | 1,222 | 0,319 |
| -0,560 | 1,227 | 0,331 | 1,179 | 0,307 |
| -0,400 | 1,180 | 0,317 | 1,133 | 0,294 |
| -0,240 | 1,129 | 0,302 | 1,084 | 0,280 |
| -0,080 | 1,076 | 0,287 | 1,033 | 0,267 |
| 0,080 | 1,023 | 0,273 | 0,982 | 0,253 |
| 0,160 | 0,996 | 0,266 | 0,957 | 0,247 |
| 0,240 | 0,971 | 0,259 | 0,958 | 0,242 |
| 0,280 | 0,960 | 0,256 | 0,965 | 0,240 |
| 0,320 | 0,952 | 0,253 | 0,973 | 0,238 |
| 0,400 | 0,952 | 0,247 | 0,989 | 0,236 |
| 0,560 | 0,978 | 0,239 | 1,019 | 0,238 |
| 0,721 | 1,005 | 0,237 | 1,046 | 0,242 |
| 0,881 | 1,030 | 0,239 | 1,072 | 0,247 |
| 1,041 | 1,054 | 0,242 | 1,096 | 0,251 |
| 1,201 | 1,076 | 0,246 | 1,117 | 0,256 |