**Structural Engineering and Mechanics**, *Vol. 74, No. 4 (2020) 471-479*
DOI: https://doi.org/10.12989/sem.2020.74.4.471


# Effect of the laser pulse on transient waves in a non-local thermoelastic medium under Green-Naghdi theory

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*(Received October 17, 2019, Revised December 4, 2019, Accepted December 20, 2019)*

**Abstract.** This paper aims to study the effect of the elastic nonlocality on the transient waves in a two-dimensional thermoelastic medium influenced by thermal loading due to the laser pulse. The bounding plane surface is heated by a non-Gaussian laser beam. The problem is discussed under the Eringen's nonlocal elasticity model and the Green-Naghdi (G-N) theory with and without energy dissipation. The normal mode analysis method is used to get the exact expressions for the physical quantities which illustrated graphically by comparison and discussion. The effects of nonlocality and different values of time on the displacement, the stresses, and the temperature were made numerically. All the computed results obtained have been depicted graphically and explained.

**Keywords:** nonlocal; laser pulse; Green-Naghdi theory; normal mode; ramp parameter

## 1. Introduction

The non-classical theories of thermoelasticity so-called generalized thermoelasticity have been developed to remove the paradox of physically impossible phenomenon of an infinite velocity of thermal signals in the conventionally coupled thermoelasticity, Lord-Shulman theory (1967) and Green-Lindsay theory (1972). In the decade of the 1990s, Green and Naghdi (1992) (G-N) have formulated three models (I, II, III) of thermoelasticity for homogeneous and isotropic material. The model I of (G-N) theory after linearization reduced to the classical thermo-elasticity theory. The model II of (G-N) theory (1993) does not suction the dissipation of the thermoelastic energy. In this model, the constitutive equations are derived by starting with the reduced energy equation and by including the thermal displacement gradient among the constitutive variables.

Chandrasekharaiah (1996) used the Laplace method to study the one-dimensional thermal wave propagation in a half- space based on the (G-N) theory of type II due to a sudden application of the temperature of the boundary. The reflection of plane waves from electro-magneto-thermo-elastic half-space with a dual-phase-lag model was explained by Abd-Alla *et al*. (2016). Othman *et al*. (2014) studied the effect of gravitational field and temperature-dependent properties on two-temperature thermoelastic medium with voids under G-N theory. The effect of rotation and inclined load on transversely isotropic magneto-thermoelastic solid was studied by Lata and Kaur (2019). Also, the effect of heat source and gravity on a fractional-order fiber-reinforced thermoelastic medium was investigated by Jain *et al.* (2018). The wave propagation in a two-temperature fiber-reinforced magneto-thermoelastic medium with the three-phase-lag model was studied by Said and Othman (2016). Marin and Öchsner (2017)_explained the effect of a dipolar structure on the Holder stability in (G–N) thermoelasticity. The disturbances produced in a half-space by the application of a mechanical point load and thermal source acting on a boundary of the half-space is investigated by Sharma and Chauhan (2001). Model III of (G-N) theory confesses a dissipation of energy, where the constitutive equations are derived starting with a reduced energy equation. It includes the thermal displacement gradient, in addition to the temperature gradient among it is independent constitutive variables. The theory of thermo-elasticity concerned with (G-N) theory has been the aim of many research papers (Othman and Jahangir 2015, Othman and Atwa 2012, Othman *et al*. 2015, Abd-Elaziz *et al*. 2019, Kumari and Mukhopadhyay 2016, Kothari and Mukhopadhyay 2012, Mukhopadhyay *et al*. 2011, Marin *et al*. 2017, Marin *et al*. 2019, Shirvan 2017, Esfahani *et al*. 2017, Rashidi *et al*. 2017, Rashidi *et al*. 2018, Yousif *et al*. 2019, Asadollahi *et al*. 2018, Bhatti and Lu 2019).

The theory of nonlocal elasticity has been studied by many researchers (see Altan 1984, Chirita 1976). The nonlocal elasticity theories characterized by the presence of nonlocality residuals of fields (like body force, mass, entropy, internal energy, etc.) and determined these residuals, along with the constitutive laws, with the help of suitable thermodynamic restrictions was developed by Edelen and Laws (1971), Edelen *et al*. (1971), Eringen and Edelen (1972). The concept of non-locality has been extended to various other fields by (Eringen 1972; 1981, 1990; 1991), McCay and Narasimhan (1981). New non-conventional methods for quantitative concepts of

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http://www.techno-press.com/journals/sem&subpage=7
ISSN: 1225-4568 (Print), 1598-6217 (Online)

anomalous rheology was studied by Jun (2019a,b). Jun *et al*. (2019) investigated new mathematical models in anomalous visco-elasticity from the derivative with respect to another function view point. Jun (2018) explained new rheological problems involving general fractional derivatives with non-singular power-law kernels. Jun *et al*. (2017) studied new rheological models within local fractional derivative. In the nonlocal theory of elasticity, the stress at any reference point $x$ within a continuous body depends not only on the strain at that point but also significantly influenced by the strains at all other points $x$ of the continuous body. Thus, the nonlocal stress forces act as remote action forces. These types of forces are frequently encountered in the atomic theory of lattice dynamics. Nonlocal continuum mechanics is now well established and is being applied to the problems of wave propagation; see Khurana and Tomar (2016), Sarkar and Tomar (2019). Using nonlocal continuum mechanics for modeling, the analysis of nanostructures has been made by several researchers, for example, Narendar (2012), Narendar and Gopalakrishnan (2010).

Very rapid thermal processes under the action of an ultra-short laser pulse are interesting from the standpoint of thermoelasticity because they require deformation fields and an analysis of the coupled temperature. This means that the laser pulse energy absorption results in a localized temperature increase, which causes thermal expansion and generates rapid movements in the structure elements, thus cause the rise of vibrations. These effects make materials susceptible to the diffusion of heat by conduction. The ultra-short lasers are those with pulse duration ranging from nanoseconds to femtoseconds. The high intensity, energy flux, and ultra-short duration laser beam have studied situations where very large thermal gradients or an ultra-high heating rate may exist on the boundaries, this in the case of ultra-short-pulsed laser heating (see Bromwich, 1898). The microscopic two-step models that are parabolic and hyperbolic are useful for modifying the material as thin films. When a metal film is heated by a laser pulse, a thermoelastic wave is generated due to thermal expansion near the surface. Wang and Xu (2002) investigated the stress wave induced by Pico and femto-second laser pulses in a semi-infinite metal by expressing the laser pulse energy as a Fourier series. Othman and Song (2009) studied the effect of rotation on 2-D thermal shock problems for a generalized magneto-thermoelasticity half-space under three theories.

The present article aims to determine the distributions of the displacement components, the stresses and the temperature in a homogeneous, isotropic, thermoelastic medium under influence of the laser pulse in the case of the absence and the presence of the gravity and two values of the time. The model is illustrated in the context of (G-N) theory of types II and III. Expressions for the physical quantities is obtained using the normal mode analysis and represented graphically.

## 2. Formulation of the problem

Consider a homogeneous, linear; isotropic thermally conducting nonlocal elastic half-space ($x\geq 0$) in the rectangular Cartesian coordinate system $(x,y,z)$ having originated on the surface $z=0$. In the used equations, a dot denotes differentiation with respect to time, while a comma denotes the material derivative. For two-dimensional problems assume the dynamic displacement vector as $u=(u,v,0)$ all the considered quantities will be functions of the time variable $t$, and of the coordinates $x$ and $y$.

Following Green and Naghdi (1992), the field equations and the constitutive relations of a linear, homogenous, isotropic thermally conducting non-local elastic medium without body forces in the context of the G-N theory of type III can be written as

$$\mu u_{i,jj}+(\lambda+\mu)u_{j,ij}-\beta T_{,i}=\rho(1-\xi^2\nabla^2)\,u_{i,tt}, \tag{1}$$

$$kT_{,ii}+k^{*}T_{,iit}=\rho C_e T_{,tt}+\beta T_0 e_{,tt}-\rho Q_{,t}, \tag{2}$$

$$(1-\xi^2\nabla^2)\sigma_{ij}=(\lambda u_{s,s}-\beta T)\delta_{ij}+2\mu e_{ij}, \tag{3}$$

$$e_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i}), \tag{4}$$

where $\lambda$, $\mu$ are the Lame' constants, $\xi(=a_0e_0)$ is the elastic nonlocal parameter having dimension of length, $a_0$, $e_0$, respectively are an internal characteristic length and a material constant (see Eringen and Edelen, 1972) for details, $T$ is the temperature distribution, $\beta=(3\lambda+2\mu)\alpha_t$ such that $\alpha_t$ is the coefficient of thermal expansion, $\rho$ is the density, $C_e$ is the specific heat, $k$ is the thermal conductivity, $k^*$ is the material constant characteristic of the theory, $T_0$ is the reference temperature chosen so that $|(T-T_0)/T_0|<<1$ $e$ is the dilation, $e_{ij}=(u_{i,j}+u_{j,i})/2$, the strain tensor components, $\sigma_{ij}$ are the stress tensor components, $\delta_{ij}$ is the Kronecker delta and $Q$ is the heat input of the laser pulse. In the above equations, $I,j,s=x,y$. When $k^*\to 0$, Eq. (2) reduces to the heat condition equation in (G-N) theory of type II.

The plate surface is illuminated by the laser pulse given by the heat input as Abd-Elaziz and Othman (2019)

$$Q(x,y,t)=\frac{I_0\gamma t}{2\pi r^2t_0^2}\exp\left(-\frac{y^2}{r^2}-\frac{t}{t_0}-\gamma x\right) \tag{5}$$

where $I_0$ is the energy absorbed, $t_0$ is the pulse rise time, $r$ is the beam radius, $\gamma$ is constant.

If we restrict our analysis in plan strain problem parallel to the $x$–$y$ plane, then the basic governing equations (1)-(5) are simplified to

$$\mu\nabla^2u+(\lambda+\mu)\frac{\partial e}{\partial x}-\beta\frac{\partial T}{\partial x}=\rho(1-\xi^2\nabla^2)\frac{\partial^2u}{\partial t^2}, \tag{6}$$

$$\mu\nabla^2v+(\lambda+\mu)\frac{\partial e}{\partial y}-\beta\frac{\partial T}{\partial y}=\rho(1-\xi^2\nabla^2)\frac{\partial^2v}{\partial t^2}, \tag{7}$$

$$k\nabla^2T+k^*\frac{\partial}{\partial t}\nabla^2T=\rho C_e\frac{\partial^2T}{\partial t^2}+\beta T_0\frac{\partial^2e}{\partial t^2}-\rho\frac{\partial Q}{\partial t}, \tag{8}$$

where

$$\nabla^2\equiv\partial^2/\partial x^2+\partial^2/\partial y^2.$$

To facilitate the solution of the problem, introduce the following dimensionless variables

$$(x',y',\xi') = \frac{\omega_1^*}{c_1}(x,y,\xi),\ (u',v') = \frac{\omega_1^*}{c_1}(u,v),\quad \sigma'_{ij} = \frac{\sigma_{ij}}{\mu},$$
$$\theta' = \frac{T}{T_0},\ t' = \omega_1^* t,\quad Q' = \frac{Q}{\omega_1^* T_0 C_e},\ c_1^2 = \frac{\lambda+2\mu}{\rho},\quad \omega_1^* = \frac{\rho C_e c_1^2}{k}.$$

Equations (6)-(8), with the help of the above non-dimensional variables will be rewritten into the non-dimensional form with dropping primes for convenience

$$\nabla^2 u + b_1 \frac{\partial e}{\partial x} - b_2 \frac{\partial \theta}{\partial x} = b_3(1-\xi^2\nabla^2)\frac{\partial^2 u}{\partial t^2}, \tag{9}$$

$$\nabla^2 v + b_1 \frac{\partial e}{\partial y} - b_2 \frac{\partial \theta}{\partial y} = b_3(1-\xi^2\nabla^2)\frac{\partial^2 v}{\partial t^2}, \tag{10}$$

$$\varepsilon_3 \nabla^2 \theta + \varepsilon_2 \frac{\partial}{\partial t}\nabla^2\theta = \frac{\partial^2\theta}{\partial t^2} + \varepsilon_1 \frac{\partial^2 e}{\partial t^2} - \frac{\partial Q}{\partial t}. \tag{11}$$

Here,

$$b_1 = \frac{\lambda+\mu}{\mu},\quad b_2 = \frac{\beta T_0}{\mu},\ b_3 = 1+b_1,\quad \varepsilon_1 = \frac{\beta}{\rho C_e},$$
$$\varepsilon_2 = \frac{k^*\omega_1^*}{\rho C_e c_1^2},\quad \varepsilon_3 = \frac{k}{\rho C_e c_1^2},$$

where $\varepsilon_1$, $\varepsilon_2$ and $\varepsilon_3$ are the coupling constants. Using the expressions, relating the displacement components $u(x,y,t)$ and $v(x,y,t)$ to the scalar potential function $\psi_1(x,y,t)$ and the vector potential function $\psi_2(x,y,t)$ in the dimensionless form as follows:

$$u = \frac{\partial \psi_1}{\partial x} + \frac{\partial \psi_2}{\partial y},\quad v = \frac{\partial \psi_1}{\partial y} - \frac{\partial \psi_2}{\partial x}, \tag{12}$$

$$e = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \nabla^2 \psi_1 \tag{13}$$

Using (12) and (13) into Eqs. (9)-(11), we get

$$b_3[(1+\xi^2\frac{\partial^2}{\partial t^2})\nabla^2 - \frac{\partial^2}{\partial t^2}]\psi_1 - b_2\theta = 0, \tag{14}$$

$$[(1+b_3\xi^2\frac{\partial^2}{\partial t^2})\nabla^2 - b_3\frac{\partial^2}{\partial t^2}]\psi_2 = 0, \tag{15}$$

$$-\varepsilon_1\frac{\partial^2}{\partial t^2}\nabla^2\psi_1 + (\varepsilon_3 + \varepsilon_2\frac{\partial}{\partial t})\nabla^2\theta - \frac{\partial^2\theta}{\partial t^2} = -\frac{\partial Q}{\partial t}. \tag{16}$$

The non-zero stress components of interest are

$$(1-\xi^2\nabla^2)\sigma_{xx} = b_1 e + (\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}) - b_2\theta, \tag{17}$$

$$(1-\xi^2\nabla^2)\sigma_{yy} = b_1 e - (\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}) - b_2\theta, \tag{18}$$

$$(1-\xi^2\nabla^2)\sigma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}. \tag{19}$$

## 3. The normal mode analysis

We can decompose the solution of physical quantities in terms of the normal mode as the following:

$$[\psi_1,\psi_2,\theta](x,y,t) = [\psi_1^*,\psi_2^*,\theta^*](x)\exp[i(\omega t + a y)], \tag{20}$$

where $[\psi_1^*,\psi_2^*,\theta^*](x)$ are the amplitude of the physical quantities, $\omega$ is the angular frequency, $\mathrm{i} = \sqrt{-1}$ and $a$ is the wave number.

On using (20), the Eqs. (14)-(16) will take the form

$$[\mathrm{D}^2 - B_1]\psi_1^* - B_2\theta^* = 0, \tag{21}$$

$$[\mathrm{D}^2 - B_3]\psi_2^* = 0, \tag{22}$$

$$B_4[\mathrm{D}^2 - a^2]\psi_1^* + [\mathrm{D}^2 - B_5]\theta^* = \frac{B_6 I_0\gamma}{2\pi r^2 t_0^2}(1-\frac{t}{t_0})\exp[-(\frac{y^2}{r^2} + \frac{t}{t_0} + \gamma x + i\omega t + iay)]. \tag{23}$$

where

$$B_1 = a^2 - \frac{\omega^2}{1-\xi^2\omega^2},\quad B_2 = \frac{b_2}{(1+b_1)(1-\xi^2\omega^2)},$$
$$B_3 = a^2 - \frac{b_3\omega^2}{1-b_3\xi^2\omega^2},\ B_4 = \frac{\varepsilon_1\omega^2}{\varepsilon_3 + i\varepsilon_2\omega},\ B_5 = a^2 - \frac{\omega^2}{\varepsilon_3 + i\varepsilon_2\omega},$$
$$B_6 = -\frac{1}{\varepsilon_3 + i\varepsilon_2\omega},\ \text{and}\ \mathrm{D} = \frac{\mathrm{d}}{\mathrm{d}x}.$$

Eliminating $\theta^*$ between Eqs. (21) and (23), we obtain the following differential equation satisfied by $\psi_1^*$:

$$[\mathrm{D}^4 - L_1\mathrm{D}^2 + L_2]\psi_1^* = L_3(1-\frac{t}{t_0})\exp[-(\frac{y^2}{r^2} + \frac{t}{t_0} + \gamma x + i\omega t + iay)], \tag{24}$$

Where

$$L_1 = B_1 + B_5 - B_2B_4,\qquad L_2 = B_1B_5 - a^2B_2B_4,$$
$$L_3 = \frac{B_2B_6I_0\gamma}{2\pi r^2 t_0^2}.$$

Similarly, $\theta^*$ satisfies the following differential equation

$$[\mathrm{D}^4 - L_1\mathrm{D}^2 + L_2]\theta^* = L_4(1-\frac{t}{t_0})\exp[-(\frac{y^2}{r^2} + \frac{t}{t_0} + \gamma x + i\omega t + iay)], \tag{25}$$

where

$$L_4 = \frac{B_6I_0\gamma(\gamma^2 - B_1)}{2\pi r^2 t_0^2}.$$

Eqs. (24) and (25) can be factored as

$$(D^2-k_1^2)(D^2-k_2^2)\{\psi_1^*,\theta^*\} = (L_3,L_4)(1-\frac{t}{t_0})\exp[-(\frac{y^2}{r^2}+\frac{t}{t_0}+\gamma x+i\omega t+iay)], \quad (26)$$

where $k_n\ (n=1,2)$ are given by

$$k_1=\sqrt{\frac{L_1+\sqrt{L_1^2-4L_2}}{2}},\ k_2=\sqrt{\frac{L_1-\sqrt{L_1^2-4L_2}}{2}},\ \mathrm{Re}(k_n)>0. \quad (27)$$

The general solutions for $\psi_1(x,y,t)$ and $\theta(x,y,t)$, bounded as $x\to\infty$, are given by

$$\psi_1(x,y,t)=\sum_{n=1}^{2}R_n\exp(-k_nx+i\omega t+iay)+L_3gf(x,y,t), \quad (28)$$

$$\theta(x,y,t)=\sum_{n=1}^{2}H_nR_n\exp(-k_nx+i\omega t+iay)+L_4gf(x,y,t). \quad (29)$$

Here,

$$H_n=\frac{(k_n^2-B_1)}{B_2},\ g=\frac{1}{\gamma^4-L_1\gamma^2+L_2},$$

$$f(x,y,t)=(1-\frac{t}{t_0})\exp[-(\gamma x+\frac{y^2}{r^2}+\frac{t}{t_0})],$$

and $R_n$ ($n$=1,2) are some constants to be determined from the boundary conditions of the present problem.

For $x\to\infty$, the solution of Eq. (22) can be written as

$$\psi_2(x,y,t)=R_3\exp(-k_3x+i\omega t+iay), \quad (30)$$

where $k_3=\sqrt{B_3}>0$ and $R_3$ is another constant as defined above.

To obtain the components of the displacement vector, substitute from (28) and (30) in (12) and we get

$$u(x,y,t)=[-\sum_{n=1}^{2}k_nR_n\exp(-k_nx) + iaR_3\exp(-k_3x)]\exp(i\omega t+iay)-L_3g\gamma f(x,y,t), \quad (31)$$

$$v(x,y,t)=[ia\sum_{n=1}^{2}R_n\exp(-k_nx) + k_3R_3\exp(-k_nx)]\exp(i\omega t+iay)-\frac{2L_3g}{r^2}yf(x,y,t). \quad (32)$$

Inserting from (29), (31) and (32) in the Eqs. (16)-(19), and after detail calculations, we get the non-zero stress components in a non-local thermoelasticmedium in the following forms:

$$\sigma_{xx}(x,y,t)=\sum_{n=1}^{3}M_{1n}R_n\exp(-k_nx)\exp(iay+i\omega t)+I_1f(x,y,t), \quad (33)$$

$$\sigma_{yy}(x,y,t)=\sum_{n=1}^{3}M_{2n}R_n\exp(-k_nx)\exp(iay+i\omega t)+I_2f(x,y,t), \quad (34)$$

$$\sigma_{xy}(x,y,t)=\sum_{n=1}^{3}M_{3n}R_n\exp(-k_nx)\exp(iay+i\omega t)+yI_3f(x,y,t), \quad (35)$$

Where

$$M_{1n}=\frac{b_1(k_n^2-a^2)+k_n^2-b_2H_n}{[1-\xi^2(k_n^2-a^2)]},\ M_{2n}=\frac{b_1(k_n^2-a^2)-k_n^2-b_2H_n}{[1-\xi^2(k_n^2-a^2)]},$$

$$M_{3n}=\frac{-2iak_n}{[1-\xi^2(k_n^2-a^2)]},\qquad M_{13}=\frac{a^2-2ia}{[1-\xi^2(k_3^2-a^2)]},$$

$$M_{23}=-M_{13},\qquad M_{33}=-\frac{k_3^2-a^2}{[1-\xi^2(k_3^2-a^2)]},$$

$$I_1=\frac{g}{[1-\xi^2(\gamma^2-a^2)]}[b_1L_3(\gamma^2-\frac{2}{r^2}+\frac{4y^2}{r^4})+L_3\gamma^2-b_2L_4],$$

$$I_2=\frac{g}{[1-\xi^2(\gamma^2-a^2)]}[b_1L_3(\gamma^2-\frac{2}{r^2}+\frac{4y^2}{r^4})-L_3\gamma^2-b_2L_4],$$

$$I_3=\frac{4L_3g\gamma}{r^2[1-\xi^2(\gamma^2-a^2)]}.$$

## 4. The boundary conditions

In this section, we shall consider the following boundary conditions (in the non-dimensional form) on the surface $x$=0 to determine the constants $R_n$ ($n$=1,2,3):

(1) The thermal boundary condition:

A ramp-type thermal shock is applied to the boundary of the surface $x$=0 in the form

$$\theta(0,y,t)=\theta_0\delta(y)h(t), \quad (36)$$

where $\theta_0$ is a constant temperature, $\delta(y)$ is the Dirac-delta function and $h(t)$ is a prescribed function of $t$, given by

$$h(t)=\begin{cases}0 & t\le 0\\ \frac{t}{\tau_0} & 0<t\le\tau_0\\ 1 & t>\tau_0\end{cases} \quad (37)$$

(2) The mechanical boundary conditions:

The surface $x$=0 is taken to be traction-free which in turn means

$$\sigma_{xx}(0,y,t)=\sigma_{xy}(0,y,t)=0 \quad (38)$$

By using the normal as defined in (20), the above boundary conditions reduce to

$$\theta^*(y)=\theta_0\left(\frac{1-e^{-\omega\tau_0}}{\omega^2\tau_0}\right)=P(\omega,\tau_0), \quad (39)$$

$$\sigma_{xx}^*(y)=\sigma_{xy}^*(y)=0 \quad (40)$$

Substituting the expressions of the considered variables in the above boundary conditions, we can obtain the following equations satisfied by the constants $R_n$:

$$\sum_{n=1}^{2} H_n R_n = P, \tag{41}$$

$$\sum_{n=1}^{3} M_{1n} R_n = 0, \tag{42}$$

$$\sum_{n=1}^{3} M_{3n} R_n = 0, \tag{43}$$

After applying the inverse of the matrix method, we obtain the three constants $R_n$ ($n$=1,2,3) as follows

$$\begin{pmatrix} R_1 \\ R_2 \\ R_3 \end{pmatrix} = \begin{pmatrix} H_1 & H_2 & 0 \\ M_{11} & M_{12} & M_{13} \\ M_{31} & M_{32} & M_{33} \end{pmatrix}^{-1} \begin{pmatrix} P \\ 0 \\ 0 \end{pmatrix}. \tag{44}$$

With (44), the expressions (29) and (31)-(35) yield the temperature, displacements, and the stress components analytically for the present problem.

## 5. Numerical results and discussion

For numerical computations, following Dhaliwal and Singh (1980) the magnesium material was chosen for purposes of numerical evaluations. All the units of parameters used in the calculation are given in SI units. The constants of the problem were taken as

$$\lambda = 2.17\times10^{10} N/m^2, \quad \mu = 3.278\times10^{10} N/m^2,$$

$$\beta = 2.68\times10^{6} N/m^2 \cdot K, C_e = 1.04\times10^{3} J/kg\cdot K, \quad T_0 = 298\,K,$$

$$\omega_1^* = 5.338\times10^{11}/s, \quad \alpha_t = 1.78\times10^{-5} K^{-1},$$

The laser pulse parameters are

$$I_0 = 10^2 J/m^2, \; r = 0.2\,\mu m, \; \gamma = 25/m, \; t_0 = 10\,ns.$$

The nonlocal parameter is taken from Sarkar and Tomar (2019) as $a_0 = 0.5\times10^{-9}$ $m$, $e_0$=0.39.

The comparisons were carried out for

$k^* = 100W/m\cdot K$, $a = 0.5$, $\omega = 2.9$, $y = 2.0$, $t = 0.9$, $\theta_0 = 1.0$

These values are used for the distribution of the real parts of the temperature, displacement components, and the stresses with the distance $x(0\le x\le 4.0)$ for (G–N) theory of both types II and III in different values of the non-local parameter ($\xi$ = 0.00, 0.05) Here, ($\xi$ = 0) stand for the case when the nonlocality of the medium considered and for nonlocal medium, we take the value of the nonlocal parameter ($\xi$ =0.05) which is quite admissible (see Sarkar and Tomar, 2019).

Fig. 1 represents the distribution of the temperature $\theta$ in the case of ($\xi$ =0) (i.e. for local medium) and ($\xi$ =0.05) (i.e. for nonlocal medium) in the context of both types II and III of (G–N) theory. It is noticed that the distribution of $\theta$ decreases with the distance from the boundary of the half-space. At a larger distance from the boundary plane, the temperature getting more and more closure to zero, and finally, it becomes zero. This phenomenon ensures the fact of the finite speed of the thermal signal, which is the main essence of non-classical thermoelasticity. Moreover, a significant difference in temperature near the boundary plane is observed for GN-II and GN-III theory. The larger value of the temperature is noticed GN-II model as compared to GN-III model. In addition, for nonlocal medium, temperature found smaller than the local medium. One may conclude from here, that the nonlocality of the medium diminishes the magnitude in the temperature field. Fig. 2 represents the distribution of the displacement $u$ for the same set of parameters. The nonlocality of the medium acts to decrease the magnitude of $u$. Larger magnitude in $u$ is found for the GN-II model as compared than GN-III model. It is revealed in Fig. 3 that, in non-local medium, stress component $\sigma_{xx}$ has a smaller magnitude than that of in a local medium. Additionally, for GN-III model, magnitudes in $\sigma_{xx}$ observed smaller as compared to the GN-II model. Fig. 4 is drawn to understand the effect of nonlocality on stress component $\sigma_{yy}$ for both the said models. From this figure, the presence of nonlocality decreases the magnitude of $\sigma_{yy}$ for both GN models. For the GN-III model, we found smaller magnitudes in the major portion of the half-space. Maximum variations for stress components $\sigma_{xx}$, $\sigma_{yy}$ are notices at the boundary plane ($x$=0) and both the stress components reduce to zero magnitudes at a large distance apart from the boundary. The behavior of the stress component $\sigma_{xy}$ is revealed in Fig. 5. This figure ensures that our theoretical boundary condition [Eq. (38)] have been satisfied as $\sigma_{xy}$ becomes zero at ($x$=0) and $\sigma_{xy}$ becomes zero as $x$ increases. The nonlocal parameter $\xi$ makes the stress component $\sigma_{xy}$ small. Along with this, for GN-III model, we noticed a larger magnitude in $\sigma_{xy}$ as compared to the GN-II model. Figs. 6-10 are drawn to study the behavior of all the physical fields at different times. For the computational purpose, we choose the non-dimensional time as $t$= 0.3 and $t$= 0.5 It is revealed from these five figures that at a comparatively larger time, larger magnitudes in all the studied fields are noticed. From Figs. 6-9 we found greater magnitudes for the GN-II model than GN-III model in all the field quantities. The only exception is found for $\sigma_{xy}$ in Fig. 10. Here we observed that larger magnitudes are found for GN-III models than GN-II model.

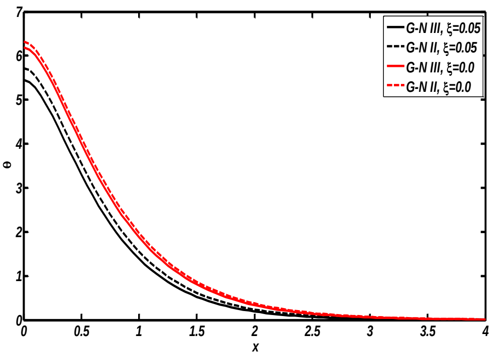


Fig. 1 Distribution of the temperature $\theta$ against $x$ when $t$ = 0.3.

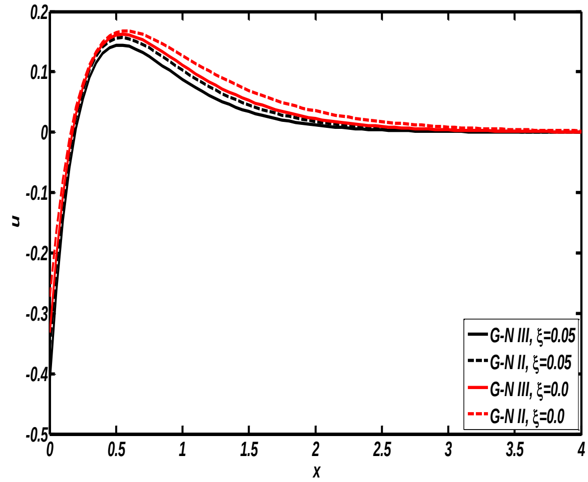


Fig. 2 Distribution of the displacement $u$ against $x$ when $t$ = 0.3.

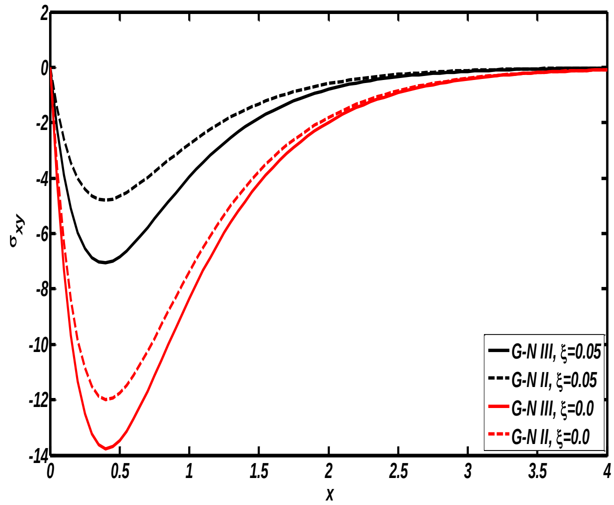


Fig. 5 Distribution of the stress $\sigma_{xy}$ against $x$ when $t$=0.3

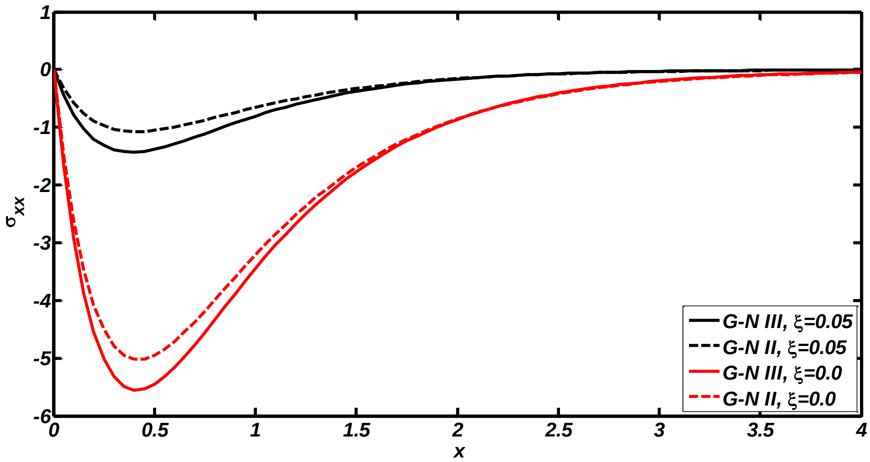


Fig. 3 Distribution of the stress $\sigma_{xx}$ against $x$ when $t$=0.3

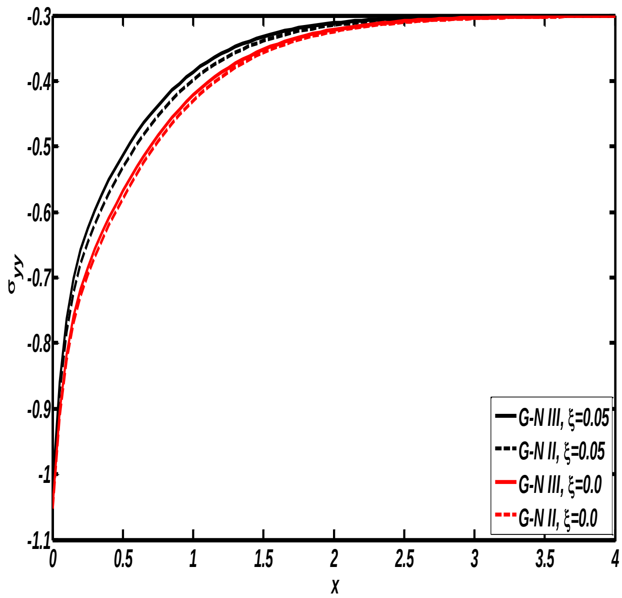


Fig. 4 Distribution of the stress $\sigma_{yy}$ against $x$ when $t$=0.3

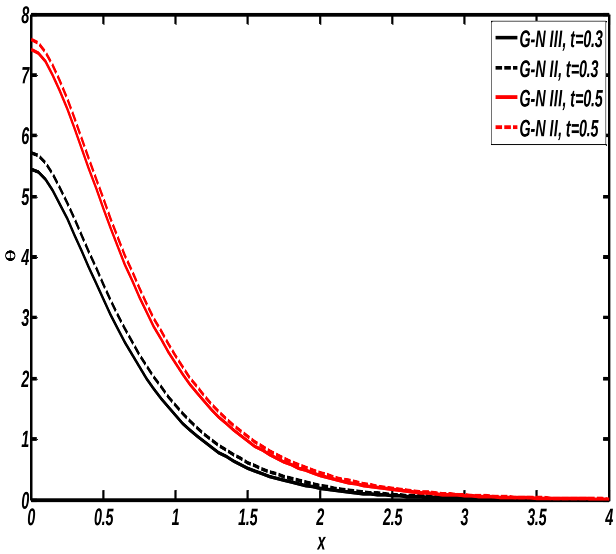


Fig. 6 Distribution of the temperature $\theta$ against $x$ for different time $t$ when $t$=0.05

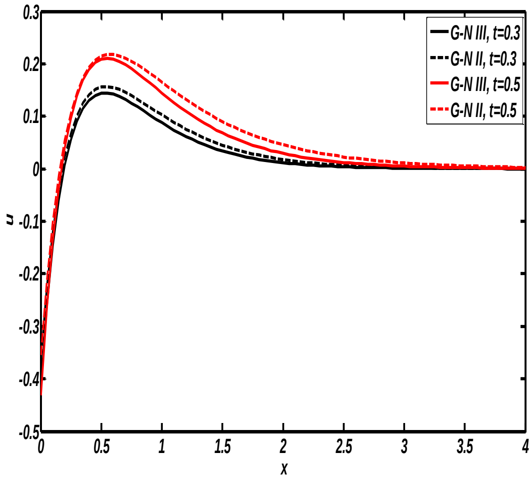


Fig. 7 Distribution of the displacement $u$ against $x$ for different time $t$ when $\xi$=0.05

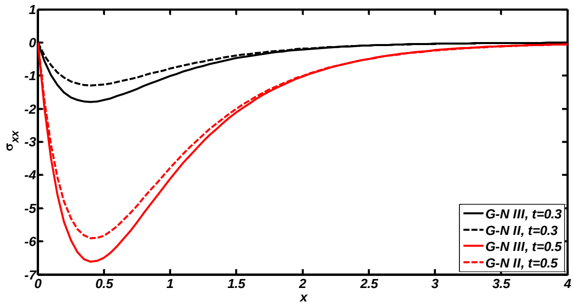


Fig. 8 Distribution of the stress $\sigma_{xx}$ against $x$ for different time $t$ when $\xi = 0.05$

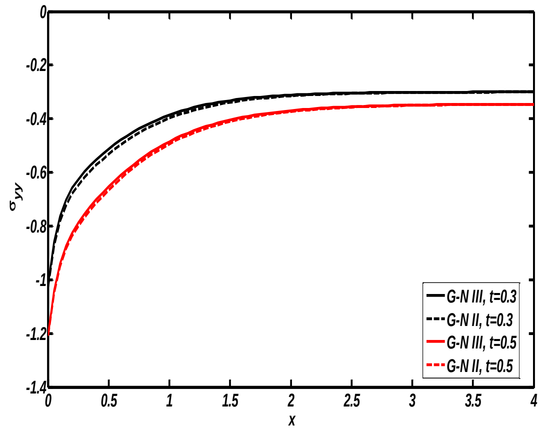


Fig. 9 Distribution of the stress $\sigma_{yy}$ against $x$ for different time $t$ when $\xi$=0.05

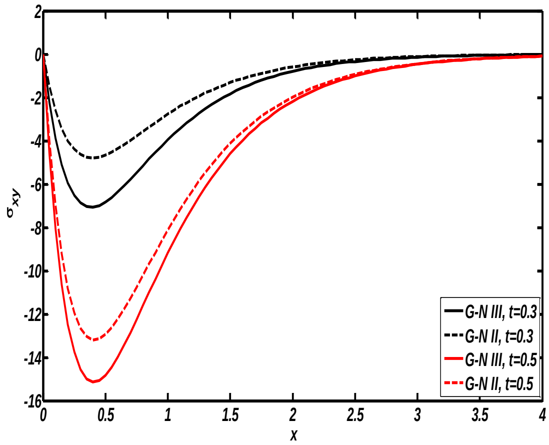


Fig. 10 Distribution of the stress $\sigma_{xy}$ against $x$ for different time $t$ when $\xi$=0.05

## 6. Conclusion

The results of the present work can be summarized as:

1. The values of all physical quantities converge to zero by increasing the distance $x$, and all functions are continuous.
2. The nonlocalities of the medium have a significant role in the considered physical quantities.
3. The laser pulse has significant influences on the distribution of the considered physical quantities.
4. All the field quantities change majorly depending on time.

## Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

## Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

## Acknowledgments

The research described in this paper was not financially supported by the Natural Science Foundation.

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