# Dimensional analysis of base-isolated buildings to near-fault pulses

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**Abstract.** In this paper the dynamic behavior of an isolated building subjected to idealized near-fault pulses is investigated. The building is represented with a simple 2-DOF model. Both linear and non-linear behavior of the isolation system is considered. Using dimensional analysis, in conjunction with closed form mathematical idealized pulses, appropriate dimensionless parameters are defined and self-similar curves are plotted on dimensionless graphs, based on which various conclusions are reached. In the linear case, the role of viscous damping is examined in detail and the existence of an optimum value of damping along with its significant variation with the number of half-cycles is shown. In the nonlinear case, where the behavior of the building depends on the amplitude of the excitation, the benefits of dimensional analysis are evident since the influence of the dimensionless II-terms is easily examined. Special consideration is given to the normalized strength of the non-linear isolation system that appears to play a complex role which greatly affects the response of the 2-DOF. In the last part of the paper, a comparison of the responses to idealized pulses between a linear fixed-base SDOF and the respective isolated 2-DOF with both linear and non-linear damping is conducted and it is shown that, under certain values of the superstructure and isolation system characteristics, the use of an isolation system can amplify both the normalized acceleration and displacement of the superstructure.

Keywords: seismic response; near-fault pulses; base-isolation; dimensional analysis; non-linear analysis; earthquake engineering

# 1. Introduction

Base isolation has been one of the most effective and widely implemented seismic protection systems for the last 25 years. This system is intended to simultaneously reduce interstory drifts and floor accelerations through a reduction of stiffness and an increase in damping (Kelly 1997). Therefore, base-isolated buildings are characterized by relatively large fundamental periods.

In recent years the records from near-fault earthquakes (e.g. the 1995 Kobe Earthquake, the 1999 Chi-chi Taiwan Earthquake) have shown that the ground motions near the faults of major earthquakes may contain long period velocity pulses that can amplify the displacement demands of the isolation system. Although during the 1994 Northridge and the 1995 Kobe Earthquakes several isolated buildings with Lead Rubber Bearing (LRB) performed well and base isolation was effective in reducing the response (Nagarajaia and Sun 2000, Nagarajaiah and Sun 2001, Buckle et al. 2002), several researchers have pointed out the vulnerability of base-isolated buildings to large velocity pulses contained in near-source ground motions (Buckle and Mayes 1990, Heaton et al. 1995, Hall et al. 1995, Yang and Agrawal 2002, Castaldo and Tubaldi 2018). Under these motions there may appear such large displacement demands that could result in the failure of the isolation

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Copyright © 2020 Techno-Press, Ltd. http://www.techno-press.com/journals/sem&subpage=7 systems (Yang and Agrawal 2002, Nagarajaiah and Ferrell 1999). This topic has been challenging and, consequently, various studies have been reported (Makris 1997, Makris and Chang 2000, Jangid and Kelly 2001, Liao et al. 2004, Jangid 2005, Panchal and Jangid 2008, Ariga et al. 2006, Alhan and Gavin 2004, Ramallo et al. 2002, Shen et al. 2004, Rao and Jangid 2001, Kampas and Makris 2012, 2013, Makris 2014, Konstantinidis and Nikfar 2015, Chen et al. 2017). Ariga et al. (2006) revealed that the longperiod ground motions recorded in Japan have the intensity to bring in resonance base-isolated high-rise buildings with long period components and that careful treatment is required in the structural design of these buildings. Shen et al. (2004) investigated the near-fault effects on the performance of a seismically isolated bridge. They revealed (i) an amplified seismic response when the pulse period was close to the effective period and (ii) a beneficial effect of bearing nonlinearity on the acceleration response.

Several researchers have suggested the use of supplemental energy dissipative mechanisms (to protect structures from near-fault earthquakes) (Hall 1999, Hall and Ryan 2000, Zhang and Iwan 2002, Sahasrabudhe and Nagarajaiah 2005), while others have shown that the addition of damping may reduce the isolator displacement but it may also increase the floor accelerations and interstorey drifts (Alhan and Gavin 2004, Inaudi and Kelly 1993, Sadek and Mohraz 1998). Jangid and Kelly (2001) reported the existence of a particular value of isolation damping that minimizes superstructure acceleration for a given structural system under near-fault motion. Some

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Fig. 1 Ground motions time histories (displacement, velocity, acceleration) given by the mathematical model of Mavroeidis & Papageorgiou for Tp=2s, A=0.5m/s, v=0 and different values of  $\gamma$ 

researchers have investigated various passive isolation systems and dissipative mechanisms and showed the existence of optimum values of the parameters of the isolation system for near-fault earthquakes (Jangid 2005, Alhan and Gavin 2004, Jangid 2007), while others have proposed the use of semi-active dampers (Makris 1997, Ramallo *et al.* 2002) to effectively protect structures from near-fault ground motions.

Many of the aforementioned studies made use of recorded ground motions (Jangid and Kelly 2001, Jangid 2005, Ariga et al. 2006, Alhan and Gavin 2004, Ramallo et al. 2002, Jangid 2007), while others additionally used simple pulses along the recorded data (Makris 1997, Panchal and Jangid 2008, Makris and Chang 2000, Shen et al. 2004, Mazza and Vulcano 2009) since such an approach can lead to a better understanding of near-fault phenomena. Makris and Chang (2000) found that there is a resemblance between the structural response for near-fault ground motions and the one caused by a harmonic pulse only for buildings with moderate to large periods. Using the same harmonic pulses in conjunction with dimensional analysis, Makris and Black (2004) and Makris and Psychogios (2006) managed to specify dimensionless parameters which uncover the physics of the behavior of rigid-plastic, elastoplastic and bilinear SDOF systems (Makris and Black 2004, Makris and Psychogios 2006) and the physics of the behavior of yielding structures with first-mode dominated responses (Makris and Psychogios 2006), respectively.

In this study, the behavior of a base-isolated building with both linear and non-linear behavior of the isolation system, under idealized near-fault pulses, using dimensional analysis is investigated. A 2-DOF model is used, where for simplicity the superstructure will be assumed to remain in the elastic range during the earthquake excitation. The aim of this study is to specify dimensionless variables for the 2-DOF, similar to the ones found in (Makris and Black 2004)for the SDOF, that will allow to plot dimensionless graphs in order to shed light on the dynamic behavior of a base-isolated building subjected to pulse-type excitations resembling near-fault ground motions.

#### 2. Analytical model for near-fault ground motions

Base isolation has been one of the most effective and widely implemented seismic protection systems for the last 25 years. This system is intended to simultaneously reduce interstory drifts and floor accelerations. Since the early study of Jacobsen and Ayre (1958) that examined the behavior of a SDOF under different types of acceleration pulses, several other researchers have used simple pulses in their studies (e.g. Hall et al., 1995, Makris & Chang, 2000, Mylonakis & Reinhorm, 2001, Mylonakis & Voyagaki, 2006). In recent years, that near-fault seismic ground motions comprise an active research topic for seismologists and earthquake engineers, various analytical models for the representation of near-fault ground motions have been introduced (Makris and Chang 2000, Menun and Fu 2002, Mavroeidis and Papageorgiou 2003). These mathematical models can approximate the main kinematic characteristics of near-source ground motions and have the potential to facilitate the study of the response of structures subjected to these motions. The minimum number of parameters is 2, that is: the duration  $T_p$  and either the acceleration amplitude ap or the velocity amplitude  $v_p$ . The present study employs both the simple harmonic pulses (used in Makris and Chang 2000, Makris and Black 2004, Makris and Psychogios 2006 among others] that employ only the 2 input parameters T<sub>p</sub>



Fig. 2 Base isolated system and corresponding displacements

and  $a_p$  (or  $v_p$ ), as well as the more sophisticated model of Mavroeidis and Papageorgiou (2003, 2004) that involves 4 input parameters which are the pulse period T<sub>p</sub>, the velocity pulse amplitude A, the number  $\gamma$  and the phase v of the half cycles. However, because of limited space only the graphs from the latter model are presented. This model is also selected because of the large set of related near-fault ground motions that were used for its calibration and verification. The models are described by Equations (1) and (2). It must also be clarified that the pulse velocity is represented with v<sub>p</sub> according to Makris and Black (2004) and with A according to Mavroeidis and Papageorgiou (2003, 2004). Figure 1 depicts the ground displacement,  $d_g(t)$ , velocity  $v_g(t)$  velocity and acceleration time history  $a_g(t)$  given by the model of Mavroeidis and Papageorgiou (2003, 2004) for A= 0,5 m/s,  $T_p$ = 2s, v=0 and  $\gamma$ =1,  $\gamma$ =2,  $\gamma$ =3.

The Type A harmonic pulse is expressed by Makris and Black (2004):

$$a_g(t) = \frac{\omega_p \cdot v_p}{2} \sin(\omega_p \cdot t), \qquad 0 \le t \le T_p$$
$$v_g(t) = \frac{v_p}{2} (1 - \cos(\omega_p \cdot t)), \qquad 0 \le t \le T_p \qquad (1)$$

$$d_g(t) = \frac{v_p}{2} \left( t - \frac{\sin(\omega_p \cdot t)}{\omega_p} \right), \qquad 0 \le t \le T_p$$

The pulse suggested by Mavroeidis and Papageorgiou (2003, 2004) is given by:

# 3. Structural model

In order to study the behavior of a base isolated building and to examine the response of both the isolation system and the superstructure, a model with at least two degrees-offreedom (2-DOF) is suggested (Kelly, 1997). Therefore, the present study uses a 2-DOF isolated structure with a linear superstructure, a linear restoring force with stiffness  $k_b$  and both linear and non-linear isolation systems. The mass of the building superstructure and the mass of the base floor above the isolation system are demoted by m and m<sub>b</sub>, respectively. In both cases, the system is characterized by the parameters:  $T_s$ ,  $\xi s$ ,  $\lambda$ ,  $T_b$  and additionally  $b_y \xi_b$  for the linear isolation system with viscous damping or by the yielding force Q and the yielding displacement u<sub>y</sub> for the non-linear isolation system, defined as:

$$T_s = \frac{2\pi}{\omega_s} \tag{3}$$

and 
$$\omega_s = \sqrt{\frac{K_s}{m}}$$
 (4)

$$2\xi_s \omega_s = \frac{c_s}{m} \tag{5}$$

and 
$$2\xi_b \omega_b = \frac{c_b}{m + m_b}$$
 (6)

$$v_{g}(t) = \begin{cases} A \frac{1}{2} \cdot \left[ 1 - \cos\left(\frac{2\pi \cdot f_{p}}{\gamma} \cdot t\right) \right] \cdot \cos\left[2\pi \cdot f_{p} \cdot t - \pi \cdot \gamma + v\right], & 0 \le t \le \frac{\gamma}{f_{p}} \text{ with } \gamma > 1 \\ 0, \text{ otherwise} \end{cases}$$

$$\alpha_{g}(t) = \begin{cases} \frac{A \cdot \pi \cdot f_{p}}{\gamma} \cdot \sin\left(\frac{2\pi \cdot f_{p}}{\gamma} \cdot t\right) \cdot \cos\left[2\pi \cdot f_{p} \cdot t - \pi \cdot \gamma + v\right], & 0 \le t \le \frac{\gamma}{f_{p}} \text{ with } \gamma > 1 \\ 0, \text{ otherwise} \end{cases}$$

$$(2)$$

$$\lambda = \frac{m}{m + m_b} \tag{7}$$

$$T_b = \frac{2\pi}{\omega_b} \tag{8}$$

and 
$$\omega_b = \sqrt{\frac{K_b}{m + m_b}}$$
 (9)

The equations of motion for the 2-DOF shown in Figure 2 is given:

$$\begin{bmatrix} 1 & \lambda \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \ddot{u}_{b}(t) \\ \ddot{u}_{s}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 2\xi_{s}\omega_{s} \end{bmatrix} \begin{bmatrix} \dot{u}_{b}(t) \\ \dot{u}_{s}(t) \end{bmatrix} + \begin{bmatrix} \omega_{b}^{2} & 0 \\ 0 & \omega_{s}^{2} \end{bmatrix} \begin{bmatrix} u_{b}(t) \\ u_{s}(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} a(t) = -\begin{bmatrix} 1 \\ 1 \end{bmatrix} \ddot{u}_{g}(t)$$
(10)

where:

 $u_s$  = relative displacement of the superstructure  $u_b$  = relative displacement of the isolation system

$$a(t) = \frac{P(t)}{m + m_b} \tag{11}$$

and P(t) is the dissipation force (given by:)

$$P(t) = c_b \cdot \dot{u}_b(t) \tag{12}$$

for viscous of damping or

$$P(t) = Q \cdot z(t) \tag{13}$$

for hysteretic damping, where the hysteretic dimensionless quantity z(t) is given in

$$u_{y} \cdot \dot{z} + \gamma \cdot \dot{u}_{b}(t) \cdot z \cdot |z|^{n-1}$$
  
+  $\beta \cdot \dot{u}_{b}(t) \cdot |z|^{n} - \dot{u}_{b}(t) = 0$  (14)

The Equations (13) and (14) describe the Bouc-Wen model (Wen 1976) for elastoplastic behavior with smooth transition by setting  $\beta = \gamma = 0.5$  and n = 20 (Wen 1976). The response of the 2-DOF structure is computed numerically in MATLAB using standard ordinary differential equation solvers (MATLAB 2005). In this study it is assumed that  $\xi_s=2\%$ .

# 4. Dimensional Analysis of a 2-Dof Model with Linear Isolation System

In this paper the dimensional analysis (Langhaar 1951, Barenblatt 1996) used also by Makris and Black (2004) for a linear, rigid-plastic, elastoplastic and bilinear SDOF oscillator is employed in order to investigate the response of a 2-DOF isolated model. By using the harmonic pulses expressed by Equation (1), adequately described by the parameters  $T_p$  (or  $\omega_p$ ) and  $v_p$  (or  $a_p$ ), one may specify the minimum number of parameters that describe the problem. Therefore, the maximum relative displacement of the superstructure,  $u_s$ , the maximum relative displacement of the isolation system,  $u_b$ , and the maximum structural acceleration, as, as well, will be functions of the following seven variables:

$$u_{s,max}, u_{b,max}, a_{s,max} = f(m, m_b, K_s, K_b, \xi_b, \omega_p, v_p) \quad (15)$$

It is noted that the function f is different for  $u_{s,max}$ ,  $u_{b,max}$ ,  $a_{s,max}$  but the variables are the same.

The variables appearing in Equation (15),  $u_{s,max}$ ,  $m_i$ ,  $K_i$ ,  $\omega_p$ ,  $v_p$  involve all three reference quantities, that is, the mass [M], length [L] and time [T]. According to Buckingham's  $\Pi$ -theorem (Langhaar 1951, Barenblatt 1996), the number of independent dimensionless  $\Pi$ -terms for the 2-DOF model with a linear isolation system will be 8 (variables)-3(reference dimensions) = 5. The  $\Pi$ -terms are:

$$\Pi_1 = \frac{u_{s,max} \cdot \omega_p}{v_p}, \frac{u_{b,max} \cdot \omega_p}{v_p}, \frac{a_{s,max}}{\omega_p \cdot v_p}$$
(16)

$$\Pi_1 = \Phi(\Pi_2, \Pi_3, \Pi_4, \Pi_5) \tag{17}$$

$$\Pi_2 = \frac{\omega_b}{\omega_p} \left( = \frac{T_p}{T_b} \right) \tag{18}$$

$$\Pi_3 = \lambda = \frac{m}{m + m_b} \tag{19}$$

$$\Pi_4 = \frac{K_b}{K_s} \tag{20}$$

$$\Pi_5 = \xi_b \tag{21}$$

At this point, it is worth mentioning that the  $\Pi$ -terms that describe the response of 2-DOF include the  $\Pi$ -terms used by Makris and Black (2004) for the response of the linear SDOF but additional dimensionless parameters  $\Pi_3$  and  $\Pi_4$ .

As mentioned before, for the mathematical model proposed by Mavroeidis and Papageorgiou (2003) there exist two additional parameters v and  $\gamma$  which will be set as  $\Pi_6 = v$  and  $\Pi_7 = \gamma$ . In the next graphs use will be made of this model. Makris and Black (2004) showed that for the harmonic pulses the self-similar solutions of the SDOF oscillator scale better with the peak pulse acceleration ap rather than with the peak pulse velocity, indicating that peak pulse acceleration is a superior intensity measure of the pulse excitation. In this paper however, the peak velocity pulse (A=v<sub>p</sub>) for normalization will be used, as it was originally presented in (Mavroeidis *et al.* 2004).

Figure 3 shows the variation of the normalized response  $(u_b, u_s, a_s)$  of the base-isolated 2-DOF structure with viscous damping as a function of  $\Pi_2=T_p/T_b$  for various values of v and  $\gamma$ , as was initially done for the SDOF (Mavroeidis *et al.* 2004). This figure shows that the parameter v has a significant effect only on the normalized  $u_b$  and  $u_s$  when  $\gamma$  approaches the value of 1 and then only for small values of the term  $\Pi_2$  (<0.7). In the other cases v has an insignificant effect and this influence diminishes as  $\gamma$  increases. In addition, the increase of  $\gamma$ , which means more half-cycles in the pulse, results in an increase of the peak normalized response characteristics of the 2-DOF attributed to resonance, a fact that is in agreement with the respective observations made for the SDOF (Mavroeidis *et al.* 2004, Chopra 2007). Finally, it is observed that as  $\gamma$ 



Fig. 3 Variation of the normalized response (ub, us, as) of the base-isolated 2-DOF structure with viscous damping as a function of  $\Pi 2=Tp/Tb$  for various values of v and  $\gamma$ 

increases the peak value of the normalized acceleration, as occurs for smaller values of  $\Pi_2$ . For example for  $\gamma \approx 1$  the peak value of normalized displacement as is achieved for  $\Pi_2 \approx 1.8$ s, but for  $\gamma = 3$  the peak value of the normalized as corresponds to  $\Pi_2 \approx 1.15$ s. It is worth mentioning that the graphs of the normalized  $u_b$  and as of the 2-DOF plotted against  $T_p/T_b$  look similar to the ones of the linear SDOF plotted against  $T_p/T_s$  in the paper of Mavroeidis and Papageorgiou (2004).

Figure 4 illustrates the influence of the dimensionless terms  $\Pi_3$ ,  $\Pi_4$ ,  $\Pi_5$  on the normalized response ( $u_b$ , $u_s$ , $a_s$ ) of the base-isolated 2-DOF structure with viscous damping presented as function of  $\Pi_2$  for  $\gamma \approx 1$  and v=0. In particular, it is shown that the dimensionless term  $\Pi_3$  does not influence the normalized displacement of the isolation system (u<sub>b</sub>) and the total acceleration of the superstructure (a<sub>s</sub>), but to a small degree it influences the normalized displacement of the superstructure (u<sub>s</sub>) for values of  $\Pi_2 < 2.5$ by reducing the normalized  $(u_s)$  when  $\Pi_3$  decreases . Similarly, the dimensionless term  $\Pi_4$  has a minor effect on the normalized response characteristics  $(u_b, a_s)$ , but a great effect on (us). For example, for  $\Pi_2=1$  an increase of  $\Pi_4$ from 0.1 to 0.2 results in a 50% reduction of the normalized us. As for the dimensionless term  $\Pi_5$ , it seems to have a more complex role since an increase of  $\Pi_5$  always decreases the normalized  $(u_b)$ , but it can both increase or decrease the normalized response characteristics  $(u_s, a_s)$  depending on the value of  $\Pi_2$ . Specifically, in the case of  $\gamma \approx 1$  for approximately  $0.8 < \Pi_2 < 4$ , the normalized response characteristics  $(u_s,a_s)$  seem to decrease as  $\Pi_5$  increases but for approximately  $\Pi_2 = <0.7$  the opposite happens.

From Figure 4 it can be concluded that the dimensionless parameters that play a key role on the dynamic behavior of the base-isolated 2-DOF structure with viscous damping under idealized near-fault pulses are the terms  $\Pi_2$  and  $\Pi_5$ , while the normalized (u<sub>s</sub>) is also greatly affected by  $\Pi_4$  and much less by  $\Pi_3$ . In all these cases, however, the influence is significant only for values of  $\Pi_2$  <3. These observations were made using the idealized pulse with  $\gamma \approx 1$  but we have verified them also for greater values of  $\gamma$  (= 2 and 3) which, however, will not be presented here because of limited space.

In Figure 5 the influence of the dimensionless term  $\Pi_5$ on the normalized response  $(u_b, u_s, a_s)$  of the base-isolated 2-DOF structure with viscous damping presented as function of  $\Pi_2$  for v=0 and  $\gamma=1, 2, 3$  is examined in detail. It is shown that the influence of the dimensionless term  $\Pi_5$  on the normalized response  $(u_b, u_s, a_s)$  of the 2-DOF is greater as  $\gamma$  increases, a remark similar to the one for the SDOF (Jacobsen and Ayre 1958, Mavroeidis et al. 2004, Chopra 2007). For example, an increase of  $\Pi_5$  from 5% to 30% leads to 40% reduction of the peak normalized as for  $\gamma \approx 1$ and 53% reduction for  $\gamma=3$ . In addition, it seems that the effect of the dimensionless term  $\Pi_5$  is similar for the different values of  $\gamma=1, 2, 3$ , since for all three cases there exists a range of values of  $\Pi_2$  where an increase of  $\Pi_5$  will decrease the normalized response characteristics  $(u_s, a_s)$  and another range of values of  $\Pi_2$  where  $\Pi_5$  will have the opposite effect on (u<sub>s</sub>,a<sub>s</sub>). This behavior implies the existence of an optimum value of the dimensionless term  $\Pi_5$ , namely, the viscous damping  $\xi_b$ , depending on the value of  $\Pi_2$ , that will minimize the normalized response



Fig. 4 Influence of the dimensionless terms  $\Pi 3$ ,  $\Pi 4$ ,  $\Pi 5$  on the normalized response (ub, us, as) of the base-isolated 2-DOF structure with viscous damping presented as functions of  $\Pi 2=Tp/Tb$  for  $\gamma=1$  and v=0

y = 2



 $\gamma = 3$ 



Fig. 5 Influence of the dimensionless terms  $\Pi 5=\xi b$  on the normalized response (ub, us, as) of the base-isolated 2-DOF structure with viscous damping presented as functions of  $\Pi 2=Tp/Tb$  for v=0 and  $\gamma=1,2,3$ .



Fig. 6 Variation of optimum  $\xi$ b as a function of  $\Pi 2=Tp/Tb$  for  $\Pi 4=0.1$ , v=0,  $\gamma=1,2,3$  and various values of  $\Pi 3$ 

characteristics of the 2-DOF. The existence of the optimum  $\xi_b$  has also been observed by other researchers (Jangid and Kelly 2001, Alhan and Gavin 2004). In this study the influence of the dimensionless  $\Pi$ -terms on the optimum  $\xi_b$  is further investigated.

Based on the aforementioned remarks, it is considered very useful to plot the variation of the optimum  $\xi_b$  as a function of  $\Pi_2$  for  $\gamma=1, 2, 3$  and also examine how the other parameters of the system,  $\Pi_3$  and  $\Pi_4$  influence the value of optimum  $\xi_b$ . This is depicted in Figures 6 and 7. In these figures a minimum value of  $\Pi_5$ , min $\xi b=1\%$  and a maximum value max  $\xi b=70\%$  is considered for each value of  $\Pi 2$  in the range of 0.25 and 2 (which is of most practical interest), and all the damping values between min $\xi_b$  and max $\xi_b$  are applied in order to find the optimum  $\xi_b$  for each one of the normalized response  $(u_b, u_s, a_s)$  separately, of the baseisolated 2-DOF. Thus, Figure 6 illustrates that the optimum  $\xi_b$  for minimizing the normalized (u<sub>b</sub>) is always the max $\xi_b$ (except some few small values of  $\Pi_2 < 0.4$  for  $\gamma=3$ ) such a trend is in agreement with the previous conclusions, that is, the normalized (u<sub>b</sub>) decreases as the damping  $\xi_b$  increases. On the other hand, the optimum  $\xi_b$  for minimizing the normalized ( $u_s$  and  $a_s$ ) is different from the max $\xi_b$  for a certain range of values of  $\Pi_2$ . For instance, in the case of the pulse with  $\gamma \approx 1$ , for approximately  $1.1 < \Pi_2 < 2$  the optimum  $\xi_b$  is equal to the max $\xi_b$  (=70% in these study), while  $\xi_b$  for  $\Pi_2 < 0.4$  is equal to the min $\xi_b$  (=1%). For 0.4<  $\Pi_2 < 1.1$  the optimum  $\xi_b$  is somewhere between the min $\xi_b$  and the max $\xi_b$ , increasing monotonically from the min $\xi_b$  to the max $\xi_b$  as the normalized frequency  $\Pi_2$  increases from 0.4 to 1.1.

Moreover, as  $\gamma$  increases, the range of values of  $\Pi_2$  where the optimum  $\xi_b$  is somewhere between the min $\xi_b$  and the max $\xi_b$  is reduced; thus, being approximately 0.4<  $\Pi_2$ <1.1 for  $\gamma \approx 1$  and  $0.4 < \Pi_2 < 0.8$  for  $\gamma = 3$ . In addition, the increase of  $\gamma$  also leads to an increase of the value of optimum  $\xi_b$ . For example, for  $\Pi_2=0.75$  in the case of  $\gamma\approx 1$  and  $\gamma=3$  the optimum  $\xi_b$  is approximately 18% and 52%, respectively. Thus, for a certain value of  $\Pi_2$  a value of  $\xi_b$  may be optimum for reducing ( $u_s$ , $a_s$ ), when for example  $\gamma \approx 1$ , but it is not the optimum for a different  $\gamma$ , a fact that makes even more complex the selection of a suitable viscous damping for the isolation system in a building near a fault. Also, from Figure 6 and 7, respectively, it can be concluded that the dimensionless parameter  $\Pi_3$  has an immaterial effect on the optimum  $\xi_b$ , but the dimensionless parameter  $\Pi_4$  has a greater effect than  $\Pi_3$  in the range of values of  $\Pi_2$  where the optimum  $\xi_b$  is between the min $\xi_b$  and the max $\xi_b$ . In all cases it is obvious that the optimum  $\xi_b$  and the variation of the optimum  $\xi_b$ , as a function of the normalized frequency  $\Pi_2$ , is the same for the normalized displacement and the acceleration of the superstructure  $(u_s \text{ and } a_s)$ . These examples have demonstrated that the optimum  $\xi_b$  is highly dependent on  $\Pi_2$  and, also, on the number of half-cycles  $\gamma$ in a certain range of the  $\Pi_2$  variation.

# 5. Dimensional Analysis of a 2-Dof Model with Non-Linear Isolation System

It is known that when damping emanates from nonlinear mechanisms, such as friction or yielding, the response



Fig. 7 Variation of optimum  $\xi$ b as a function of  $\Pi 2=Tp/Tb$  for  $\Pi 3=0.5$ , v=0,  $\gamma=1,2,3$  and various values of  $\Pi 4$ 

depends on the amplitude of the excitation (Alhan and Gavin 2004) which implies that different normalized response curves of the base-isolated 2DOF structure would be obtained for different values of the peak ground velocity A of the pulse. Therefore, it would not be easy to deduct general conclusions for a nonlinear isolation system. Nevertheless, Makris and Black (2004), showed that the use of dimensionless II-terms defined from dimensional analysis have the great advantage to provide self-similar solutions independent of the peak ground velocity A (or acceleration) of the pulse, even in the case of non-linear damping. Therefore, using dimensional analysis, as presented for the linear isolation system, it is found that the dimensionless II-terms that describe the 2DOF with the bilinear isolation system (where we set  $\xi_b=0$ ) for the simple harmonic pulses are:

$$\Pi'_{1} = \frac{u_{s,max} \cdot \omega_{p}}{v_{p}}, \frac{u_{b,max} \cdot \omega_{p}}{v_{p}}, \frac{a_{s,max}}{\omega_{p} \cdot v_{p}}$$
(22)

$$\Pi'_{1} = \Phi(\Pi_{2}, \Pi_{3}, \Pi_{4}, \Pi_{5})$$
(23)

$$\Pi'_{5} = \frac{Q}{(m+m_{b}) \cdot \omega_{p} \cdot V_{p}}$$
(24)

$$\Pi_6 = \frac{u_y \cdot \omega_p}{v_p} \tag{25}$$

It should be mentioned that for the case of the mathematical model of Mavroeidis and Papageorgiou (2003), the dimensionless terms  $\Pi 7 = v$  and  $\Pi 8 = \gamma$  should be added. Notice that in Equations (22), (23) and (24) prime has been used to differentiate the  $\Pi i(i=1,5)$  terms from the ones expressed by Equations (16), (17) and (21).

Figure 8 shows the variation of the normalized response (u<sub>b</sub>,u<sub>s</sub>,a<sub>s</sub>) of the base-isolated 2-DOF structure with nonlinear damping as a function of  $\Pi_2$  for various values of v and  $\gamma$ . Also, Figure 10 demonstrates that the parameter v has a minor effect on the normalized response (ub,us,as) of the base-isolated 2-DOF structure for small values of the dimensionless term  $\Pi'_5(=0.1)$ , except for some small values of the dimensionless term  $\Pi_2 < 0.7$ , where the normalized (u<sub>b</sub>) is affected by the variation of v. However, for larger values of  $\Pi'_{5}(=0.5)$  the parameter v seems to have a greater influence on the normalized response characteristics  $(u_{b}, a_{s})$ . This influence is diminished as the parameter  $\gamma$  increases. In addition, it is observed that for small values of the dimensionless term  $\Pi'_5=0,1$  as  $\gamma$  increases, there appear larger peak normalized response characteristics  $(u_b, u_s, a_s)$ , as in the case of the linear isolation system. It should be noted this is not the case for larger value of  $\Pi'_5=0.5$ , which implies that, as the dimensionless term  $\Pi'_5$  increases, the effect of  $\gamma$  diminishes.

Figure demonstrates the influence of the 9 dimensionless terms  $\Pi_3$ ,  $\Pi_4$ ,  $\Pi'_5$ ,  $\Pi_6$  on the normalized response (u<sub>b</sub>, u<sub>s</sub>, a<sub>s</sub>) of the base-isolated 2-DOF structure with nonlinear damping presented as functions of  $\Pi_2$  for  $\gamma \approx 1$  and v=0. In particular, it is shown that for both small and large values of the dimensionless term  $\Pi'_5$ (=0.1 and 0.5, respectively) the dimensionless term  $\Pi_3$  does not influence the normalized displacement of the isolation system  $(u_b)$  and the total acceleration of the superstructure (a<sub>s</sub>), but it affects the normalized displacement of the superstructure (u<sub>s</sub>) to a small degree and only for values of  $\Pi_2$  <2.5, by reducing the normalized (u<sub>s</sub>) when  $\Pi_3$ decreases. Similarly, the second column of the same figure shows the negligible effect of the dimensionless term  $\Pi_4$  on



Fig. 8 Variation on the normalized response (ub, us, as) of the base-isolated 2-DOF structure with non-linear damping as a function of  $\Pi 2=Tp/Tb$  for various values of v and  $\gamma$ 

 $\gamma \approx 1$ 



Fig. 9 Influence of the dimensionless terms  $\Pi 3$ ,  $\Pi 4$ ,  $\Pi' 5$ ,  $\Pi 6$  on the normalized response (ub, us, as) of the base-isolated 2-DOF structure with non-linear damping presented as functions of  $\Pi 2=Tp/Tb$  for  $\gamma=1$  and v=0



Fig. 10 Influence of the dimensionless terms  $\Pi$ '5 on the normalized response (ub, us, as) of the base-isolated 2-DOF structure with non-linear damping presented as functions of  $\Pi$ 2=Tp/Tb for characteristic values of  $\Pi$ 3,  $\Pi$ 4,  $\Pi$ 6 and v=0,  $\gamma$ =1,2,3

the normalized response characteristics  $u_b$  and  $a_s$  for small values of the dimensionless term  $\Pi'_5 \leq 0,1$  and again only for values of  $\Pi_2 < 2.5$ . However, for  $\Pi'_5 = 0.5$  one can observe that  $\Pi_4$  has a noticeable influence on the normalized superstructure acceleration as for  $\Pi_2$  2.5 which implies that the greater the value of  $\Pi'_5$  is, the greater the

influence of  $\Pi_4$  on as. Irrespective of the value of  $\Pi_5$ , though, for values of  $\Pi_2 < 2.5$  the dimensionless term  $\Pi_4$  has a decisive effect on the normalized  $u_s$   $b_y$  causing a significant increase of  $(u_s)$  when it increases. As for the dimensionless term  $\Pi'_5$  it seems to be the dominant factor in influencing the normalized response of the 2-DOF. It is quite interesting the fact that the dimensionless term  $\Pi_5$  can either increase or decrease the normalized response  $(u_s, a_s)$ depending on the value of  $\Pi_2$ .

It is interesting to note that there exists a range of values of the dimensionless term  $\Pi_2$  around the value of  $\Pi_2=0.5$ where an increase in the normalized strength  $\Pi'_5$  for certain values of  $\Pi'_5$ , increases the normalized isolation displacement u<sub>b</sub>, a bahavior that could be regarded as 'counterintuitive'. In the third column of Figure 10 particularly, in the range of  $\Pi'_2=0.5$  greater values of u<sub>b</sub> are observed for  $\Pi'_5=0.7$  and 0.9 than those for  $\Pi'_5=0.5$ . Finally, the fourth column of Figure 10 illustrates the insignificant effect of the normalized yielding displacement  $\Pi_6$  of the nonlinear isolation system on the normalized maximum response of the 2-DOF (u<sub>b</sub>,u<sub>s</sub>,a<sub>s</sub>) for small values of  $\Pi_5 \leq 0.1$ . This observation shows that the conclusion made by Makris and Black (2004) using a bilinear SDOF that 'under a strong earthquake an isolated bridge will exhibit the same maximum displacement, regardless if it is supported on lead-rubber bearings or friction pendulum bearings that exhibit the same strength and offer the same isolation period', can be also extended to a base-isolated building when  $\Pi'_5 \leq 0.1$ . For  $\Pi'_5 = 0.5$ . Although the effect of  $\Pi_6$  on us is still insignificant, its effect on u<sub>b</sub> and as becomes noticeable. In particular, it is worth noting that for  $\Pi_2$  2 the normalized response (u<sub>b</sub>, a<sub>s</sub>) increases as  $\Pi_6$  increases but for  $\Pi_2$  2 the opposite occurs. This trend reveals that for large values of  $\Pi'_5$  the dimensionless term  $\Pi_6$  can increase or decrease the normalized response (u<sub>b</sub>, a<sub>s</sub>) depending on the value of  $\Pi_2$ .

From Figure 9 it can be concluded that the dimensionless parameters that primarily affect the dynamic behavior of the base-isolated 2-DOF structure with nonlinear damping under idealized near-fault pulses are the terms  $\Pi_2$  and  $\Pi'_5$  while the normalized (u<sub>s</sub>) is also greatly affected by  $\Pi_4$  and less by  $\Pi_3$ . However, as  $\Pi'_5$  increases,  $\Pi_6$  also begins also to have a noticeable effect on u<sub>b</sub> and a<sub>s</sub>. Even though these observations were made using the idealized pulse with  $\gamma \approx 1$  the authors have verified them also for greater values of  $\gamma$  (= 2 and 3).

Figure 10 presents the influence of the dimensionless term on the normalized response  $(u_b, u_s, a_s)$  for the different



Fig. 11 Influence of the dimensionless terms  $\Pi 6$  on the normalized response (ub, us, as) of the base-isolated 2-DOF structure with non-linear damping presented as functions of  $\Pi 2=Tp/Tb$  for characteristic values of  $\Pi 3$ ,  $\Pi 4$ ,  $\Pi 3$  and v=0,  $\gamma=1,2,3$ 

number of half-cycles of the idealized pulses ( $\gamma$ =1, 2, 3). The significant role of  $\Pi'_5$  is verified, irrespective of the number of half-cycles. For all values of  $\gamma$  there exists a range of values of  $\Pi_2$ , where an increase of  $\Pi'_5$  will increase the normalized response ( $u_s, a_s$ ) and another range of values of  $\Pi_2$  where  $\Pi'_5$  will have the opposite effect on ( $u_s, a_s$ ). Figure 10 also reveals the existence of an optimum value of the normalized strength  $\Pi'_5$ , depending on the value of  $\Pi_2$ , that will minimize the normalized response characteristics of the 2-DOF. The last conclusion is in agreement with the findings of other researchers that used recorded ground motions and the dimensionless term (Alhan and Gavin 2004, Jangid 2007) or other parameterization for the nonlinear isolation system (Jangid 2005) in their studies.

In Figure 11 the influence of the dimensionless term on the normalized response (u<sub>b</sub>,u<sub>s</sub>,a<sub>s</sub>) for different numbers of half-cycles of the idealized pulses ( $\gamma$ =1, 2, 3) is presented. The role of  $\Pi_6$  described previously is verified here, irrespective of the number of half-cycles. In addition, this figure clearly shows that the range of values of  $\Pi_2$ , where an increase of  $\Pi_6$  results in the increase of u<sub>b</sub> and a<sub>s</sub>, becomes smaller as  $\gamma$  increases, thus, being approximately  $\Pi_2 < 1.8$  for  $\gamma \approx 1$  and  $\Pi_2 < 1.2$  for  $\gamma = 3$ . This means that there exists an optimum value of  $\Pi_6$  depending again on the value of  $\Pi_2=T_p/T_b$  that can minimize the normalized response of the 2-DOF, especially in the case of large values of  $\Pi_5$ .

Use of Figures 10 and 11 can help one choose the

appropriate combination of values of  $\Pi'_5$  and  $\Pi_6$ . Therefore, the appropriate yield strength and yield displacement of the isolation system that will most effectively minimize the response of the 2-DOF under an idealized near-fault pulse characterized by  $\omega_p$  and A.

#### 6. Comparison of the Fixed SDOF Model with the Isolated 2-Dof Model

The last part of this study examines that response of a fixed base building (SDOF) as well as the same building that is base isolated with linear or non-linear damping, both subjected to idealized pulses. As it is known, the dimensionless parameter  $T_p/T_s$  plays a key role on the dynamic behavior of the SDOF (Mavroeidis et al. 2004), and respectively, the dimensionless parameter  $T_p/T_b$  plays a key role on the dynamic behavior of the isolated 2-DOF. However, using the parameter  $T_p/T_s$  allows plotting the response of both structures on the same graph. Thus, Figure 12 presents the normalized response characteristics  $(u_s, a_s)$  of the fixed SDOF and the isolated 2-DOF as a function of the dimensionless parameter  $T_p/T_s$  ,where Ts is the period of the fixed SDOF. Inspection of this figure shows the vulnerability of the isolated building compared to the fixed one, as first revealed by other researchers, e.g., 7, 8. In particular, it is observed that, depending on the value of the parameter  $T_p/T_s$  and the values of the dimensionless terms  $\Pi_3$ ,  $\Pi_4$ ,  $\Pi_5$ ,  $\Pi_6$  that define the characteristics of the isolation



Fig. 12 Comparison of the response of (i) the fixed base structure; (ii) the base isolated structure with viscous damping and (iii) the base isolated structure with nonlinear damping

system, for the same superstructure and idealized pulse, base isolation can have a beneficial or a detrimental effect on the response of the structure. For instance, for  $\gamma=2$ , and  $T_p/T_s \le 2$  the isolation system reduces significantly the normalized acceleration  $a_s$ , but for  $T_p/T_s>3$  the opposite occurs and, even for highly damped systems with 30% viscous damping, the normalized acceleration of the structure as is amplified compared to the fixed structure. In addition, it is noted the fact that even the normalized displacement of the isolated superstructure us for certain values of T<sub>p</sub>/T<sub>s</sub> and for certain values of the characteristics of the isolation system can be greater than the respective one of the fixed-base building. This is clearly seen, for example for  $\gamma=3$ , where for  $T_p/T_s>2.5$  the 2-DOF with viscous damping  $\Pi_5=5\%$  and the one with nonlinear damping with  $\Pi'_5=0.1$  and  $\Pi_6=0.1$  that give greater normalized displacement u<sub>s</sub> than the fixed base SDOF.

# 7. Conclusions

In this study the dynamic behavior of a 2-DOF baseisolated structure under idealized near-fault pulses is investigated. Both the linear and non-linear behavior of the isolation system is considered and the suitable dimensionless parameters of the isolation system are defined. Using the dimensional analysis in conjunction with closed form mathematical idealized pulses, self-similar curves are plotted on dimensionless graphs and various conclusions are drawn: Respectively to the case of SDOF (Mavroeidis *et al.* 2004), it is also observed that for the base-isolated 2-DOF structure the parameter v has a minor effect on the peak normalized response of the structure for both the linear and the non-linear system, except for the case where  $\gamma$  approaches unity and then only for small values of the term  $\Pi_2 < 0.7$ . The effect of v diminishes as the parameter  $\gamma$  increases. In addition, an increase of  $\gamma$  results in larger peak normalized response for both the linear and non-linear isolation systems with small normalized yield strength  $\Pi'_5 \leq 0.1$ ; however, the effect of  $\gamma$  diminishes as increases.

For the linear isolation system, it is revealed that: (a) the dimensionless parameters that play a significant role on the dynamic behavior of the base-isolated 2-DOF structure under idealized near-fault pulses are the pulse-to- base isolation period ratio  $\Pi_2$  and the viscous damping  $\Pi_5$ , while (b) the peak normalized relative displacement of the superstructure is also greatly affected by the isolation-tostructure stiffness ratio  $\Pi_4$  and much less by the mass ratio  $\Pi_3$ . However, the influence of  $\Pi_4$  and  $\Pi_5=\xi_b$  on the superstructure displacement is significant only for  $\Pi_2=T_p/T_b$ <3. Regarding the peak normalized isolation displacement and superstructure acceleration; they are both insensitive to the stiffness and mass ratios. Furthermore, it is shown that the viscous damping  $\xi_b$  can have either a beneficial or a detrimental effect on the peak normalized displacement us and the acceleration as of the superstructure, depending on the value of  $\Pi_2$ . In addition, the existence of an optimum value of the viscous damping  $\xi_b$  that was first revealed by

For the nonlinear isolation system, the dimensionless parameters that play a major role on the dynamic behavior of the base-isolated 2-DOF structure under idealized nearfault pulses are: (a) the pulse-to-base isolation period ratio  $\Pi_2$  and (b) the normalized yield strength  $\Pi'_5$ , while the normalized superstructure relative displacement is also primarily affected by the stiffness ratio  $\Pi_4$  and much less by the mass ratio  $\Pi_3$ . The normalized yield displacement  $\Pi_6$ has a very small effect on the response of the 2-DOF for small values of  $\Pi_5 \leq 0.1$ , and only for greater values of  $\Pi_5$ the term  $\Pi_6$  begins to have a noticeable effect on the peak normalized isolation displacement and on the superstructure acceleration. This effect can be either detrimental or beneficial to the normalized isolation displacement and superstructure acceleration, depending on the value of  $\Pi_2$ . Respectively, to the effect of damping on linear isolation systems, on non-linear systems the increase of the normalized strength  $\Pi_5$  can also either increase or decrease normalized the superstructure displacement and acceleration depending on the value of  $\Pi_2$ , a fact that implies the existence of an optimum normalized strength  $\Pi_5$ for each value of  $\Pi_2$ . In addition, for the isolated 2-DOF, there exists a range of values of the dimensionless term  $\Pi_2$ around the value of  $\Pi_2=0.5$  and a certain range of values of the normalized strength  $\Pi_5$ , where an increase in the normalized strength  $\Pi_5$  increases the normalized isolation displacement ub. Knowing the influence of the normalized yield strength and yield displacement  $\Pi_5$  and  $\Pi_6$  on a response of the base-isolated building, as a function of the normalized period  $\Pi_2$  and the number of half-cycles  $\gamma$ , can help one to choose the optimum combination of values of  $\Pi_5$  and  $\Pi_6$  for specific values of  $T_p/T_b$  and  $\gamma$ , as well as the appropriate yield strength and yield displacement of the isolation system that will most effectively minimize the response characteristics of the 2-DOF under a certain velocity pulse.

Finally, a comparison of the responses between a linear fixed-base SDOF and an isolated 2-DOF with both linear and non-linear damping has shown that, under certain values of the superstructure and isolation system characteristics, the use of an isolation system can amplify both the normalized acceleration as and the displacement of the superstructure  $u_s$ . This fact points out the need for careful design of base-isolated buildings that can be potentially subjected to strong velocity pulses.

In this study a simple 2-DOF model has been used for the base-isolated building. The simpler case of a non isolated 2-dof system as well as more complex and detailed models accounting for torsional response, for example models with more stories that could also account for more physical phenomena should be developed and studied to thoroughly investigate the response of real structural systems. Definitely, more research is required in this field.

#### References

- Alhan, C. and Gavin, H. (2004), "A parametric study of linear and non-linear passively damped seismic isolation systems for buildings", *Eng. Struct.*, **26**, 485-497, https://doi.org/10.1016/j.engstruct.2003.11.004.
- Ariga, T., Kanno, Y. and Takewaki, I. (2006), "Resonant Behaviour of Base-Isolated High-Rise Buildings under longperiod ground motions", *Struct. Design Tall Spec. Build.*, 15, 325-338, https://doi.org/10.1002/tal.298.
- Buckle, I.G. and Mayes, R.L. (1990), "Seismic isolation history, application and performance—A world view", *Earthq. Spectra*, **6**, 161-201. https://doi.org/10.1193/1.1585564
- Buckle, I., Nagarajaiah, S. and Ferrell, K. (2002), "Stability of elastomeric isolation bearings: Experimental study", *J. Struct. Eng.*, **128**(1), 3-11. https://doi.org/10.1061/(ASCE)0733-9445(2002)128:1(3)
- Barenblatt, G. I. (1996), *Scaling, Self-Similarity, and Intermediate Asymptotics*, Cambridge University Press, Cambridge, United Kingdom.
- Castaldo, P. and Tubaldi, E. (2018), "Influence of ground motion characteristics on the optimal single concave sliding bearing properties for base-isolated structures", *Soil Dyn. Earthq. Eng.*, 104, 346-364, https://doi.org/10.1016/j.soildyn.2017.09.025.
- Chen, H., Tan, P., Ma, H. and Zhou, F. (2017). "Response spectrum analysis considering non-classical damping in the baseisolated benchmark building", *Struct. Eng. Mech.*, 64(4), 473-485, https://doi.org/10.12989/sem.2017.64.4.473.
- Chopra, A.K. (2007), *Dynamics of Structures: Theory and Applications to Earthquake Engineering* (3<sup>rd</sup> edition), Prentice-Hall, New Jersey, USA.
- Mazza, F. and Vulcano, A. (2009), "Nonlinear Response of RC Framed Buildings with Isolation and Supplemental Damping at the Base Subjected to Near-Fault Earthquakes", *J. Earthquake Eng.*, **13**, 690-715, https://doi.org/10.1080/13632460802632302.
- Hall, J.F., Heaton, T.H., Halling, M.W. and Wald, D.J. (1995), "Near-source ground motion and its effects on flexible buildings", *Earthq. Spectra*, **11**(4), 569-605, https://doi.org/10.1193/1.1585828.
- Hall, J.F. (1999), "Discussion on the role of damping in seismic isolation", *Earthq. Eng. Struct. Dyn.*, **28**, 1717-1720, https://doi.org/10.1002/(SICI)1096-9845(199912)28:12<1717::AID-EQE889>3.0.CO;2-G.
- Hall, J.F. and Ryan, K.L. (2000), "Isolated buildings and the 1997 UBC near-source factors", *Earthq. Spectra*, **16**(2), 393-411, https://doi.org/10.1193/1.1586118.
- Heaton, T.H., Hall, J.F., Wald, D.J. and Halling, M.W. (1995), "Response of high-rise and base-isolated buildings to a hypothetical MW 7.0 blind thrust earthquake", *Science*, **267**(5195), 206–211, 10.1126/science.267.5195.206.
- Inaudi, J.A. and Kelly, J.M. (1993), "Optimum damping in linear isolation systems", *Earthq. Eng. Struct. Dyn.*, **22**(7), 583-598, 10.1002/eqe.4290220704.
- Jacobsen, L.S. and Ayre, R.S. (1958), *Engineering Vibrations*, McGraw-Hill, New York, USA.
- Jangid, R.S. and Kelly, J.M. (2001), "Base isolation for near-fault motions", *Earthquake Eng. Struct. Dyn.*, **30**(5), 691-707, https://doi.org/10.1002/eqe.31.
- Jangid, R.S. (2005), "Optimum friction pendulum system for near-fault motions", *Eng. Struct.*, **27**(3), 349-359, https://doi.org/10.1016/j.engstruct.2004.09.013.
- Jangid, R.S. (2007), "Optimum lead-rubber isolation bearings for near-fault motions", *Eng. Struct.*, 2503-2513, https://doi.org/10.1016/j.engstruct.2006.12.010.
- Kampas, G. and Makris, N. (2012), "Time and frequency domain identification of seismically isolated structures: advantages and limitations", *Earthq. Struct.*, **3**(3), 249-270,

https://doi.org/10.12989/eas.2012.3 3.249.

- Kampas, G. and Makris, N. (2013), "The engineering merit of the 'effective period' of bilinear isolation systems, *Earthq. Struct.*", 4(4), 397-428, https://doi.org/10.12989/eas.2013.4.4.397.
- Kelly, J.M. (1997), *Earthquake-resistant Design with Rubber*, 2nd ed., Springer, London, United Kingdom.
- Konstantinidis, D., Nikfar, F. (2015), "Seismic response of sliding equipment and contents in base-isolated buildings subjected to broadband ground motions" *Earthquake Eng. Struct. Dyn.*, 44(6), 865-887, https://doi.org/10.1002/eqe.2490.
- Langhaar, H.L. (1951), Dimensional Analysis and Theory of Models, Wiley, New York, USA.
- Liao, W.I., Loh, C.H. and Lee, B.H. (2004), "Comparison of dynamic response of isolated and non-isolated continuous girder bridges subjected to near-fault ground motions", *Eng. Struct.*, 26, 2173–2183. https://doi.org/10.1016/j.engstruct.2004.07.016.
- Makris, N. (1997), "Rigidity-plasticity-viscosity: Can electrorheological dampers protect base-isolated structures from nearsource ground motions?" *Earthquake Eng. Struct. Dyn.*, 26, 571–591, https://doi.org/10.1002/(SICI)1096-9845(199705)26:5<571::AID-EQE658>3.0.CO;2-6.
- Makris, N. (2014), "A half-century of rocking isolation", *Earthq. Struct.*, **7**(6), 1187-1221, https://doi.org/10.12989/eas.2014.7.6.1187.
- Makris, N. and Chang, S.P. (2000). "Response of damped oscillators to cycloidal pulses", *J. Eng. Mech.*, **126**, 123-131, https://doi.org/10.1061/(ASCE)0733-9399(2000)126:2(123).
- Makris, N. and Chang, S.P. (2000), "Effect of viscous, viscoplastic and friction damping on the response of seismic isolated structures", *Earthq. Eng. Struct. Dyn.*, **29**, 85-107, https://doi.org/10.1002/(SICI)1096-9845(200001)29:1<85::AID-EQE902>3.0.CO;2-N.
- Makris, N. and Black, C.J. (2004), "Dimensional Analysis of Rigid-Plastic and Elastoplastic Structures under Pulse-Type Excitations", J. Eng. Mech., ASCE, **130**(9), 1006-1018, https://doi.org/10.1061/(ASCE)0733-9399(2004)130:9(1006).
- Makris, N. and Black, C.J. (2004). "Dimensional Analysis of Bilinear Oscillators under Pulse-Type Excitations", *J. Eng. Mech.*, ASCE, **130**(9), 1019-1031, https://doi.org/10.1061/(ASCE)0733-9399(2004)130:9(1019).
- Makris, N. and Black, C.J. (2004), "Evaluation of Peak Ground Velocity as a "Good" Intensity Measure for Near-Source Ground Motions", *J. Eng. Mech.*, ASCE, **130**(9), 1032-1044, https://doi.org/10.1061/(ASCE)0733-9399(2004)130:9(1032).
- Makris, N. and Psychogios, T. (2006), "Dimensional response analysis of yielding structures with first-mode dominated response", *Earthq. Eng. Struct. Dyn.*, **35**, 1203–1224, https://doi.org/10.1002/eqe.578.
- MATLAB (2005), The Language of Technical Computing, Version 7, The Mathworks Inc.
- Mavroeidis, G.P. and Papageorgiou, A.S. (2003), "A mathematical representation of near-fault ground motions", *Bull. Seismol. Soc. Am.*, **93**, 1099-1131, https://doi.org/10.1785/0120020100.
- Mavroeidis, G.P., Dong, G. and Papageorgiou, A.S. (2004), "Near-fault ground motions, and the response of elastic and inelastic single-degree-of freedom (SDOF) systems", *Earthquake Eng. Struct. Dyn.*, **33**, 1023-1049, https://doi.org/10.1002/eqe.391.
- Menun, C. and Fu, Q. (2002), "An analytical model for near-fault ground motions and the response of SDOF systems", *Proceedings of the 7th U.S. National Conference on Earthquake Engineering*, Boston, Massachusetts, July 21-25.
- Mylonakis, G. and Reinhorn, A.M. (2001), "Yielding oscillator under triangular ground acceleration pulse", *J. Earthq. Eng.*, **5**(2), 225-251, https://doi.org/10.1080/13632460109350393.
- Mylonakis, G. and Voyagaki, E. (2006), "Yielding oscillator to simple pulse waveforms: numerical analysis and closed-form solutions", *Earthq. Eng. Struct. Dyn.*, **35**(15), 1949-1974, https://doi.org/10.1002/eqe.615.

- Nagarajaiah, S. and Ferrell, K. (1999), "Stability of elastomeric seismic isolation bearings", *J. Struct. Eng.*, **125**(9), 946–954, https://doi.org/10.1061/(ASCE)0733-9445(1999)125:9(946).
- Nagarajaiah, S. and Sun, X. (2000), "Response of base isolated USC hospital building in Northridge earthquake", *J. Struct. Eng.*, **126**(10), 1177–1186, https://doi.org/10.1061/(ASCE)0733-9445(2000)126:10(1177).
- Nagarajaiah, S. and Sun, X. (2001), "Base isolated FCC building: Impact response in Northridge earthquake", J. Struct. Eng., 127(9), 1063–1074, https://doi.org/10.1061/(ASCE)0733-9445(2001)127:9(1063).
- Panchal, V.R. and Jangid, R.S. (2008), "Variable friction pendulum system for near-fault ground motions", *Struct Control Health Monit.*, 15(4), 568-584, https://doi.org/10.1002/stc.216.
- Ramallo, J.C., Johnson, E.A. and Spencer, B.F.Jr. (2002), "Smart Base Isolation Systems", *J. Eng. Mech.*, **128**(10), 1088-1099, https://doi.org/10.1061/(ASCE)0733-9399(2002)128:10(1088).
- Rao, P.B. and Jangid, R.S. (2001). "Performance of sliding systems under near-fault motions", *Nucl. Eng. Des.*, 203(2-3), 259-272, https://doi.org/10.1016/S0029-5493(00)00344-7.
- Sadek, F. and Mohraz, B. (1998), "Semi-active control algorithms for structures with variable dampers", *J. Eng. Mech.*, **124**(9), 981– 990, https://doi.org/10.1061/(ASCE)0733-9399(1998)124:9(981).
- Sahasrabudhe, S. and Nagarajaiah, S. (2005), "Experimental study of sliding base-isolated Buildings with magnetorheological dampers in near-fault earthquakes", J. Struct. Eng., 131(7), 1025-1034, https://doi.org/10.1061/(ASCE)0733-9445(2005)131:7(1025).
- Shen, J., Tsai, M-H., Chang, K.C. and Lee, G.C. (2004), "Performance of a seismically Isolated Bridge under Near-Fault Earthquake Ground Motions", *J. Struct. Eng.*, **130**(6), 861-868.
- Wen, Yi-Kwei (1976), "Method for Random Vibration of Hysteretic Systems", J. Eng. Mech., 102(2), 249-263.
- Yang. J.N. and Agrawal, A.K. (2002), "Semi-active hybrid control systems for nonlinear buildings against near-field earthquakes", *Eng. Struct.*, 24(3), 271-280. https://doi.org/10.1016/S0141-0296(01)00094-3.
- Zhang, Y. and Iwan, W.D. (2002), "Protecting base-isolated structures from near-field ground motion by tuned interaction damper", *J. Eng. Mech.*, **128**(3), 287-295, https://doi.org/10.1061/(ASCE)0733-9399(2002)128:3(287).

CC

#### Nomenclature

The following symbols and abbreviations are used in this paper:

#### Greek Letters

- $\gamma$  Number characterizing the shape of the pulse
- $\lambda$  Dimensionless parameter defined in eq. 19
- $\xi_{h}$  Damping parameter defined in eq. 6
- $\xi_s$  Damping parameter defined in eq. 5
- $\pi$  Archimedes' constant
- $\omega_b$  Circular frequency defiend in eq. 9

 $\omega_s$  Circular frequency defiend in eq. 4

# Latin Letters

- A Velocity pulse amplitude
- a Function of time defined in eq. 11
- $a_g$  Ground acceleration
- $c_s$  Damping parameter defined in eq. 5
- DOF Degree of freedom
  - $d_g$  Ground displacement
  - $K_b$  Lateral stiffness of isolators
  - *K<sub>s</sub>* Lateral stiffness of superstructure
  - m Mass of superstructure
  - $m_b$  Mass of foundation
- SDOF Single degree of freedom
  - $T_b$  Period defined in eq.8
  - $T_s$  Period defined in eq.3
  - $\mathbf{u}_{\mathbf{b}}$  relative displacement of the isolation system
  - us relative displacement of the superstructure
  - v<sub>g</sub> Ground velocity
  - v<sub>p</sub> Velocity amplitude