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Approximate calculation of the static analysis of a lifted stay cable in super-long span cable-stayed bridges

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Abstract. The sag effect of long stay cables is one of the key factors restricting further increase in the span of cable-stayed bridges. Based on the formerly proposed concept of long stay cables lifted by an auxiliary suspension cable in cross-strait cable-stayed bridges, corresponding static approximate calculations and analytical theory based on catenary and parabolic cable configurations are established. Taking a main span 1400 m cable-stayed bridge as the research object, three typical lifting conditions and the whole process of auxiliary cable lifting are analyzed and discussed. The results show that the sag effect is effectively reduced. The support efficiency is only improved when the cables are lifted above the original cable chord. Reduction of the horizontal component force of the cable is limited. The equivalent elastic modulus and the vertical support stiffness of the lifted cables are significantly increased with increased horizontal projection length and not sensitive to the change of the lifting point position. The scheme of lifting the cable to the chord midpoint is more economical because of the less steel required for the auxiliary suspension cable, but its effect on improving the vertical support efficiency is limited. The support efficiency is better when the cable is lifted to the cable end tangential to the original cable chord, but the lifting force and the cross-sectional area of the auxiliary suspension cable are doubled. The approximate calculation results of the lifted cables are very close to the numerical analysis results, which verifies the applicability of the approximation method proposed in this study. The results of parabolic approximation calculations are approximately equal to that of catenary cable geometry. As the parabolic approximation analysis theory of lifted cables is more convenient in mathematical processing, it is feasible to use parabolic approximation analysis theory as the analytical method for the conceptual design of lifted cables of super-long span cable-stayed bridges.

Keywords: cable-stayed bridge; cable sag; approximate calculation analysis; equivalent elastic modulus; cable support efficiency

1. Introduction

The cable-stayed bridge has become one of the fastest-developing, innovating and the most competitive bridge types in modern bridge engineering due to its superior spanning ability, reasonability in terms of mechanical performance and structural behavior, novel structural form and beautiful geometry (Gimsing 2005). As one of the most important members of the cable-stayed bridge, the cable takes up most of the structural loads. However, due to the characteristics of small stiffness and low strength-to-density ratio of the cable, the sag effect is very prominent. The sag effect caused by the self-weight of a long stay cable is one of the main factors limiting the further development of the span of the cable-stayed bridge (Starossek 1996). With the increased span of cable-stayed bridges, the cable's own weight and horizontal projection length increase, such that the sag effect becomes more obvious and there is a significant decrease in cable stiffness, which leads to an increase in the deflection and stress of the girder. In addition, the decrease in the cable inclination at the far pylon also leads to decreased vertical support efficiency of the cable. Therefore, how to effectively reduce the sag effect of a super-long stay cable has become another important topic for improving the span of the cable-stayed bridge (Virlogeux 1999, Tang 2017).

Currently, there are two main means for reducing the cable sag effect. One is to use high strength-to-density ratio materials for the cable, such as the carbon-fiber-reinforced polymer (CFRP) stay cables scheme (Christoffersen *et al*. 1999, Noisternig 2000, Wu and Wang 2008) and the CFRP-steel composite stay cable scheme (Xiong *et al*. 2011). The other is to improve the arrangement of the cable system, such as the cable-stayed and suspension hybrid system with cables connected to vertical hangers (Zhang 2007, Zhang and Yu 2015, Sun *et al*. 2016) and a cable net system with secondary cross cables (Gimsing 1980, Gimsing and Georgakis 2012). Recently, a lifted stay cables suspended by an auxiliary cable concept in cross-strait cable-stayed bridges has been put forward for reducing the sag of super-long stay cables (Tang 2006, Tang 2014). Taking external cables at midspan as an example, as shown in Fig. 1, diagonal auxiliary lifting cables (blue line) are suspended by the auxiliary suspension cable (magenta line) that lifts the external cables (red line) at their midpoint to achieve the purpose of reducing the sag effect of the long stay cable. From relevant studies (Tang 2006, Tang 2014), a lifted stay cable can improve the sag effect, support efficiency and stiffness of the lifted stay cables which allow for further improvement to the span limit of cable-stayed bridges.

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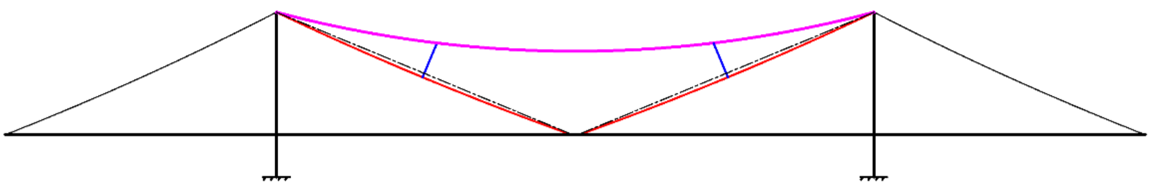


Fig. 1 Diagram of auxiliary lifting-suspended cable concept of super-long span cable-stayed bridges

However, the above auxiliary lifting-suspended cable system is only in the conceptual design stage and has yet to be applied in practice. There have been no further studies regarding the extent to which the above proposed concept scheme might improve the sag effect, support efficiency and stiffness of the lifted stay cables. Therefore, it is necessary to establish a reasonable and feasible approximate calculation method for studying the problem.

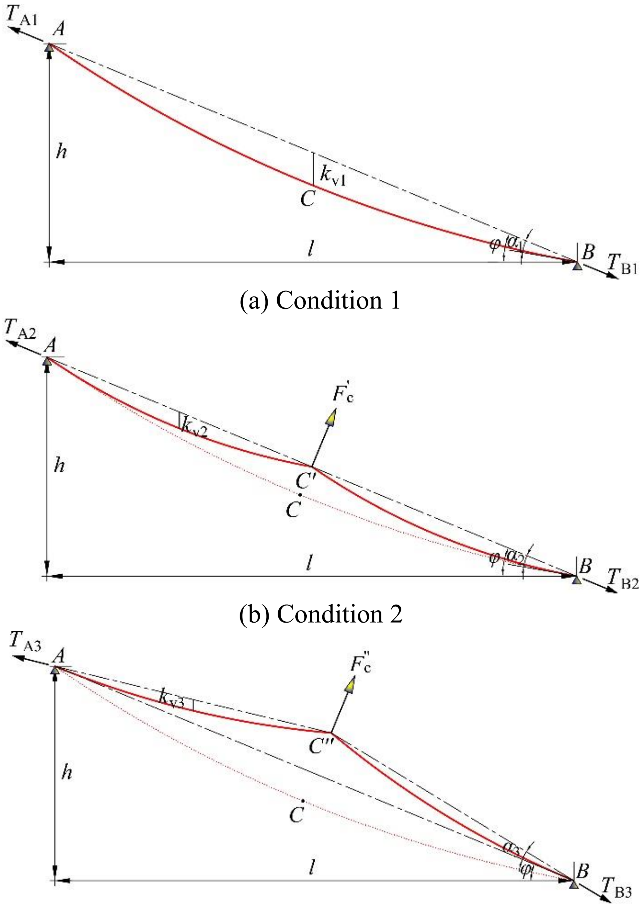
In this study, for the cable-suspended system concept of cable-stayed bridges, the corresponding approximate calculations of the static analysis based on catenary and parabolic cable configurations were established for three typical lifting conditions of lifted cable and the whole lifting process. Then, taking a self-anchored cable-stayed bridge with a main span of 1400 m as a study case, the lifting force of the auxiliary lifting cable, vertical sag of the lifted cable, support efficiency and horizontal force were calculated and numerically analyzed. The advantages and disadvantages of the two typical lifting schemes were then compared. The cross-sectional area of the auxiliary suspension cable was preliminarily estimated. And finally, the equivalent elastic modulus and vertical support stiffness of the lifted cable were discussed.

2. Approximate calculations of the static analysis of the lifted cable

2.1 Approximate analysis of typical lifting conditions

As each load or supporting condition leads to a specific cable curve, the transition from one load or supporting condition to another implies a change of the cable geometry. In order to establish the approximate condition equation of the lifted cable, it is necessary to study the cable in each condition separately. For the lifted cable, three typical lifting conditions are first considered. First, the auxiliary cable is not used (Fig. 2(a)); second, the auxiliary cable is used to lift the stay cable to the midpoint of the original cable chord (Fig. 2(b)); and third, the auxiliary cable is used to lift the stay cable to the position where the end of the cable is tangent to the original cable chord (Fig. 2(c)).

For an inclined stay cable, the sag-creating transverse load has an intensity of $g_{\mathrm{cb}}\cos\varphi$ (Fig. 3). As the chord-wise load intensity along the chord length $g_{\mathrm{cb}}\sin\varphi$ makes little contribution to the transverse deformation of the cable sag, the deformational characteristics of the inclined cable will be very close to those of a horizontal cable with the same chord length c, but subjected to a vertical dead load $g_{\mathrm{cb}}\cos\varphi$ (Gimsing and Georgakis 2012).



(a) Condition 1

(b) Condition 2

(c) Condition 3

Fig. 2 Three typical lifting conditions of the lifted cable

Based on the above equivalent horizontal stay cable theory, the above three basic conditions are respectively equivalent to the equivalent horizontal stay cables (Fig. 4). In condition 1, the cable is supported at points $A(0, 0)$ and $B(2a, 0)$, only carrying its own weight $g'_{\mathrm{cb}}(=g_{\mathrm{cb}}\cos\varphi)$ (Fig. 4(a)). In condition 2, the cable is supported at points $A(0, 0)$ and $B(2a, 0)$, with the cable midpoint lifted to $C'(a, 0)$, carrying its own weight g'_{cb} and concentrated force F_c' acting on the point $C'(a, 0)$ (Fig. 4(b)). In condition 3, the end of the lifted cable is tangent to the original cable chord and supported at points $A(0, 0)$ and $B(2a, 0)$, with the cablemidpoint lifted to $C''(a, a\tan\alpha_3)$, carrying its own weight and concentrated force F_c'' acting on the point C'' (Fig. 4(c)).

2.1.1 Approximate calculation of condition 1

<u>*Catenary cable geometry*</u>

For condition 1, the cable geometry is a catenary under the cable self-weight (Fig. 4(a)). According to the above-

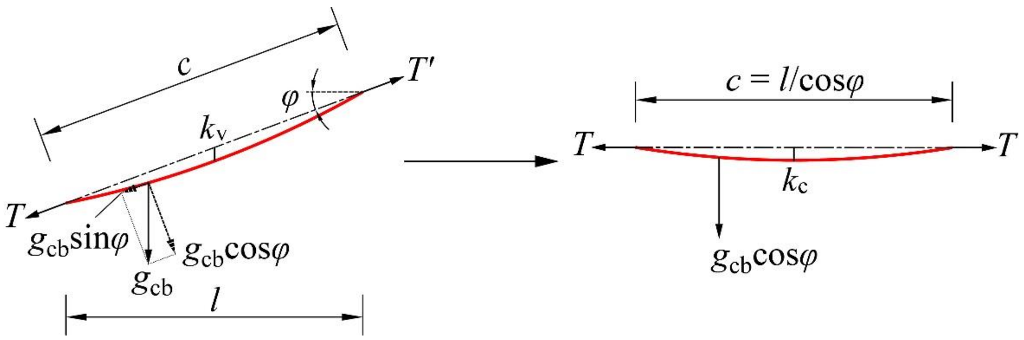
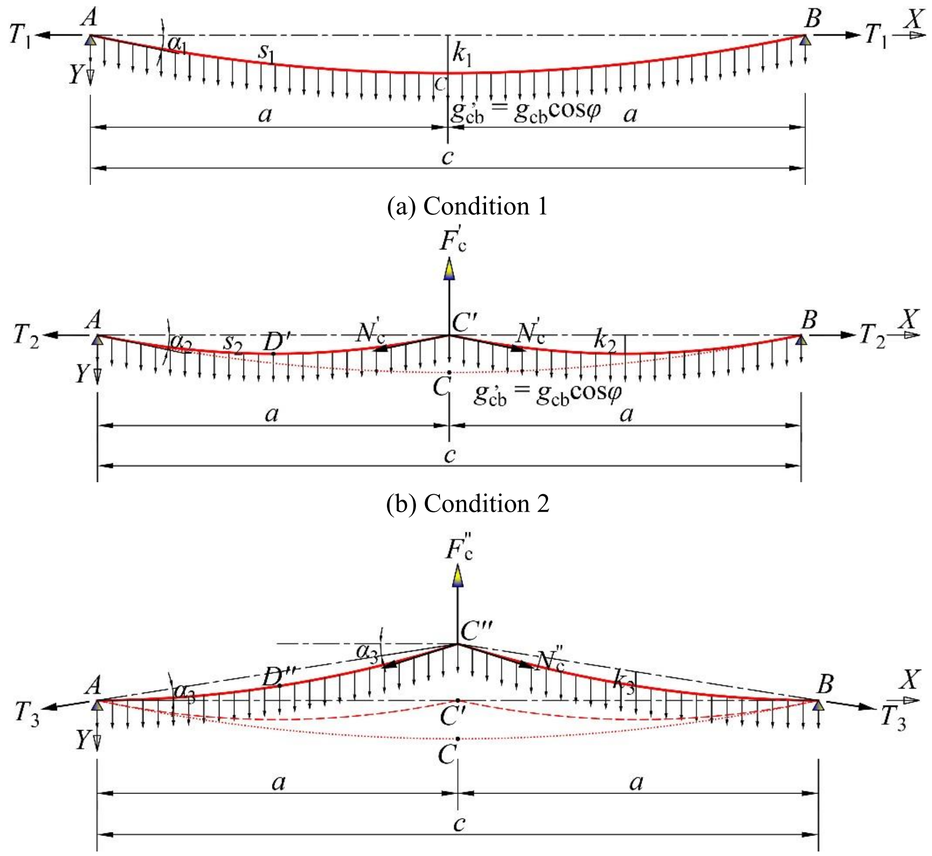


Fig. 3 Inclined stay cable and equivalent horizontal stay cable



(a) Condition 1

(b) Condition 2

(c) Condition 3

Fig. 4 Equivalent horizontal condition of three typical lifting conditions

mentioned equivalent horizontal stay cable theory, the cable geometry in condition 1 is

$$y_{1,\,\mathrm{cat}}=\frac{T_{1,\,\mathrm{cat}}}{g'_{\mathrm{cb}}}\left\{\cosh\left(\frac{g'_{\mathrm{cb}}}{T_{1,\,\mathrm{cat}}}a\right)-\cosh\left[\frac{g'_{\mathrm{cb}}}{T_{1,\,\mathrm{cat}}}(x-a)\right]\right\} \quad (1)$$

where $T_{1,\mathrm{cat}}$ is the chord force of the equivalent horizontal stay cable for condition 1 (without auxiliary cable lifting).

The cable sag $k_{1,\mathrm{cat}}$ is

$$k_{1,\,\mathrm{cat}}=\frac{T_{1,\,\mathrm{cat}}}{g'_{\mathrm{cb}}}\left[\cosh\left(\frac{g'_{\mathrm{cb}}}{T_{1,\,\mathrm{cat}}}a\right)-1\right] \quad (2)$$

The cable end inclination angle $\alpha_{1,\mathrm{cat}}$ is

$$\alpha_{1,\,\mathrm{cat}}=\mathrm{arcsinh}\left(\frac{g'_{\mathrm{cb}}}{T_{1,\,\mathrm{cat}}}a\right)=\ln\left[\frac{g'_{\mathrm{cb}}}{T_{1,\,\mathrm{cat}}}a+\sqrt{1+\left(\frac{g'_{\mathrm{cb}}}{T_{1,\,\mathrm{cat}}}a\right)^2}\right] \quad (3)$$

The cable curve length $s_{1,\mathrm{cat}}$ from supporting point A to $C(a, k_{1,\mathrm{cat}})$ is

$$s_{1,\,\mathrm{cat}}=\int_0^a\sqrt{1+\left(\frac{\mathrm{d}\,y_{1,\,\mathrm{cat}}}{\mathrm{d}\,x}\right)^2}\,\mathrm{d}\,x=\frac{T_{1,\,\mathrm{cat}}}{g'_{\mathrm{cb}}}\sinh\left(\frac{g'_{\mathrm{cb}}}{T_{1,\,\mathrm{cat}}}a\right) \quad (4)$$

In this study of lifted cable under different load conditions, each is characterized by a different elongation Δs of the cable from the unstressed condition.

$$\Delta s=\int_0^a \varepsilon(x)\frac{\mathrm{d}s}{\mathrm{d}x}\mathrm{d}x=\frac{1}{E_{\mathrm{cb}}A_{\mathrm{cb}}}\int_0^a H(x)\left[1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right]\mathrm{d}x \quad (5)$$

where $H(x)$ is horizontal force, which here is $H(x) = H =$ Const. Then, the elongation of the half-span equivalent horizontal stay cable in condition 1 Δs_1 is obtained from the above equation.

The expression for $\Delta s_{1,\mathrm{cat}}$ is derived after substituting $H=T_{1,\mathrm{cat}}$ and $\mathrm{d}y_{1,\mathrm{cat}}/\mathrm{d}x=\sinh\left[g'_{\mathrm{cb}}(x-a)/T_{1,\mathrm{cat}}\right]$ into Eq. (5) and rearranged as

$$\Delta s_{1,\,\mathrm{cat}}=\frac{T_{1,\,\mathrm{cat}}^2}{4g'_{\mathrm{cb}}E_{\mathrm{cb}}A_{\mathrm{cb}}}\left[\sinh\left(\frac{2g'_{\mathrm{cb}}}{T_{1,\,\mathrm{cat}}}a\right)+\frac{2g'_{\mathrm{cb}}}{T_{1,\,\mathrm{cat}}}a\right] \quad (6)$$

<u>*Approximate parabolic cable configuration*</u>

For condition 1, the corresponding approximation analysis is equivalent to the quadratic parabolic geometry and the parabolic approximation curve equation of the equivalent horizontal stay cable is

$$y_{1,\,\mathrm{par}}=\frac{k_{1,\,\mathrm{par}}}{a^2}x(2a-x) \quad (7)$$

where $k_{1,\mathrm{par}}$ is the parabolic approximation cable sag and there is

$$k_{1,\,\mathrm{par}}=\frac{g'_{\mathrm{cb}}a^2}{2T_{1,\,\mathrm{par}}} \quad (8)$$

The cable end inclination angle $\alpha_{1,\mathrm{par}}$ is

$$\alpha_{1,\,\mathrm{par}}=\arctan\left(\frac{2k_{1,\,\mathrm{par}}}{a}\right) \quad (9)$$

The cable curve length $s_{1,\mathrm{par}}$ from supporting point A to $C(a,\ k_{1,\mathrm{par}})$ is obtained after integration and arrangement to yield

$$s_{1,\,\mathrm{par}}=a\left\{\begin{array}{l}\dfrac{1}{2}\sqrt{1+4\left(\dfrac{k_{1,\,\mathrm{par}}}{a}\right)^2}\\ +\dfrac{a}{4k_{1,\,\mathrm{par}}}\ln\left[2\left(\dfrac{k_{1,\,\mathrm{par}}}{a}\right)+\sqrt{1+4\left(\dfrac{k_{1,\,\mathrm{par}}}{a}\right)^2}\right]\end{array}\right\} \quad (10)$$

Expanding the above equation by the Taylor expansion with only the first two items yields the approximate expression of $s_{1,\mathrm{par}}$ as

$$s_{1,\,\mathrm{par}}=a\left[1+\frac{2}{3}\left(\frac{k_{1,\,\mathrm{par}}}{a}\right)^2-\frac{2}{5}\left(\frac{k_{1,\,\mathrm{par}}}{a}\right)^4+\frac{4}{7}\left(\frac{k_{1,\,\mathrm{par}}}{a}\right)^6-\cdots\right]\approx a\left[1+\frac{2}{3}\left(\frac{k_{1,\,\mathrm{par}}}{a}\right)^2\right] \quad (11)$$

For a general stay cable (k/a < 0.02), the difference between the approximate value of the curve length from the above equation and the exact value from Eq. (10) is much less than 0.001‰. Thus, the approximate expression (Eq.(11)) gives adequate results, meeting the accuracy requirements of actual projects.

The half-span elongation expression for $\Delta s_{1,\mathrm{par}}$ of the parabola based equivalent horizontal cable in condition 1 is derived after substituting $H=T_{1,\mathrm{par}}=g'_{\mathrm{cb}}a^2/(2k_{1,\mathrm{par}})$ and $\mathrm{d}y_{1,\mathrm{par}}/\mathrm{d}x=2k_{1,\mathrm{par}}(x-a)/a^2$ into Eq. (5) as

$$\Delta s_{1,\,\mathrm{par}}=\frac{g'_{\mathrm{cb}}a^2}{E_{\mathrm{cb}}A_{\mathrm{cb}}}\left(\frac{a}{2k_{1,\,\mathrm{par}}}+\frac{2k_{1,\,\mathrm{par}}}{3a}\right) \quad (12)$$

2.1.2 Approximate calculation of condition 2

<u>*Catenary cable geometry*</u>

For condition 2, the catenary cable geometry for a half-span equivalent horizontal stay cable is (Fig. 4(b))

$$y_{2,\,\mathrm{cat}}=\frac{T_{2,\,\mathrm{cat}}}{g'_{\mathrm{cb}}}\left\{\cosh\left(\frac{g'_{\mathrm{cb}}}{2T_{2,\,\mathrm{cat}}}a\right)-\cosh\left[\frac{g'_{\mathrm{cb}}}{T_{2,\,\mathrm{cat}}}\left(x-\frac{a}{2}\right)\right]\right\} \quad (13)$$

The sag of half-span horizontal cable $k_{2,\mathrm{cat}}$ is

$$k_{2,\,\mathrm{cat}}=\frac{T_{2,\,\mathrm{cat}}}{g'_{\mathrm{cb}}}\left[\cosh\left(\frac{g'_{\mathrm{cb}}}{2T_{2,\,\mathrm{cat}}}a\right)-1\right] \quad (14)$$

The end inclination angle of half-span horizontal cable $\alpha_{2,\mathrm{cat}}$ is

$$\begin{aligned}\alpha_{2,\,\mathrm{cat}}&=\operatorname{arcsinh}\left(\frac{g'_{\mathrm{cb}}}{2T_{2,\,\mathrm{cat}}}a\right)\\&=\ln\left[\frac{g'_{\mathrm{cb}}}{2T_{2,\,\mathrm{cat}}}a+\sqrt{1+\left(\frac{g'_{\mathrm{cb}}}{2T_{2,\,\mathrm{cat}}}a\right)^2}\right]\end{aligned} \quad (15)$$

The cable curve length $s_{2,\mathrm{cat}}$ from supporting point A to $D'(a/2,\ k_{2,\mathrm{cat}})$ is

$$\begin{aligned}s_{2,\,\mathrm{cat}}&=\int_0^{\frac{a}{2}}\sqrt{1+\left(\frac{\mathrm{d}y_{2,\,\mathrm{cat}}}{\mathrm{d}x}\right)^2}\,\mathrm{d}x\\&=\frac{T_{2,\,\mathrm{cat}}}{g'_{\mathrm{cb}}}\sinh\left(\frac{g'_{\mathrm{cb}}}{2T_{2,\,\mathrm{cat}}}a\right)\end{aligned} \quad (16)$$

The cable curve elongation $\Delta s_{2,\mathrm{cat}}$ from supporting point A to $D'(a/2,\ k_{2,\mathrm{cat}})$ is

$$\begin{aligned}\Delta s_{2,\,\mathrm{cat}}&=\frac{T_{2,\,\mathrm{cat}}}{E_{\mathrm{cb}}A_{\mathrm{cb}}}\int_0^{\frac{a}{2}}\left[1+\left(\frac{\mathrm{d}y_{2,\,\mathrm{cat}}}{\mathrm{d}x}\right)^2\right]\mathrm{d}x\\&=\frac{T_{2,\,\mathrm{cat}}^2}{4g'_{\mathrm{cb}}E_{\mathrm{cb}}A_{\mathrm{cb}}}\left[\sinh\left(\frac{g'_{\mathrm{cb}}}{T_{2,\,\mathrm{cat}}}a\right)+\frac{g'_{\mathrm{cb}}}{T_{2,\,\mathrm{cat}}}a\right]\end{aligned} \quad (17)$$

Referring to Figs. 4(a)-(b), the condition compatibility equation is

$$s_1-2s_2=\Delta s_1-2\Delta s_2 \quad (18)$$

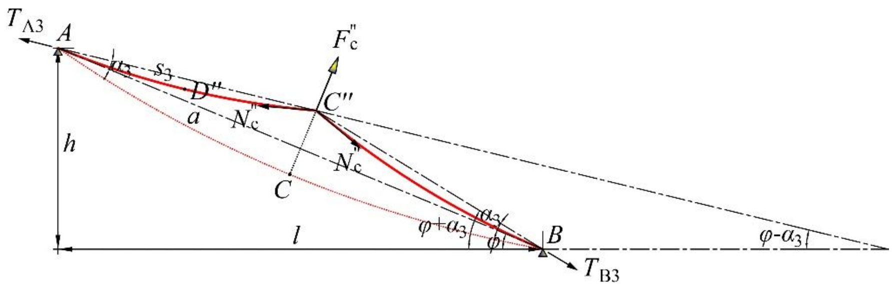


Fig. 5 Lifted cable geometry relationship in condition 3

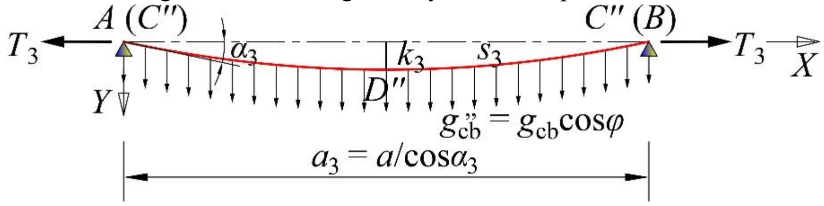


Fig. 6 Equivalent horizontal stay cable in condition 3

The following condition compatibility equation of the catenary cable is derived after substituting Eqs. (4), (6), (16) and (17) into the above equation and rearranged to

$$\begin{aligned} &T_{1,\,\mathrm{cat}}\sinh\left(\frac{g'_{\mathrm{cb}}}{T_{1,\,\mathrm{cat}}}a\right)-2T_{2,\,\mathrm{cat}}\sinh\left(\frac{g'_{\mathrm{cb}}}{2T_{2,\,\mathrm{cat}}}a\right)= \\ &\frac{T_{1,\,\mathrm{cat}}^2}{4E_{\mathrm{cb}}A_{\mathrm{cb}}}\left[\sinh\left(\frac{2g'_{\mathrm{cb}}}{T_{1,\,\mathrm{cat}}}a\right)+\frac{2g'_{\mathrm{cb}}}{T_{1,\,\mathrm{cat}}}a\right]-\frac{T_{2,\,\mathrm{cat}}^2}{2E_{\mathrm{cb}}A_{\mathrm{cb}}}\left[\sinh\left(\frac{g'_{\mathrm{cb}}}{T_{2,\,\mathrm{cat}}}a\right)+\frac{g'_{\mathrm{cb}}}{T_{2,\,\mathrm{cat}}}a\right] \end{aligned} \tag{19}$$

Solving Eq. (19) obtains the chord force $T_{2,\mathrm{cat}}$ of the half-span equivalent horizontal stay cable in condition 2. According to Eq. (14), the half-span horizontal cable sag $k_{2,\mathrm{cat}}$ in condition 2 is subsequently obtained and, then, the vertical sag of the inclined stay cable is $k_{v2,\mathrm{cat}} = k_{2,\mathrm{cat}}/\cos\varphi$. According to Eq. (15), the end inclination angle $\alpha_{2,\mathrm{cat}}$ of the half-span equivalent horizontal stay cable in condition 2 is obtained and then the auxiliary lifting force $F'_{c,\mathrm{cat}}$ determined according to the balance condition at the lifting point.

Approximate parabolic cable configuration

For condition 2, the corresponding approximation analysis is equivalent to the quadratic parabolic geometry and the parabolic approximation curve equation of the half-span equivalent horizontal stay cable is

$$y_{2,\,\mathrm{par}}=\frac{4k_{2,\,\mathrm{par}}}{a^2}x(a-x) \tag{20}$$

where $k_{2,\mathrm{par}}$ is the half-span horizontal cable sag and there is

$$k_{2,\,\mathrm{par}}=\frac{g'_{\mathrm{cb}}a^2}{8T_{2,\,\mathrm{par}}} \tag{21}$$

The half-span horizontal cable end inclination angle $\alpha_{2,\mathrm{par}}$ is

$$\alpha_{2,\,\mathrm{par}}=\arctan\left(\frac{4k_{2,\,\mathrm{par}}}{a}\right) \tag{22}$$

The approximate expression of the curve length $s_{2,\mathrm{par}}$ from supporting point A to $D'(a/2, k_{2,\mathrm{par}})$ is

$$s_{2,\,\mathrm{par}}\approx\frac{a}{2}\left[1+\frac{8}{3}\left(\frac{k_{2,\,\mathrm{par}}}{a}\right)^2\right] \tag{23}$$

The cable curve elongation $\Delta s_{2,\mathrm{par}}$ from supporting point A to $D'(a/2, k_{2,\mathrm{par}})$ is

$$\Delta s_{2,\,\mathrm{par}}=\frac{g'_{\mathrm{cb}}a^2}{4E_{\mathrm{cb}}A_{\mathrm{cb}}}\left(\frac{a}{4k_{2,\,\mathrm{par}}}+\frac{4k_{2,\,\mathrm{par}}}{3a}\right) \tag{24}$$

The following condition compatibility equation of the parabolic approximation cable is derived after substituting Eqs. (11), (12), (23) and (24) into Eq. (18) and rearranged to

$$\begin{aligned} &\left(\frac{k_{2,\,\mathrm{par}}}{a}\right)^3-\frac{g'_{\mathrm{cb}}a}{4E_{\mathrm{cb}}A_{\mathrm{cb}}}\left(\frac{k_{2,\,\mathrm{par}}}{a}\right)^2 \\ &+\frac{3}{8}\left[1-\frac{(s_{1,\,\mathrm{par}}-\Delta s_{1,\,\mathrm{par}})}{a}\right]\left(\frac{k_{2,\,\mathrm{par}}}{a}\right)-\frac{3g'_{\mathrm{cb}}a}{64E_{\mathrm{cb}}A_{\mathrm{cb}}}=0 \end{aligned} \tag{25}$$

Solving the cubic equation concerning sag ratio ($k_{2,\mathrm{par}}/a$) obtains the half-span equivalent horizontal stay cable sag $k_{2,\mathrm{par}}$ in condition 2 and, then, the vertical sag of the inclined stay cable is $k_{v2,\mathrm{par}} = k_{2,\mathrm{par}}/\cos\varphi$. According to Eq. (21), the chord force $T_{2,\mathrm{par}}$ in condition 2 is subsequently obtained. According to Eq. (22), the end inclination angle $\alpha_{2,\mathrm{par}}$ of the half-span equivalent horizontal stay cable in condition 2 is obtained and, then, the auxiliary lifting force $F'_{c,\mathrm{par}}$ is determined according to the balance condition at the lifting point.

2.1.3 Approximate calculation of condition 3

As shown in Fig. 4(c), the end of the lifted cable is tangent to the original cable chord. Considering the consistency of the approximate calculation method, the simplified analysis method of the above-mentioned equivalent horizontal stay cable is still adopted to analyze in this condition. The cable chord inclinations of the upper and lower cables from the lifting point are inconsistent (Fig. 5) and the inclination angle of the upper cable chord is ($\varphi-\alpha_3$), while the lower side is ($\varphi+\alpha_3$). Therefore, relative to C–C'' axis, the cable geometry curves on both sides are not completely symmetrical. Considering $\alpha_3 \ll \varphi$, it might have roughly approximated the two sides of the cable curve equivalent to a chord length $a_3 = a/\cos\alpha_3$ and a chord line inclination φ equivalent horizontal stay cable (Fig. 6).

Catenary cable geometry

Based on the above approximation assumption, for condition 3 (Fig. 6), the catenary cable geometry of the equivalent horizontal stay cable is

$$y_{3,\,\mathrm{cat}} = \frac{T_{3,\,\mathrm{cat}}}{g''_{\mathrm{cb}}}\left\{\cosh\left(\frac{g''_{\mathrm{cb}}}{2T_{3,\,\mathrm{cat}}}a_3\right) - \cosh\left[\frac{g''_{\mathrm{cb}}}{T_{3,\,\mathrm{cat}}}\left(x-\frac{a_3}{2}\right)\right]\right\} \tag{26}$$

where $g''_{\mathrm{cb}} \approx g_{\mathrm{cb}}\cos\varphi = g'_{\mathrm{cb}}$ and $a_3 (= a/\cos\alpha_3)$.

The sag of equivalent horizontal stay cable $k_{3,\mathrm{cat}}$ is

$$k_{3,\,\mathrm{cat}} = \frac{T_{3,\,\mathrm{cat}}}{g''_{\mathrm{cb}}}\left\{\cosh\left(\frac{g''_{\mathrm{cb}}}{2T_{3,\,\mathrm{cat}}}a_3\right) - 1\right\} \tag{27}$$

Equivalent horizontal stay cable end inclination angle $\alpha_{3,\mathrm{cat}}$ is

$$\begin{aligned}\alpha_{3,\,\mathrm{cat}} &= \operatorname{arcsinh}\left(\frac{g''_{\mathrm{cb}}}{2T_{3,\,\mathrm{cat}}}a_3\right) \\ &= \ln\left[\frac{g''_{\mathrm{cb}}}{2T_{3,\,\mathrm{cat}}}a_3 + \sqrt{1+\left(\frac{g''_{\mathrm{cb}}}{2T_{3,\,\mathrm{cat}}}a_3\right)^2}\,\right]\end{aligned} \tag{28}$$

The cable curve length $s_{3,\mathrm{cat}}$ from supporting point $A(B)$ to $D''(a_3/2, k_{3,\mathrm{cat}})$ is

$$s_{3,\,\mathrm{cat}} = \frac{T_{3,\,\mathrm{cat}}}{g''_{\mathrm{cb}}}\sinh\left(\frac{g''_{\mathrm{cb}}}{2T_{3,\,\mathrm{cat}}}a_3\right) \tag{29}$$

The cable curve elongation $\Delta s_{3,\mathrm{cat}}$ from supporting point $A(B)$ to $D''(a_3/2, k_{3,\mathrm{cat}})$ is

$$\Delta s_{3,\,\mathrm{cat}} = \frac{T^2_{3,\,\mathrm{cat}}}{4g''_{\mathrm{cb}}E_{\mathrm{cb}}A_{\mathrm{cb}}}\left[\sinh\left(\frac{g''_{\mathrm{cb}}}{T_{3,\,\mathrm{cat}}}a_3\right) + \frac{g''_{\mathrm{cb}}}{T_{3,\,\mathrm{cat}}}a_3\right] \tag{30}$$

Referring to Figs. 4(a) and 4(c), the condition compatibility equation is

$$s_1 - 2s_3 = \Delta s_1 - 2\Delta s_3 \tag{31}$$

The following condition compatibility equation of the catenary cable is derived after substituting Eqs. (4), (6), (29) and (30) into the above equation and rearranged to

$$\begin{aligned}&T_{1,\,\mathrm{cat}}\sinh\left(\frac{g'_{\mathrm{cb}}}{T_{1,\,\mathrm{cat}}}a\right) - 2T_{3,\,\mathrm{cat}}\sinh\left(\frac{g''_{\mathrm{cb}}}{2T_{3,\,\mathrm{cat}}}a_3\right) = \\ &\frac{T^2_{1,\,\mathrm{cat}}}{4E_{\mathrm{cb}}A_{\mathrm{cb}}}\left[\sinh\left(\frac{2g'_{\mathrm{cb}}}{T_{1,\,\mathrm{cat}}}a\right) + \frac{2g'_{\mathrm{cb}}}{T_{1,\,\mathrm{cat}}}a\right] - \frac{T^2_{3,\,\mathrm{cat}}}{2E_{\mathrm{cb}}A_{\mathrm{cb}}}\left[\sinh\left(\frac{g''_{\mathrm{cb}}}{T_{3,\,\mathrm{cat}}}a_3\right) + \frac{g''_{\mathrm{cb}}}{T_{3,\,\mathrm{cat}}}a_3\right]\end{aligned} \tag{32}$$

In addition to the unknown chord-wise force $T_{3,\mathrm{cat}}$, the end inclination angle $\alpha_{3,\mathrm{cat}}$ of the equivalent horizontal stay cable is also unknown and to be solved in the above equation. It can be seen from Eq. (28), $\alpha_{3,\mathrm{cat}}$ is the function of chord force $T_{3,\mathrm{cat}}$. Combining the above equation with the Eq. (28), the chord force $T_{3,\mathrm{cat}}$ and end inclination angle $\alpha_{3,\mathrm{cat}}$ are obtained. According to Eq. (27), the equivalent horizontal cable sag $k_{3,\mathrm{cat}}$ in condition 3 is subsequently obtained and, then, the auxiliary lifting force $F''_{c,\mathrm{cat}}$ of the lifted cable, of which the end is tangent to the original cable chord, is determined according to the balance condition at the lifting point.

It should be pointed out that the condition compatibility equation is not unique. The following compatibility equation could have also been used

$$s_2 - s_3 = \Delta s_2 - \Delta s_3 \tag{33}$$

Approximate parabolic cable configuration

For condition 3, the corresponding approximation analysis is equivalent to the quadratic parabolic geometry and the parabolic approximation curve equation of the equivalent horizontal stay cable is

$$y_{3,\,\mathrm{par}} = \frac{4k_{3,\,\mathrm{par}}}{a_3^2}x(a_3 - x) \tag{34}$$

where $k_{3,\mathrm{par}}$ is the horizontal cable sag and there is

$$k_{3,\,\mathrm{par}} = \frac{g''_{\mathrm{cb}}a_3^2}{8T_{3,\,\mathrm{par}}} \tag{35}$$

Equivalent horizontal stay cable end inclination angle $\alpha_{3,\mathrm{par}}$ is

$$\alpha_{3,\,\mathrm{par}} = \arctan\left(\frac{4k_{3,\,\mathrm{par}}}{a_3}\right) \tag{36}$$

The approximate expression of the curve length $s_{3,\mathrm{par}}$ from supporting point $A(B)$ to $D''(a_3/2, k_{3,\mathrm{par}})$ is

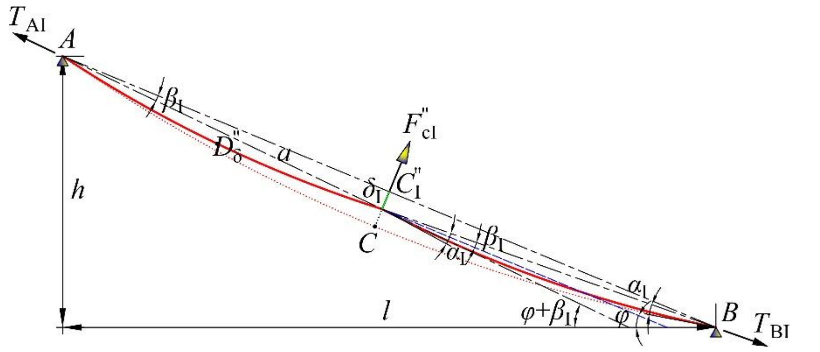
$$s_{3,\,\mathrm{par}} \approx \frac{a_3}{2}\left[1+\frac{8}{3}\left(\frac{k_{3,\,\mathrm{par}}}{a_3}\right)^2\right] \tag{37}$$

The cable curve elongation $\Delta s_{3,\mathrm{par}}$ from supporting point $A(B)$ to $D''(a_3/2, k_{3,\mathrm{par}})$ is

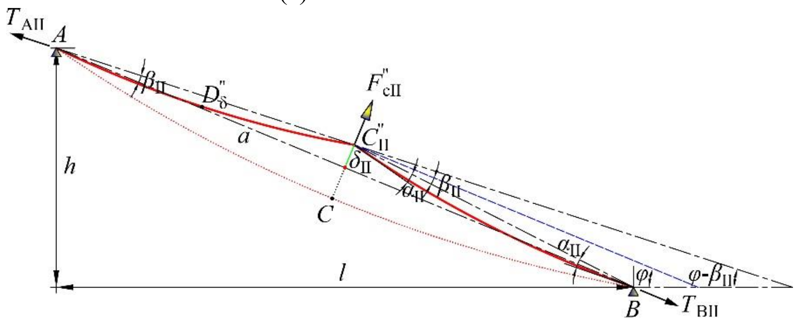
$$\Delta s_{3,\,\mathrm{par}} = \frac{g''_{\mathrm{cb}}a_3{}^2}{4E_{\mathrm{cb}}A_{\mathrm{cb}}}\left(\frac{a_3}{4k_{3,\,\mathrm{par}}} + \frac{4k_{3,\,\mathrm{par}}}{3a_3}\right) \tag{38}$$

The following condition compatibility equation of the parabolic approximation cable is derived after substituting Eqs. (11), (12), (37) and (38) into the Eq. (31) and rearranged to

$$\begin{aligned}&\left(\frac{k_{3,\,\mathrm{par}}}{a_3}\right)^3 - \frac{g''_{\mathrm{cb}}a_3}{4E_{\mathrm{cb}}A_{\mathrm{cb}}}\left(\frac{k_{3,\,\mathrm{par}}}{a_3}\right)^2 \\ &+\frac{3}{8}\left[1-\frac{(s_{1,\,\mathrm{par}} - \Delta s_{1,\,\mathrm{par}})}{a_3}\right]\left(\frac{k_{3,\,\mathrm{par}}}{a_3}\right) - \frac{3g''_{\mathrm{cb}}a_3}{64E_{\mathrm{cb}}A_{\mathrm{cb}}} = 0\end{aligned} \tag{39}$$



(a) Interval condition I



(b) Interval condition II

Fig. 7 General auxiliary cable-lifting conditions

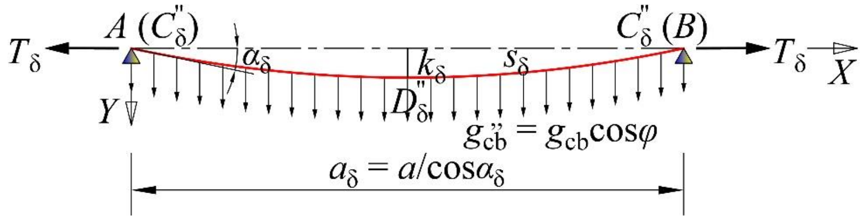


Fig. 8 Equivalent horizontal stay cable of the general lifting condition

Similar to the calculation of the catenary cable geometry analysis above in condition 3, it is also noted that, besides the unknown sag ratio ($k_{3,\text{par}}/a_3$), the end inclination angle $\alpha_{3,\text{par}}$ of the equivalent horizontal stay cable is also unknown and to be solved in the above equation. It can be seen from Eq. (36) that $\alpha_{3,\text{par}}$ is the function of sag ratio ($k_{3,\text{par}}/a_3$). Combining the above equation with Eq. (36), the sag ratio ($k_{3,\text{par}}/a_3$) and end inclination angle $\alpha_{3,\text{par}}$ are obtained. According to Eq. (35), the chord force $T_{3,\text{par}}$ in condition 3 is subsequently obtained and, then, the auxiliary lifting force $F''_{c,\text{par}}$ of the lifted cable, of which the end is tangent to the original cable chord, is determined according to the balance condition at the lifting point.

2.2 Approximate calculation of the general auxiliary cable-lifting conditions

Three typical lifting conditions of the lifted cable are introduced in the previous section. Based on the simplified approximation analysis of condition 3, the more general lifting conditions of the auxiliary cable are analyzed in this section in order to complete the approximate static calculation method of the lifted cable.

Along with whole auxiliary cable-lifting process from condition 1 (without auxiliary cable lifting) to condition 3 (lifted cable end tangent to the original cable chord), in addition to the three typical conditions mentioned above, there are also two interval conditions: interval condition I is a lifting condition between conditions 1 and 2 (Fig. 7(a)), which is a lifting condition in which the lifting point of the cable is located between C and the midpoint of the chord and the distance from the lifting point to the chord midpoint is δ_I; and interval condition II (Fig. 7(b)), which is a lifting condition in which the lifting point of the cable is located outside the chord line and the distance from the lifting point to the midpoint of the original cable chord is δ_{II}.

Similar to condition 3, the inclination angles of the upper and lower side cable chord of the interval conditions I and II are inconsistent. The inclination angle of the upper side chord of the interval condition I is ($\varphi+\beta_I$), while the lower side is ($\varphi-\beta_I$), wherein β_I = arctan(δ_I/a). The inclination angle of the upper side chord of the interval condition II is ($\varphi-\beta_{II}$), while the lower side is ($\varphi+\beta_{II}$), wherein β_{II} = arctan(δ_{II}/a). Therefore, relative to the $C-C''_I(C''_{II})$ axis, the cable curves on both sides are not completely symmetrical. Considering $\beta_I(\beta_{II})\ll\varphi$, similar to the approximation treatment of condition 3, the two side cable curves of the two interval conditions are uniformly equivalent to a chord length $a_\delta = a/\cos\beta_\delta$ and chord line inclination φ equivalent horizontal stay cable (Fig. 8), wherein $\beta_\delta = \arctan(\delta/a)$ with δ the distance from the lifting point to the midpoint of the original cable chord. It is specified as a negative value when the lifting point is below the chord midpoint and a positive one above the chord midpoint.

2.2.1 Catenary cable geometry

Based on the above approximation assumption, for the general condition shown in Fig. 8, the catenary cable geometry of the equivalent horizontal stay cable is

$$y_{\delta,\,\mathrm{cat}} = \frac{T_{\delta,\,\mathrm{cat}}}{g''_{\mathrm{cb}}}\left\{\cosh\left(\frac{g''_{\mathrm{cb}}}{2T_{\delta,\,\mathrm{cat}}}a_\delta\right) - \cosh\left[\frac{g''_{\mathrm{cb}}}{T_{\delta,\,\mathrm{cat}}}\left(x - \frac{a_\delta}{2}\right)\right]\right\} \quad (40)$$

where $g''_{\mathrm{cb}} \approx g_{\mathrm{cb}}\cos\varphi = g'_{\mathrm{cb}}$ and $a_\delta = a/\cos\beta_\delta$, with $\beta_\delta = \arctan(\delta/a)$ and δ the distance from the lifting point to the midpoint of the original cable chord. It is specified as a negative value when the lifting point is below the chord midpoint and as a positive one above the chord midpoint.

Equivalent horizontal stay cable sag $k_{\delta,\mathrm{cat}}$ is

$$k_{\delta,\,\mathrm{cat}} = \frac{T_{\delta,\,\mathrm{cat}}}{g''_{\mathrm{cb}}}\left\{\cosh\left(\frac{g''_{\mathrm{cb}}}{2T_{\delta,\,\mathrm{cat}}}a_\delta\right) - 1\right\} \quad (41)$$

Equivalent horizontal stay cable end inclination angle $\alpha_{\delta,\mathrm{cat}}$ is

$$\begin{aligned}\alpha_{\delta,\,\mathrm{cat}} &= \operatorname{arcsinh}\left(\frac{g''_{\mathrm{cb}}}{2T_{\delta,\,\mathrm{cat}}}a_\delta\right) \\ &= \ln\left[\frac{g''_{\mathrm{cb}}}{2T_{\delta,\,\mathrm{cat}}}a_\delta + \sqrt{1 + \left(\frac{g''_{\mathrm{cb}}}{2T_{\delta,\,\mathrm{cat}}}a_\delta\right)^2}\right]\end{aligned} \quad (42)$$

The cable curve length $s_{\delta,\mathrm{cat}}$ from supporting point $A(B)$ to D''_δ is

$$s_{\delta,\,\mathrm{cat}} = \frac{T_{\delta,\,\mathrm{cat}}}{g''_{\mathrm{cb}}}\sinh\left(\frac{g''_{\mathrm{cb}}}{2T_{\delta,\,\mathrm{cat}}}a_\delta\right) \quad (43)$$

The cable curve elongation $\Delta s_{\delta,\mathrm{cat}}$ from supporting point $A(B)$ to D''_δ is

$$\Delta s_{\delta,\,\mathrm{cat}} = \frac{T^2_{\delta,\,\mathrm{cat}}}{4g''_{\mathrm{cb}}E_{\mathrm{cb}}A_{\mathrm{cb}}}\left[\sinh\left(\frac{g''_{\mathrm{cb}}}{T_{\delta,\,\mathrm{cat}}}a_\delta\right) + \frac{g''_{\mathrm{cb}}}{T_{\delta,\,\mathrm{cat}}}a_\delta\right] \quad (44)$$

The condition compatibility equation is

$$s_1 - 2s_\delta = \Delta s_1 - 2\Delta s_\delta \quad (45)$$

The following condition compatibility equation of the catenary cable is derived after substituting Eqs. (4), (6), (43) and (44) into the above equation and rearranged to

$$T_{1,\,\mathrm{cat}}\sinh\left(\frac{g'_{\mathrm{cb}}}{T_{1,\,\mathrm{cat}}}a\right) - 2T_{\delta,\,\mathrm{cat}}\sinh\left(\frac{g''_{\mathrm{cb}}}{2T_{\delta,\,\mathrm{cat}}}a_\delta\right) = \frac{T^2_{1,\,\mathrm{cat}}}{4E_{\mathrm{cb}}A_{\mathrm{cb}}}\left[\sinh\left(\frac{2g'_{\mathrm{cb}}}{T_{1,\,\mathrm{cat}}}a\right) + \frac{2g'_{\mathrm{cb}}}{T_{1,\,\mathrm{cat}}}a\right] - \frac{T^2_{\delta,\,\mathrm{cat}}}{2E_{\mathrm{cb}}A_{\mathrm{cb}}}\left[\sinh\left(\frac{g''_{\mathrm{cb}}}{T_{\delta,\,\mathrm{cat}}}a_\delta\right) + \frac{g''_{\mathrm{cb}}}{T_{\delta,\,\mathrm{cat}}}a_\delta\right] \quad (46)$$

When the distance δ from the lifting point to the midpoint of the original cable chord is given, the chord force $T_{\delta,\mathrm{cat}}$ is obtained by solving the above equation. According to Eq. (41), the horizontal cable sag $k_{\delta,\mathrm{cat}}$ in a general lifting condition is subsequently obtained. According to Eq. (42), the end inclination angle $\alpha_{\delta,\mathrm{cat}}$ of the equivalent horizontal stay cable is obtained and, then, the auxiliary lifting force $F''_{\delta,\mathrm{cat}}$ acting at the cable midpoint is determined according to the balance condition at the lifting point.

2.2.2 Approximate parabolic cable configuration

For the general condition, the corresponding approximation analysis is equivalent to the quadratic parabolic geometry and the parabolic approximation curve equation of the equivalent horizontal stay cable is

$$y_{\delta,\,\mathrm{par}} = \frac{4k_{\delta,\,\mathrm{par}}}{a^2_\delta}x(a_\delta - x) \quad (47)$$

where $k_{\delta,\mathrm{par}}$ is the horizontal cable sag and there is

$$k_{\delta,\,\mathrm{par}} = \frac{g''_{\mathrm{cb}}}{8T_{\delta,\,\mathrm{par}}}a^2_\delta \quad (48)$$

Equivalent horizontal stay cable end inclination angle $\alpha_{\delta,\mathrm{par}}$ is

$$\alpha_{\delta,\,\mathrm{par}} = \arctan\left(\frac{4k_{\delta,\,\mathrm{par}}}{a_\delta}\right) \quad (49)$$

The approximate expression of the curve length $s_{\delta,\,\mathrm{par}}$ from supporting point $A(B)$ to D''_δ is

$$s_{\delta,\,\mathrm{par}} \approx \frac{a_\delta}{2}\left[1 + \frac{8}{3}\left(\frac{k_{\delta,\,\mathrm{par}}}{a_\delta}\right)^2\right] \quad (50)$$

The cable curve elongation $\Delta s_{\delta,\mathrm{par}}$ from supporting point $A(B)$ to D''_δ is

$$\Delta s_{\delta,\,\mathrm{par}} = \frac{g''_{\mathrm{cb}}a^2_\delta}{4E_{\mathrm{cb}}A_{\mathrm{cb}}}\left(\frac{a_\delta}{4k_{\delta,\,\mathrm{par}}} + \frac{4k_{\delta,\,\mathrm{par}}}{3a_\delta}\right) \quad (51)$$

The following condition compatibility equation of the parabolic approximation cable is derived after substituting Eqs. (11), (12), (50) and (51) into the Eq. (45) and rearranged to

$$\begin{aligned}&\left(\frac{k_{\delta,\,\mathrm{par}}}{a_\delta}\right)^3 - \frac{g''_{\mathrm{cb}}a_\delta}{4E_{\mathrm{cb}}A_{\mathrm{cb}}}\left(\frac{k_{\delta,\,\mathrm{par}}}{a_\delta}\right)^2 \\ &+ \frac{3}{8}\left[1 - \frac{(s_{1,\,\mathrm{par}} - \Delta s_{1,\,\mathrm{par}})}{a_\delta}\right]\left(\frac{k_{\delta,\,\mathrm{par}}}{a_\delta}\right) - \frac{3g''_{\mathrm{cb}}a_\delta}{64E_{\mathrm{cb}}A_{\mathrm{cb}}} = 0\end{aligned} \quad (52)$$

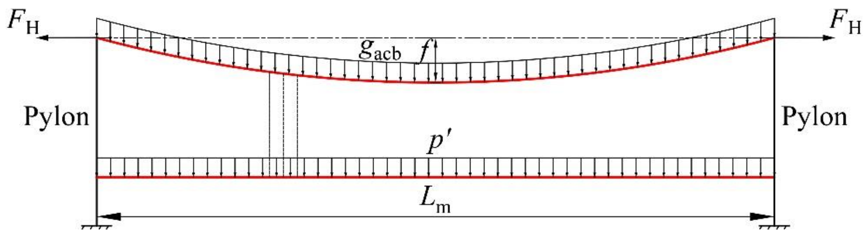


Fig. 9 Maximum force loading condition of the auxiliary suspension cable

When the distance δ from the lifting point to the midpoint of the original cable chord is given, solving the cubic equation concerning sag ratio ($k_{\delta,par}/a_\delta$) obtains the equivalent horizontal stay cable sag $k_{\delta,par}$ in a general lifting condition. According to Eq. (48), the chord force $T_{\delta,par}$ in a general lifting condition is subsequently obtained. According to Eq. (49), the end inclination angle $\alpha_{\delta,par}$ of the equivalent horizontal stay cable in a general lifting condition is obtained and, then, the auxiliary lifting force $F''_{\delta,par}$ acting at the cable midpoint is determined according to the balance condition at the lifting point.

2.3 Cross-sectional area estimation of the auxiliary suspension cable

Based on the static approximation calculation method of the lifted cable, the section area of the auxiliary suspension cable is preliminarily designed, which is based on the lifting force of the most unfavorable external cable of the cable-stayed bridge. The converted load intensity is (Fig. 9)

$$p' = \frac{F_c}{\lambda \cos\varphi} \tag{53}$$

Substituting lifting force F_c, external cable chord inclination φ and the midspan cable spacing λ into above equation allows calculation of the converted load intensity p'. It is assumed that the auxiliary suspension cable cross-sectional area is safely determined according to the uniform distribution p' loading on the whole span and the suspension cable material has the same characteristics as that of the stay cable; and, as the self-weight of the auxiliary lifting cable is small, it is neglected in the preliminary estimation of the suspension cable section area. Then, the auxiliary suspension cable is approximated by the quadratic parabola (Fig. 9) and the maximum horizontal force at the upper supporting point F_H is

$$F_H = \frac{(g_{acb} + p')L_m^2}{8f} \tag{54}$$

where g_{acb} is the dead load per unit length of the auxiliary suspension cable.

As the auxiliary suspension cable curve is assumed to be a quadratic parabola, the maximum cable force of the auxiliary suspension cable F_T is

$$F_T = F_H \frac{\sqrt{L_m^2 + 16f^2}}{L_m} = (g_{acb} + p')L_m \cdot \frac{\sqrt{L_m^2 + 16f^2}}{8f} \tag{55}$$

Then, the auxiliary suspension cable cross-sectional area A is obtained after substituting $F_T = [\sigma]_{cb}A$ and $g_{acb} = \gamma_{cb}A$ into the above equation, yielding

$$A = \frac{p'L_m\sqrt{L_m^2 + 16f^2}}{8[\sigma]_{cb} f - \gamma_{cb} L_m \sqrt{L_m^2 + 16f^2}} \tag{56}$$

The auxiliary suspension cable cross-sectional area A is preliminarily determined from the above equation.

2.4 Analysis of equivalent elastic modulus and vertical support stiffness of the lifted cable

Currently, the Ernst formula is commonly used in engineering to calculate the equivalent elastic modulus of a stay cable, but the application of this formula to a long stay cable will produce a large error and, thus, it is necessary here to discuss the axial stiffness of the lifted cable based on the approximation calculation method and catenary cable geometry.

According to the calculation assumption of the general lifting condition, the cable curves on the two sides are equivalent to a horizontal cable with chord length a_δ (Fig. 8), such that the lifted cable is approximately equivalent to a series connection of two equivalent horizontal stay cables with equal axial stiffness and length. Thus, the equivalent elastic modulus of the lifted cable is determined by the elastic modulus of the equivalent horizontal stay cable. The following two loading conditions are analyzed to study the relationship between the chord force and chord length of the equivalent horizontal stay cable (Fig. 10).

For condition A, the catenary geometry is

$$y_{\delta,a} = \frac{T_{\delta,a}}{g''_{cb}}\left\{\cosh\left(\frac{g''_{cb}}{2T_{\delta,a}}a_\delta\right) - \cosh\left[\frac{g''_{cb}}{T_{\delta,a}}\left(x - \frac{a_\delta}{2}\right)\right]\right\} \tag{57}$$

The cable curve length is

$$s_{\delta,a} = \frac{2T_{\delta,a}}{g''_{cb}}\sinh\left(\frac{g''_{cb}}{2T_{\delta,a}}a_\delta\right) \tag{58}$$

The total elongation $\Delta s_{\delta,a}$ is

$$\Delta s_{\delta,a} = \frac{T_{\delta,a}^2}{2g''_{cb}E_{cb}A_{cb}}\left[\sinh\left(\frac{g''_{cb}}{T_{\delta,a}}a_\delta\right) + \frac{g''_{cb}}{T_{\delta,a}}a_\delta\right] \tag{59}$$

For condition B, the catenary geometry is

$$y_{\delta,b} = \frac{T_{\delta,b}}{g''_{cb}}\left\{\cosh\left[\frac{g''_{cb}}{2T_{\delta,b}}(a_\delta + \Delta_\delta)\right] - \cosh\left[\frac{g''_{cb}}{T_{\delta,b}}\left(x - \frac{a_\delta + \Delta_\delta}{2}\right)\right]\right\} \tag{60}$$

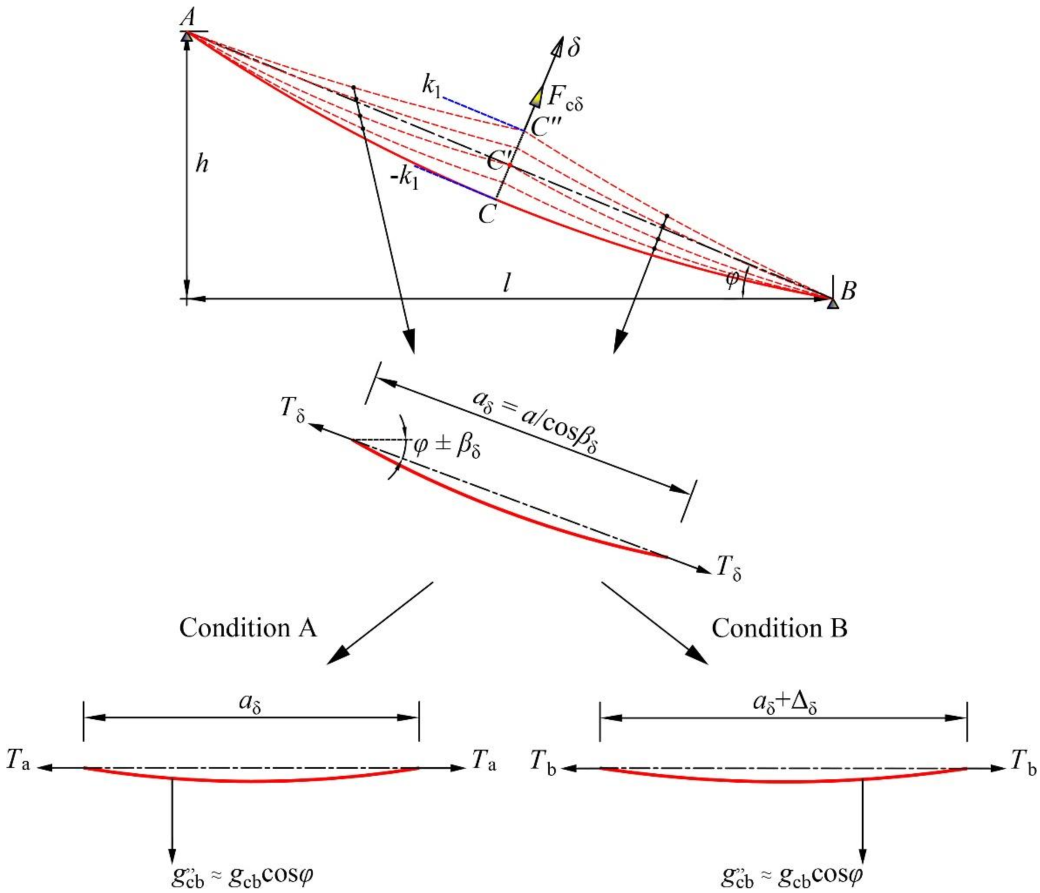


Fig. 10 Equivalent horizontal stay cables under two types of loading conditions

The cable curve length is

$$s_{\delta,\mathrm{b}} = \frac{2T_{\delta,\mathrm{b}}}{g''_{\mathrm{cb}}}\sinh\left[\frac{g''_{\mathrm{cb}}}{2T_{\delta,\mathrm{b}}}(a_\delta + \Delta_\delta)\right] \tag{61}$$

Considering $\Delta_\delta \ll a_\delta$, the above equation is replaced by

$$s_{\delta,\mathrm{b}} = \frac{2T_{\delta,\mathrm{b}}}{g''_{\mathrm{cb}}}\sinh\left(\frac{g''_{\mathrm{cb}}}{2T_{\delta,\mathrm{b}}}a_\delta\right) + \Delta_\delta\cosh\left(\frac{g''_{\mathrm{cb}}}{2T_{\delta,\mathrm{b}}}a_\delta\right) \tag{62}$$

The total elongation $\Delta s_{\delta,\mathrm{b}}$ is

$$\Delta s_{\delta,\mathrm{b}} = \frac{T^2_{\delta,\mathrm{b}}}{2g''_{\mathrm{cb}}E_{\mathrm{cb}}A_{\mathrm{cb}}}\left[\sinh\left(\frac{g''_{\mathrm{cb}}}{T_{\delta,\mathrm{b}}}a_\delta\right) + \frac{g''_{\mathrm{cb}}}{T_{\delta,\mathrm{b}}}a_\delta\right] \tag{63}$$

According to the condition compatibility equation $s_{\delta,\mathrm{b}} - s_{\delta,\mathrm{a}} = \Delta s_{\delta,\mathrm{b}} - \Delta s_{\delta,\mathrm{a}}$, substituting Eqs. (58), (59), (62) and (63) into it, the following expression after arrangement is obtained as

The elastic modulus of the equivalent horizontal stay cable $E_\delta = (T_{\delta,\mathrm{b}} - T_{\delta,\mathrm{a}})/(A_{\mathrm{cb}}\varepsilon_\delta)$ is obtained by substituting the above equation into it. Then, the equivalent elastic modulus of the lifted cable $E_{\delta,\mathrm{eq}}$ is determined by the series connection of two equal-stiffness idealized straight cable elements (Fig. 11).

$$\frac{2a}{E_{\delta,\mathrm{eq}}A_{\mathrm{cb}}} = \frac{a_\delta}{E_\delta A_{\mathrm{cb}}} + \frac{a_\delta}{E_\delta A_{\mathrm{cb}}}$$
$$\Rightarrow E_{\delta,\mathrm{eq}} = E_\delta\cos\beta_\delta \tag{65}$$

For the chord-wise cable force $T_{\delta,\mathrm{a}}$ and $T_{\delta,\mathrm{b}}$ in Eq. (64), the values are determined by considering the following two types of typical loading conditions. One is the case of a dead load condition and the other a traffic load condition, from which the cable force is thus estimated (Fig. 12). At the anchorage point of the girder, it is assumed that the cable supports dead and live loads on the girder segment within the cable spacing as well as half of the self-weight of the cable (Xiao 2016). The cable force of the i^{th} cable

$$\varepsilon_\delta = \frac{\Delta_\delta}{a_\delta} = \frac{(T_{\delta,\mathrm{b}} - T_{\delta,\mathrm{a}}) + \dfrac{1}{g''_{\mathrm{cb}}a_\delta}\left[T^2_{\delta,\mathrm{b}}\sinh\left(\dfrac{g''_{\mathrm{cb}}}{T_{\delta,\mathrm{b}}}a_\delta\right) - T^2_{\delta,\mathrm{a}}\sinh\left(\dfrac{g''_{\mathrm{cb}}}{T_{\delta,\mathrm{a}}}a_\delta\right)\right]}{2E_{\mathrm{cb}}A_{\mathrm{cb}}\cosh\left(\dfrac{g''_{\mathrm{cb}}}{2T_{\delta,\mathrm{b}}}a_\delta\right)} + \frac{\dfrac{2}{a_\delta}\left[T_{\delta,\mathrm{a}}\sinh\left(\dfrac{g''_{\mathrm{cb}}}{2T_{\delta,\mathrm{a}}}a_\delta\right) - T_{\delta,\mathrm{b}}\sinh\left(\dfrac{g''_{\mathrm{cb}}}{2T_{\delta,\mathrm{b}}}a_\delta\right)\right]}{g''_{\mathrm{cb}}\cosh\left(\dfrac{g''_{\mathrm{cb}}}{2T_{\delta,\mathrm{b}}}a_\delta\right)} \tag{64}$$

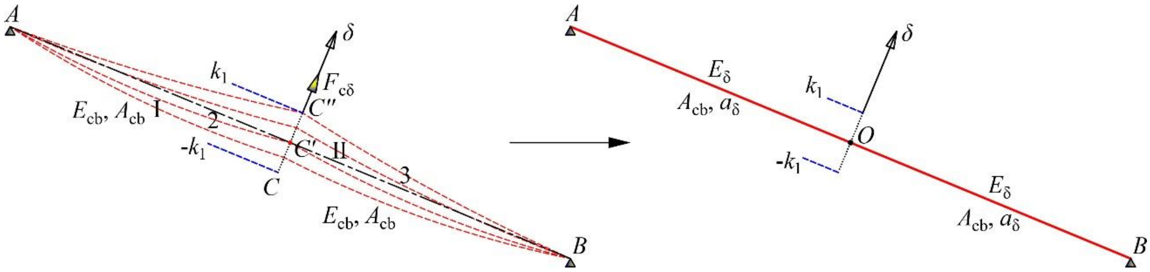


Fig. 11 Idealized straight cable elements with equivalent axial stiffness of the lifted cable

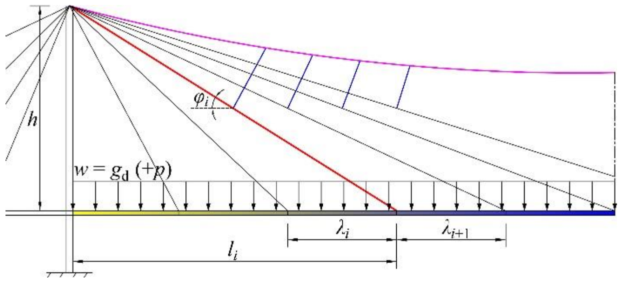


Fig. 12 Schematic diagram of loading conditions of the stay cable

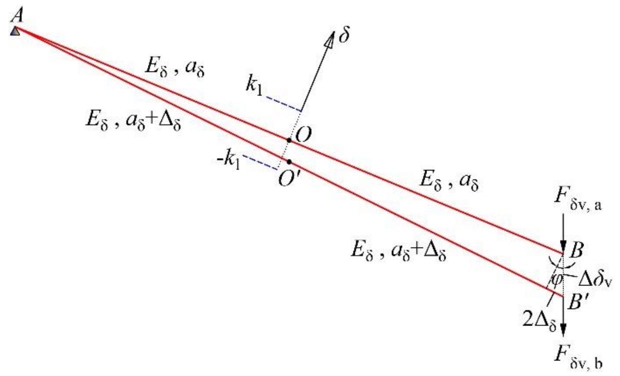


Fig. 13 Equivalent vertical support stiffness of the lifted cable

under two types of typical loading conditions is

$$\left.\begin{aligned} &\text{Condition A:} \quad T^i_{\delta,\,a} = g_d \frac{\lambda_i + \lambda_{i+1}}{4\sin\varphi_i} + \gamma_{cb} A_{cb1} \frac{l_i}{\sin 2\varphi_i} \\ &\text{Condition B:} \quad T^i_{\delta,\,b} = (g_d + p) \frac{\lambda_i + \lambda_{i+1}}{4\sin\varphi_i} + \gamma_{cb} A_{cb1} \frac{l_i}{\sin 2\varphi_i} \end{aligned}\right\} \quad (66)$$

Without considering the bending stiffness of the girder, the equivalent vertical support stiffness of the lifted cable $K_{v,\delta}$ is determined by the following expression according to the geometric relationship shown in Fig. 13.

$$K_{v,\,\delta} = \frac{\Delta F_v}{\Delta\delta_v} = \frac{(T_{\delta,\,b} - T_{\delta,\,a})\sin\varphi}{2\Delta_\delta \csc\varphi} = \frac{(T_{\delta,\,b} - T_{\delta,\,a})}{2\Delta_\delta}\sin^2\varphi \quad (67)$$

where Δ_δ is the chord-wise elongation of single-side equivalent cable.

3. Example analysis of the lifted cable of the cable-stayed bridge

3.1 Basic design parameters

In this section, a main span 1400 m cable-stayed bridge scenario (Fig. 14(a)) is taken as the engineering case and the external cable of the cable-stayed bridge taken as the research object (Nagai *et al*. 2004, Miao 2006, Zhang 2007, Xiong *et al*. 2011). Then, the static approximation analysis of the lifted cable of the cable-stayed bridge (Fig. 14(b)) is discussed. The specific design parameters of the materials and the structural system of the cable-stayed bridge are listed below.

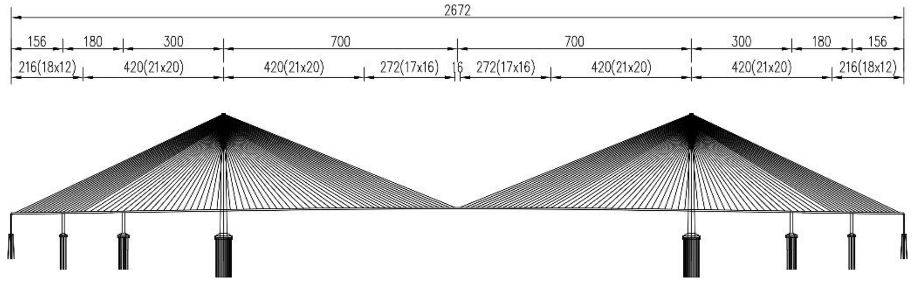
The principle parameters of the main span of the cable-stayed bridge are L_m = 1400 m, height of the pylon h = 287 m, horizontal projection length of the cable l = 692 m, height-to-span ratio of the cable-stayed bridge n = 0.2, midspan cable spacing λ = 16 m, cable strength $[\sigma]_{cb}$ = 744 MPa (f_k = 1860 MPa) and weight per unit volume of the cable γ_{cb} = 80 kN/m^3 (including corrosion protection). The stay cable is arranged with double planes, with the cross-sectional area of a single-side external cable A_{cb1} = 0.01 m^2. The dead load intensity g_d is safely set to 27 t/m. Live loads in design codes of most countries are composed of a uniformly distributed load and several concentrated forces. In super long-span bridges, especially in the preliminary design stage, concentrated forces can be safely represented as a uniformly distributed load with an amplification factor (Sun *et al*. 2016). Thus, live load intensity p, considering the multi-function traffic of long-span cable-stayed bridges and concentrated live load (for heavy trucks), with an amplification factor 1.5 (Zhang 2013), is safely set to (7 t/m). Based on the above basic design parameters, the static approximation analysis of the lifted cable of the cable-stayed bridge is carried out.

3.2 Estimation of single-side cable force

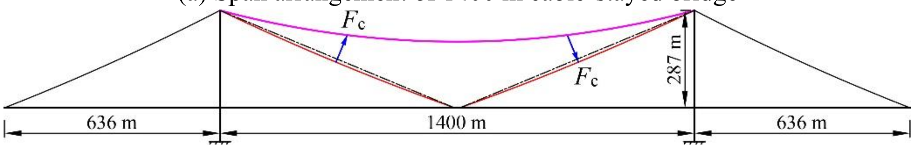
The external cable at midspan serves as the research object. The single-side cable force is estimated by the traffic load condition (Fig. 12); that is, at the girder anchorage point, it is assumed that the cable supports dead and traffic loads on the girder segment within the cable spacing as well as half of the cable self-weight. Thus, the cable force of the single-side external cable T_{cb1} is

$$T_{cb1} = (g_d + p)\frac{\lambda}{2\sin\varphi} + \gamma_{cb} A_{cb1} \frac{l}{\sin 2\varphi} \quad (68)$$

Substituting the parameters presented in Section 3.1 into the above equation obtains the single-side external cable force T_{cb1} = 7882.21 kN.



(a) Span arrangement of 1400 m cable-stayed bridge



(b) External cables and auxiliary cables

Fig. 14 Layout of external stay cable lifted by the suspension cable with the main 1400 m span self-anchored cable-stayed bridge (m)

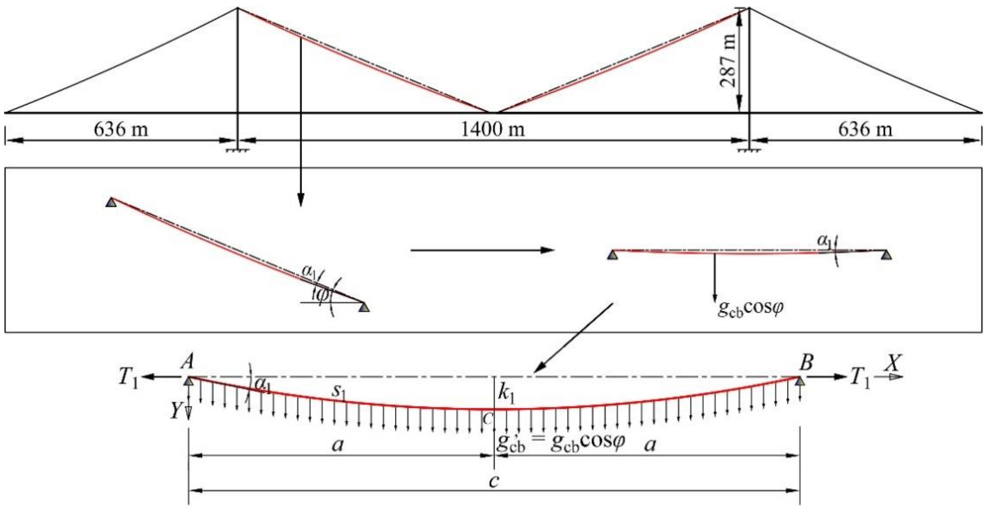


Fig. 15 The midspan external cable and its equivalent horizontal stay cable (condition 1)

3.3 Typical lifting conditions

3.3.1 Original stay cable (condition 1)

Using the aforementioned lifted cable approximation calculation method, in this example, the external cable of the cable-stayed bridge with a main span of 1400 m is equivalent to a horizontal cable with the same chord length ($c = l/\cos\varphi = 2a$) (Fig. 15), which is subjected to a vertical load intensity $g'_{cb}(=g_{cb}\cos\varphi)$.

Substituting each parameter into Eqs. (2) and (8) obtains the midspan external cable sag $k_{1,cat} = 6.578$ m and $k_{1,par} = 6.577$ m and then the vertical sags are $k_{v1,cat} = 7.121$ m and $k_{v1,par} = 7.120$ m, respectively. Substituting $k_{1,cat}$ and $k_{1,par}$ into Eqs. (3) and (9) obtains the cable end inclinations $\alpha_{1,cat} = 2.012^{\circ}$ and $\alpha_{1,par} = 2.011^{\circ}$, respectively. Then, according to Eqs. (4), (6), (11) and (12), the non-stress lengths of external cable under the catenary cable geometry $s_{01,cat} = 746.441$ m and under parabolic approximation cable geometry $s_{01,par} = 746.441$ m are obtained, respectively. By comparing the calculation results based on the catenary and parabolic cable configurations in condition 1, the parabolic approximation equivalent results are seen to be very close to those of catenary cable geometry.

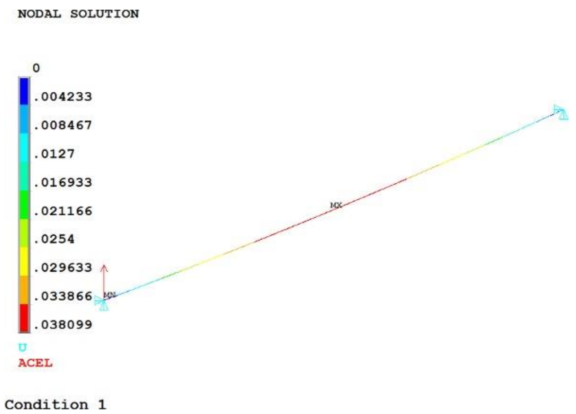


Fig. 16 Vertical deformation nephogram of the external cable (condition 1)

Results of numerical analysis show a vertical sag of the inclined cable $k_{v1,cat} = 7.120$ m, the cable end inclination angles of the anchorage point of the pylon and girder are $\alpha_{1,top} = 1.992^{\circ}$ and $\alpha_{1,bot} = 2.031^{\circ}$, respectively and the non-stress length of the external cable $s_{01} = 746.440$ m

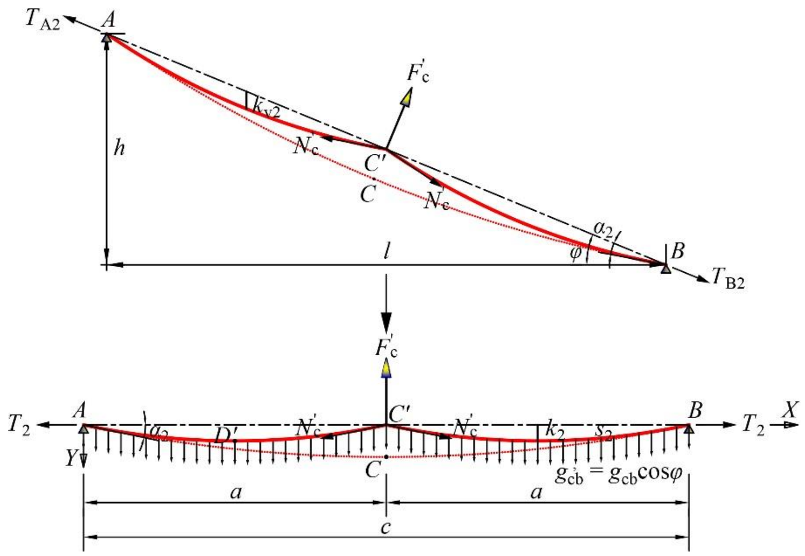


Fig. 17 Auxiliary cable-lifting scheme 1 and its equivalent horizontal stay1 cable (condition 2)

(Fig. 16). The approximate calculation results are very close to those of the numerical analysis, which meet the accuracy requirements of actual projects.

3.3.2 Auxiliary cable-lifting scheme 1 (condition 2)

The auxiliary cable-lifting scheme 1 is to lift the cable to the midpoint of the cable chord (condition 2).

Solving Eq. (19) obtains the chord force $T_{2,\mathrm{cat}}$ = 7575.770 kN of the half-span equivalent horizontal stay cable in condition 2. According to Eq. (14), the half-span equivalent horizontal stay cable sag is $k_{2,\mathrm{cat}}$ = 1.711 m and the stay cable vertical sag is then $k_{v2,\mathrm{cat}}$ = 1.852 m. According to the Eq. (15), the half-span equivalent horizontal stay cable end inclination angle is $\alpha_{2,\mathrm{cat}} = 1.047^\circ$. Then, from Eqs. (16)-(17), the non-stress length of the half-span equivalent horizontal stay cable with catenary geometry is $s_{02,\mathrm{cat}}$ = 373.221 m.

Solving cubic Eq. (25) concerning the sag ratio ($k_{2,\mathrm{par}}/a$) obtains $k_{2,\mathrm{par}}$ = 1.711 m and then the vertical sag of the inclined cable $k_{v2,\mathrm{par}}$ = 1.852 m. Substituting the value of $k_{v2,\mathrm{par}}$ into Eqs. (21)-(22), the half-span equivalent horizontal stay cable chord force $T_{2,\mathrm{par}}$ = 7575.793 kN and end inclination angle $\alpha_{2,\mathrm{par}} = 1.047^\circ$ are obtained. According to Eqs. (23)-(24), the non-stress length of the half-span equivalent horizontal stay cable under parabolic approximation is $s_{02,\mathrm{par}}$ = 373.221 m. By comparing the calculation results based on catenary and parabolic cable configurations in condition 2, the parabolic approximation equivalent results are seen to be very close to those of catenary cable geometry.

Results of numerical calculation show that the sag of the half-span external cable under the auxiliary lifting scheme 1 is reduced to k_2 = 1.712 m. The cable end inclination angles of the anchorage point of the pylon and girder are $\alpha_{2,\mathrm{top}} = 1.042^\circ$ and $\alpha_{2,\mathrm{bot}} = 1.053^\circ$, respectively (Fig. 18). The non-stress length of the half-span external cable under the auxiliary lifting scheme 1 is s_{02} = 373.222 m. The approximate calculation results are very close to those of the numerical analysis, which meet the accuracy requirements of actual projects.

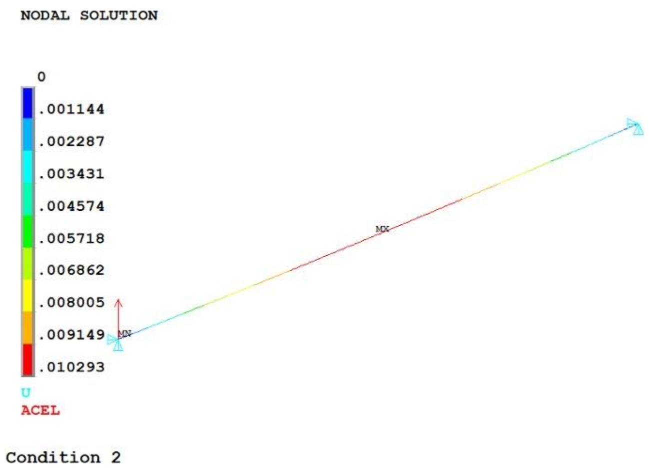


Fig. 18 Vertical deformation nephogram of the half-span external cable (condition 2)

The concentrated force acting on the midpoint of cable F_c' is determined from the force balance condition combined with the geometric relationship at the lifting point (Fig. 17).

$$F_{\mathrm{c}}' = 2N_{\mathrm{c}}' \sin\alpha_2 \tag{69}$$

where N_c' is obtained according to the force balance in the direction of the cable chord in Fig. 3, such that $N_c' = (T_2 + 2g_{\mathrm{cb}}\sin\varphi \cdot s_{02})/\cos\alpha_2$. Substituting T_2 and α_2 obtained into this equation obtains the concentrated force acting on the midpoint of the external cable under the cable geometry of the catenary as well as the parabolic approximate $F_{c,\mathrm{cat}}' = 280.995\ \mathrm{kN}$ and $F_{c,\mathrm{par}}' = 280.979\ \mathrm{kN}$. Then, the section area of the auxiliary suspension cable is obtained according to Eqs. (53) and (56), such that $A_{2,\mathrm{cat}} \approx A_{2,\mathrm{par}} = 6.04\times10^{-2}\ \mathrm{m}^2$.

By comparing the calculation results of conditions 1 and 2, the sag of the auxiliary cable-lifting scheme 1, in which the cable is lifted at the stay cable midpoint to the chord line midpoint, decreases from the original cable sag k_1 = 6.58 m to the lifted cable k_2 = 1.71 m, such that the sag effect decreases significantly. The cable end inclination angle is

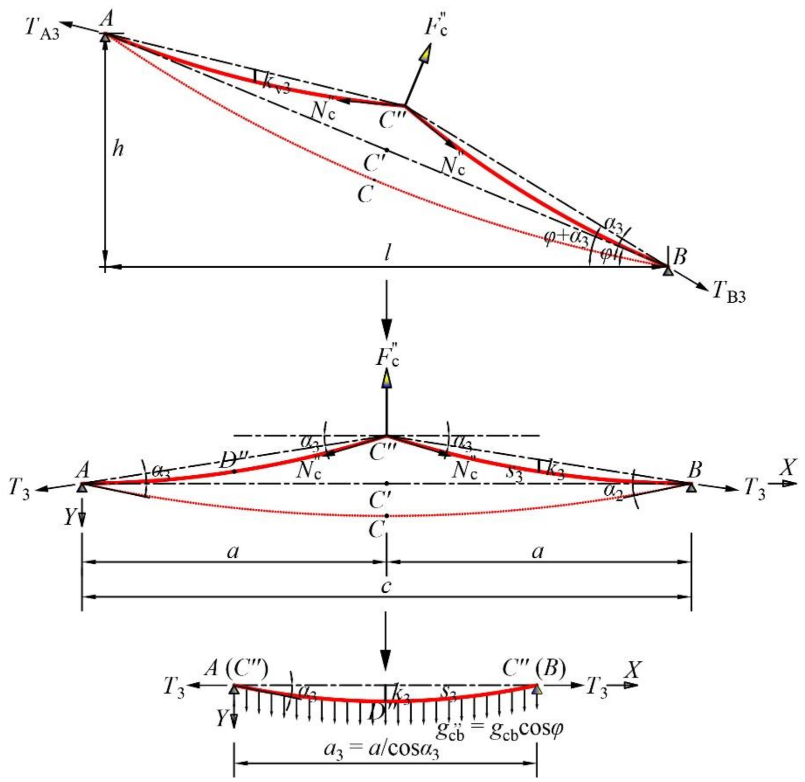


Fig. 19 Auxiliary cable-lifting scheme 2 and its equivalent horizontal stay cable (condition 3)

reduced by half, from the original value $\alpha_1 = 2.01^\circ$ to $\alpha_2 = 1.05^\circ$ after lifting. The cross-sectional area of the auxiliary suspension cable is about 0.06 m^2.

3.3.3 Auxiliary cable-lifting scheme 2 (condition 3)

The auxiliary cable-lifting scheme 2 is to lift the cable to the cable end tangent to the original cable chord (condition 3).

Combining Eqs. (28) and (32) to resolution obtains the chord force $T_{3,\mathrm{cat}}$ = 7883.426 kN of the equivalent horizontal stay cable in condition 3 and the equivalent horizontal stay cable end inclination angle $\alpha_{3,\mathrm{cat}} = 1.006^\circ$. According to Eq. (27), the equivalent horizontal stay cable sag is $k_{3,\mathrm{cat}}$ = 1.645 m and, according to Eqs. (29)-(30), the non-stress length of the equivalent horizontal stay cable with catenary geometry is $s_{03,\mathrm{cat}}$ = 373.221 m. In addition, the distance from the lifting point, where the cable end is tangent to the original cable, to the midpoint of the original cable chord is $\delta_{3,\mathrm{cat}} = a\tan\alpha_{3,\mathrm{cat}} = 6.577\ \mathrm{m} \approx k_{1,\mathrm{cat}}$.

Combining Eqs. (36) and (39) to resolution yields the sag ratio $k_{3,\mathrm{par}}/a_3 = 0.0044$ and cable end inclination angle $\alpha_{3,\mathrm{par}} = 1.006^\circ$. Then, the equivalent horizontal stay cable sag is obtained as $k_{3,\mathrm{par}}$ = 1.645 m and substituted into Eq. (35) to obtain the equivalent horizontal stay cable chord force $T_{3,\mathrm{par}}$ = 7883.418 kN. According to Eqs. (37)-(38), the non-stress length of the equivalent horizontal stay cable under parabolic approximation is $s_{03,\mathrm{par}}$ = 373.221 m. The distance from the lifting point, where the cable end is tangent to the original cable, to the midpoint of the original cable chord is $\delta_{3,\mathrm{par}} = a\tan\alpha_{3,\mathrm{par}} = 6.577\ \mathrm{m} \approx k_{1,\mathrm{par}}$. The parabolic approximate equivalent calculation results are very close to those of the catenary cable geometry.

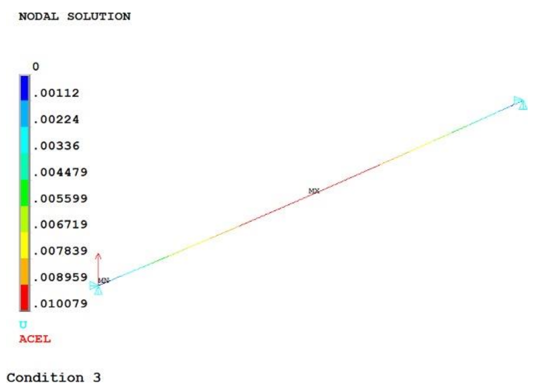


Fig. 20 Vertical deformation nephogram of the external cable (condition 3)

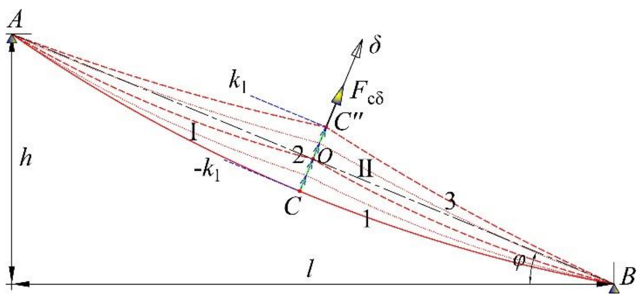
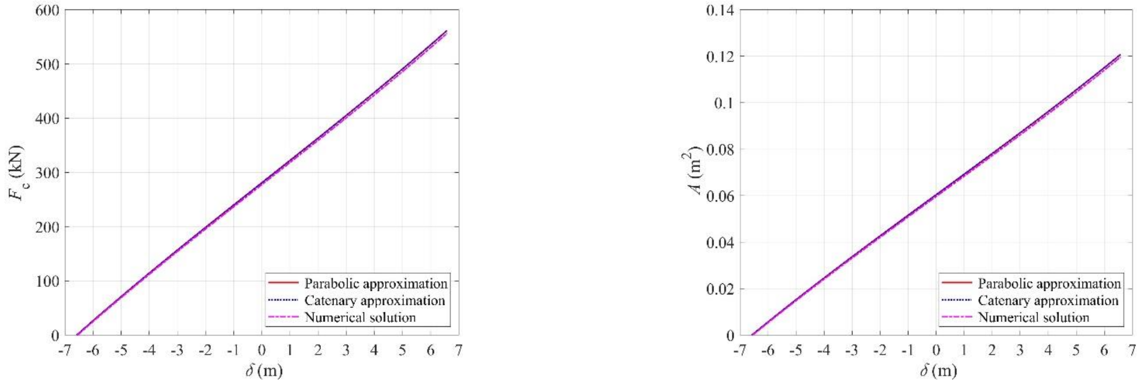


Fig. 21 Schematic diagram of the general conditions of auxiliary cable lifting

Results of numerical analysis show that the sag of the stay cable under the auxiliary lifting scheme 2 is reduced to k_3 = 1.643 m and the cable end inclination angles of the anchorage point of the pylon and girder are $\alpha_{3,\mathrm{top}} = 1.00^\circ$ and $\alpha_{3,\mathrm{bot}} = 1.01^\circ$, respectively and the non-stress length of



(a) $F_c - \delta$ (b) $A - \delta$

Fig. 22 Relationship between the auxiliary cable-lifting force and the auxiliary suspension cable cross-sectional area and the distance δ from the lifting point to the chord line midpoint

external cable under the auxiliary lifting scheme 2 is s_{03} = 373.229 m (Fig. 20). The approximate calculation results are very close to those from the numerical analysis, which meet the accuracy requirements of actual projects.

The concentrated force acting on the cable midpoint F_c'' is determined from the force balance condition combined with the geometric relationship at the lifting point (Fig. 19)

$$F_c'' = 2N_c'' \sin 2\alpha_3 \tag{70}$$

in which N_c'' is obtained according to the force balance in the cable chord direction in Fig. 3 and thus $N_c'' = (T_3 + 2g_{cb}\sin\varphi \cdot s_{03})/\cos\alpha_3$. Substituting T_3 and α_3, obtained above, into this equation obtains the concentrated force acting on the cable midpoint under the cable geometry of the catenary and the parabolic approximation, as $F_{c,\text{cat}}'' = 561.661\ kN$ and $F_{c,\text{par}}'' = 561.632\ kN$, respectively. Then, the section area of the auxiliary suspension cable is obtained according to Eqs. (53) and (56), such that $A_{3,\text{cat}} \approx A_{3,\text{par}} = 0.121$ m^2.

By comparing the calculation results of the two typical lifting schemes, the cable sag of the auxiliary cable-lifting scheme 2, which makes the cable end tangent to the original cable chord, is reduced to about 1.65 m from the 1.71 m of the auxiliary cable-lifting scheme 1. The sag effect reduction is not obvious and the lifting force acting at the cable midpoint is increased from 281 kN under auxiliary cable-lifting scheme 1 to 562 kN under lifting scheme 2, such that the lifting force is doubled. The section area of the auxiliary suspension cable in auxiliary cable-lifting scheme 2 is about 0.12 m^2, with the cross-sectional area doubled compared to that of lifting scheme 1.

3.4 General conditions of the auxiliary cable-lifting process

Three typical conditions of the auxiliary cable lifting are discussed here in detail. In this section, the general conditions of the cable-lifting process from condition 1 to condition 3 are analyzed and discussed. The distance δ from the lifting point to the midpoint of the original cable chord is a variable (Fig. 21), with the cable chord midpoint (original point C') as the origin O. If the cable-lifting point is below the chord midpoint, δ is negative, if the lifting point is higher than the chord midpoint, δ is positive and if the lifting point is at the origin O, then δ = 0 (condition 2). From the calculation results of the last section, it is found that the distance from the cable-lifting point, where the cable end is tangent to the original cable chord, to the midpoint of the original chord (condition 3) is nearly equal to the sag of the original inclined non-lifted cable (condition 1), such that the general conditions of the auxiliary cable-lifting process is discussed by selecting the lifting process within the range from condition 1 ($\delta = -k_1$) to condition 3 ($\delta = k_1$).

3.4.1 Relationship between auxiliary cable-lifting force and auxiliary suspension cable cross-sectional area and δ

The relationship between the auxiliary cable-lifting force and the required auxiliary suspension cable cross-sectional area and the distance at the corresponding lifting position from the lifting point to the midpoint of the original chord line δ during the lifting process is shown in Fig. 22.

It is observed from the Fig. 22 that the relationship of the auxiliary cable-lifting force and the required auxiliary suspension cable cross-sectional area of the numerical analysis and the approximate calculation results are both close to linear with δ, with the curves of both basically consistent. The results of the lifting forces and cross-sectional area of auxiliary suspension cables in condition 2 (δ = 0) and condition 3 ($\delta = k_1$) are consistent with the results calculated by the typical auxiliary cable-lifting schemes mentioned above. The results of the parabolic equivalent approximation are close to those of the catenary cable.

3.4.2 Relationship between the lifted cable vertical sag and δ

In the general conditions of cable lifting, the vertical sags of the upper and lower cables are determined by the following expressions, according to the geometric relationship shown in Fig. 7

$$\left.\begin{aligned} &\text{for upper cable,} \quad k_{\text{vt}} = k_\delta \sec(\varphi - \beta_\delta) \\ &\text{for lower cable,} \quad k_{\text{vb}} = k_\delta \sec(\varphi + \beta_\delta) \end{aligned}\right\} \tag{71}$$

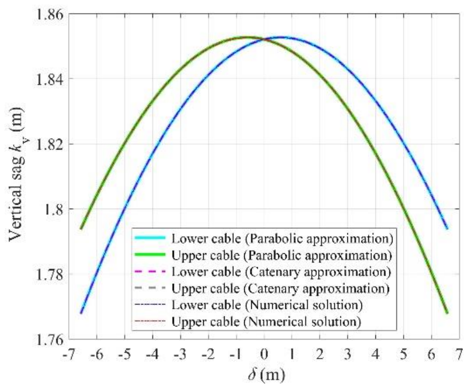


Fig. 23 Relationship between the vertical sags of the upper and lower cables and the distance δ from the lifting point to the chord line midpoint

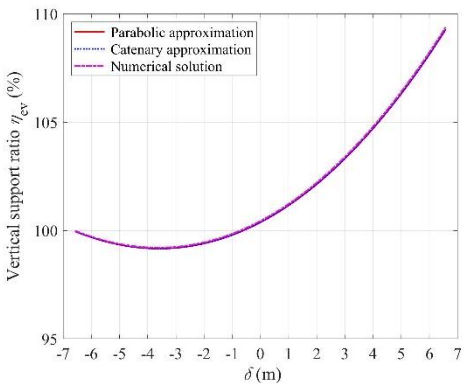


Fig. 24 Relationship between the vertical support efficiency ratio and the distance δ from the lifting point to the chord line midpoint

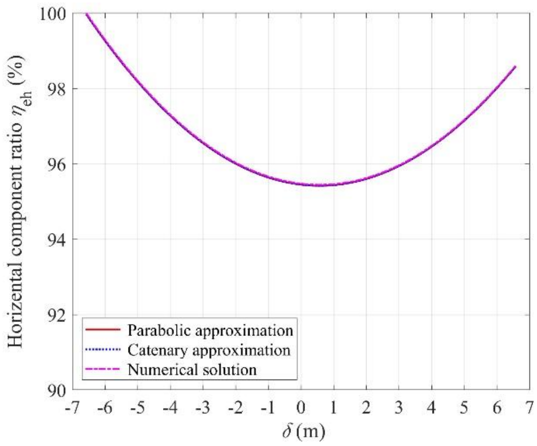


Fig. 25 Relationship between the horizontal component ratio of the cable force and the distance δ from the lifting point to the chord line midpoint

where k_δ is the sag of the corresponding equivalent horizontal stay cable. Then, the relationship between the vertical sags of the upper and lower side cables and the distance δ from the lifting point to the chord line midpoint during the lifting process are obtained from the above equation (Fig. 23).

It is observed from the Fig. 23 that the numerical analysis results are close to the approximate calculation results. Variations in the vertical sags of the upper and lower side cables are minor during the lifting process. When the cable is lifted to the chord line midpoint in condition 2 (δ = 0), the vertical sags of the upper and lower side cables are nearly the same, which is basically consistent with the actual situation. And the vertical sag values are consistent with the calculation results of the typical lifting scheme mentioned above. During the process of cable lifting, the vertical sag values of the upper and lower side cables show a symmetrical relationship relating to (δ = 0).

3.4.3 Relationship between vertical support efficiency and δ

For long-span stay cables, the cable end inclination at the girder anchorage point is reduced due to an obvious sag effect, which significantly reduces the vertical support efficiency, especially for the near midspan stay cables. To discuss the change relationship of the vertical support efficiency during the lifting process of the cable, the vertical support efficiency ratio $\eta_{ev,\delta}$ of the cable is defined as

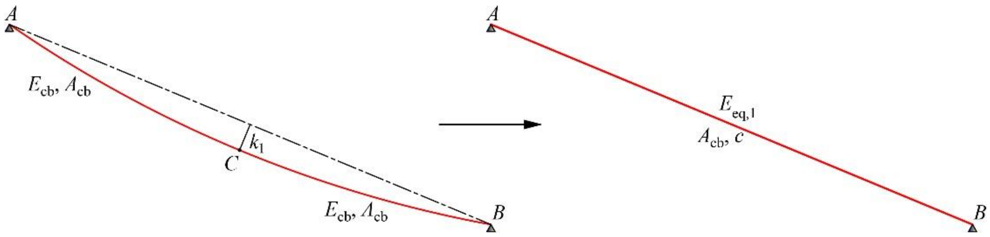
$$\eta_{ev,\delta} = \frac{F_{v\delta}}{F_{v1}} \times 100\% \tag{72}$$

where $F_{v\delta}$ is the vertical force component of the cable at the distance of auxiliary cable lifting δ from the lifting point to the chord line midpoint and F_{v1} the vertical force component of the original non-lifted cable at the girder anchorage point. Then, the relationship between the vertical support efficiency of the cable and the distance δ from the lifting point to the chord line midpoint during the lifting process is calculated using Eq. (72) (Fig. 24).

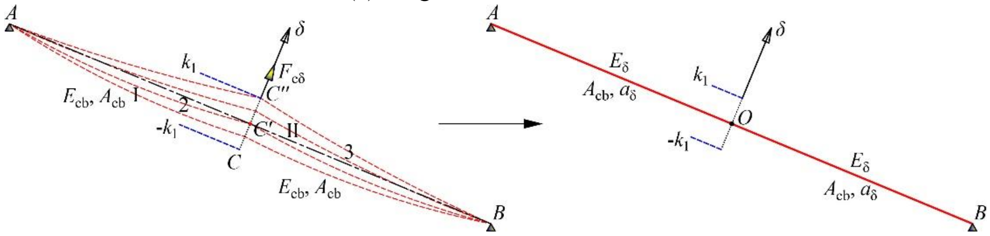
It is observed from the Fig. 24 that the vertical support efficiency of the cable does not increase linearly with the lifting force during the lifting process, but decreases slightly when the lifting position is below the chord line midpoint and then increases when the cable is lifted to the chord line midpoint (condition 1), such that the vertical support efficiency is basically the same as that of the original non-lifted cable. After that, with further elevation of the lifting point, the vertical support efficiency of the cable increases significantly and reaches a maximum in condition 3 (the lifted cable end tangent to the chord line). This means that the cable must be lifted above the cable chord to achieve the purpose of improving the vertical support efficiency by auxiliary cable lifting. In addition, the support efficiency change curve of the approximate calculation results is well consistent with that of the numerical analysis and the results of the parabolic equivalent approximation are close to the results of catenary cable geometry.

3.4.4 Relationship between cable force horizontal component and δ

As the span of the cable-stayed bridge increases, due to the accumulation of the horizontal component of the cable force, a large axial pressure is formed in the girder near the pylon and the excessive axial force of the girder becomes a factor in determining the span limit (Zhao *et al*. 2019). To discuss the change relationship of the horizontal component force during the auxiliary cable lifting, the horizontal component ratio $\eta_{eh,\delta}$ of the cable force is defined as



(a) Original non-lifted cable



(b) Lifted cables

Fig. 26 Cables and idealized straight cable elements with equivalent axial stiffness

$$\eta_{eh,\delta} = \frac{F_{h\delta}}{F_{h1}} \times 100\% \tag{73}$$

where $F_{h\delta}$ is the cable force horizontal component at the girder anchorage point at the distance of auxiliary cable lifting δ from the lifting point to the chord midpoint and F_{h1} the horizontal force component of the girder anchorage point of the original non-lifted cable. Therefore, variation between the cable force horizontal component during the lifting process and the distance from the lifting point to the chord midpoint is calculated using Eq. (73) (Fig. 25).

It is observed from the Fig. 25 that, during the lifting process, when the lifting point is below the chord line midpoint, the cable horizontal component decreases. When the cable is lifted to about $\delta = 1$ m outside the chord line midpoint, the cable horizontal component reaches a minimum and, then, with further elevation of the lifting point, the cable horizontal component gradually increases. The maximum value is reached when the cable is elevated to condition 3 (lifted cable to the cable end tangent to the chord line), but it is still slightly smaller than for the non-lifted cable (condition 1). The horizontal component variation shows that the lifted cable has limited effect on reducing the horizontal component. In addition, the variation of the horizontal component force curve calculated by the approximate calculation method is well consistent with that of the numerical analysis results and the results of the parabolic equivalent approximation are close to the results of catenary cable geometry.

3.5 Equivalent elastic modulus and vertical support stiffness of the lifted cable

3.5.1 Relationship between the equivalent elastic modulus and δ

According to the analysis in Section 2.6, the equivalent elastic modulus $E_{\delta,eq}$ of the lifted cable is determined by the series connection of the idealized stiffness of the straight cable elements with same section area, modulus of elasticity

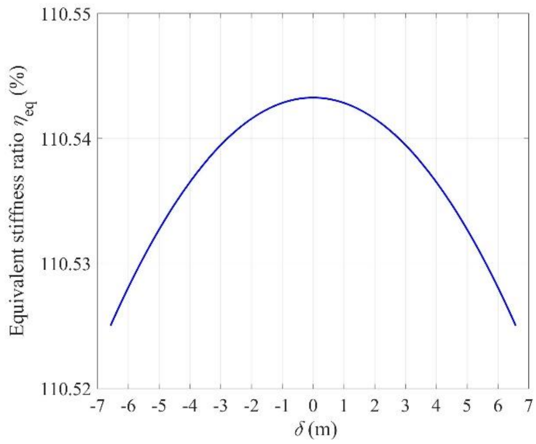


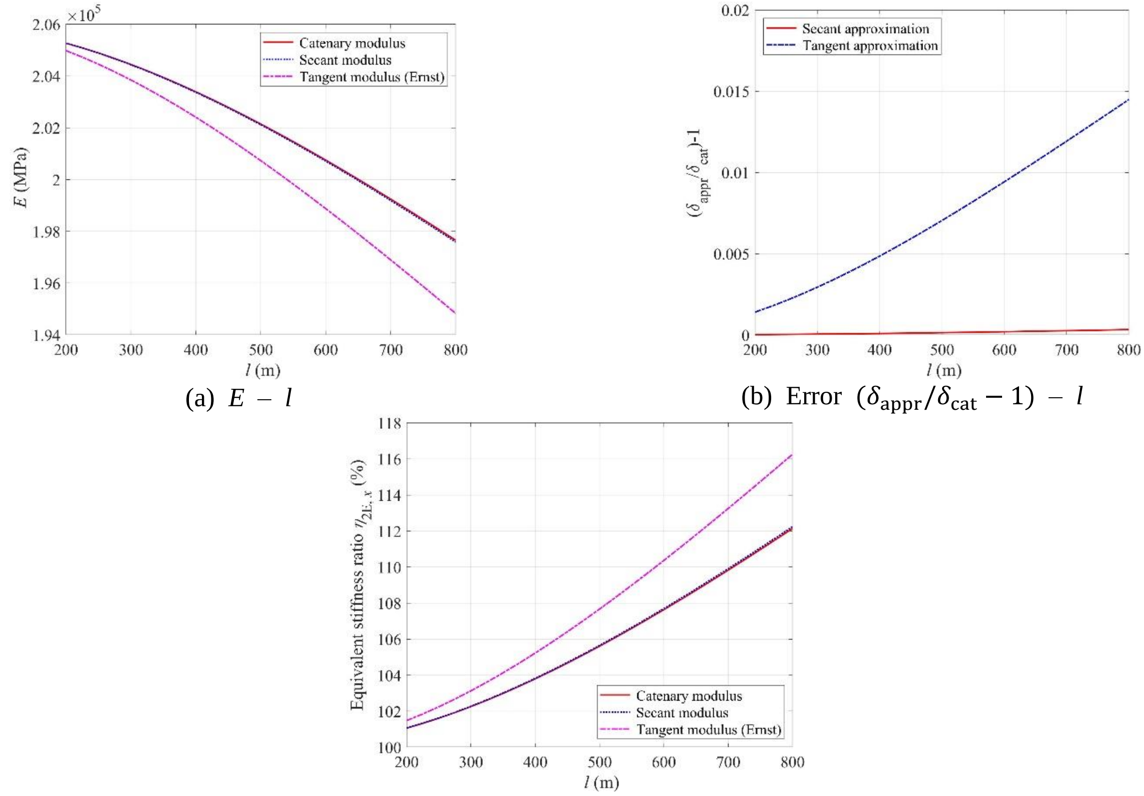
Fig. 27 Relationship between the equivalent elastic modulus ratio of the lifted cable and the distance δ from the lifting point to the chord line midpoint

E_δ and the length a_δ (Fig. 26(b)); thus Eq. (65) is established. To discuss the effect of auxiliary cable lifting on axial stiffness, the equivalent elastic modulus ratio $\eta_{eq,\delta}$ of the lifted cable and the original non-lifted cable (condition 1) is defined as

$$\eta_{eq,\delta} = \frac{E_{\delta,eq}}{E_{eq,1}} \times 100\% \tag{74}$$

The relationship between the equivalent elastic modulus of the lifted cable and the distance from the lifting point to the chord line midpoint δ is shown in Fig. 27.

It is observed from the Fig. 27 that the axial stiffness of the lifted cable is increased by about 10% over that of the original non-lifted cable, but the equivalent stiffness change of the lifted cable is not obvious for different δ. It should be pointed out that, unlike the continuous process of the cable lifting discussed above, the axial stiffness change of the lifted cables and non-lifted cable is discussed here under two different boundary conditions. Therefore, the equivalent elastic modulus ratio of the lifted cables does not begin to change from 100%. This also shows that the



(a) $E - l$ (b) Error $(\delta_{\mathrm{appr}}/\delta_{\mathrm{cat}} - 1) - l$

(c) $\eta_{2\mathrm{E},x} - l$

Fig. 28 Relationship between the equivalent elastic modulus of the lifted cables and the horizontal projection length

increase of axial stiffness of lifted cables is mainly affected by the change in cable boundary conditions after lifting and is not sensitive to the change of the lifting point position.

3.5.2 Relationship between the equivalent elastic modulus of the lifted cable and the horizontal projection length of the cable

From the analysis discussed in the above section, the equivalent axial stiffness of the lifted cable does not change significantly relative to δ and, thus, the relationship between the equivalent elastic modulus of the lifted cable and the horizontal projection length is analyzed and discussed here, using the typical lifting condition 2 (δ = 0). The discussion range of horizontal projection length of stay cables is selected from 200 to 800 m and the height-to-span ratio of the cable-stayed bridge n = 0.2, with other parameters the same as before. To discuss the relationship between the axial stiffness of the lifted cables and the horizontal projection length, the equivalent elastic modulus ratio $\eta_{2\mathrm{E},x}$ of the lifted cables and the original non-lifted cable is defined as

$$\eta_{2\mathrm{E},x} = \frac{E_{2,x}}{E_{1,x}} \times 100\% \tag{75}$$

The relationship between the equivalent elastic modulus ratio of the lifted cable and the horizontal projection length is shown in Fig. 28. In addition, the secant and tangent moduli (Ernst formula) using the parabolic approximation and the error $(\delta_{\mathrm{appr}}/\delta_{\mathrm{cat}} - 1)$ by application of the tangent or secant modulus are also shown.

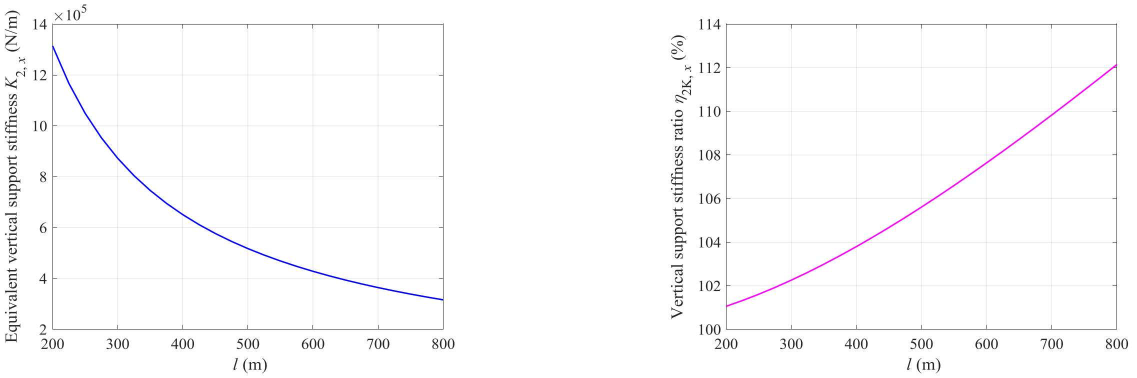
It is observed from the Fig. 28 that the equivalent elastic modulus ratio of the lifted cables increases significantly with increased horizontal projection length while the equivalent elastic modulus value decreases with increased horizontal projection length. When the horizontal projection length is 200 m, the axial stiffness increases by only 1% while, when the horizontal projection length is 800 m, the axial stiffness increases by 12%, which indicates that the lifted cable, using the auxiliary cable-lifting scheme has an obvious increasing effect on the stiffness of the super-long stay cables. In addition, comparing the results of correct catenary solution and secant and tangent moduli based on the parabolic approximation show that the secant modulus is a good approximation of the catenary solution, while the application of tangent modulus (Ernst formula) will produce a large error.

Equivalent vertical support stiffness of lifted cables is obtained using Eq. (67). To discuss the relationship between the vertical support stiffness of the lifted cables and the horizontal projection length, the equivalent vertical stiffness ratio $\eta_{2\mathrm{K},x}$ of the lifted cable and original non-lifted cable is defined as

$$\eta_{2\mathrm{K},x} = \frac{K_{2,x}}{K_{1,x}} \times 100\% \tag{76}$$

The relationship between the equivalent vertical support stiffness ratio of the lifted cable and the horizontal projection length is shown in Fig. 29.

It is observed from the Fig. 29 that the equivalent vertical support stiffness ratio of the lifted cables increases



(a) $K_{2,x} - l$ (b) $\eta_{2K,x} - l$

Fig. 29 Relationship between the equivalent vertical support stiffness of the lifted cables and the horizontal projection length

significantly with increased horizontal projection length while the equivalent support stiffness value decreases with increased horizontal projection length, which is similar to the variation in the equivalent elastic modulus ratio relationship. When the horizontal projection length is 200 m, the vertical support stiffness increases by only 1% while, when the horizontal projection length is 800 m, the vertical support stiffness increases by 12%. This also shows that the lifted cable, using the auxiliary cable-lifting scheme has an obvious increasing effect on the vertical support stiffness of the super-long cables.

5. Conclusions

The results of this study can be summarized as follows:

- The approximate calculation results of a lifted stay cable based on catenary and parabolic cable configurations in three typical lifting conditions and the whole process of auxiliary cable lifting are very close to the numerical analysis results, which verifies the applicability of the approximation method proposed in this paper.
- The scheme of lifting the cable to the chord line midpoint is more economical due to the less steel required for the auxiliary suspension cable, but its effect on improving the vertical support efficiency is limited. While the support efficiency is better with the cable lifted to the cable end tangential to the original cable chord, the lifting force and cross-sectional area of the auxiliary suspension cable are doubled.
- The sag effect is effectively reduced using auxiliary lifting-suspended cables to lift long stay cables, but the support efficiency can only be improved when the cables are lifted above the original cable chord. Reduction of the cable horizontal component force is limited when the cable is lifted.
- The equivalent elastic modulus and the vertical support stiffness of the lifted cables are significantly increased with the increase of the horizontal projection length and not sensitive to the change of the lifting point position.
- The results of parabolic approximation calculations are approximately equal to that of catenary cable geometry. As the parabolic approximation analysis theory of lifted cables is more convenient in mathematical processing, it is feasible to use parabolic approximation analysis theory as the analytical method for the conceptual design of lifted cables of super-long span cable-stayed bridges.

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Notation

The following main variables are used in this paper:

A cross-sectional area of the auxiliary suspension cable;

A_{cb} cross-sectional area of the cable;

A_{cb1} cross-sectional area of a single-side cable;

E_{cb} elastic modulus of the cable;

$E_{eq,1}$ equivalent elastic modulus of the original non-lifted cable;

E_{δ} elastic modulus of the equivalent horizontal stay cable;

$E_{\delta,eq}$ equivalent elastic modulus of the lifted cable;

F_{H} horizontal component of cable force of the auxiliary suspension cable;

F_{T} cable force of the auxiliary suspension cable;

F_c' concentrated force acting on the midpoint of cable;

F_{h1} horizontal force component of the girder anchorage point of the original non-lifted cable;

$F_{h\delta}$ horizontal component of the cable at the girder anchorage point at the distance of auxiliary cable lifting δ from the lifting point to the chord midpoint;

F_{v1} vertical force component of the original non-lifted cable at the girder anchorage point;

$F_{v\delta}$ vertical force component of the cable at the distance of auxiliary cable lifting δ from the lifting point to the chord line midpoint;

H horizontal component of cable force;

$K_{v,\delta}$ equivalent vertical support stiffness of the lifted cable;

T chord-wise cable force;

$[\sigma]_{cb}$ allowable stress of the cable;

a half the length of the original cable chord;

c chord length of the original cable;

g_{acb} dead load per unit length of the auxiliary suspension cable;

g_{cb} dead load per unit length of the cable;

g_{d} dead load of the girder;

h height of the pylon;

k inclined cable sag;

k_{v} vertical sag of the inclined cable;

k_{δ} sag of the corresponding equivalent horizontal stay cable;

l horizontal projection length of the cable;

n	height-to-span ratio of the cable-stayed bridge;
p	live load intensity on the girder;
p'	converted load intensity on the suspension cable;
s	cable curve length;
w	load of the girder per meter of track;
α	inclination angle of the cable end;
δ	distance from the lifting point to the midpoint of the original cable chord;
Δ	chord-wise elongation of the cable;
Δs	elongation of the cable from the unstressed condition;
Δ_δ	chord-wise elongation of single-side equivalent cable;
$\eta_{2\mathrm{E},x}$	equivalent elastic modulus ratio;
$\eta_{2\mathrm{K},x}$	equivalent vertical stiffness ratio of the lifted cable and original non-lifted cable;
$\eta_{\mathrm{eh},\delta}$	horizontal component ratio of the lifted cable and original non-lifted cable;
$\eta_{\mathrm{eq},\delta}$	equivalent elastic modulus ratio of the lifted cable and original non-lifted cable;
$\eta_{\mathrm{ev},\delta}$	vertical support efficiency of the lifted cable and original non-lifted cable;
γ_{cb}	weight per unit volume of the cable;
λ	midspan cable spacing cable spacing; and
φ	inclination of the chord linking the two cable ends.