Structural Engineering and Mechanics, *Vol. 74, No. 4 (2020) 503-514*
DOI: https://doi.org/10.12989/sem.2020.74.4.503

Shear strength prediction for SFRC and UHPC beams using a Bayesian approach

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(Received March 13, 2019, Revised November 14, 2019, Accepted December 26, 2019)

Abstract. This study proposes prediction models for the shear strength of steel fiber reinforced concrete (SFRC) and ultra-high-performance fiber reinforced concrete (UHPC) beams using a Bayesian parameter estimation approach and a collected experimental database. Previous researchers had already proposed shear strength prediction models for SFRC and UHPC beams, but their performances were limited in terms of their prediction accuracies and the applicability to UHPC beams. Therefore, this study adopted a statistical approach based on a collected database to develop prediction models. In the database, 89 and 37 experimental data for SFRC and UHPC beams without stirrups were collected, respectively, and the proposed equations were developed using the Bayesian parameter estimation approach. The proposed models have a simplified form with important parameters, and in comparison to the existing prediction models, provide unbiased high prediction accuracy.

Keywords: SFRC; UHPC; shear strength; steel fiber; bayesian parameter estimation

1. Introduction

The shear failure of a concrete member is undesirable due to a brittle nature, and various studies have been conducted to improve the performance of shear resistance of concrete members by proposing a minimum shear reinforcement ratio requirement and developing more accurate shear strength prediction models (Wight and MacGregor 2012). In addition, studies have been conducted to improve the post-cracking behavior of concrete members (Vora and Shah 2016, Kaya and Yaman 2018, Qissab and Salman 2018, Barakat *et al*. 2019). For example, fiber reinforcing is known as a method that can greatly improve the tensile and post-cracking performance. In particular, steel fibers are most widely used for this purpose because they are easy to produce and have uniform material properties compared to other fiber types (Sharma 1986, Narayanan and Darwish 1987, Al-Ta'an and Al-Feel 1990, Ashour *et al*. 1992, Khuntia *et al*. 1999, Oh and Kim 2008, Karl *et al*. 2010, Hwang *et al*. 2013, Wille *et al*. 2014, Lim and Hong 2016). According to the studies by Sharma (1986), Narayanan and Darwish (1987), Al-Ta'an and Al-Feel (1990), Ashour *et al*. (1992), Khuntia *et al*. (1999), Hwang *et al*. (2013), and Lim and Hong (2016), steel fiber reinforced concrete (SFRC) can prevent the development and propagation of macro cracks through the pull-out action of steel fibers, and the shear performance can be greatly increased through the improved tensile strength and ductility of the concrete compared to the plain concrete without steel fiber. The post cracking behavior of SFRC depends on the properties of steel fibers, the fiber volume fraction, and the crack arresting mechanism by fibers. On the other hand, ultra-high performance concrete (UHPC) is a kind of fiber reinforced concrete, but it is distinguished from SFRC because it has a special mix design to obtain an ultra-high strength by considering the following: the strength of concrete is closely related to porosity, and if porosity become smaller, the packing density of concrete increases, which increases the strength of the concrete (Naaman and Reinhardt 2003, Prisco *et al*. 2009, Wille *et al*. 2014, Rahdar and Ghalehnovi 2016). As shown in Fig. 1, UHPC is made by mixing materials with different grain

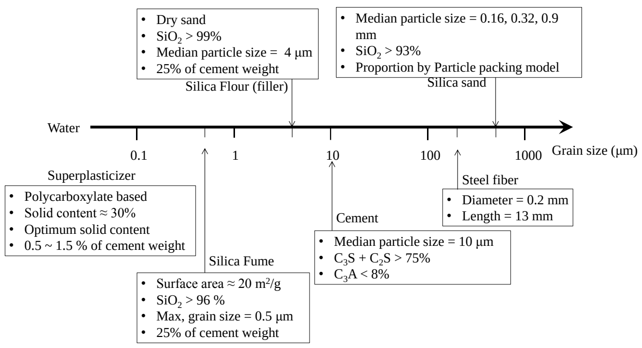

Fig. 1 Grain sizes of materials

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http://www.techno-press.com/journals/sem&subpage=7
ISSN: 1225-4568 (Print), 1598-6217 (Online)

sizes (Wille *et al*. 2012). For example, as the small grain size materials such as silica fume are placed between large grain size materials such as silica sand, the packing density of concrete increases. Fig. 2 shows the tensile behavior of SFRC and UHPC through the following stages: strain-hardening, multiple cracking, and strain-softening (Naaman and Reinhardt 2003, Prisco *et al*. 2009, Wille *et al*. 2014). In this figure, σ_{cc} and ε_{cc} are the first cracking strength and strain, respectively, σ_{cp} and ε_{cp} are the post cracking strength and strain, respectively. The SFRC mostly exhibits strain softening after the first crack occurs, and its strength decreases due to the expansion of the crack width. On the other hand, ultra-high performance fiber reinforced concrete generally shows strain-hardening and multiple cracking behaviors because the fiber's bond strength is greater than SFRC. UHPC generally have the optimum particle size distribution for maximum packing density in order to get ultra-high strength during the mix design. Therefore, UHPC has less porosity than SFRC and is denser than SFRC, which increases fiber bond strength. The strain-softening phase occurs after the strain-hardening phase because of the development of macro cracks.

Many attempts have been made to investigate the shear resistance of SFRC, but it is still difficult to fully understand the complex concrete shear mechanisms and randomness of fiber directions and bonding distributions. The shear strength of UHPC can be increased by optimum particle size distribution design, which improves the bonding between the cementitious matrix and the steel fiber, but it shows post-cracking behavior that is different from that of the existing non-ultra-high strength concrete. Various parameters, such as the fiber volume fraction, fiber type, member size, and reinforcement ratio add significant uncertainties to the prediction of shear resistance of an UHPC member. This complex behavior of an UHPC member makes the prediction of its shear strength difficult if we just use a theoretical analogy, and therefore, this study uses a statistical approach based on collected experimental observations.

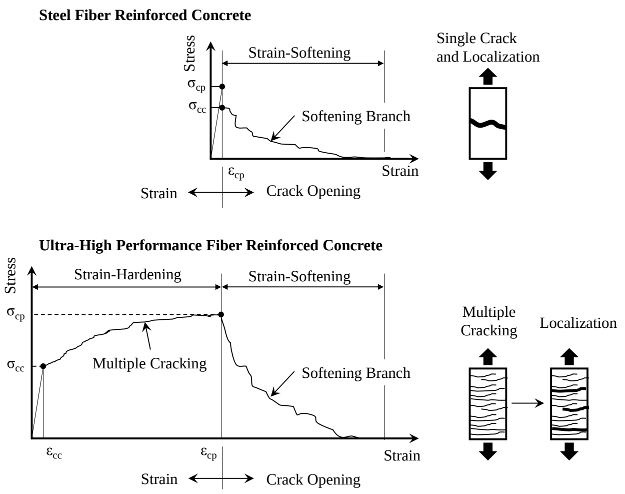


Fig. 2 Tensile behavior of steel fiber reinforced concrete (Naaman and Reinhardt 2003, Prisco *et al*. 2009, Wille *et al*. 2014)

2. Literature review

In this study, to evaluate existing concrete shear strength equations and to develop new statistical equations, a total of 126 shear test data of beams without stirrups have been collected, among which 89 were SFRC members and 37 were UHPC members. It should be noted that the test results that had flexural yielding prior to shear failure have been removed from the database. Tables 1 and 2 show the references, collected number of specimens, and the

Table 1 Parameters of shear test data for SFRC specimens

	The number of Specimens	Concrete tensile strength (f_t , MPa)	Concrete compressive strength (f_{ck} , MPa)	Shear span to depth ratio (a/d)	Reinforcement ratio (ρ_l , %)	Steel fiber volume fraction (V_f , %)	Fiber factor (F)	Shear stress (v_{test} , MPa)
Batson *et al*. (1972)	37	3.03 ~ 3.34	33.2 ~ 40.2	2.80 ~ 5.00	1.96	0.22 ~ 1.76	0.11 ~ 0.88	1.82 ~ 4.38
Narayanan and Darwish (1987)	18	2.80 ~3.76	28.4 ~51.1	2.52 ~3.52	2.00 ~5.72	0.25 ~1.00	0.19 ~1.00	1.94 ~5.00
Mansur *et al*. (1986)	7	2.39 ~3.04	20.6 ~33.4	2.80 ~3.60	1.34 ~2.00	0.50 ~0.75	0.30 ~0.45	1.52 ~2.91
Lim *et al*. (1987)	5	3.07	34.0	2.50 ~3.50	1.10 ~2.20	0.50 ~1.00	0.30 ~0.60	1.47 ~2.46
Ashour *et al*. (1992)	5	5.10 ~ 5.19	95.0 ~ 97.1	4.00 ~ 6.00	2.84 ~ 4.58	0.50 ~ 1.50	0.38 ~ 1.13	1.95 ~ 3.88
Li and Hamza (1992)	5	2.51 ~2.69	22.7 ~26.0	3.00	1.10 ~2.20	1.00	0.60 ~1.00	2.43 ~3.55
Swamy *et al*. (1993)	4	3.01 ~3.35	32.7 ~40.3	3.43 ~4.91	2.76 ~4.31	1.00	0.75	2.29 ~4.05
Swamy and Bahia (1985)	4	3.12 ~3.30	35.1 ~39.32	4.50	3.05 ~4.00	0.40 ~1.20	0.30 ~0.90	2.20 ~3.27
Karl *et al*. (2010)	4	3.14 ~3.33	35.5 ~40.0	3.02	3.46	0.50 ~2.00	0.30 ~1.20	1.81 ~3.24

Table 2 Parameters of shear test data for UHPC specimens

	The number of Specimens	Concrete tensile strength (f_t , MPa)	Concrete compressive strength (f_{ck} , MPa)	Shear span to depth ratio (a/d)	Reinforcement ratio (ρ_l , %)	Steel fiber volume fraction (V_f , %)	Fiber factor (F)	Shear stress (v_{test} , MPa)
Baby *et al*. (2013)	2	7.8, 10.0	203.0, 205.0	2.49	2.50	2.0, 2.5	0.65, 0.83	23
Lim and Hong (2016)	1	8.1	166.9	3.00	7.79	1.5	0.66	14
Telleen *et al*. (2010)	1	9.1	149.5	2.78	2.18	3.0	1.22	26
Qi *et al*. (2016)	8	3.6 ~5.4	79.2 ~113.9	2.5 ~3.75	1.65	0.5 ~ 2.0	0.16 ~0.65	11 ~23
Pourbaba *et al*. (2018)	19	21.7 ~26.6	125.0 ~137.0	0.90 ~2.80	0.20 ~7.80	2.0	0.72	8 ~25
Mészöly and Randl(2018)	6	5.0, 6.3	134.7 ~158.3	3.55	3.55	1.0, 2.0	0.38, 0.75	11~19

coverage of the parameters for the data. The database was established by combining the databases in Karl *et al*. (2010), Hwang *et al*. (2013), and Cho *et al*. (2018). In order to estimate the shear strength of SFRC and UHPC beams without stirrups, important parameters contributing to the shear strength should be identified first. All previous studies shown in Tables 1 and 2 have reported that the shear strength of SFRC and UHPC beams without stirrups can be increased significantly using steel fibers, and the effect of the fiber needs to be therefore considered. In addition, the influence of various parameters, including shear span to depth ratio, longitudinal reinforcement ratio, concrete compressive strength, fiber aspect ratio, and fiber volume fraction, has been also observed. Especially, Narayanan and Darwish (1987) and Karl *et al*. (2010) developed shear strength prediction models for SFRC beams without stirrups using these parameters. In particular, Narayanan and Darwish (1987) proposed a fiber coefficient ($F = L_f / D_f \cdot V_f \cdot d_f$) based on experimental results, and many studies used it as an important factor (Sharma 1986, Narayanan and Darwish 1987, Ashour *et al*. 1992, Oh and Kim 2008, Karl *et al*. 2010). In this expression for fiber coefficient, L_f and D_f are the length and diameter of the fiber, respectively, V_f is the steel fiber volume fraction, and d_f is a coefficient determined according to the type of fiber with the values of 1.0 for the hook type and 0.5 for the straight type, respectively.

2.1 Steel fiber reinforced concrete (SFRC)

This section introduces the existing analytical and regression equations for predicting the shear strength of SFRC members derived by the researchers including Sharma (1986), Narayanan and Darwish (1987), Ashour *et al*. (1992), Oh and Kim (2008), and Karl *et al*. (2010). Sharma (1986) investigated the effect of steel fiber to the shear strength of SFRC members through an experimental study. He confirmed that the shear strength was improved by incorporating steel fibers and observed a post-cracking strength improvement. By statistically utilizing the experimental observations and by adopting the ACI544-88 equation as a base form, a prediction model was developed

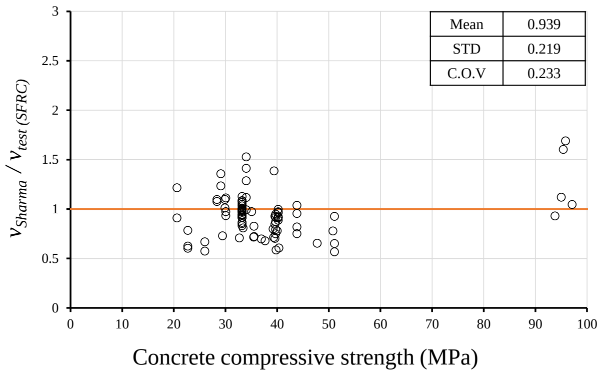


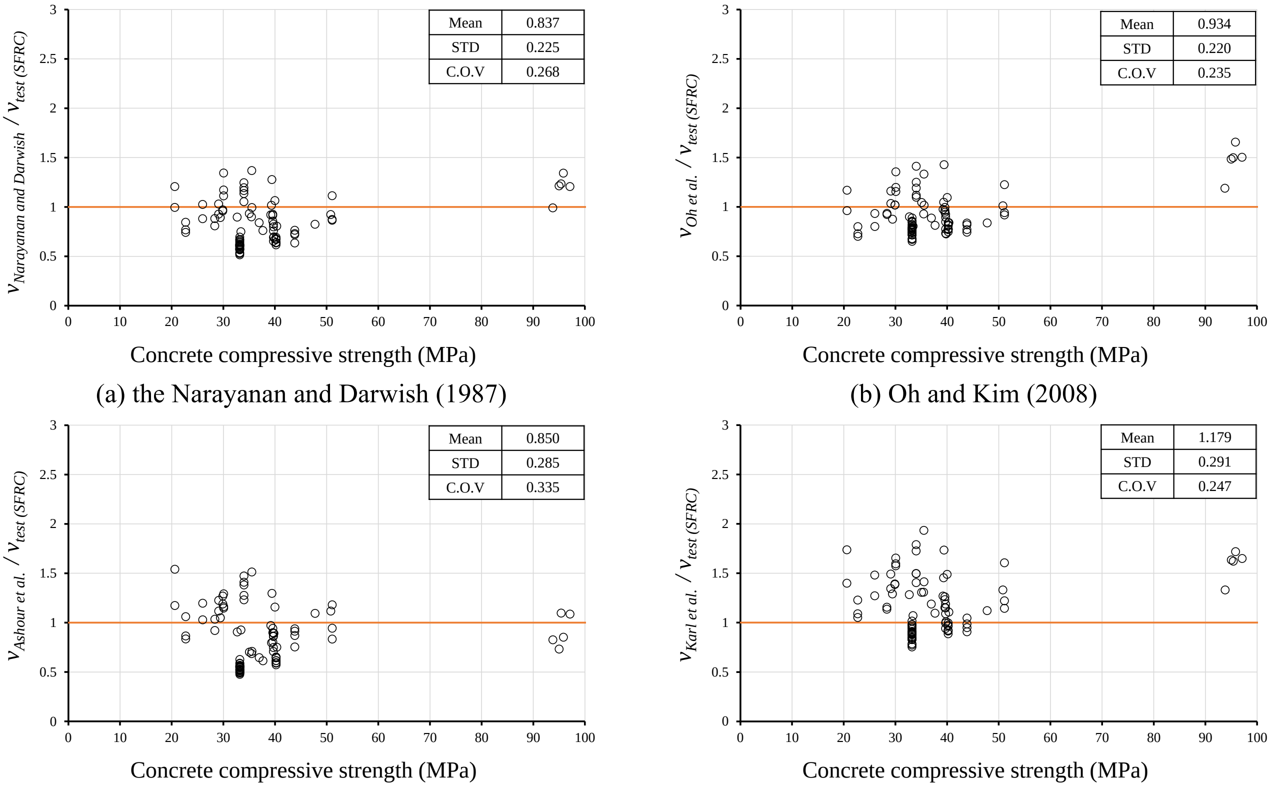
Fig. 3 Shear strength ratio between the Sharma (1986)'s equation and SFRC experimental data

considering the effects of the tensile strength and the depth to shear span ratio, as follows:

$$v_u = \frac{2}{3} f_t \left(\frac{d}{a}\right)^{0.25} \tag{1}$$

$$\text{where } f_t = 0.79\sqrt{f_c'}$$

where d / a is the depth to shear span ratio, f_t is the concrete tensile strength, and f'_c is the concrete compressive strength. Eq. (1) includes no term about the effect of steel fiber, but it is indirectly reflected in f_t. Fig. 3 shows the shear stress ratio between Eq. (1)'s prediction and SFRC experimental data in Table 1. The horizontal solid line at the shear stress ratio value of 1 indicates an unbiased prediction of the equation's prediction for the test data. As shown in Fig. 3, the accuracy of Eq. (1) is represented by an average strength ratio (v_{Sharma} / v_{test}) of 0.939 and the coefficient of variation (c.o.v.) of 0.233 for the test specimens provided in Table 1. It is widely known that the prediction of shear strength is very difficult due to the complicated shear failure mechanism. For example, for normal concrete (non-SFRC) beams without stirrups, Jung and Kim (2008) reported that the c.o.v. of the prediction of shear strength using previous studies and code equations (Okamura and Higai 1980, Zsutty, 1971) ranged from 0.21 to 0.34. Considering more uncertain and complicated mechanisms



(a) the Narayanan and Darwish (1987)

(b) Oh and Kim (2008)

(c) Ashour *et al*. (1992)

(d) Karl *et al*. (2010)

Fig. 4 Shear strength ratio between the existing equations and SFRC experimental data

of SFRC than the normal concrete, Sharma's equation in Eq. (1) is considered to estimate the shear strength of SFRC accurately.

Narayanan and Darwish (1987) conducted shear tests on 49 long-span SFRC members to confirm the influence of the splitting tensile strength of the members, the dowel action of longitudinal reinforcement, and the shear span to depth ratio to the shear strength of the members. The form of this equation is as follows:

$$v_u = e_n\left[0.24 f_{spfc} + 80\rho\frac{d}{a}\right] + 0.41\tau F$$
$$\text{where } f_{spfc} = \frac{f_c'}{20-\sqrt{F}} + 0.7 + \sqrt{F},\ \tau = 4.15 \tag{2}$$

where e_n is a coefficient taking into account the effect of arch action, which applies 1.0 for $a / d > 2.8$ and $2.8 / (a / d)$ for $a / d \leq 2.8$. f_{sfrc} is the spalling tensile strength of SFRC, ρ is the longitudinal reinforcement ratio, and τ is the average fiber matrix interfacial bond stress. As shown in Fig. 4(a), the prediction accuracy of Eq. (2) can be represented by the average ratio error of 0.837 and the c.o.v. of 0.268 for the database in Table 1.

Ashour *et al*. (1992) conducted an experimental study on high-strength fiber reinforced concrete with over 90 MPa for 18 specimens. Their proposed model considers the longitudinal reinforcement ratio and the steel fiber volume fraction as follows:

$$v_u = \left(0.7\sqrt{f_c'} + 7F\right)\frac{d}{a} + 17.2\rho\frac{d}{a} \tag{3}$$

As shown in Fig. 4(b), the accuracy of this equation is represented by the average ratio error of 0.850 and the c.o.v. of 0.335, for the database in Table 1.

Oh and Kim (2008) established a database by collecting 77 experimental data for SFRC beams from the literature. Based on the database, they proposed a formula using statistical analysis as follows:

$$v_u = \left(0.2e_o + 0.25F\right)\sqrt{f_c'} + 75\rho\frac{d}{a} \tag{4}$$

where e_o is a coefficient taking into account the effect of arch action, which applies 1.0 for $a / d > 2.5$ and $2.5 / (a / d)$ for $a / d \leq 2.5$. As shown in Fig. 4(c), the accuracy of this equation is represented by the average ratio error of 0.934 and the c.o.v. of 0.235, for the database in Table 1.

Karl *et al*. (2010) expanded the database collected by Oh and Kim (2008) by carrying out 4 more experiments and proposed an equation using statistical analysis as follows:

$$v_u = 3\left(\frac{d}{a}\rho f_c'\right)^{1/3} + 0.41\tau F$$
$$\text{where } \tau = 6.8 \tag{5}$$

As shown in Fig. 4(d), the accuracy of this equation is represented by the average ratio error of 1.179 and the c.o.v. of 0.247, for the database in Table 1.

The equations reviewed above were mostly derived based on the regression analyses based on a collected database on shear test results of SFRC members, and they are applicable to the range of the parameters covered in the database. However, it is unsure if they can be applied to the outside range of the database, for example, to the fiber reinforced ultra-high strength concrete members with over 150 MPa concrete compressive strength.

2.2 Ultra-high-performance fiber reinforced concrete (UHPC)

There are very limited proposed models for the shear strength prediction of a UHPC member. The models existing are from the design standard of the French Society of Civil Engineers (AFGC 2013) and the design standard presented by Japan's Society of Civil Engineers (JSCE 2008). In both standards, the shear strength of a UHPC member is calculated considering the shear strengths of concrete (V_c), steel fiber (V_f) and shear reinforcement (V_s) as follows:

$$V_u = V_c + V_f + V_s \tag{6}$$

In the AFGC-SETRA design standard (AFGC 2013), V_c is calculated as follows.

$$V_c = \frac{0.21}{\gamma_{cf}\gamma_E} k\sqrt{f_c'}\, b_w d \tag{7}$$

where γ_{cf} is provided to be 1.3 as a partial safety factor for steel fibers, and γ_E is a safety coefficient. In this study, the safety factors were taken as 1 to exclude the effect of these factors when verifying the accuracy of the equation. k is calculated as $1{+}3\sigma_{cp} / f'_c$ if the compressive stress by prestress (σ_{cp}) is greater than 0 and is calculated as $1{+}0.7\sigma_{cp} / f_{ctk,0.05}$ if σ_{cp} is smaller than 0. b_w and d are the width of the member and the depth of longitudinal reinforcement, respectively. where $f_{ctk,0.05}$ is the characteristic axial tensile strength corresponding to the 5% fractile (MPa).

V_f is calculated as follows:

$$V_f = \frac{A_{fv}\sigma_{Rd,f}}{\tan\theta_{cr}} \tag{8}$$

where A_{fv} is the area of the fiber effect (A_{fv} = 0.9$b_w d$ or $b_0 d$ for a rectangular or T-section, and A_{fv} = 0.8(0.9d)^2 for a circular section), and θ_{cr} is the angle of principal compression stress with a value of at least 30 degrees. In the case of strain-softening or low strain-hardening UHPC, $\sigma_{Rd,f}$ is calculated as the residual tensile strength as follows:

$$\sigma_{Rd,f} = \frac{1}{K\gamma_{cf}} \frac{1}{w_{\lim}} \int_0^{w_{\lim}} \sigma_f(w)\, dw \tag{9}$$

where K is a coefficient that represents the direction of the fiber (K_{global} = 1.25), w_{lim} = max(w_u,w_{max}) where w_u is the crack width at the ultimate limit state (ULS), and w_{max} is the threshold maximum crack width for the serviceability limit states (SLS). In the case of high strain-hardening UHPC, $\sigma_{Rd,f}$ is calculated as follows:

$$\sigma_{Rd,f} = \frac{1}{K\gamma_{cf}} \times \frac{1}{\varepsilon_{\lim} - \varepsilon_{el}} \int_{el}^{\varepsilon_{\lim}} \sigma_f(\varepsilon)\, d\varepsilon \tag{10}$$

where ε_{lim} is $max(\varepsilon_u , \varepsilon_{max})$, ε_u is the ultimate strain attained at the ULS. The residual tensile stress after cracking ($\sigma_{Rd,f}$) can be obtained by integrating the stress-crack width curve (σ-w curve) obtained from material tests. In addition, V_s can be calculated as follows.

$$V_s = \frac{A_v}{s} z f_{yt} \cot\theta \tag{11}$$

where A_v is the sectional area of shear reinforcement, s is the spacing of shear reinforcement, z is the inner lever arm, and f_{yt} is the yield strength of shear reinforcement.

In the JSCE design standard (JSCE 2008), V_c is calculated as follows:

$$V_c = \phi_b 0.18\sqrt{f_c'}\, b_w d \tag{12}$$

where ϕ_b is the component strength reduction factor, which is provided as 0.77. In this study, the reduction factor was taken as 1 to exclude the effect of these factors when verifying the accuracy of the equation. V_{fb} can be calculated as follows:

$$V_{fb} = \phi_b \left(\frac{f_{vd}}{\tan\beta_u} \right) b_w z \tag{13}$$

where β_u is the angle of the diagonal crack with the value greater than 30 degrees, z is the distance between the locations of the compressive force and the tensile force, which has a suggested value of d/1.15. f_{vd} is the average tensile stress acting perpendicular to a diagonal crack and calculated as follows:

$$f_{vd} = \frac{1}{w_v} \int_0^{w_v} \phi_c \sigma_k(w)\, dw \tag{14}$$

where w_v takes the greater value between 0.3 mm and the crack width at the maximum load, ϕ_c is the material strength reduction factor, and $\sigma_k(w)$ represents the tensile stress corresponding to the crack width (w) on tensile softening curve after a crack occurs. Both the AFGC and JSCE equations need the information of the maximum crack width and the tensile stress at the maximum cracks but these values are often not available except through experiments, and thus, they are difficult to use in design practice. In this study, these equations were calculated using assumed values for the residual tensile stress, ($\sigma_{Rd,f}$). In this study, these equations were calculated using assumed values for the residual tensile stress($\sigma_{Rd,f}$), resulting in somewhat higher C.O.V of 0.618 and 0.623, respectively, as shown in Figs. 5 and 6. As shown in Table 2, the tensile strengths of the 19 UHPC specimens tested by Pourbaba *et al*. ranged from 21.7 to 26.6 MPa, which are considered to be exceptionally high tensile strengths. In AFGC and JSCE, the shear strengths are calculated based on the residual tensile stress, while the other equations reflect the effect of tensile strength on the shear strength of the members indirectly, which is why the shear strengths of those specimens were overestimated by AFGC and JSCE.

Figs. 7, 8, 9, 10 and 11 show the maximum shear stress ratio between the predictions using the previous studies and the observations from the UHPC experimental data used in this study. As seen in the figures, the predictions of the previous studies overall underestimate the shear strengths for the test data with a range from 29.6% to 58.5%. Those equations were originally proposed and verified for the shear strength of SFRC beams with normal compressive strength of concrete or at most under 100 MPa. The UHPC beams usually have the concrete compressive strength over 100 MPa and sometimes even higher than 200 MPa, and therefore, the equations underestimated their shear strengths. The accuracy of the existing equations has been measured in terms of the coefficient of variation (c.o.v.). The Sharma's equation showed the c.o.v. of 23.7% while the other equations proposed by Narayanan and Darwish (1987), Ashour *et al*. (1992), Oh and Kim (2008), and Karl *et al*. (2010) showed the c.o.v. in the range of 36-58% for the same test data. This high accuracy of Sharma's equation for UHPC data is quite interesting because Sharma's equation has the simplest form among the SFRC equations considered in this study, and it does not include separate terms reflecting the effects of steel fibers such as bond strength, dimension, and volume fraction. Nevertheless, it is considered to reflect the influence of steel fibers appropriately by considering only the concrete compressive strength increased by the steel fibers.

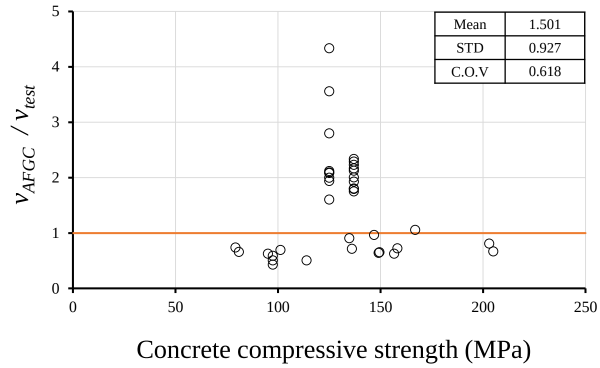


Fig. 5 Shear strength ratio between the AFGC (2013) equation and UHPC experimental data

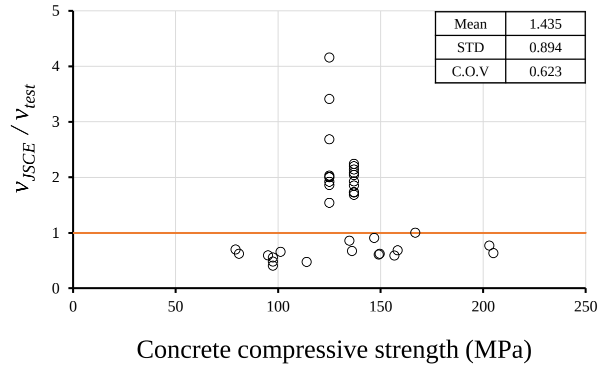


Fig. 6 Shear strength ratio between the JSCE (2008) equation and UHPC experimental data

3. Bayesian parameter estimation

To develop a new probabilistic prediction model or to calibrate or revise existing deterministic model to a probabilistic model, the Bayesian parameter estimation method can be used, which was firstly used in developing capacity models for RC columns (Gardoni 2002) and the seismic demand models for RC bridges (Gardoni *et al*. 2003). This study uses this method to develop a shear capacity prediction model, and it has the following mathematical form:

$$C\left(\mathbf{x},\mathbf{\Theta}\right)=c_d\left(\mathbf{x}\right)+\gamma\left(\mathbf{x},\mathbf{\theta}\right)+\sigma\varepsilon \tag{15}$$

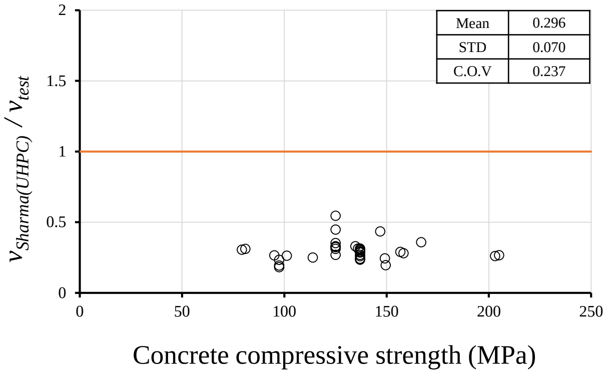


Fig. 7 Shear strength ratio between the Sharma (1986)'s equation and UHPC experimental data

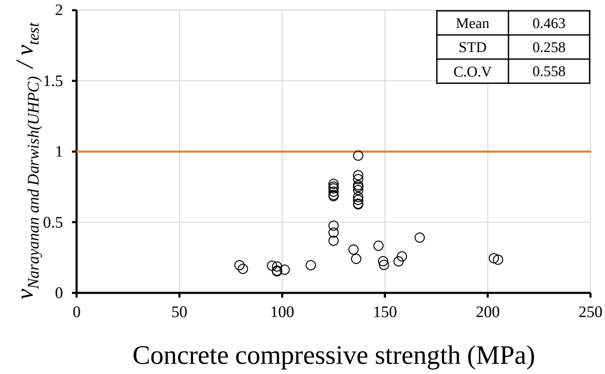


Fig. 8 Shear strength ratio between the Narayanan and Darwish (1987)'s equation and UHPC experimental data

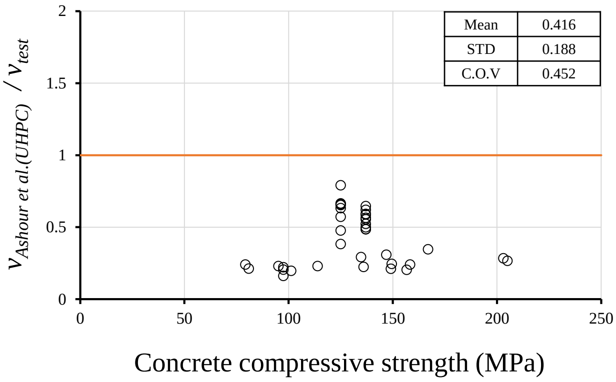


Fig. 9 Shear strength ratio between the Ashour *et al*. (1992)'s equation and UHPC experimental data

where **x** a vector of input parameter values; $\boldsymbol{\Theta} = (\boldsymbol{\theta}, \sigma)$ is a set of unknown parameters that provide the best fit to the experiments; c_d (**x**) is an existing deterministic prediction model or this term can become zero when there is no existing deterministic model; γ (**x**, $\boldsymbol{\theta}$) is a bias correction term that minimizes the bias and scatter of the overall prediction model, which includes **x** and $\boldsymbol{\theta} = [\theta_1, \theta_2, \ldots, \theta_p]^T$; ε is an error term after the bias correction, which is represented by a standard normal random variable; and σ represents the magnitude of the remaining error. This model is constructed under the following two assumptions: (i) the homoscedasticity assumption, which means that the model variance σ is constant over all input parameters **x**; and (ii) the normality assumption, which means that the error term ε follows the standard normal distribution.

This study first assumes that the bias-correction function γ (**x**, $\boldsymbol{\theta}$) is a linear function, which is expressed as the summation of a suitable set of p explanatory functions h_i (**x**), i=1, …, p, as follows:

$$\gamma(\mathbf{x},\boldsymbol{\theta}) = \sum_{i=1}^{p} \theta_i h_i(\mathbf{x}) \tag{16}$$

As many prediction models have a form that is the product of multiple terms, the above equation is modified by applying the natural logarithms to all the terms, satisfying the homoscedasticity assumption, as follows (Song *et al*. 2010)

$$\ln\left[C(\mathbf{x},\boldsymbol{\Theta})\right] = \ln\left[c_d(\mathbf{x})\right] + \sum_{i=1}^{p} \theta_i h_i(\mathbf{x}) + \sigma\varepsilon \tag{17}$$

where the explanatory terms h_i (**x**) are now the lognormal functions of input parameters.

The model can be fully developed when finding the model parameters $\boldsymbol{\Theta} = (\boldsymbol{\theta}, \sigma)$ that provide the best fit with the experiments. To estimate these model parameters, the following Bayesian updating rule is used in this study (Box and Tiao 1992):

$$f(\boldsymbol{\Theta}) = \kappa L(\boldsymbol{\Theta})\, p(\boldsymbol{\Theta}) \tag{18}$$

where $p(\boldsymbol{\Theta})$ is the prior function of $\boldsymbol{\Theta}$, $L(\boldsymbol{\Theta})$ is the likelihood function constructed using the experimental observations; and $\kappa = [\int L(\boldsymbol{\Theta}) p(\boldsymbol{\Theta}) d(\boldsymbol{\Theta})]^{-1}$ is the normalizing constant. In this equation, the prior function can be taken as the non-informative function as follows (Gardoni 2002):

$$p(\sigma) \propto \frac{1}{\sigma} \tag{19}$$

when we have failure data only with no upper or lower bound data, the likelihood function can be defined as follows (Gardoni 2002, Song *et al*. 2010):

$$L(\boldsymbol{\Theta}) \propto \prod_{\text{failure data}} \left\{ \frac{1}{\sigma} \phi\left[\frac{\ln[C_i] - \ln[C_d(\mathbf{x}_i)] - \gamma(\mathbf{x}_i, \boldsymbol{\theta})}{\sigma} \right] \right\} \tag{20}$$

where $\varphi(\cdot)$ and $\Phi(\cdot)$ are the probability density function (PDF) and the cumulative density function (CDF) of the

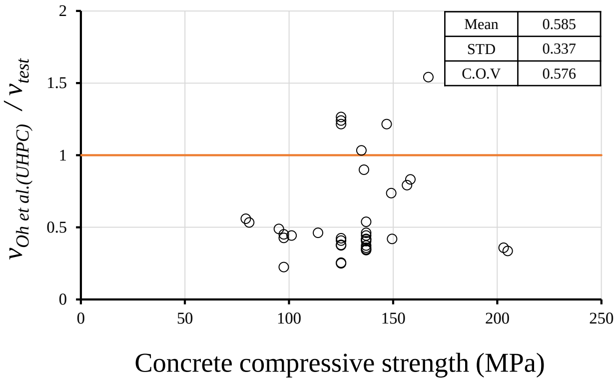


Fig. 10 Shear strength ratio between the Oh and Kim (2008)'s equation and UHPC experimental data

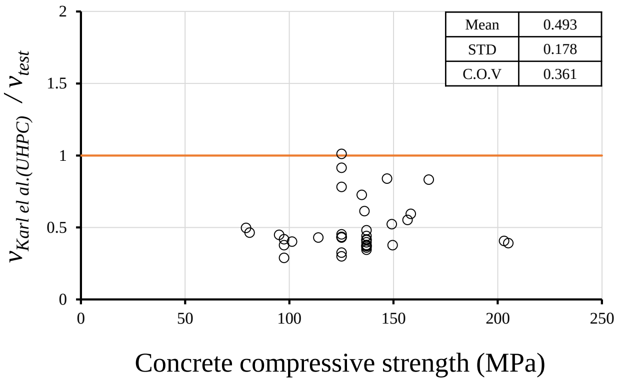


Fig. 11 Shear strength ratio between the Karl *et al.* (2010)'s equation and UHPC experimental data

standard normal distribution, respectively. The calculation of the normalizing constant in Eq. (18) requires the implementation of multifold integrals, and this is also required to calculate the posterior mean vector $\mathbf{M}_\Theta$, and covariance matrix $\sum_{\Theta\Theta} = \int \Theta\Theta^T f(\Theta) d(\Theta) - \mathbf{M}_\Theta \mathbf{M}_\Theta^T$. This study adopts an importance sampling technique for this multifold integral calculation, in which the sampling density function is centered at the maximum likelihood point to expedite the convergence (Gardoni 2002).

The Bayesian parameter estimation method provides a stepwise equation simplification procedure by identifying and removing unimportant explanatory terms one-by-one. The unimportance of each explanatory terms is represented by its posterior c.o.v., and the term with the greatest c.o.v. becomes the first candidate to be removed in the equation. The identified most unimportant term can be removed if its removal does not change the overall error of the equation significantly. This removal process is repeated until the overall error change is within an acceptable level. This stepwise equation simplification process enables us to identify informative terms in a systematic manner without performing multiple regression analyses considering all possible combinations of explanatory terms.

4. Proposed models

In this study, although the equation developed by Oh and Kim (2008) and Sharma (1986) showed similar accuracies for SFRC members, we propose equations based on the Sharma's equation (1986) due to the following reasons: (i) the Oh and Kim's equation (2008) has a more complexity than the Sharma's equation, and (ii) the coefficient taking into account the effect of arch action, *e*, is specifically best fitted to the SFRC database, and if the equation is applied to the UHPC database, the c.o.v. value significantly increases from 0.235 to 0.576 showing a huge decrease in the accuracy. By correcting or excluding the constant bias, the Sharma's equation successfully predicted strengths of the UHPC members in the database with the c.o.v. value of 0.237.

As aforementioned, the shear strength of UHPC is increased by optimum particle size distribution design, which improves the bonding between the cementitious matrix and the steel fiber and shows post-cracking behavior that is different from that of the existing non-ultra-high strength concrete. Since the compressive strengths of UHPC and SFRC have very different effects on the shear strength, it is difficult to develop a unified model using both the SFRC and UHPC databases. Fig. 12 shows the result of the Bayesian parameter estimation using both SFRC and UHPC data. As shown in this figure, the SFRC and UHPC data show two separated trends, and to fully address this, two separate equations are proposed for SFRC and UHPC, respectively.

To further develop Sharma's equation for SFRC, the first trial form of the equation is proposed as follows:

$$v_{modified\ sharma(SFRC)} = \frac{2}{3} f_t \left(\frac{d}{a}\right)^{0.25} + 2^{0.333} F^{2.293} \tag{21}$$

where $f_t = 0.79\sqrt{f_c'}$

Eq. (21) has the form of the Sharma's equation in Eq. (1) plus the fiber term consisting of a constant term and a proposed fiber factor calculated based on the concrete compressive strength, which further considers the effects of the fiber volume fraction, fiber length and diameter, and the fiber type. As shown in Fig. 13, Eq. (21) shows the average ratio error of 1.042 and the c.o.v. of 0.200, which shows the best accuracy compared to all of the reviewed equations above for the SFRC database. This result proves that the addition of the fiber factor is important, and it has a significant effect on the shear strength. This is because the dowel action of steel fiber inhibits the expansion of crack widths (Karl *et al*. 2010, Hwang *et al*. 2013). In addition to the fiber factor, longitudinal reinforcement has been known to be an important factor that affects the shear strength of concrete. (Narayanan and Darwish 1987, Ashour *et al*. 1992, Oh and Kim 2008, Karl *et al*. 2010, Hwang *et al*. 2013)

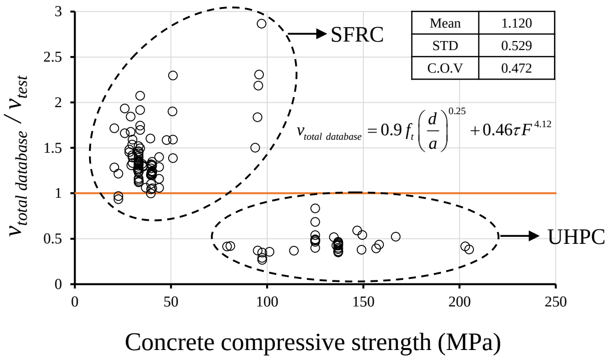


Fig. 12 Shear strength ratio between total database model using the Bayesian parameter estimation method and SFRC & UHPC experimental data

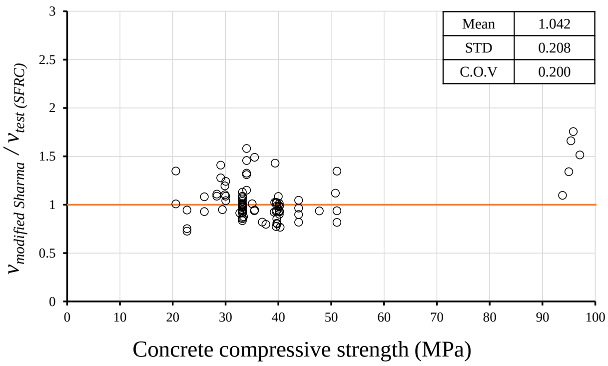


Fig. 13 Shear strength ratio between the modified Sharma's equation and SFRC experimental data

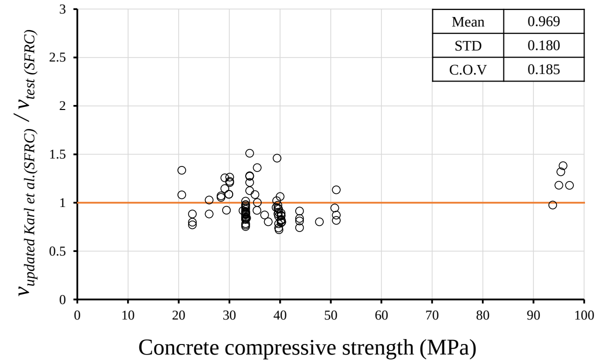


Fig. 14 Shear strength ratio between the Karl *et al*.'s model using the Bayesian parameter estimation method and SFRC experimental data

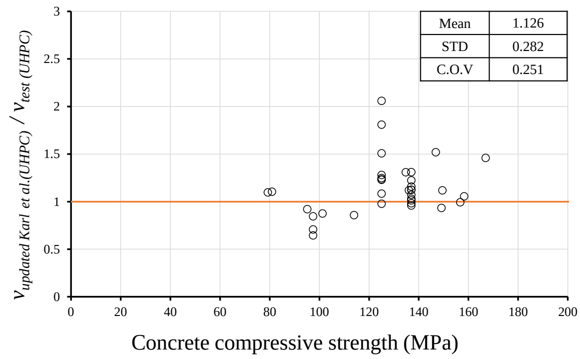
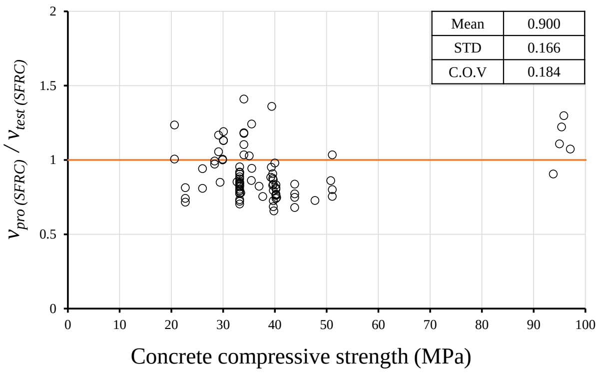


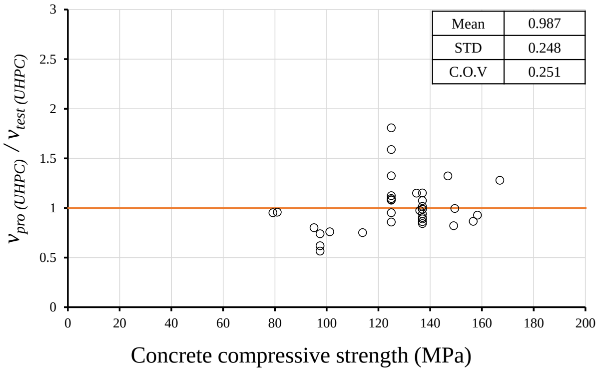
Fig. 15 Shear strength ratio between the Karl et al.'s model using the Bayesian parameter estimation method and UHPC experimental data

After considering the effect of longitudinal reinforcement ratio, Eq. (21) has a similar form to the Karl *et al*.'s model in Eq. (5), which is updated using the Bayesian parameter estimation method. The form obtained through this process for the SFRC database is shown as follows:

$$v_{updated\ Karl\ et\ al.(SFRC)} = 2^{1.491} \left(\frac{d}{a}\right)^{0.287} \rho^{0.233} f_{ck}^{0.265} + 2^{0.186} F^{1.582} \tag{22}$$



(a) Proposed equation for SFRC



(b) Proposed equation for UHPC

Fig. 16 Shear strength ratio between the proposed equation and experimental data

As shown in Fig. 14, the accuracy of this equation is represented through the average ratio error of 0.969 and the c.o.v. of 0.185. This equation shows better accuracy than all the equations reviewed in this study. Many researchers observed that a higher longitudinal reinforcement ratio induces higher shear strength because of increased dowel action of longitudinal reinforcement (Ashour *et al.* 1992, Swamy *et al.* 1993). Eq. (22) is more accurate than Eq. (21) because it takes into account the effect of the dowel action of longitudinal reinforcement. This equation is determined to be the final shear strength prediction model of SFRC without stirrups in this study.

For the UHPC shear strength prediction model, Eq. (22) was updated based on UHPC database as follows:

$$v_{updated\ Karl\ et\ al.(UHPC)} = 2^{2.122}\left(\frac{d}{a}\right)^{0.308} \rho^{-0.113} f_{ck}^{0.190} + 2^{4.107} F^{1.233} \quad (23)$$

The accuracy of this equation is represented through the average ratio error of 1.356 and the c.o.v. of 0.259. Eq. (23) shows better accuracy than the AFGC-SETRA and JSCE design standard, but the power term of longitudinal reinforcement ratio has a negative value. Swamy and Bahia (1985) reported that the ratio of the shear strength increase is greater at lower longitudinal reinforcement ratio but becomes smaller at higher longitudinal reinforcement ratio because of the decreasing rate of the dowel action. This bridging effect also occurs by steel fibers. Dinh (2009) observed that the effect of the fiber factor on shear strength in SFRC was greater than the longitudinal reinforcement ratio and longitudinal reinforcement has no significant effect on shear strength. In particular, UHPC has a higher bonding performance of steel fiber than SFRC, which has a greater effect on shear strength. Therefore, this negative number is regarded as a mechanically not-acceptable value caused during the statistical analysis, because steel fiber has a greater effect on shear strength than longitudinal reinforcement. In this study, in order to avoid a negative power term of the longitudinal reinforcement ratio, the Bayesian parameter estimation was performed by modifying the power terms as follows:

Table 3 Classification of concrete according to strength

Classification	Range
Conventional concrete	< 65
High strength concrete	65 to 100
Very-High strength concrete	101 to 150
Ultra-High strength concrete	<150

$$v_{updated\ Karl\ et\ al.(UHPC)} = 2^{3.915}\left(\frac{d}{a}\rho\right)^{0.198} f_{ck}^{0.197} + 2^{2.541} F^{4.107} \quad (24)$$

As shown in Fig. 15, the accuracy of this equation is represented through the average ratio error of 1.126 and the c.o.v. of 0.251. It is noted that the c.o.v. value has even been decreased from 0.259 to 0.251 confirming that the choice of the form was reasonable.

Considering all the equations developed above, the final equations are proposed by calibrating and simplifying the power terms of the updated Karl *et al.*'s equation and adding a term representing the bond strength of fiber (τ) as shown in Fig. 16. The bond strength of fiber (τ) was taken to be 6.8, which was used by Karl *et al.* (2010). Therefore, Eqs. (22) and (24) for SFRC and UHPC members are simplified, respectively, as follows:

$$v_{pro(SFRC)} = 2.8\left(\frac{d}{a}\rho f_c^{'}\right)^{0.25} + 0.15\tau F^{1.5} \quad \text{for SFRC} \quad (25)$$

where $\tau = 6.8$

$$v_{pro(UHPC)} = 13\left(\frac{d}{a}\rho f_c^{'}\right)^{0.2} + 0.8\tau F^{4} \quad \text{for UHPC} \quad (26)$$

where $\tau = 6.8$

In terms of the concrete compressive strength, it is recommended that Eq. (25) is used for concrete strength under 100 MPa, and Eq. (26) is used for concrete strength over 100 MPa. This is based on the data distribution in the SFRC and UHPC databases used in this study, as shown in Fig. 12. Although Rahman *et al.* (2005), Sohail *et al.* (2015), and Wang *et al.* (2016) suggested the classification of UHPC to be the concrete strength over 150 MPa as in Table 3, if we define UHPC based on the aforementioned special material design to maximize the packing density, the practical range of UHPC starts from around 100 MPa as shown in the UHPC database used in this study (Qi *et al.*

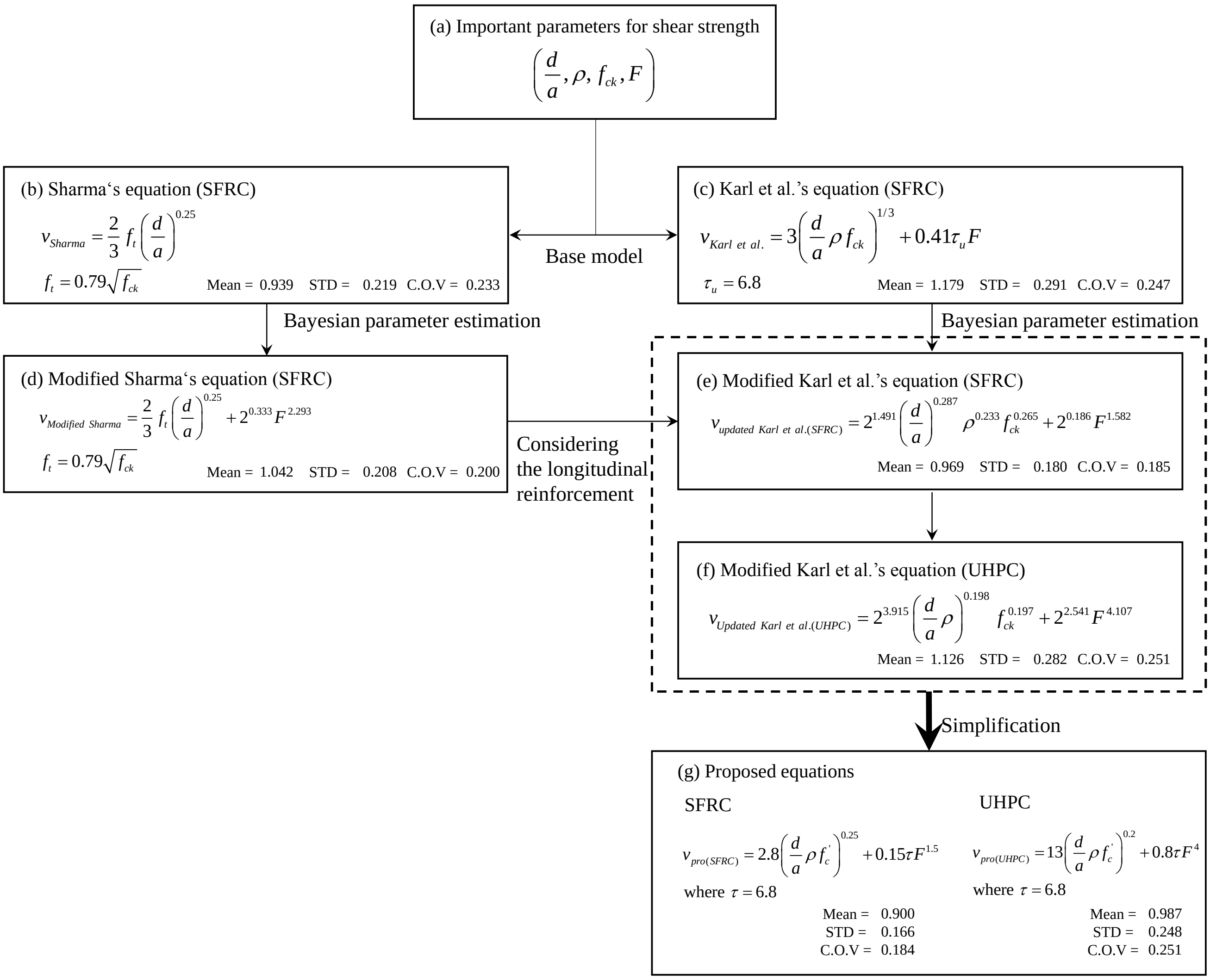


Fig. 17 Update process of the proposed equation

2016, Pourbaba *et al*. 2018, Mészöly and Randl 2018). As shown in Fig. 16(b), Eq. (26) very well predicts the shear strength of UHPC beams with the concrete compression strength of 100 to 150 MPa. Eq. (25) can evaluate the shear strength of SFRC beams with the concrete strength under 100 MPa, as shown in Fig. 16(a). However, careful usage of this equation is required especially for beams with concrete strength in the range of 60 to 90 MPa as the data in this range is limited.

The process of the equation development in this study is summarized in Fig. 17. In this figure, as shown in (a), the shear span to depth ratio, longitudinal reinforcement ratio, concrete compressive strength, and fiber factor were determined as the main factors from the literature review. As shown in (b) and (c), the shear prediction model proposed in this study used Sharma's and Karl *et al*.'s equations as the base models. As shown in (d), because the fiber factor has an effect to the shear strength, the equation was constructed by including the fiber factor in the Bayesian parameter estimation process using the SFRC database. (e) is the result of updating the Karl's model using the Bayesian parameter estimation and the SFRC database. Because the fiber factor and the longitudinal reinforcement have great influence on the shear strength, their inclusion increased the accuracy of t he prediction, and the (e) was decided as the finally proposed form for the shear strength prediction model for SFRC beams without stirrups. Because the compressive strengths of UHPC and SFRC have different effects on the shear strength due to their different resistance mechanisms, the shear strength prediction model of UHPC beams should have a different parameters values to express this difference. As shown in (f), the UHPC shear strength prediction equation was updated based on (e), using the UHPC database. In this study, (e) and (f) were finally selected as the shear strength prediction model for SFRC and UHPC beams, respectively, and as shown in (g), the shear strength prediction models for SFRC and UHPC beams without stirrups were simplified, respectively. Table 4 shows the summary of the biases and accuracies of all the equations including those in the literature and proposed ones in this study. The proposed equations for SFRC and UHPC show much better accuracy than other researchers' equations for SFRC and the equations in design standards for UHPC.

Table 4 Comparison of accuracy of shear strength prediction models covered in this study for SFRC and UHPC databases

SFRC			
	mean	std	c.o.v.
Sharma (1986)	0.939	0.219	0.233
Narayanan and Darwish (1987)	0.837	0.225	0.268
Ashour *et al*. (1992)	0.850	0.285	0.335
Oh and Kim (2008)	0.934	0.220	0.235
Karl *et al*. (2010)	1.179	0.291	0.247
Proposed equation	0.900	0.166	0.184
UHPC			
	mean	std	c.o.v.
AFGC (2013)	1.501	0.927	0.618
JSCE (2008)	1.435	0.894	0.623
Proposed equation	0.987	0.248	0.251

5. Conclusions

In this study, to statistically analyze existing concrete shear strength prediction equations and to develop new prediction equations, 89 and 37 experimental data for SFRC and UHPC beams without stirrups were collected, respectively. The proposed equations were developed using the Bayesian parameter estimation approach based on the equation form selected, combined, and modified from the existing equations provided in the literature. The following conclusions were obtained in this study:

• The analysis of the previous equations for SFRC showed that the Sharma's equation was simple and provided the most accurate shear strength compared to the other equations. When the effect of the fiber term was additionally considered in the Sharma's equation and the parameters of the equation was updated using the Bayesian estimation, the updated equation showed a very good accuracy with the c.o.v. of 0.200.

• The shear strength increases as the longitudinal reinforcement increases because of the dowel action of re-bars. After reflecting the longitudinal reinforcement ratio to the proposed equation and using the Bayesian parameter estimation, the proposed equation showed the c.o.v. of 0.185 for the SFRC test data, providing the best accuracy.

• Another model was developed for UHPC that was different from that for SFRC beams. Based on the database of the UHPC members, the proposed model showed the c.o.v. of 0.251, which gives a better accuracy than the equations in AFGC-SETRA and JSCE.

Acknowledgments

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (No. 2018R1A4A1025953)

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