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# Minimum cost strengthening of existing masonry arch railway bridges

Amin Rafiee*

*Department of Civil Engineering, University of Maragheh, Maragheh, Iran*

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**Abstract.** The preservation of historic masonry-arch railway bridges is of paramount importance due to their economic benefits. These bridges which belong to past centuries may nowadays be expected to carry loads higher than those for which they were designed. Such an increase in loads may be because of increase in transportation speed or in the capacity of freight-wagons. Anyway, adequate increase in their load-carrying-capacity through structural-strengthening is required. Moreover, the increasing costs of material/construction urge engineers to optimize their designs to obtain the minimum-cost one. This paper proposes a novel numerical optimization method to minimize the costs associated with strengthening of masonry-arch railway bridges. To do so, the stress/displacement responses of Sahand-Goltappeh bridge are assessed under ordinary train pass as a case study. For this aim, 3D-Finite-Element-Model is created and calibrated using experimental test results. Then, it is strengthened such that following goals are achieved simultaneously: (1) the load-carrying-capacity of the bridge is increased; (2) the structural response of the bridge is reduced to a certain limit; and, (3) the costs needed for such strengthening are minimized as far as possible. The results of the case study demonstrate the applicability/superiority of the proposed approach. Some economic measures are also recommended to further reduce the total strengthening cost.

**Keywords:** numerical optimization; arch bridge; cost minimization; railway network; structural strengthening; train speed

## 1. Introduction

On contrary to their oldness, masonry arch bridges are nowadays playing an important role in the railway transportation network in many countries. For instance, in Europe, masonry arch bridges compose almost 40% of the in-service railway bridge infrastructure (Bieñ *et al*. 2007). Thereby, the preservation of these bridges is of paramount importance due to their economic benefits and their role in the daily life of the majority of the population. These historic bridges which belong to past centuries may nowadays be expected to carry loads higher than those for which they were designed. This increase in loads might be required for different reasons. It may be either due to increase in transportation speed or because of addition in the capacity of freight wagons. In both cases, the train loads applied to the track system are increased in each axle. Anyway, adequate increase in the load-carrying-capacity of these bridges through structural strengthening is required. To this end, it is imperative to assure whether these structures present the required levels of safety and/or strength or if it is necessary to perform strengthening /replacement measures (Moreira *et al*. 2016). At the moment, not only most of ancient masonry arch bridges are expected to carry loads which are higher than those they were built for, but also the lack of sufficient maintenance and the degradation of structural material induced by time and utilization, have weakened them (Moreira *et al*. 2016). Therefore, in such a condition, to achieve a sufficient safety level, the goal of a structural strengthening procedure for such antique bridges may be not only to prevent the excessive increase in their structural response under increased loads, but also to reduce the response to a certain level.

On the other hand, the available funds for maintenance and repairs are limited. Therefore, the increasing costs of structural material and construction urge structural engineers to optimize their designs. In this way, the so-called minimum cost design can be obtained. It is also necessary to accurately assess the safety/strength level of the existing railway bridges, and if it is needed, adequate costly effective strengthening measures are applied. In particular, the investigations on the assessment of masonry bridges are notable (Altunisik *et al*. 2015a,b, Cakir and Seker 2015, Rovithis and Pitilakis 2016, Sayin 2016, Breccolotti *et al*. 2018).

Currently, it is common to use numerical optimization methods for solving engineering optimization problems. One of the well-known categories of numerical optimization methods is meta-heuristic algorithms. Genetic Algorithm (GA) (Goldberg 1989), Particle Swarm Optimization (PSO) (Kennedy and Eberhart 1995) and Harmony Search (HS) (Lee and Geem 2005) are some popular optimization algorithms which have successfully been used in seeking for solutions for various engineering optimization and reliability-based design problems (Cheng *et al*. 2008, Mun and Geem 2009, Azar *et al*. 2015, Ayyıldız and Çetinkaya 2016, Hamza *et al*. 2017, Kamboj *et al*. 2017, Hadidi and coworkers 2014, 2015, 2016, 2017, Zou *et al*. 2018, Yu *et al*. 2018, Islam *et al*. 2018). It seems that these algorithms can also be used to minimize the costs

*Corresponding author, Professor
E-mail: a.rafiee@tabrizu.ac.ir


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associated with structural strengthening of masonry arch railway bridges. Moreover, numerical soft computing methods have vigorously been used in different disciplines for analysis, design and behavior appraisal of bridge structures (Choi *et al*. 2017, Gao *et al*. 2017, Jansseune and De Corte 2017, Xiong *et al*. 2018, Dong 2018, Onat 2019). Meanwhile, in the literature, the use of such methods for optimal structural strengthening of ancient bridge structures has not yet been dealt with.

In this paper, a novel numerical optimization algorithm is proposed to minimize the costs associated with structural strengthening of masonry arch railway bridges. As mentioned earlier, the numerical optimization algorithms (e.g. meta-heuristics) seem to be applicable to minimum cost structural strengthening of masonry arch railway bridges. The aim of this contribution is mainly to show the real application of meta-heuristics (like which proposed herein) to such kind of engineering optimization problems. To do so, the structural response values of the Sahand-Goltappeh bridge (as a case study of historic masonry arch in-service railway bridges located in Iran) is assessed under ordinary train pass using both experimental and analytical/numerical investigations. Then, the numerical model of the bridge is structurally strengthened such that following three goals are achieved simultaneously. First, the bridge is submitted to loads which are higher than those it has been built for, and so, it is needed that the load-carrying-capacity of the bridge is increased, too. Second, in order for the bridge to achieve a sufficient safety level, not only the excessive increase in its structural response under increased loads should be prevented, but also should be reduced, and so, it is required that the structural response of the bridge is reduced to a certain limit. Third, due to the limitations on available funds the strengthening measures should be costly effective, and so, the costs needed for such strengthening should be minimized as far as possible.

To solve the above optimization problem aimed at mentioned goals, a novel optimization algorithm is proposed which is described in Section 2. In Section 3 of the paper, the common methods for structural strengthening of masonry arch bridges are briefly reviewed. Section 4 is devoted to the numerical modeling and minimum cost structural strengthening of the Sahand-Goltappeh bridge as a case study. Finally, in Section 5 the paper closes with some concluding remarks.

Before starting with Section 2, it seems very important to highlight the inspiration of this work. Hence, some more studies are reviewed in the upcoming Subsection.

### *1.1 Related studies*

In the previous sentences, the background of present study is represented dealing with PSO as a kind of the most representative swarm intelligence algorithm. However, to further improve the literature review, some latest excellent papers of swarm intelligence are reviewed herein.

In order for the reader to be more familiar with the swarm intelligence optimization algorithms and their engineering applications, the study of following papers is suggested: Cui *et al*. (2017); Feng and Wang (2018); Feng *et al*. (2017, 2018); Guo *et al*. (2014); Liu *et al*. (2017); Rizk-Allah *et al*. (2018); Wang *et al*. (2013, 2014a-f, 2016a-i, 2017, 2018, 2019a-b); Wang and Yi (2018); Wang (2018); Wang and Tan (2019); and, Yi *et al*. (2016).

## 2. The numerical optimization algorithm

In this study, a new numerical optimization algorithm is proposed for cost minimization. In this algorithm, called JPSO hereafter, PSO and Jaya (Rao 2016) algorithms are hybridized to improve each other. Indeed, in this algorithm the advantages of both the PSO and Jaya methods are used simultaneously. At the same time, this hybridization eliminates the disadvantages of crude PSO and Jaya algorithms.

In fact, in each iteration of crude Jaya, only the data regarding current global best and worst solutions are used as previous knowledge (gained through previous generations) in upcoming computations. In this way, the best solutions of previous iterations play no role anymore in optimization process. Meanwhile, the so-called global optimum may locate at the vicinity of best solutions of previous iterations. Whereas in crude PSO, local best solutions (seen by each particle/bird) are constantly used as previous knowledge during optimization. To overcome this drawback of Jaya, by increasing the cooperation component of the search algorithm in JPSO, more previous knowledge is used in optimization process.

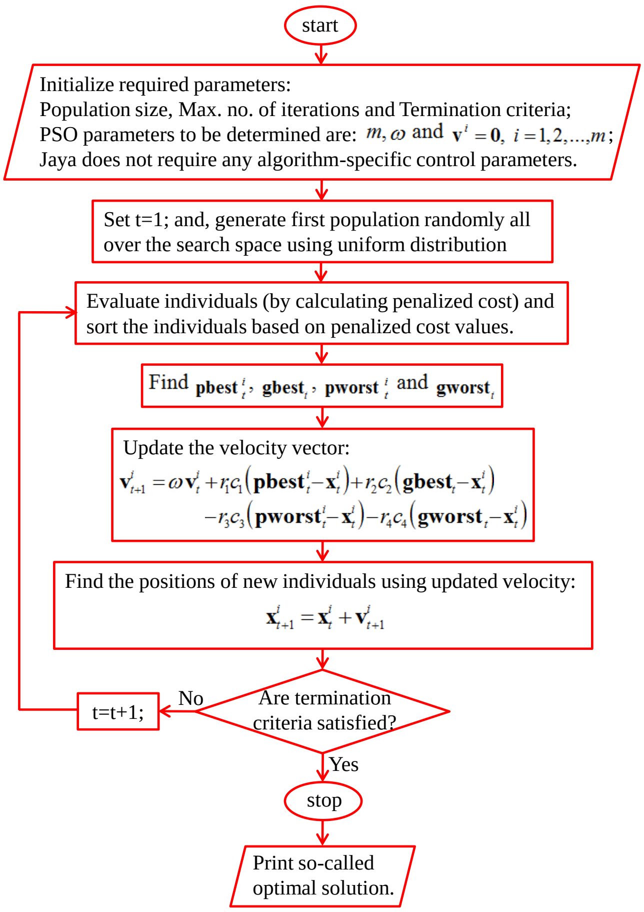



Fig. 1 The flowchart of the proposed JPSO algorithm

Conversely, in crude PSO algorithm, the global and local best solutions are all the previous knowledge used during optimization. While, worst solutions play no role in this process. In order for the algorithm to avoid worst solutions and to increase convergence speed, the use of these worst solutions may be effective. Inspiring from Jaya, in JPSO, the use of worst solutions together with best ones is suggested. Doing so, this drawback of PSO may be overcome, such that, betters solutions are obtained within small number of iterations.

Concisely, JPSO seeks for optimum solution by approaching global and local bests and by taking aloof from global and local worsts, simultaneously. The algorithm possesses the merits of both the PSO and Jaya, eliminating their drawbacks. The self-explanatory flow-chart of Fig. 1 describes the JPSO algorithm. More details about PSO and Jaya algorithms can best be found in (Kennedy and Eberhart 1995) and (Rao 2016), respectively.

In Fig. 1, it is clear that $\mathbf{v}_t^i$ is the velocity vector for *i*-th particle (out of *m* number of particles) at *t*-th iteration; $0.8 < \omega < 1.4$ is inertia term (introduced in Shi and Eberhart (1998)). Moreover, $\mathbf{pbest}_t^i$ and $\mathbf{gbest}_t$ are, respectively, local and global best solutions ever seen (up to *t*-th iteration) by *i*-th particle and by all the particles. Also, $\mathbf{pworst}_t^i$ and $\mathbf{gworst}_t$ are their worst counterparts. The scalars $r_1$, … $r_4$ are uniform random numbers from the interval 0 to1. The multipliers $c_1$, … $c_4$ should be equal to 2 (like the social scaling factors in Kennedy and Eberhart (1995)) for the birds to overfly the target half the time.

## 3. Common methods for structural strengthening of masonry arch bridges

In this part of the paper, some common methods for structural strengthening of masonry arch railway bridges are briefly reviewed. These methods have already been used for strengthening of railway bridges in Iran. One of the very old and less effective methods for this purpose is the use of strengthening elements (e.g. using rolled steel profiles in group) beneath the arch of the bridge. In this way, the elements added to the bottom face of the arch (intrados) contribute to the transmission of the arch loads to the foundation and increases the load bearing capacity of the bridge. Moreover, grout injections are used to fill the cracks and voids. This approach has been applied to strengthen some railway bridges in Iran, many years ago. Fig. 2 shows the Kondolaj bridge, a railway bridge which has been strengthened using the above approach.

Some other traditional strengthening techniques consist of some of following items (Triantafillou and Fardis 1997, D'Ambrisi *et al*. 2015): (i) stitching cracks/weak areas using metallic/brick elements or concrete zones; (ii) applying reinforced grouted perforations to increase cohesion/tensile strength of the masonry; (iii) jacketing by shotcrete/cast-in-situ concrete; (iv) post-tensioning (external/internal) using steel ties to integrate structural system; (v) injecting cementitious mortar at the vicinity of foundation; (vi) casting reinforced concrete elements (at the intrados/extrados); and (vii) insertion of steel bars.

Fig. 2 The strengthened Kondolaj railway bridge

Such an approach has also been followed for structural strengthening of Iranian railway bridges, examples of which are the bridge 471 and the bridge 475 (Fig. 3). For these two bridges the strengthening procedure consists of three main items. The first item is the injection of cementitious mortar to the zones near and beneath the foundation. Fig. 4-(a) shows the application of this item to the bridge 471.

The second and third items are external jacketing by making use of shotcrete or by cast-in-situ concrete. For this aim, horizontal and vertical bars (as bar grid) are placed at the perimeter of the piers, and then, these grids are embedded inside cast-in-situ concrete. Figs. 4-(b) and 4-(c) show these measures. The second item is shown in Fig. 4-(b) for the bridge 471. Finally, by means of shotcrete an internal concrete arch is added to the intrados of the masonry arch to help the bridge to transmit the arch loads to the foundation. Doing so, the load bearing capacity of the bridge is increased. This latter item is also shown in Fig. 4-(c) for the bridge 471. As it is seen from this figure, the grids have been embedded inside cast-in-situ concrete, and after that, in order for the bridge piers to have masonry outward appearance, revetment is applied.

As another alternative, masonry arch bridges can be strengthened using Fiber Reinforced Polymer (FRP) sheets. FRPs are produced from fibers of Carbon, Glass, Aramid, etc. In such an approach for strengthening, sheets of FRP are attached (e.g. using epoxy) to the face of structural element wherever needed to increase its strength.

During the last two decades numerous experimental/theoretical researches have been devoted to study the use of FRPs in the construction of new structures or strengthening of existing ones (D'Ambrisi *et al*. 2014, Castillo *et al*. 2018, D'Altri *et al*. 2018, Siwowski *et al*. 2018, Kim 2019). Meanwhile, the use of FRPs in the strengthening of masonry arches is also notable (De Lorenzis *et al*. 2007, Briccoli Bati and coworkers 2007, 2008, Oliveira *et al*. 2010). The advantages of FRPs are their light weight, high strength/stiffness, resistance to corrosion, flexibility, and rapidity of application. Whereas, the moderate matrix heat and fire resistance due to the matrix (epoxy resin) low glass transition temperature, difficulty of application at low temperatures, impossibility of application on humid surfaces (other than cementitious ones), and lack of vapor permeability are disadvantages of the FRPs (D'Ambrisi *et al*. 2015). Exhaustive details/reviews of the application of FRPs for the strengthening of masonry structures/arches can best be found in (Carozzi *et al*. 2018, Parghi and Alam 2018).

(a) The bridge 471

(b) The bridge 475

Fig. 3 The strengthened railway bridges

(a) The first item

(b) The second item

(c) The third item

Fig. 4 The strengthening items for the bridge 471

## 4. Numerical modeling and minimum cost structural strengthening of railway bridges

In this section, the Sahand-Goltappeh masonry arch railway bridge which is located at the northwest part of Iran, is studied as a case study. The bridge is numerically modeled under train pass, and then, the analysis results are calibrated using experimental results. After that, the bridge is structurally strengthened using finite elements software. Finally, using the proposed HSJ algorithm the optimum values for design variables of such strengthening are sought.

### *4.1 Finite-element modeling of Sahand-Goltappeh bridge*

This bridge, shown in Fig. 5, has seven spans and the length of each span is 10 meters. The numbering of spans starts from left hand. The bridge has a width of 4 meters together with a total length of 92 meters and its height from the river bed to the arch intrados (near arch crown) is about 13 meters. For all the arch numbers (1 through 7) following values define the geometry: the radius of the intrados face of the arch is equal to 5 *m*; the thickness of the arch ring (arch barrel) is equal to 80 *cm* at arch crown. The dimension of all the piers is 2 *m* alongside the bridge length and corresponding value for the abutments is 5 *m*.

The mechanical properties of the materials of different parts of the bridge, obtained from experimental tests, are listed in Table 1. These values have been modified based on model calibration using train pass displacement results.

In order to analyze the stresses/strains and deformations/displacements of the different parts of the bridge under train pass, 3D finite element model of the bridge is created using solid C3D8R elements in ABAQUS. Fig. 6 shows this model in which the mechanical properties of Table 1 are used. In order for the analysis results to be reliable, the constructional details of the bridge are tried to be modeled as far as possible. These details include the correct modeling of gaps/watercourses, uneven surface of the bridge (which is evened using ballast layer). Accounting for such details makes the model and its load carrying mechanism more and more realistic. Moreover, since finite element model is calibrated by experimental test results, the

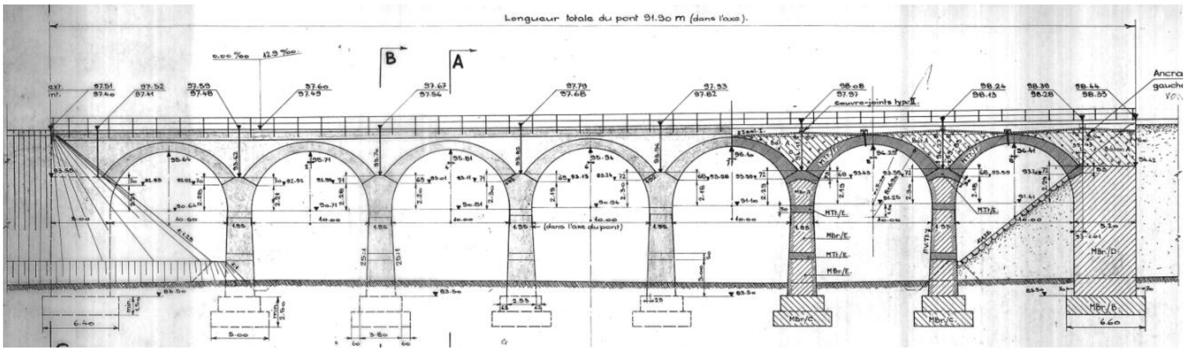


Fig. 5 The Sahand-Goltappeh masonry arch railway bridge elevation

Table 1 Mechanical properties of materials used in Sahand-Goltappeh bridge model

| Material used in | Elastic modulus (*GPa*) | Compressive Strength (*MPa*) | Poisson's ratio | Weight per unit volume (*kN/m^3*) |
|---|---|---|---|---|
| Arch rings | 10 | 21 | 0.167 | 24 |
| Piers/Abutments | 20 | 22 | 0.167 | 24 |
| Backing/Fill | 10 | 12 | 0.167 | 24 |
| Ballast | 0.2 | - | 0.2 | 24 |

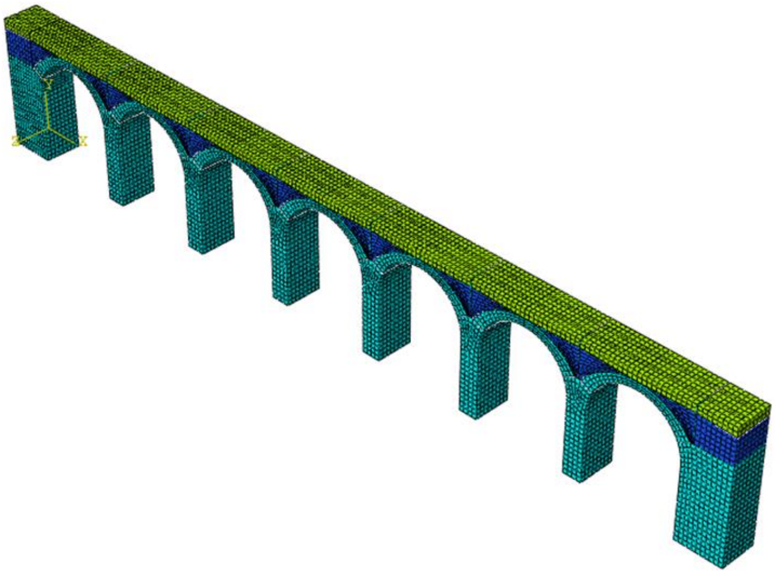


Fig. 6 The 3D finite element model of the Sahand-Goltappeh bridge

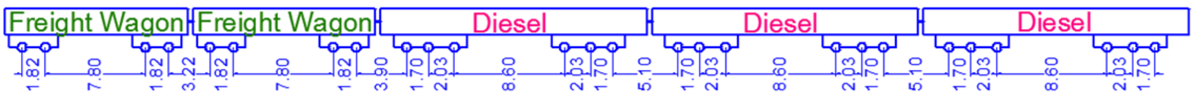


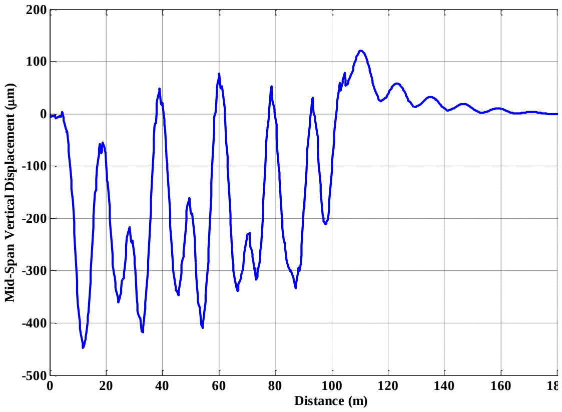

Fig. 7 The axles arrangement for the passing train

contact between the various types of elements is indirectly treated.

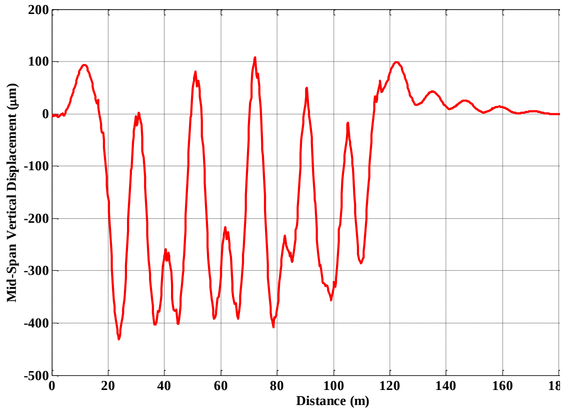
The train loads should also be modeled in a proper manner. To this end, assume that the passing train consists of three GT26 locomotives together with two freight wagons. Fig. 7 shows all the axle-to-axle distances of such a train in meters. It is also simply assumed that the axial load applied by each axle to the rail tracks is equal to 200 *kN* and assume that the speed of the train produces 1.4 impact multiplier.

The results of the structural analysis of the bridge under the train pass are shown in Fig. 8 for some of the arches of the bridge. Indeed, the vertical displacements of the arch crowns during train pass are shown in this figure, for example, for arch numbers 1, 2, 4 and 7. In this figure, the abscissas show the locations of the first axle of the train at different times during train pass.

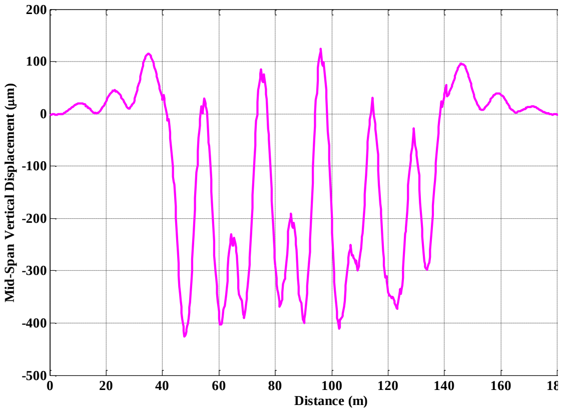
It should be noted that the length of the bridge is 92 meters and the distance between the first and the last axles of the train is 88.38 meters. Hence, Fig. 8 shows the vertical



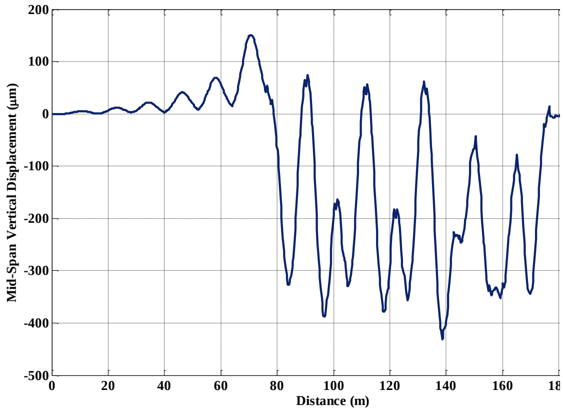

(a) The arch 1




(b) The arch 2

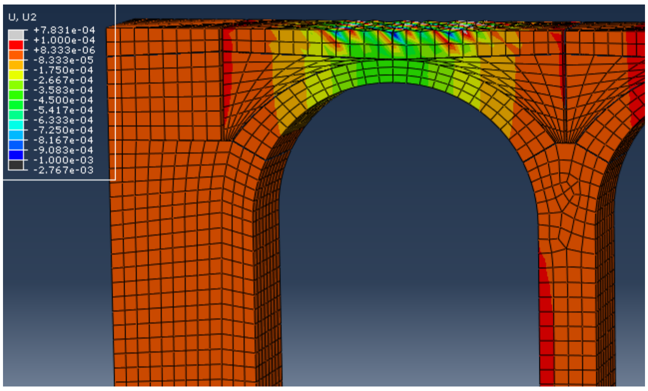



(c) The arch 4

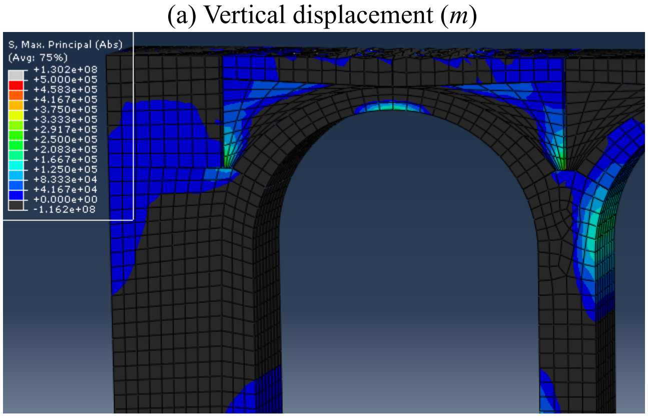



(d) The arch 7

Fig. 8 The mid-span vertical displacements of the bridge ($\mu m$)

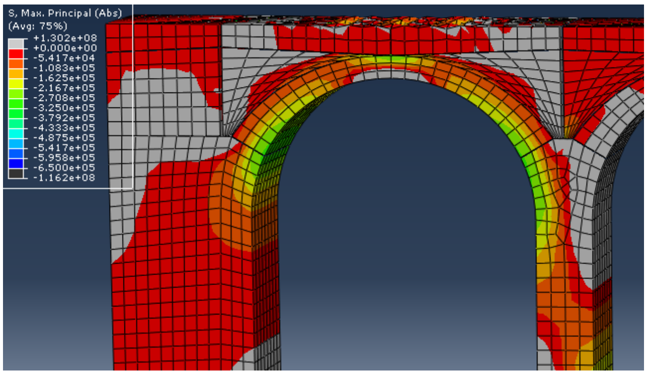



(a) Vertical displacement ($m$)




(b) Tensile stress (maximum principal, $Pa$)

Fig. 9 The analysis results at the vicinity of arch 1 (at the moment of maximum displacement occurrence)




(c) Compressive stress (maximum principal, $Pa$)

Fig. 9 (*Continued*)

displacements of the central point of mentioned arches at a time window, which starts at the time when the first axle enters and ends when the last axle exits.

Among the displacement histories of seven arches, the maximum vertical displacement of the arch 1 is the greatest one. This maximum value of 448 $\mu m$ occurs when the second axle of the train reaches at the center of the first span (Fig. 8a). The contours of vertical displacement, tensile and compressive stress (maximum principal) are, respectively, shown in Figs. 9a-c.

The maximum allowable tensile/compressive stress for masonry structures, based on the Doc. 308 (Iranian Doc. 308), is as follows

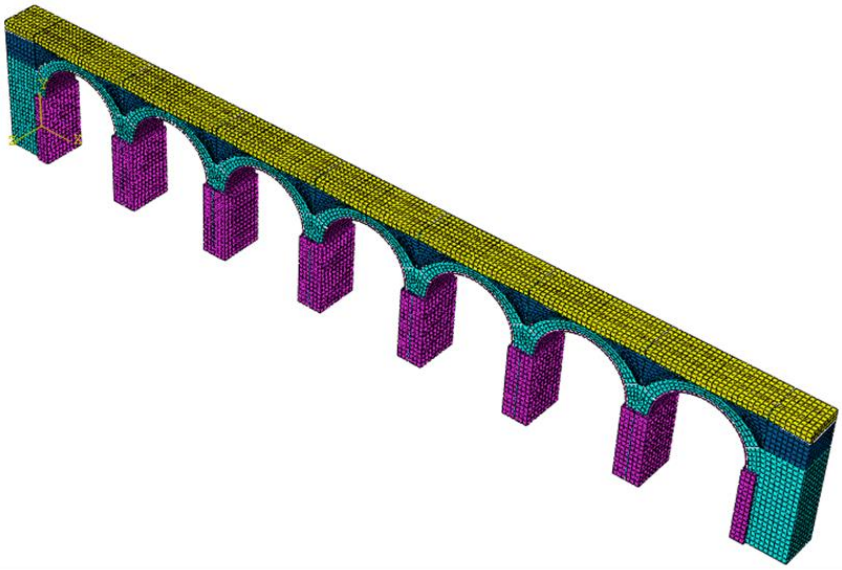


Fig. 10 The 3D finite element model of the Sahand-Goltappeh bridge (after strengthening)

$$\sigma_C^{allow} = 0.45 f_c^{'} = 9.45, \;\; \sigma_T^{allow} = 0.135\sqrt{f_c^{'}} = 0.62 \;\; (MPa) \quad (1)$$

As it is also seen from Fig. 9, the stresses are less than above bounds, all over the bridge.

### *4.2 Finite-element modeling of numerically strengthened Sahand-Goltappeh bridge*

In this subsection the bridge is numerically strengthened using finite elements approach. The strengthening procedure is analogous to which was described for the bridge 471 (Section 3). For this aim, two layers of bar grid are placed at the perimeter of each pier/abutment, and then, these grids are embedded in cast-in-situ concrete, and finally, an additional concrete arch ring is imposed on the intrados face of each masonry arch ring. Fig. 10 shows the numerical model of the strengthened bridge. In this figure, the bar grids are not visible because of being embedded in concrete. However, as mentioned, two layers of bar grid have been placed at the perimeter of each pier/abutment.

In order to show the effectiveness of such strengthening in reducing the structural response values of the bridge under train pass, these values are shown in Figs. 11-12. These figures are obtained assuming exactly the same train pass conditions used in preparing Figs. 8-9. Meanwhile, the strengthening system added to the primary structure of the bridge is composed in a manner as follows. The diameters of all the bars are assumed to be 10 *mm*. The spacing between both the horizontal and vertical bars is assumed to be 25 *cm*. The spacing between the internal and external layers of bar grid is also assumed to be 20 *cm*. Moreover, considering a concrete cover of 5 *cm* for both of the bar layers, the thickness of the concrete added to the perimeter of each pier/abutment is assumed to be 35 *cm*. Finally, an additional concrete arch ring with a thickness of 24 *cm* is attached to intrados face of each masonry arch ring. This additional arch works in combined with the mentioned system of bar and concrete to strengthen the bridge.

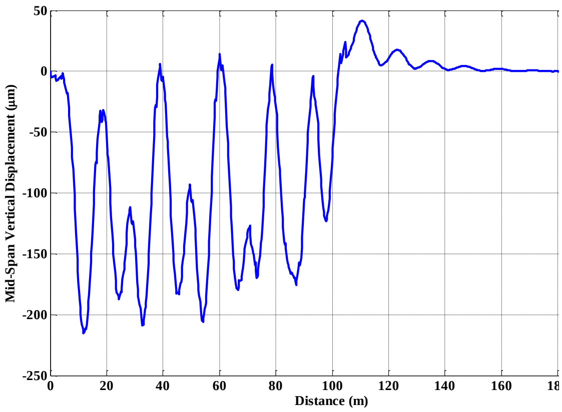
By comparing Fig. 11 with Fig. 8, it is clear that the strengthening is significantly effective in reducing the vertical displacements of the bridge. Among the seven arches of the strengthened bridge, the maximum mid-span vertical displacement, during train pass, belongs to the arch 1. This maximum value occurs when the second axle of the train reaches the mid-span location of the arch 1 (as was the case for original bridge prior to strengthening). This maximum value is equal to 215 $\mu m$. Indeed, the strengthening reduces the maximum displacement by 52%. Moreover, the comparison between Fig. 12 and Fig. 9 shows the effectiveness of strengthening in reducing the maximum principal stresses. The comparisons show that the stress values are reduced by about half.

### *4.3 Minimum-cost strengthening of Sahand-Goltappeh bridge*

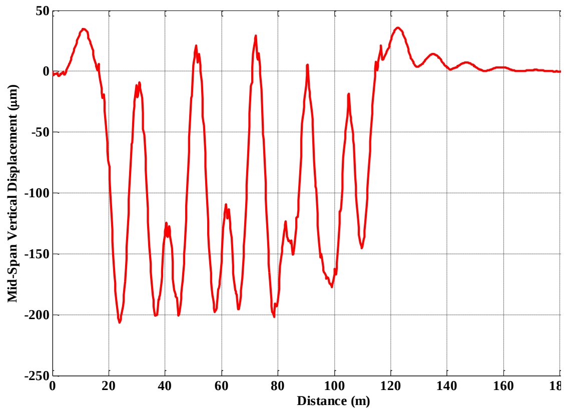
In this subsection, the computational framework for minimum-cost strengthening of masonry arch railway bridges is introduced and used for Sahand-Goltappeh bridge (as a case study) to increase train's pass speed and/or its axial load. This framework uses the proposed optimization algorithm (Section 2).

In the previous subsection, the effectiveness of the strengthening was demonstrated by assuming some values, for example, for the spacing and diameters of bars, the thickness of added arch ring, etc. Now, to show the application of proposed framework, an example is solved. For this aim, it is assumed that all the parameters have predefined values, and only, the values of three design variables remain to be determined through optimization. These three variables are assumed to be the diameter of horizontal (shear) bars, the diameter of vertical (bending) bars and the thickness of added concrete arch ring.

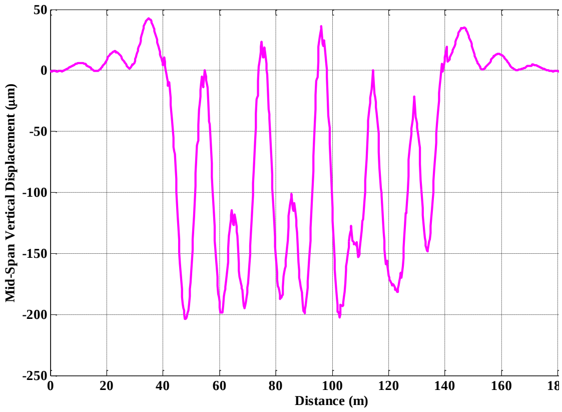
The discrete design space in which the optimum values of design variables are sought is as follows. The thickness of added arch ring, diameter of horizontal bars and that of vertical ones are, respectively, assumed to be chosen from the *Sec-Shell*, *Sec-HBar* and *Sec-VBar* lists, as follows



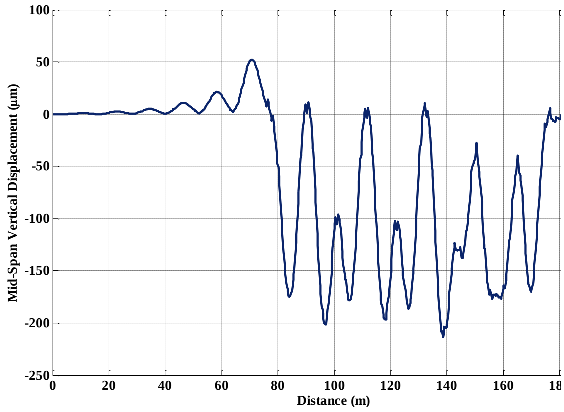

(a) The arch 1




(b) The arch 2

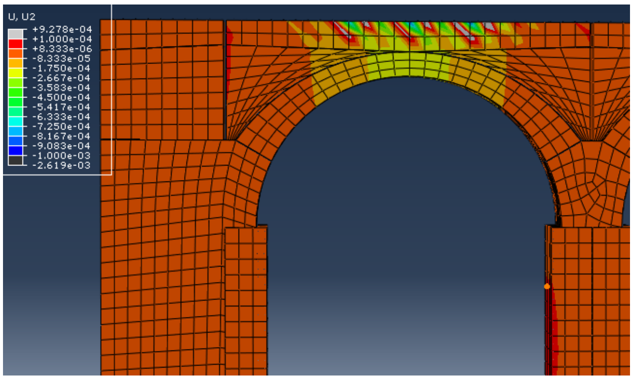



(c) The arch 4

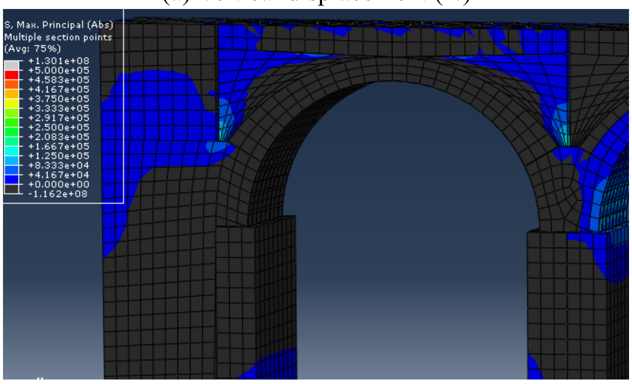



(d) The arch 7

Fig. 11 The mid-span vertical displacements of the bridge (after strengthening, $\mu m$)

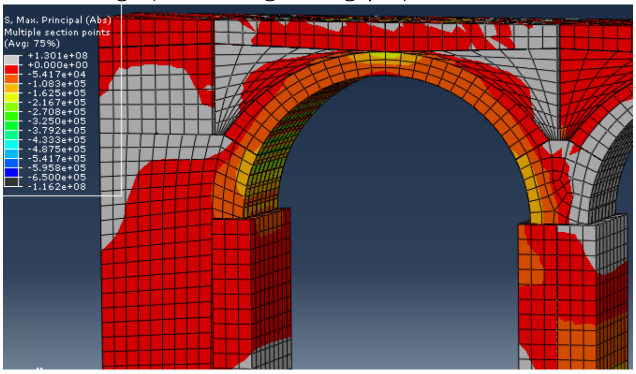


(a) Vertical displacement ($m$)



(b) Tensile stress (maximum principal, $Pa$)

Fig. 12 The analysis results at the vicinity of arch 1 (when maximum displacement occurs; after strengthening)



(c) Compressive stress (maximum principal, $Pa$)

Fig. 12 (*Continued*)

$$Sec-Shell = \{10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34\}\ cm \tag{2}$$

$$Sec-HBar = \{6, 8, 10, 12, 14, 16, 18, 20\}\ mm \tag{3}$$

$$Sec-VBar = \{12, 14, 16, 18, 20, 22, 25, 28, 30\}\ mm \tag{4}$$

Moreover, as the goal of the problem is to minimize the cost of the bridge strengthening; therefore, the objective function of the optimization problem is such a cost. To determine this cost, the Iranian prices list for road, railway and airport construction services (2018) is used in this study.

Furthermore, the goal of the bridge strengthening is, herein, to increase the train pass speed. Since, this increase level is not supposed to be so high; hence, such an increase manifests itself as an increase in the axial load of the train and/or in the impact multiplier. So, in the example under consideration, it is assumed that the increase in the train speed results in a 40% increase in loads applied to the bridge by each axle of the train. Doing so, the stress/deformation values are expected to be increased significantly, and, these response values should then be limited properly.

In the subsection 4.1, the Sahand-Goltappeh bridge was numerically modeled under ordinary train pass. Among the displacement histories of seven arches, the maximum vertical displacement of the arch 1, before strengthening, was the greatest one (448 $\mu m$, Fig. 8a). If the applied loads increases by 40%; then, the structural response values are also supposed to increase considerably. So the vertical displacement will, out and away, be more than 448 $\mu m$.

Let us assume now that the cost associated with the bridge strengthening is to be minimized, such that, the maximum vertical displacements of none of the arch centers do exceed 300 $\mu m$, during the new condition of the train pass. That is to say, the maximum vertical displacements of arch centers of the bridge not only should not be increased after the strengthening but also should be decreased, under the new train pass condition which applies 40% more axial loads to the bridge. This is because the uncertainties regarding the load-carrying-capacity of these historic bridges are of paramount importance and the strengthened bridge should be reliable, as well.

The minimum-cost design problem, described above, is, in fact, a constrained optimization problem. Such a problem can simply be replaced with a suitable unconstrained one. To this end, it is usual to use a proper penalty function which penalizes the value of objective function when the algorithm goes towards unfeasible domain of the search space. In this way, the algorithm is directed towards the feasible domain. Here, such a penalty function is defined as follows

$$P(U) = 1 + 10 \times \max\ (0, \frac{U}{300} - 1) \tag{5}$$

wherein, $U$ (in $\mu m$) is the maximum vertical displacement of the arch centers of the bridge during train pass. This function is multiplied to the objective function (*i.e.* the cost of the bridge strengthening) to give the value of the penalized cost.

Finally, three optimization algorithms, namely, the PSO, the Jaya and the proposed JPSO algorithms are used to solve the minimum-cost design. The optimum results and the performance of the algorithms are compared, as well. In order for the algorithms to be compared in terms of efficiency, all the conditions are set to be the same for all of them. Then, the number of iterations is 13; the population size of each iteration is 12; and, the candidate solutions of the first population are the same for all of the algorithms. Fig. 13 shows the convergence histories of these algorithms.

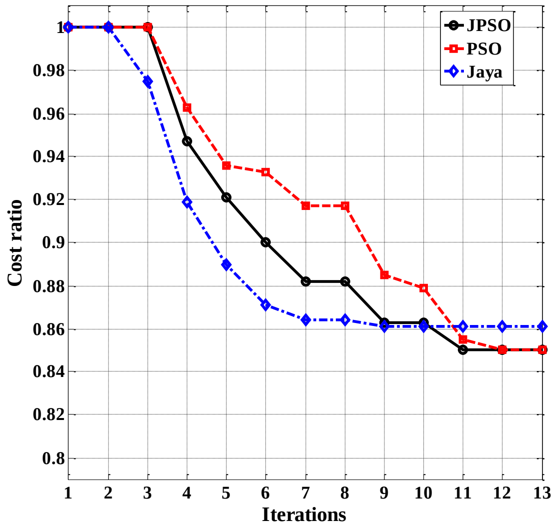



Fig. 13 The convergence histories of PSO, Jaya and JPSO

As it is clear for this figure, the JPSO finds the same optimal solution within less number of iterations compared to PSO. Also, in comparison with Jaya, the JPSO finds better solution within the same number of iterations. This shows the superiority of JPSO over PSO and Jaya. The correctness of such a conclusion was checked by repeating the optimization process using different seeds of random sets, and, the similarity of the results showed the efficiency of the proposed algorithm.

It should be noted that the cost values are obtained based on Iranian prices list (2018) in IRR (Iranian Rials) units. Then, in order for the results to be unit-less, these costs are divided by the cost of the best feasible solution of the first population. The final solution obtained by the JPSO uses following optimal values for the design variables: 6 *mm* for the diameters of horizontal bars; 12 *mm* for the diameters of vertical bars; and, 26 *cm* for the thickness of the added concrete arch ring. For such a design, the $U$ is 299.4 $\mu m$. Furthermore, Fig. 13 shows that the strengthening cost is reduced by 14.7%, if one compares the strengthening cost of the final optimal solution with that of the first solution (*i.e.* the best feasible solution of the first population).

## 5. Conclusions

The historic masonry arch bridges are crucial element of railway transportation network, worldwide. Nowadays, these bridges should be not only preserved from degradation but also structurally strengthened because of being expected to carry loads higher than those for which they were built. To minimize the costs associated with such a strengthening, a computational framework was proposed; which uses a novel optimization algorithm, called JPSO. Then, as a case study, the framework was applied to the Sahand-Goltappeh bridge; aimed simultaneously at: (1) the load-carrying-capacity of the bridge is increased; (2) the structural response of the bridge is reduced to a certain limit; and, (3) the costs needed for such strengthening are minimized as far as possible.

It should be noted that, the effect of the repetitive nature of the loading has not been taken into account, herein. However, this important issue should be accounted for in future studies. The results of this study can be summarized as follows:

• A novel computational framework is introduced for minimum-cost strengthening of masonry arch railway bridges. Such an application of optimization algorithms have not been paid attention in the literature.

• The proposed JPSO algorithm is more efficient than PSO; so that, it finds the same optimum result within less number of function evaluations needed in PSO.

• The JPSO performs also better than Jaya; so that, it finds better optimal solution within the same number of iterations processed in Jaya.

• For the bridges with the geometry similar to the studied bridge, the peek values of compressive/tensile stresses during train pass occur mainly at the vicinity of arch springing parts, near the joining points of arches with piers/abutments, at the vicinity of arch crown locations and also at the bottom of piers/abutments near foundation. So, strengthening at these parts should be special. For instance, it should be reinforced with closer bars as done in connections of special-moment-frames; and/or, thicker strengthening elements (e.g. the added cast-in-situ concrete) should be applied compared to the other parts.

• According to the results of investigations done for the studied bridge, it seems that the thickness of concrete arch added to the arches of the bridge may be chosen unequal. In fact, some of the arches of the bridge are more critical than the others, in terms of stress. For example, for the studied bridge, arch numbers 1 and 7 are more critical. Generally, for bridges with different geometry and different train pass conditions, critical arches should be detected.

• Finally, to decrease the cost of strengthening of masonry arch bridges, it is recommended to diagnose the critical parts of maximum stress, and then, focus on the strengthening of these parts. In this way, there is no necessity for the use of extra strengthening measures for locations where are not critical. Doing so, the cost of strengthening is further reduced.

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