Structural Engineering and Mechanics, *Vol. 74, No. 3 (2020) 365-380*
DOI: https://doi.org/10.12989/sem.2020.74.3.365

An efficient shear deformation theory with stretching effect for bending stress analysis of laminated composite plates

Soufiane Abbas[1], Soumia Benguediab[2,3], Kada Draiche*[2,4], Ahmed Bakora[2] and Mohamed Benguediab[1]

[1]*Laboratory of Materials and Reactive Systems (LMRS), University of Sidi Bel Abbes, Faculty of Technology, Mechanical Engineering Department, Algeria*
[2]*Material and Hydrology Laboratory, University of Sidi Bel Abbes, Faculty of Technology, Civil Engineering Department, Algeria*
[3]*Université Dr Tahar Moulay, Faculté de Technologie, Département de Génie Civil et Hydraulique, BP 138 Cité En-Nasr 20000 Saida, Algérie*
[4]*Department of Civil Engineering, Ibn Khaldoun University, BP 78 Zaaroura, 14000 Tiaret, Algeria*

(Received March 14, 2019, Revised November 19, 2019, Accepted December 12, 2019)

Abstract. The focus of this paper is to develop an analytical approach based on an efficient shear deformation theory with stretching effect for bending stress analysis of cross-ply laminated composite plates subjected to transverse parabolic load and line load by using a new kinematic model, in which the axial displacements involve an undetermined integral component in order to reduce the number of unknowns and a sinusoidal function in terms of the thickness coordinate to include the effect of transverse shear deformation. The present theory contains only five unknowns and satisfies the zero shear stress conditions on the top and bottom surfaces of the plate without using any shear correction factors. The governing differential equations and its boundary conditions are derived by employing the static version of principle of virtual work. Closed-form solutions for simply supported cross-ply laminated plates are obtained applying Navier's solution technique, and the numerical case studies are compared with the theoretical results to verify the utility of the proposed model. Lastly, it can be seen that the present outlined theory is more accurate and useful than some higher-order shear deformation theories developed previously to study the static flexure of laminated composite plates.

Keywords: stress analysis; cross-ply laminated plates; parabolic load; line load; static flexure

1. Introduction

The composite materials are more recognized for their attractive mechanical properties and their many advantages, such as very high strength and stiffness coupled with a very low density, good resistance to corrosion, better noise insulation and have good impact toughness than those composed of most conventional metallic materials. Due to these major desirable features, the commercial use of laminated composites has been expanding rapidly over the past three decades, becoming the preferred material system in various specific applications, especially the aerospace and submarine structures, mechanical engineering, biomedical products and also in the civil engineering sector for structural strengthening techniques in the restoration of buildings affected by natural hazards and other destructive forces. The nature of these inhomogeneous materials makes them very adaptable to each area and it is possible to choose for each structure the best compromise cost/weight/mechanical strength. Their increased use has underlined the need for understanding correctly their static and dynamic responses for the continual enhancement of their performance.

However, one important point must be noticed that the stiffness of laminate varies from point to another depending on the properties of its constituents and different fiber orientations, making their analysis more complicated than the analysis of homogeneous isotropic ones. For this reason, several theories have been developed in this field to study and investigate the laminated composite plates in many cases to exploit their strength in all industrial sectors.

The equivalent single-layer (ESL) laminate theories are those in which a heterogeneous laminated plate is treated as a statically equivalent, single layer having an anisotropic constitutive behaviour, reducing the 3-D continuum problem to a 2-D problem (Carlos *et al.* 1999) by setting suitable assumptions about the kinematics of deformation or the stress state through the thickness of the laminate. The simplest model of these theories is the classical laminated plate theory (CLPT), which is an extension to Kirchhoff's (1850) isotropic plate theory. The most important assumption of this theory is that the normal to the mid-plane before deformation remains straight and normal at mid-surface after deformation. However it is not valid for the analysis of thick plate due to neglect of the effects of transverse shear deformation and rotary inertia. The different applications of this theory are presented by Love (1944), Timoshenko and Krieger (1959), Timoshenko and Gere (1961), Szilard (1974), Ugural (1981) and many others. The next accurate theory in the hierarchy of ESL laminated plate theories is the first-order shear deformation theory

*Corresponding author, Ph.D.
E-mail: kdraiche@yahoo.fr

http://www.techno-press.com/journals/sem&subpage=7
ISSN: 1225-4568 (Print), 1598-6217 (Online)

(FSDT) developed to overcome this serious limitation of CLPT by applying Reissner (1945) and Mindlin (1951) formulations to thick laminated plates. The FSDT extends the kinematics of the CLPT by considering a gross transverse shear deformation in its kinematical assumption, involves that the normal to the mid-plane remains straight but not normal at mid-surface after deformation due to the shear effect. According to this theory, the transverse shear strain is assumed to be constant in thickness direction, which requires the introduction of a shear correction factor in order to account for the difference between the constant shear stress and the parabolic variation of shear stress across the thickness. The Mindlin's theory has been extended to laminated anisotropic plates by Yang, *et al*. (1966), Whitney (1969), Whitney and Pagano (1970), Wang (1997), Civalek (2008), Ferreira *et al*. (2009), Avcar (2015, 2016 and 2019), Baltac (2018), Alimirzaei *et al*. (2019), Draoui *et al*. (2019) and several authors.

Therefore, many higher-order shear deformation theories (HSDTs) have been developed over the last years to avoid the use of shear correction factors and are based on the hypothesis of nonlinear stress variation through thickness. For instance Reddy (1984) has developed a higher-order shear deformation theory by considering a parabolic variation of the transverse shear strains through thickness for the analysis of laminated composite plates under sinusoidal and uniform loads. The theory contains the same number of dependent variables as in the first-order shear deformation theory, but satisfies the zero tangential traction boundary conditions on the top and bottom surfaces of the plate without using the shear correction factor. An efficient standard plate theory with only five variational unknowns suitable for the bending, buckling and free vibration analysis of sandwich and symmetric cross-ply laminated composite plates with simply supported edges conditions, is investigated by Touratier (1991) based on the kinematical approach in which the shear is represented by a certain sinusoïdal function. Soldatos and Timarci (1993) proposed a transverse shear deformation theory rest on a unified formulation of laminated composite by introducing into the shell displacement approximation certain general functions of the transverse coordinate for the static and dynamic analysis of cross-ply laminated cylindrical shells. A similar method was used later by Aydogdu (2006) for the bending, free vibration and buckling analysis of simply supported symmetric cross-ply rectangular plates. Various shear deformation theories were compared in this study and the obtained results shows that the parabolic and the hyperbolic shear deformation theories yield more accurate predictions for the natural frequencies and the critical buckling loads. A global higher-order theory and several sets of the governing equations of truncated approximate theories have been applied by Matsunaga (2000) for the vibration and buckling analysis of a simply supported multilayered elastic plate subjected to in-plane stresses. Zenkour (2004) investigated the bending response of symmetric and antisymmetric cross-ply laminated plates subjected to sinusoidal non-uniform thermal or thermo-mechanical loads by using a unified shear deformation plate theory. In fact, the analytical solutions for buckling and free vibration analysis of simply supported symmetric and antisymmetric cross-ply thick composite plates resting on elastic foundation and subjected to in-plane loads are presented by Akavci (2007) using a new hyperbolic displacement model and the Navier technique. Kim *et al*. (2009) developed a two variable refined plate theory for the static bending and buckling behaviours of antisymmetric cross-ply and angle-ply laminated composite plates, in which a parabolic distribution of the transverse shear strains is considered to satisfy the zero traction boundary conditions on the surfaces of the plate without using shear correction factors. Mantari *et al*. (2012) developed a finite element model based on the new trigonometric layerwise shear deformation theory for the flexure analysis of thick laminated composite and sandwich plates subjected to a transverse uniform load. Moreover, the discrete element selected is a four-nodded quadrilateral with seven degrees of freedom per node. Sayyad and Ghugal (2013) applied an equivalent single layer trigonometric shear deformation theory to investigate the effect of transverse shear deformation, transverse normal strain and local stress concentration on inplane normal and transverse shear stresses through the thickness of orthotropic and laminated plates. A refined hyperbolic shear deformation theory of plates has been developed by Nedri *et al*. (2014) to investigate the free vibration of simply supported laminated plates resting on elastic foundations. A major advantage of this theory is a considerably smaller number of unknown variables, as against five or more in other higher order shear deformation theories. A new trigonometric zigzag theory with a particularly secant function has been proposed by Sahoo and Singh (2014) to analyze the static structural behaviour of laminated composite and sandwich plates. This theory considers shear strain shape function assuming the non-linear distribution of in-plane displacement across the thickness. Mahi *et al*. (2015) proposed a new hyperbolic shear deformation theory with five degrees of freedom for bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates. The in-plane displacements of this theory use a combination of hyperbolic tangent function and polynomial ones. In the same year, Kar *et al*. (2015) investigated the bending behaviour of laminated composite flat panel subjected to hygro-thermo-mechanical loading by using a higher-order shear deformation theory, in which the nonlinear geometric term is introduced into the Green-Lagrange formulation. Sayyad *et al*. (2016) have proposed an exponential shear deformation theory (ESDT) to study the thermal stress response of cross-ply laminated composite plates under thermal load varying linearly across the thickness of plate.

Recently, Behera and Kumari (2018) investigated the free vibration of Levy-type rectangular laminated plates using an efficient zig-zag theory. Belbachir *et al*. (2019) analyzed the bending behavior of anti-symmetric cross-ply laminated plates under nonlinear thermal and mechanical loadings. However, the results demonstrate that the proposed plate theory is able to produce more accurate results than the FSDT and other HSDTs with higher number of unknowns. More recently, many studies and

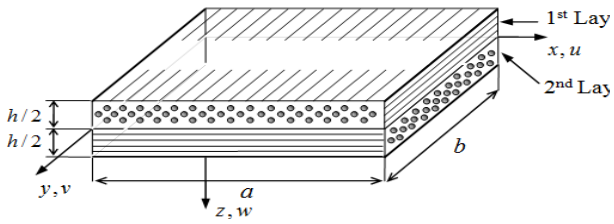

Fig. 1 Coordinate system and geometry of antisymmetric cross-ply laminated plate

investigations related to the static and dynamic analysis of laminated composite plates and beams by using the higher-order shear deformation theories (HSDTs) have been carried out and available in literature (Cetkovic and Vuksanovic 2011, Rezaiee-Pajand *et al*. 2012, Sherafat *et al*. 2013, Ahmed 2014, Sahoo *et al*. 2016, Chikh *et al*. 2017, Bouazza *et al*. 2017, Vo *et al*. 2017, Sayyad and Ghugal 2017, Sehoul *et al*. 2017, Zamani *et al*. 2017, Singh *et al*. 2017, Zine *et al*. 2018, Katariya *et al*. 2018, Belkacem *et al*. 2018, Hirwani *et al*. 2018ab, Salami and Dariushi 2018, Nor Hafizah *et al*. 2018, Avcar and Mohammed 2018, Joshan *et al*. 2018, Hirwani and Panda 2018, Katariya and Panda 2019abc, Abualnour *et al*. 2019, Addou *et al*. 2019, Berghouti *et al*. 2019, Bouanati *et al*. 2019, Boukhlif *et al*. 2019, Boulefrakh *et al*. 2019, Mahmoud and Tounsi 2019, Bourada *et al*. 2019, Boutaleb *et al*. 2019, Chaabane *et al*. 2019, Hirwani and Panda 2019, Mehar and Panda 2019, Hellal *et al*. 2019, Batou *et al*. 2019, Sahoo *et al*. 2019, Hadji *et al*. 2019, Sahla *et al*. 2019, Boussoula *et al*. 2020).

The present paper is devoted to the development of an accurate numerical approach using a novel higher-order shear deformation plate theory with stretching effect based on a new kinematic model for static bending behaviour of cross-ply laminated composite plates subjected to two different types of loading, namely transverse parabolic load and line load. In general, the proposed theory contains only five-unknown variables and satisfies the zero traction boundary conditions on the top and bottom surfaces of the plate without the need for special treatment of shear correction factors. A Navier-type analytical method was used to solve the governing differential equations. To prove the efficiency of the present theory, the numerical results of displacements and stresses of simply supported cross-ply laminated composite plates are calculated and compared with the available published results.

2. Theoretical formulation

2.1. Plate under consideration

In this analysis, a cross-ply laminated composite plate is assumed of uniform thickness h, length a, and width b as depicted in Fig. 1. It is composed of N number of orthotropic layers, which are perfectly bonded together. The plate occupies the region $0 \le x \le a$, $0 \le y \le b$, $-h/2 \le z \le h/2$ in Cartesian coordinate system. Let the plate be subjected to a mechanical load q(x, y) acting strictly at the upper surface ($z = -h/2$).

The present model is developed based on some assumptions and features of the proposed trigonometric shear deformation theory with stretching effect, the displacement components at any point on the plate can be expressed in a simpler form as

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\frac{\partial w_0}{\partial x} + k_1 f(z)\int \theta(x,y)dx \\ v(x,y,z) &= v_0(x,y) - z\frac{\partial w_0}{\partial y} + k_2 f(z)\int \theta(x,y)dy \\ w(x,y,z) &= w_0(x,y) + g(z)\phi_z(x,y) \end{aligned} \tag{1}$$

In which $u_0(x,y)$, $v_0(x,y)$, $w_0(x,y)$, $\theta(x,y)$ and $\phi_z(x,y)$ signify the displacement functions of the middle surface of the plate. The constants k_1 and k_2 depends on the geometry. Also *f*(*z*) is the representative shape function that denotes the distribution of transverse shear stress or strain along the plate thickness. In this study, we take into consideration that

$$f(z) = \left(\frac{h}{\pi}\right)\sin\left(\frac{\pi z}{h}\right), \quad g(z) = \frac{df(z)}{dz} \tag{2}$$

Based on the relationship from linear theory of elasticity, the infinitesimal normal and transverse shear strains associated with the displacement field in Eq. (1) are commonly expressed in the following form

$$\begin{aligned} &\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} \varepsilon_x^1 \\ \varepsilon_y^1 \\ \gamma_{xy}^1 \end{Bmatrix} + f(z)\begin{Bmatrix} \varepsilon_x^2 \\ \varepsilon_y^2 \\ \gamma_{xy}^2 \end{Bmatrix}, \\ &\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z)\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix}, \quad \varepsilon_z = g'(z)\varepsilon_z^0 \end{aligned} \tag{3}$$

so that virtual strains are known in terms of the virtual displacements

$$\begin{aligned} &\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial x} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} \varepsilon_x^1 \\ \varepsilon_y^1 \\ \gamma_{xy}^1 \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}, \\ &\begin{Bmatrix} \varepsilon_x^2 \\ \varepsilon_y^2 \\ \gamma_{xy}^2 \end{Bmatrix} = \begin{Bmatrix} k_1\theta \\ k_2\theta \\ k_1\dfrac{\partial}{\partial y}\displaystyle\int \theta dx + k_2\dfrac{\partial}{\partial x}\int \theta dy \end{Bmatrix}, \end{aligned} \tag{4a}$$

$$\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} k_2\displaystyle\int \theta dy + \dfrac{\partial \phi_z}{\partial y} \\ k_1\displaystyle\int \theta dx + \dfrac{\partial \phi_z}{\partial x} \end{Bmatrix}, \quad \varepsilon_z^0 = \phi_z \tag{4b}$$

The integrals adopted in the previous relations shall be resolved by a Navier solution and can be determined by

$$\frac{\partial}{\partial y}\int \theta dx = A'\frac{\partial^2\theta}{\partial x\partial y}, \quad \frac{\partial}{\partial x}\int \theta dy = B'\frac{\partial^2\theta}{\partial x\partial y}, \quad \int \theta dx = A'\frac{\partial\theta}{\partial x}, \quad \int \theta dy = B'\frac{\partial\theta}{\partial y} \tag{5}$$

where A' and B' are determined according to the type of solution employed, in this case via Navier procedure. Thus, the coefficients A', B', k_1 and k_2 are expressed by

$$A' = -\frac{1}{\alpha^2}, \quad B' = -\frac{1}{\beta^2}, \quad k_1 = \alpha^2, \quad k_2 = \beta^2 \tag{6}$$

where the parameters α and β are defined as

$$\alpha = \frac{m\pi}{a}, \quad \beta = \frac{n\pi}{b} \tag{7}$$

2.2 Constitutive equations

The constitutive equations relate the stress components to strain components. For the linear elastic range, these equations represent the generalized Hooke's law. In the case of a three-dimensional orthotropic laminate it can be noted that the fiber orientations do not coincide with the global coordinates of the plate, the stress-strain relationships for each layer are given by

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{Bmatrix}^{(k)} = \begin{bmatrix}\overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{13} & 0 & 0 & 0\\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{23} & 0 & 0 & 0\\ \overline{Q}_{13} & \overline{Q}_{23} & \overline{Q}_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & \overline{Q}_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & \overline{Q}_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & \overline{Q}_{66}\end{bmatrix}^{(k)} \begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{Bmatrix}^{(k)} \tag{8}$$

in which, $\{\sigma_x, \sigma_y, \tau_{xy}, \tau_{yz}, \tau_{xz}\}$ and $\{\varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{yz}, \gamma_{xz}\}$ are the stresses and the strains vectors with respect to the laminate coordinate system (x, y, z). Whereas $\overline{Q_{ij}}$ are the transformed elastic constants of the k^{th} orthotropic layer expressed as

$$\begin{aligned}
\overline{Q}_{11}^{k} &= Q_{11}\cos^4\theta_k + 2(Q_{12}+2Q_{66})\sin^2\theta_k\cos^2\theta_k + Q_{22}\sin^4\theta_k\\
\overline{Q}_{12}^{k} &= (Q_{11}+Q_{22}-4Q_{66})\sin^2\theta_k\cos^2\theta_k + Q_{12}(\sin^4\theta_k+\cos^4\theta_k)\\
\overline{Q}_{13}^{k} &= Q_{13}\cos^2\theta_k + Q_{23}\sin^2\theta_k\\
\overline{Q}_{22}^{k} &= Q_{11}\sin^4\theta_k + 2(Q_{12}+2Q_{66})\sin^2\theta_k\cos^2\theta_k + Q_{22}\cos^4\theta_k\\
\overline{Q}_{23}^{k} &= Q_{13}\sin^2\theta_k + Q_{23}\cos^2\theta_k\\
\overline{Q}_{33}^{k} &= Q_{33}\\
\overline{Q}_{44}^{k} &= Q_{44}\cos^2\theta_k + Q_{55}\sin^2\theta_k\\
\overline{Q}_{55}^{k} &= Q_{55}\cos^2\theta_k + Q_{44}\sin^2\theta_k\\
\overline{Q}_{66}^{k} &= (Q_{11}+Q_{22}-2Q_{12}-2Q_{66})\sin^2\theta_k\cos^2\theta_k + Q_{66}(\sin^4\theta_k+\cos^4\theta_k)
\end{aligned} \tag{9}$$

where θ_k is the angle of material axes with the reference coordinate axes of each layer and Q_{ij} are the reduced stiffness coefficients that can be defined in the case of transverse normal strain is different to zero $(\varepsilon_z \neq 0)$ by the following expressions

$$\begin{gathered}
Q_{11} = \frac{E_1(1-\nu_{23}\nu_{32})}{\Delta},\ Q_{12} = \frac{E_1(\nu_{21}+\nu_{31}\nu_{23})}{\Delta},\ Q_{13} = \frac{E_1(\nu_{31}+\nu_{21}\nu_{32})}{\Delta}\\
Q_{22} = \frac{E_2(1-\nu_{13}\nu_{31})}{\Delta},\ Q_{23} = \frac{E_2(\nu_{32}+\nu_{12}\nu_{31})}{\Delta},\ Q_{33} = \frac{E_3(1-\nu_{12}\nu_{21})}{\Delta};\\
Q_{44} = G_{23},\quad Q_{55} = G_{13},\quad Q_{66} = G_{12},\\
\Delta = 1-\nu_{12}\nu_{21}-\nu_{23}\nu_{32}-\nu_{31}\nu_{13}-2\nu_{21}\nu_{32}\nu_{13}
\end{gathered} \tag{10}$$

where E_i, G_{ij} and ν_{ij} are the Young's moduli, shear moduli and Poisson's ratio, respectively.

2.3. Governing equations

The governing equations and associated boundary conditions of the present trigonometric shear deformation plate theory with stretching effect are derived using static version of principle of virtual work. The principle of virtual work is applied in the following analytical form

$$\int_{-h/2}^{h/2}\int_A \begin{pmatrix}\sigma_x\delta\varepsilon_x + \sigma_y\delta\varepsilon_y + \sigma_z\delta\varepsilon_z\\ +\tau_{xy}\delta\gamma_{xy} + \tau_{yz}\delta\gamma_{yz} + \tau_{xz}\delta\gamma_{xz}\end{pmatrix} dAdz - \int_A q(x,y)\delta w dA = 0 \tag{11}$$

where δ is a variational operator, A and $q(x,y)$ are the top surface of the plate and the transverse distributed load, respectively. By substituting the expressions for virtual strains given in Eq. (3) into Eq. (11), the principle of virtual work can be expressed in terms of the stress resultants as

$$\int_A \begin{bmatrix}N_x\delta\varepsilon_x^0 + N_y\delta\varepsilon_y^0 + N_z\delta\varepsilon_z^0 + N_{xy}\delta\gamma_{xy}^0\\ +M_x^b\delta\varepsilon_x^1 + M_y^b\delta\varepsilon_y^1 + M_{xy}^b\delta\gamma_{xy}^1 + M_x^s\delta\varepsilon_x^2\\ +M_y^s\delta\varepsilon_y^2 + M_{xy}^s\delta\gamma_{xy}^2 + S_{yz}^s\delta\gamma_{yz}^0 + S_{xz}^s\delta\gamma_{xz}^0 - q\delta w\end{bmatrix} dA = 0 \tag{12}$$

where N, M^b, M^s and S^s are the stress resultants that generally vary from point to point in a loaded plate. These variations are governed by the static conditions of equilibrium. These stress resultants are defined by the following integrations over the thickness of the plate

$$\begin{Bmatrix}N_x, & N_y, & N_{xy}\\ M_x^b, & M_y^b, & M_{xy}^b\\ M_x^s, & M_y^s, & M_{xy}^s\end{Bmatrix} = \sum_{k=1}^{N}\int_{z_k}^{z_{k+1}} (\sigma_x, \sigma_y, \tau_{xy}) \begin{Bmatrix}1\\ z\\ f(z)\end{Bmatrix} dz, \quad N_z = \sum_{k=1}^{N}\int_{z_k}^{z_{k+1}} g'(z)\,\sigma_z dz \tag{13a}$$

$$\left(S_{xz}^s, S_{yz}^s\right) = \sum_{k=1}^{N}\int_{z_k}^{z_{k+1}} \left(\tau_{xz}, \tau_{yz}\right) g(z)\, dz \tag{13b}$$

Substituting Eq. (3) into constitutive equations given in Eq. (8) and subsequent results into Eqs. (13), the stress resultants of the proposed theory can be represented in terms of virtual strains according to the following constitutive equations

$$
\begin{aligned}
N_x &= A_{11}\varepsilon_x^0 + A_{12}\varepsilon_y^0 + B_{11}\varepsilon_x^1 + B_{12}\varepsilon_y^1 + B_{11}^s\varepsilon_x^2 + B_{12}^s\varepsilon_y^2 + X_{13}\varepsilon_z^0,\\
N_y &= A_{12}\varepsilon_x^0 + A_{22}\varepsilon_y^0 + B_{12}\varepsilon_x^1 + B_{22}\varepsilon_y^1 + B_{12}^s\varepsilon_x^2 + B_{22}^s\varepsilon_y^2 + X_{23}\varepsilon_z^0,\\
N_z &= X_{13}\varepsilon_x^0 + X_{23}\varepsilon_y^0 + Y_{13}\varepsilon_x^1 + Y_{23}\varepsilon_y^1 + Y_{13}^s\varepsilon_x^2 + Y_{23}^s\varepsilon_y^2 + Z_{33}\varepsilon_z^0,\\
N_{xy} &= A_{66}\gamma_{xy}^0 + B_{66}\gamma_{xy}^1 + B_{66}^s\gamma_{xy}^2,\\
M_x^b &= B_{11}\varepsilon_x^0 + B_{12}\varepsilon_y^0 + D_{11}\varepsilon_x^1 + D_{12}\varepsilon_y^1 + D_{11}^s\varepsilon_x^2 + D_{12}^s\varepsilon_y^2 + Y_{13}\varepsilon_z^0,\\
M_y^b &= B_{12}\varepsilon_x^0 + B_{22}\varepsilon_y^0 + D_{12}\varepsilon_x^1 + D_{22}\varepsilon_y^1 + D_{12}^s\varepsilon_x^2 + D_{22}^s\varepsilon_y^2 + Y_{23}\varepsilon_z^0,\\
M_{xy}^b &= B_{66}\gamma_{xy}^0 + D_{66}\gamma_{xy}^1 + D_{66}^s\gamma_{xy}^2,\\
M_x^s &= B_{11}^s\varepsilon_x^0 + B_{12}^s\varepsilon_y^0 + D_{11}^s\varepsilon_x^1 + D_{12}^s\varepsilon_y^1 + H_{11}^s\varepsilon_x^2 + H_{12}^s\varepsilon_y^2 + Y_{13}^s\varepsilon_z^0,\\
M_y^s &= B_{12}^s\varepsilon_x^0 + B_{22}^s\varepsilon_y^0 + D_{12}^s\varepsilon_x^1 + D_{22}^s\varepsilon_y^1 + H_{12}^s\varepsilon_x^2 + H_{22}^s\varepsilon_y^2 + Y_{23}^s\varepsilon_z^0,\\
M_{xy}^s &= B_{66}^s\gamma_{xy}^0 + D_{66}^s\gamma_{xy}^1 + H_{66}^s\gamma_{xy}^2,\quad S_{yz}^s = A_{44}^s\gamma_{yz}^0,\quad S_{xz}^s = A_{55}^s\gamma_{xz}^0
\end{aligned}
\tag{14}
$$

where $A_{ij}, B_{ij}, D_{ij},...$ etc. used in Eq. (14) are the plate stiffness coefficients can be defined as

$$
\left(A_{ij}, B_{ij}, D_{ij}, B_{ij}^s, D_{ij}^s, H_{ij}^s\right) = \sum_{k=1}^{N}\int_{z_k}^{z_{k+1}} \bar{Q}_{ij}^{(k)}\left(1, z, z^2, f(z), z\,f(z), f^2(z)\right)dz,\quad i,j=1,2,6 \tag{15a}
$$

$$
\left(X_{ij}, Y_{ij}, Y_{ij}^s\right) = \sum_{k=1}^{N}\int_{z_k}^{z_{k+1}} \bar{Q}_{i3}^{(k)} g'(z)\left(1, z, f(z)\right)\,dz,\quad i=1,2 \tag{15b}
$$

$$
Z_{33} = \sum_{k=1}^{N}\int_{z_k}^{z_{k+1}} \bar{Q}_{33}^{(k)}\left[g'(z)\right]^2 dz, \tag{15c}
$$

$$
A_{ij}^s = \sum_{k=1}^{N}\int_{z_k}^{z_{k+1}} \overline{Q}_{ij}^{(k)}\left[g(z)\right]^2 dz,\quad i,j=4,5 \tag{15d}
$$

By substituting strains and stresses expressions from Eqs. (4) and (8) into Eq. (12) and integrating by parts and setting the coefficients of $\delta u_0, \delta v_0, \delta w_0, \delta\theta$ and $\delta\varphi_z$ to zero separately, the governing differential equations are obtained

$$
\begin{aligned}
\delta\, u_0 &: \quad \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0\\
\delta\, v_0 &: \quad \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0\\
\delta\, w_0 &: \quad \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x\partial y} + \frac{\partial^2 M_y^b}{\partial y^2} + q = 0\\
\delta\, \theta &: \quad -k_1 M_x^s - k_2 M_y^s - \left(k_1 A' + k_2 B'\right)\frac{\partial^2 M_{xy}^s}{\partial x\partial y}\\
&\qquad + k_1 A'\frac{\partial S_{xz}^s}{\partial x} + k_2 B'\frac{\partial S_{yz}^s}{\partial y} = 0\\
\delta\, \phi_z &: \quad \frac{\partial S_{xz}^s}{\partial x} + \frac{\partial S_{yz}^s}{\partial y} - N_z = 0
\end{aligned}
\tag{16}
$$

Next, by substituting Eqs. (4) and (14) into Eq. (16), the governing differential equations of the present theory can be rewritten in terms of displacement variables $(u_0, v_0, w_0, \theta, \varphi_z)$ as follows

$$
\begin{aligned}
&A_{11}\frac{\partial^2 u_0}{\partial x^2} + A_{66}\frac{\partial^2 u_0}{\partial y^2} + \left(A_{12}+A_{66}\right)\frac{\partial^2 v_0}{\partial x\partial y} - B_{11}\frac{\partial^3 w_0}{\partial x^3} - \left(B_{12}+2B_{66}\right)\frac{\partial^2 w_0}{\partial x\partial y^2}\\
&+\left(k_1B_{11}^s + k_2B_{12}^s\right)\frac{\partial\theta}{\partial x} + \left(k_1A' + k_2B'\right)B_{66}^s\frac{\partial^3\theta}{\partial x\partial y^2} + X_{13}\frac{\partial\phi_z}{\partial x} = 0
\end{aligned}
\tag{17a}
$$

$$
\begin{aligned}
&\left(A_{12}+A_{66}\right)\frac{\partial^2 u_0}{\partial x\partial y} + A_{22}\frac{\partial^2 v_0}{\partial y^2} + A_{66}\frac{\partial^2 v_0}{\partial x^2} - \left(B_{12}+2B_{66}\right)\frac{\partial^3 w_0}{\partial x^2\partial y} -\\
&B_{22}\frac{\partial^3 w_0}{\partial y^3}\left(k_1B_{12}^s + k_2B_{22}^s\right)\frac{\partial\theta}{\partial y} + \left(k_1A' + k_2B'\right)B_{66}^s\frac{\partial^3\theta}{\partial x^2\partial y} + X_{23}\frac{\partial\phi_z}{\partial y} = 0
\end{aligned}
\tag{17b}
$$

$$
\begin{aligned}
&B_{11}\frac{\partial^3 u_0}{\partial x^3} + \left(B_{12}+2B_{66}\right)\frac{\partial^3 u_0}{\partial x\partial y^2} + B_{22}\frac{\partial^3 v_0}{\partial y^3} + \left(B_{12}+2B_{66}\right)\frac{\partial^3 v_0}{\partial x^2\partial y}\\
&-2\left(D_{12}+2D_{66}\right)\frac{\partial^4 w_0}{\partial x^2\partial y^2} - D_{11}\frac{\partial^4 w_0}{\partial x^4} - D_{22}\frac{\partial^4 w_0}{\partial y^4} + \left(k_1D_{11}^s + k_2D_{12}^s\right)\frac{\partial^2\theta}{\partial x^2}\\
&+2\left(k_1A' + k_2B'\right)D_{66}^s\frac{\partial^4\theta}{\partial x^2\partial y^2} + \left(k_1D_{12}^s + k_2D_{22}^s\right)\frac{\partial^2\theta}{\partial y^2} + Y_{13}\frac{\partial^2\phi_z}{\partial x^2} + Y_{23}\frac{\partial^2\phi_z}{\partial y^2} + q = 0
\end{aligned}
\tag{17c}
$$

$$
\begin{aligned}
&-\left(k_1B_{11}^s + k_2B_{12}^s\right)\frac{\partial u_0}{\partial x} - \left(k_1B_{12}^s + k_2B_{22}^s\right)\frac{\partial v_0}{\partial y} - \left(k_1A' + k_2B'\right)B_{66}^s\left(\frac{\partial^3 u_0}{\partial x\partial y^2} + \frac{\partial^3 v_0}{\partial x^2\partial y}\right)\\
&+\left(k_1D_{11}^s + k_2D_{12}^s\right)\frac{\partial^2 w_0}{\partial x^2} + 2\left(k_1A' + k_2B'\right)D_{66}^s\frac{\partial^4 w_0}{\partial x^2\partial y^2} + \left(k_1D_{12}^s + k_2D_{22}^s\right)\frac{\partial^2 w_0}{\partial y^2}\\
&-\left(k_1^2H_{11}^s + k_2^2H_{22}^s + 2k_1k_2H_{12}^s\right)\theta + \left(k_2B'\right)^2 A_{44}^s\frac{\partial^2\theta}{\partial y^2} - \left(k_1A' + k_2B'\right)^2 H_{66}^s\frac{\partial^4\theta}{\partial x^2\partial y^2}\\
&+\left(k_1A'\right)^2 A_{55}^s\frac{\partial^2\theta}{\partial x^2} + k_1A'A_{55}^s\frac{\partial^2\phi_z}{\partial x^2} + k_2B'A_{44}^s\frac{\partial^2\phi_z}{\partial y^2} - \left(k_1Y_{13}^s + k_2Y_{23}^s\right)\phi_z = 0
\end{aligned}
\tag{17d}
$$

$$
\begin{aligned}
&-X_{13}\frac{\partial u_0}{\partial x} - X_{23}\frac{\partial v_0}{\partial y} + Y_{13}^s\frac{\partial^2 w_0}{\partial x^2} + Y_{23}^s\frac{\partial^2 w_0}{\partial y^2} - \left(k_1Y_{13}^s + k_2Y_{23}^s\right)\theta + k_1A'A_{55}^s\frac{\partial^2\theta}{\partial x^2}\\
&+k_2B'A_{44}^s\frac{\partial^2\theta}{\partial y^2} + A_{55}^s\frac{\partial^2\phi_z}{\partial x^2} + A_{44}^s\frac{\partial^2\phi_z}{\partial y^2} - Z_{33}\phi_z = 0
\end{aligned}
\tag{17e}
$$

3. Illustrative examples

In order to prove the efficiency of the present theory, a simply supported square laminated composite plate is considered for the detailed numerical study. The plate is subjected to two different types of mechanical loadings $q(x,y)$ acting at the top surface (i.e., $z=-h/2$) of the plate. According to Navier solution technique, the transverse load is most commonly represented in double trigonometric series as

$$
q(x,y) = \sum_{m=1,3,5}^{\infty}\sum_{n=1,3,5}^{\infty} q_{mn}\sin(\alpha\, x)\sin(\beta\, y) \tag{18}
$$

In which the parameters α and β are already defined in Eq. (7) and q_{mn} is the coefficient of Fourier expansion of load. The values of this coefficient for different loading cases are obtained by using the following equation

$$
q_{mn} = \frac{4}{ab}\int_0^a\int_0^b q(x,y)\sin(\alpha\, x)\sin(\beta\, y)\,dxdy \tag{19}
$$

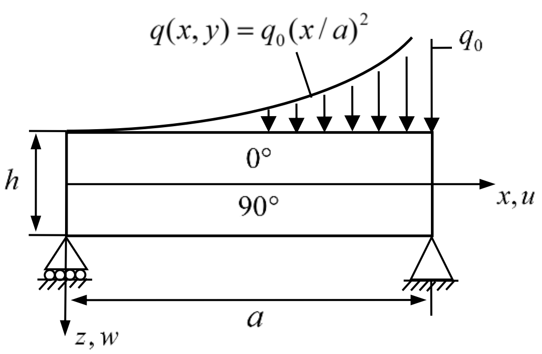

Fig. 2 Simply supported laminated plates subjected to transverse parabolic load

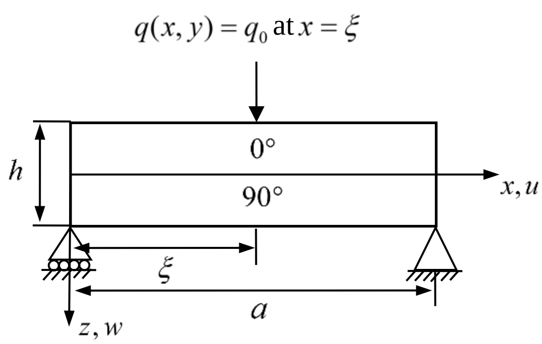

Fig. 3 Simply supported laminated plates subjected to transverse line load

Example 1: In this example a square laminated composite plate is simply supported on all edges and subjected to transverse parabolic load $q(x,y)=q_0(x/a)^2$, as presented in Fig. 2. In this case the coefficient q_{mn} from Eq. (19) can be rewritten in the form

$$q_{mn}=\frac{4q_0}{a^3b}\int_0^a x^2\sin(\alpha\,x)dx\int_0^b\sin(\beta\,y)dy=\frac{4q_0}{\pi^2mn}I,\qquad m,n=1,3,5... \tag{20a}$$

Where

$$I=\left(\frac{2}{m\pi}\sin m\pi+\frac{2}{m^2\pi^2}\cos m\pi-\cos m\pi-\frac{2}{m^2\pi^2}\right)(1-\cos m\pi) \tag{20b}$$

Example 2: In this example a square laminated composite plate is simply supported on all edges and subjected to transverse line load at the centre ($x=a/2$), as given in Fig. 3. In this case the magnitude of the coefficient q_{mn} is expressed as shown below

$$q_{mn}=\frac{2q_0}{b}\sin\left(\frac{m\pi\xi}{a}\right) \tag{21}$$

where ξ and q_0 are the distance of line load from y-axis and the intensity of transverse load, respectively.

According to the present model, the bending stress analysis of a simply supported cross-ply laminated composite plate is achieved by utilizing Navier solution technique. The following simply supported boundary conditions are assumed at the side edges of the laminated composite plate.

$$\text{at edges } (x=0,\ a):\quad v_0=w_0=N_x=M_x^b=M_x^s=\theta=0 \tag{22a}$$

$$\text{at edges } (y=0,\ b):\quad u_0=w_0=N_y=M_y^b=M_y^s=\theta=0 \tag{22b}$$

The solution of the unknown displacement variables satisfying the boundary conditions given by Eq. (22), and can be expressed in the double-Fourier sine series as

$$\begin{Bmatrix}u_0\\v_0\\w_0\\\theta\\\phi_z\end{Bmatrix}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{Bmatrix}U_{mn}\cos(\alpha\,x)\sin(\beta\,y)\\V_{mn}\sin(\alpha\,x)\cos(\beta\,y)\\W_{mn}\sin(\alpha\,x)\sin(\beta\,y)\\\Theta_{mn}\sin(\alpha\,x)\sin(\beta\,y)\\\Phi_{mn}\sin(\alpha\,x)\sin(\beta\,y)\end{Bmatrix} \tag{23}$$

where $U_{mn},V_{mn},W_{mn},\Theta_{mn}$ and Φ_{mn} are unknown coefficients. Substituting Eqs. (20) or (21) (depending on the load case) and (23) into the governing Eq. (17) yields an algebraic equations which can be written in compact matrix form as follows

$$[K]\{\Delta\}=\{F\} \tag{24}$$

where $\{\Delta\},\{F\}$ and $[K]$ are the vector of unknowns, the force vector and the stiffness matrix, respectively. These terms can be defined as follows

$$\{\Delta\}=\{U_{mn},V_{mn},W_{mn},\Theta_{mn},\Phi_{mn}\}^{Tr} \tag{25a}$$

$$\{F\}=\{0,0,q_{mn},0,0\}^{Tr} \tag{25b}$$

And the components of the symmetric stiffness matrix $[K]$ are given as follows

$$\begin{aligned}
&k_{11}=\alpha^2A_{11}+\beta^2A_{66},\quad k_{12}=\alpha\beta\,A_{12}+\alpha\beta A_{66},\\
&k_{13}=-\alpha^3B_{11}-\alpha\beta^2B_{12}-2\alpha\beta^2B_{66},\\
&k_{14}=-k_1\alpha B_{11}^s-k_2\alpha B_{12}^s+\left(k_1A'+k_2B'\right)\alpha\beta^2B_{66}^s,\\
&k_{15}=-\alpha X_{13},k_{22}=\beta^2A_{22}+\alpha^2A_{66},\\
&k_{23}=-\beta^3B_{22}-\alpha^2\beta B_{12}-2\alpha^2\beta B_{66},\\
&k_{24}=-k_1\beta B_{12}^s-k_2\beta B_{22}^s+\left(k_1A'+k_2B'\right)\alpha^2\beta B_{66}^s,\\
&k_{25}=-\beta X_{23},
\end{aligned} \tag{26}$$

$$k_{33} = \alpha^4 D_{11} + 2\alpha^2\beta^2 D_{12} + \beta^4 D_{22} + 4\alpha^2\beta^2 D_{66},$$
$$k_{34} = k_1\left(\alpha^2 D_{11}^s + \beta^2 D_{12}^s\right) + k_2\left(\alpha^2 D_{12}^s + \beta^2 D_{22}^s\right) - 2\left(k_1 A' + k_2 B'\right)\alpha^2\beta^2 D_{66}^s,$$
$$k_{35} = \alpha^2 Y_{13} + \beta^2 Y_{23},$$
$$k_{44} = -\left(k_1 A' + k_2 B'\right)^2 \alpha^2\beta^2 H_{66}^s + k_1\left(k_1 H_{11}^s + k_2 H_{12}^s\right) + k_2\left(k_1 H_{12}^s + k_2 H_{22}^s\right) + \left(k_2 B'\right)^2 \beta^2 A_{44}^s + \left(k_1 A'\right)^2 \alpha^2 A_{55}^s,$$
$$k_{45} = k_1 Y_{13}^s + k_2 Y_{23}^s + k_2 B'\beta^2 A_{44}^s + k_1 A'\alpha^2 A_{55}^s,$$
$$k_{55} = \beta^2 A_{44}^s + \alpha^2 A_{55}^s + Z_{33}$$

4. Numerical results and discussions

In this section, some illustrative examples are investigated for verification the accuracy and validity of the present formulations based on the proposed trigonometric shear deformation plate theory with stretching effect for the bending stress analysis of simply supported cross-ply laminated plates subjected to transverse parabolic load and line load. The transverse displacement uses a cosine function in thickness coordinate to introduce the influence of transverse normal strain. The results obtained for different values of side-to-thickness ratio *a/h* of composite plate are listed and compared in Tables 1–8 with their counterparts based on the classical plate theory (CPT) of Kirchhoff (1850), FSDT of Mindlin (1951), HSDT of Reddy (1984) and quasi-3D trigonometric shear deformation plate theory (TSDT) developed by Ghugal and Sayyad (2013), which included both transverse shear and normal deformations. For the simplicity, the results obtained for displacements, stresses are presented in the following non-dimensional forms

$$\bar{u}\left(0,\frac{b}{2},-\frac{h}{2}\right) = \frac{h^2 E_2}{q_0 a^3} u, \quad \bar{w}\left(\frac{a}{2},\frac{b}{2},0\right) = \frac{100 h^3 E_2}{q_0 a^4} w,$$
$$\bar{\sigma}_x\left(\frac{a}{2},\frac{b}{2},-\frac{h}{2}\right) = \frac{h^2}{q_0 a^2}\sigma_x, \quad \bar{\sigma}_y\left(\frac{a}{2},\frac{b}{2},-\frac{h}{2}\right) = \frac{h^2}{q_0 a^2}\sigma_y,$$
$$\bar{\tau}_{xy}\left(0,0,-\frac{h}{2}\right) = \frac{h^2}{q_0 a^2}\tau_{xy}, \ \bar{\tau}_{xz}\left(0,\frac{b}{2},0\right) = \frac{h}{q_0 a}\tau_{xz}, \tag{27}$$
$$\bar{\tau}_{yz}\left(\frac{a}{2},0,0\right) = \frac{h}{q_0 a}\tau_{yz}$$

The following properties of graphite-epoxy for laminated composite plates in the above examples are used.

$$E_1/E_2 = 25,\ E_3/E_2 = 1,\ G_{12}/E_2 = G_{13}/E_2 = 0.5,$$
$$G_{23}/E_2 = 0.2, \quad \nu_{12} = \nu_{13} = \nu_{23} = 0.25 \tag{28}$$

Example 1: The first example is carried out for simply supported two-layered antisymmetric (0°/90°) square laminated composite plate subjected to the transverse parabolic load. Both layers are the same thickness and the material properties given previously. Table 1 shows the in-plane and transverse displacements $\bar{u},\bar{w}$ and stresses $(\bar{\sigma}_x,\bar{\sigma}_y,\bar{\tau}_{xy})$ for various values of side-to-thickness ratio *a/h*. The obtained results are compared with those predicted by CPT, FSDT, HSDT and quasi-3D TSDT. From the examination of Table 1, it can be observed that the numerical results of transverse displacements obtained by using the present quasi-3D shear deformation plate theory are exactly matching with the results of quasi-3D TSDT solution given by Ghugal and Sayyad (2013), but a marginal difference is observed as compared to Reddy's theory due to the neglect of the thickness stretching effect (ε_z=0); Moreover, it should be noted that the results for the non-dimensional in-plane normal stresses $\bar{\sigma}_x,\bar{\sigma}_y$ and in-plane shear stress $\bar{\tau}_{xy}$ decrease with increasing value of side-to-thickness ratio *a/h*, as well as the CPT and FSDT underestimate these stresses compared to those obtained by the present theory, quasi-3D TSDT and HSDT of Reddy for all ratios. It is also observed that, for thin square laminated plate (*a/h* = 100) all theories predict the same results for the in-plane and transverse displacements and in-plane normal stress; this is due to neglect of transverse shear and normal deformations.

The variations of in-plane normal and transverse shear stresses $\bar{\sigma}_x,\bar{\tau}_{xz}^{CR}$ (computed using constitutive equations) through the thickness of (0°/90°) square laminated plate subjected to parabolic load for thickness ratio 4 are plotted in Figs. 4 and 5, respectively.It can be seen that the variation of in-plane normal and transverse shear stresses obtained by present theory for thick laminated square plate subjected to parabolic load is in good agreement with quasi-3D TSDT of Ghugal and Sayyad (2013).The comparison of the non-dimensional transverse shear stressesfor simply supported square laminated plate subjected to parabolic load is reported in Table 2 for three values of the thickness ratio (*a/h* = 4, 10, 100). It is pointed out that the numerical results obtained by using the present theory are much closer to the quasi-3D TSDT and HSDT solutions. Moreover, the transverse shear stress predicted by constitutive relations $\bar{\tau}_{xz}^{CR},\bar{\tau}_{yz}^{CR}$ of shear deformation theories is less than obtained using equations of equilibrium of the theory of elasticity $\bar{\tau}_{xz}^{EE},\bar{\tau}_{yz}^{EE}$ (computed using equilibrium equations).

Example 2: In the second example, a simply supported two-layered antisymmetric cross-ply square laminated composite plate subjected to the line load is considered. Tables 3 and 4 show the comparison of in-plane and transverse displacements and stresses for various thickness ratio *a/h*. From the examination of Table 3, it is remarked that the present theory is in excellent agreement while predicting the in-plane displacements $\bar{u}$ and in-plane normal stress $\bar{\sigma}_x$ as compared to those presented by Reddy (1984) and Ghugal and Sayyad (2013), whereas CPT and FSDT underestimate the same, especially for the cases of simply supported thick and moderately thick square laminated plates. However, the variation of in-plane normal stress $\bar{\sigma}_x$ and transverse shear stress $\bar{\tau}_{yz}^{CR}$ through the thickness, obtained using constitutive relations of two-layer

Table 1 Comparison of inplane displacement $\bar{u}$, transverse displacement $\bar{w}$, in-plane normal stresses $\bar{\sigma}_x$, $\bar{\sigma}_y$ and in-plane shear stress $\bar{\tau}_{xy}$ for simply supported two-layered (0°/90°) square laminated plate subjected to parabolic load

a/h	Theory	Model	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$
			$(-h/2)$	(0)	$(-h/2)$	$(-h/2)$	$(-h/2)$
4	Present	TSDT	0.0059	0.8799	0.3475	0.0410	0.0340
	Ref [(a)]	TSDT	0.0059	0.8790	0.3454	0.0406	0.0354
	Reddy (1984)	HSDT	0.0059	0.8991	0.3491	0.0385	0.0328
	Mindlin (1951)	FSDT	0.0044	0.8794	0.3043	0.0370	0.0313
	Kirchhoff (1850)	CPT	0.0044	0.5018	0.3071	0.0376	0.0288
10	Present	TSDT	0.0047	0.5630	0.3143	0.0387	0.0300
	Ref [(a)]	TSDT	0.0047	0.5627	0.3134	0.0385	0.0306
	Reddy (1984)	HSDT	0.0047	0.5656	0.3131	0.0376	0.0304
	Mindlin (1951)	FSDT	0.0044	0.5620	0.3061	0.0373	0.0299
	Kirchhoff (1850)	CPT	0.0044	0.5018	0.3071	0.0376	0.0288
100	Present	TSDT	0.0044	0.5021	0.3081	0.0383	0.0288
	Ref [(a)]	TSDT	0.0044	0.5021	0.3081	0.0383	0.0288
	Reddy (1984)	HSDT	0.0044	0.5024	0.3072	0.0376	0.0298
	Mindlin (1951)	FSDT	0.0044	0.5023	0.3071	0.0376	0.0288
	Kirchhoff (1850)	CPT	0.0044	0.5018	0.3071	0.0376	0.0288

[(a)] Results taken from reference Ghugal and Sayyad (2013)

Table 2 Comparison of transverse shear stresses $\bar{\tau}_{xz}, \bar{\tau}_{yz}$ for simply supported two-layered (0°/90°) square laminated plate subjected to parabolic load

a/h	Theory	Model	$\bar{\tau}_{xz}^{CR}$	$\bar{\tau}_{xz}^{EE}$	$\bar{\tau}_{yz}^{CR}$	$\bar{\tau}_{yz}^{EE}$
			(0)	(0)	(0)	(0)
4	Present	TSDT	0.1931	0.3491	0.1677	0.2505
	Ref [(a)]	TSDT	0.1956	0.3466	0.1707	0.2444
	Reddy (1984)	HSDT	0.2008	0.3292	0.1721	0.2402
	Mindlin (1951)	FSDT	0.1656	0.2540	0.1347	0.2540
	Kirchhoff (1850)	CPT	—	0.2594	—	0.2115
10	Present	TSDT	0.2207	0.2716	0.1827	0.2154
	Ref [(a)]	TSDT	0.2269	0.2703	0.1903	0.2141
	Reddy (1984)	HSDT	0.2253	0.2681	0.1878	0.2138
	Mindlin (1951)	FSDT	0.1681	0.2578	0.1378	0.2578
	Kirchhoff (1850)	CPT	—	0.2594	—	0.2115
100	Present	TSDT	0.2398	0.2590	0.1925	0.2113
	Ref [(a)]	TSDT	0.2421	0.2589	0.1993	0.2113
	Reddy (1984)	HSDT	0.2368	0.2593	0.1948	0.2114
	Mindlin (1951)	FSDT	0.1691	0.2594	0.1390	0.2594
	Kirchhoff (1850)	CPT	—	0.2594	—	0.2115

[(a)] Results taken from reference Ghugal and Sayyad (2013)

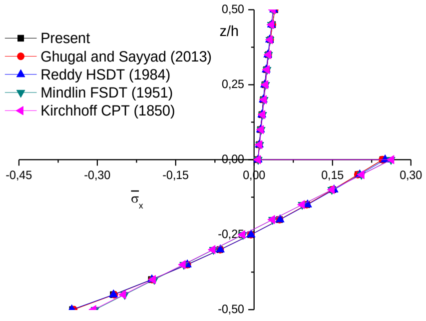

Fig. 4 Variation of in-plane normal stress $\bar{\sigma}_x$ through the thickness of (0°/90°) square laminated plate subjected to parabolic load for thickness ratio 4 at ($x = a/2$, $y = b/2$, z)

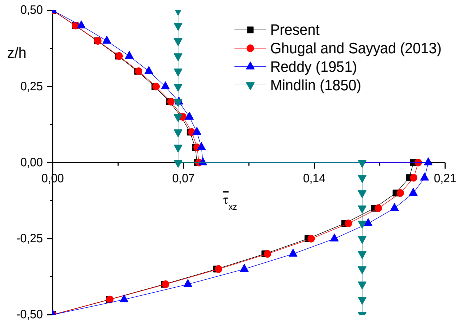

Fig. 5 Variation of transverse shear stress $\bar{\tau}_{xz}^{CR}$ through the thickness of (0°/90°) square laminated plate subjected to parabolic load for thickness ratio 4 at ($x = 0$, $y = b/2$, z)

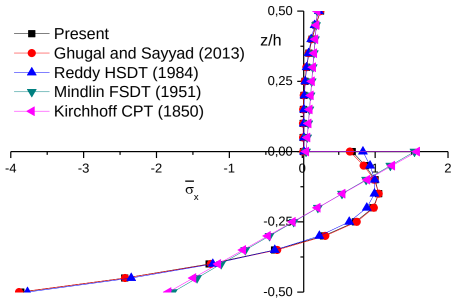

Fig. 6 Variation of in-plane normal stress $\bar{\sigma}_x$ through the thickness of (0°/90°) square laminated plate subjected to line load for thickness ratio 4 at ($x = a/2$, $y = b/2$, z)

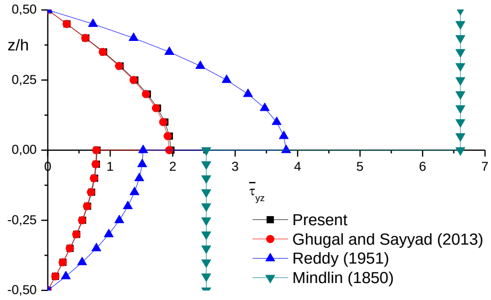

Fig. 7 Variation of transverse shear stress $\bar{\tau}_{yz}^{CR}$ through the thickness of (0°/90°) square laminated plate subjected to line load for thickness ratio 4 at ($x = a/2$, $y = 0$, z)

cross-ply square laminated plate subjected to line load for thickness ratio 4 are shown in Figs. 6 and 7, respectively. It is evident from the obtained results that the present computations are in an excellent agreement with the quasi-3D solutions provided by Ghugal and Sayyad (2013). Therefore, the results of Reddy deviate considerably, compared to the models of both quasi-3D theories due to neglect of transverse normal deformation (see Fig. 7). In fact, the present theory predicts excellent values of transverse shear stress as the obtained equilibrium equations are used.

Example 3: This example is extended from previous one, the analytical method of the present theory is checked for the bending stress analysis of simply supported three-layered symmetric (0°/90°/0°) square laminated composite plate subjected to the transverse parabolic load. Numerical values of non-dimensional displacements and stresses are presented in Tables 5 and 6 for various values of thickness ratio *a/h*. It is clear that the in-plane displacement $\bar{U}$ and in-plane normal stress $\bar{\sigma}_x$ obtained by present theory again agree well with previous solutions (Reddy 1984, Ghugal and Sayyad 2013) for all thickness ratios. However, it can be noticed that the present theory underestimates the values of transverse displacement $\bar{w}$, in-plane normal stress $\bar{\sigma}_y$ and in-plane shear stress $\bar{\tau}_{xy}$ for thick and moderately thick cross-ply (0°/90°/0°) square laminated plates compared to other shear deformation theories but it is in good agreement for the same lamination scheme of thin plates. This is explained by the kinematic model used in the present theory with only five unknowns. Besides, it can be found that this model is not suitable for the bending analysis of symmetric laminated composite plates and yield a significant difference in the numerical results compared to those of other higher-order theories, in which the displacement field involve more than five unknowns. On the other hand, it may be noted that the CPT shows more error in the results due to the neglect of transverse shear and normal deformations.

Table 6 shows that the values transverse shear stresses $\bar{\tau}_{xy}$, increase with the increase of the thickness ratio as the derived constitutive relations and equations of equilibrium are used.

Example 4: The last example is devoted to the analysis of three-layer symmetric (0°/90°/0°) square laminated composite plate with simply supported boundary conditions and subjected to the line load. Tables 7 and 8 display the comparison of displacements and stresses for various thickness ratio *a/h*. It can be observed that the non-dimensional inplane displacement $\bar{u}$ and in-plane normal stress $\bar{\sigma}_x$ obtained by present theory and quasi-3D TSDT developed by Ghugal and Sayyad (2013) and HSDT given by Reddy are in good agreement with each other for all

Table 3 Comparison of inplane displacement $\bar{u}$, transverse displacement $\bar{w}$, in-plane normal stresses $\bar{\sigma}_x, \bar{\sigma}_y$ and in-plane shear stress $\bar{\tau}_{xy}$ for simply supported two-layered (0°/90°) square laminated plate subjected to line load

a/h	Theory	Model	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$
			$(-h/2)$	(0)	$(-h/2)$	$(-h/2)$	$(-h/2)$
4	Present	TSDT	0.0191	3.8587	3.7158	0.1937	0.1063
	Ref [(a)]	TSDT	0.0187	3.8590	3.6899	0.1975	0.1062
	Reddy (1984)	HSDT	0.0190	4.0287	3.5476	0.1655	0.1123
	Mindlin (1951)	FSDT	0.0157	4.1089	1.7611	0.1451	0.1005
	Kirchhoff (1850)	CPT	0.0164	2.1186	1.8153	0.1402	0.0984
10	Present	TSDT	0.0168	2.4219	2.4489	0.1631	0.1015
	Ref [(a)]	TSDT	0.0167	2.4221	2.4453	0.1636	0.1024
	Reddy (1984)	HSDT	0.0167	2.4430	2.3719	0.1487	0.0968
	Mindlin (1951)	FSDT	0.0162	2.4372	1.8077	0.1408	0.1002
	Kirchhoff (1850)	CPT	0.0164	2.1186	1.8153	0.1402	0.0984
100	Present	TSDT	0.0164	2.1204	1.8253	0.1427	0.0982
	Ref [(a)]	TSDT	0.0164	2.1204	1.8262	0.1427	0.0983
	Reddy (1984)	HSDT	0.0164	2.1219	1.8255	0.1403	0.0930
	Mindlin (1951)	FSDT	0.0164	2.1218	1.8168	0.1401	0.0985
	Kirchhoff (1850)	CPT	0.0164	2.1186	1.8153	0.1402	0.0984

[(a)] Results taken from reference Ghugal and Sayyad (2013)

Table 4 Comparison of transverse shear stresses $\bar{\tau}_{xz}, \bar{\tau}_{yz}$ for simply supported two-layered (0°/90°) square laminated plate subjected to line load

a/h	Theory	Model	$\bar{\tau}_{xz}^{CR}$	$\bar{\tau}_{xz}^{EE}$	$\bar{\tau}_{yz}^{CR}$	$\bar{\tau}_{yz}^{EE}$
			(0)	(0)	(0)	(0)
4	Present	TSDT	0.4085	0.4647	1.9714	31.4078
	Ref [(a)]	TSDT	0.4207	0.4514	1.9572	31.5259
	Reddy (1984)	HSDT	0.4113	0.4502	3.2237	29.5337
	Mindlin (1951)	FSDT	0.2953	0.4343	6.6067	9.8324
	Kirchhoff (1850)	CPT	—	0.4535	—	9.5442
10	Present	TSDT	0.3994	0.4645	4.8608	21.9233
	Ref [(a)]	TSDT	0.4303	0.4572	4.7606	21.9704
	Reddy (1984)	HSDT	0.4246	0.4531	6.0254	18.7668
	Mindlin (1951)	FSDT	0.3076	0.4452	6.5471	9.7839
	Kirchhoff (1850)	CPT	—	0.4535	—	9.5442
100	Present	TSDT	0.4143	0.4547	8.6607	9.5217
	Ref [(a)]	TSDT	0.4576	0.4528	8.9480	9.5414
	Reddy (1984)	HSDT	0.4468	0.4534	8.7615	9.5805
	Mindlin (1951)	FSDT	0.3163	0.4534	6.2980	9.5734
	Kirchhoff (1850)	CPT	—	0.4535	—	9.5442

[(a)] Results taken from reference Ghugal and Sayyad (2013)

Table 5 Comparison of inplane displacement $\bar{u}$, transverse displacement $\bar{w}$, in-plane normal stresses $\bar{\sigma}_x, \bar{\sigma}_y$ and in-plane shear stress $\bar{\tau}_{xy}$ for simply supported three-layered (0°/90°/0°) square laminated plate subjected to parabolic load

a/h	Theory	Model	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$
			$(-h/2)$	(0)	$(-h/2)$	$(-h/2)$	$(-h/2)$
4	Present	TSDT	0.0048	0.6348	0.2859	0.0143	0.0256
	Ref [(a)]	TSDT	0.0047	0.8493	0.2838	0.0332	0.0359
	Reddy (1984)	HSDT	0.0047	0.8525	0.2807	0.0303	0.0356
	Mindlin (1951)	FSDT	0.0026	0.6882	0.1902	0.0251	0.0233
	Kirchhoff (1850)	CPT	0.0032	0.1976	0.2357	0.0091	0.0130
10	Present	TSDT	0.0035	0.2685	0.2427	0.0101	0.0160
	Ref [(a)]	TSDT	0.0035	0.3228	0.2427	0.0150	0.0191
	Reddy (1984)	HSDT	0.0035	0.3212	0.2418	0.0142	0.0191
	Mindlin (1951)	FSDT	0.0031	0.2845	0.2251	0.0131	0.0162
	Kirchhoff (1850)	CPT	0.0032	0.1976	0.2357	0.0091	0.0130
100	Present	TSDT	0.0032	0.1983	0.2361	0.0093	0.0131
	Ref [(a)]	TSDT	0.0032	0.1989	0.2361	0.0094	0.0131
	Reddy (1984)	HSDT	0.0032	0.1989	0.2358	0.0091	0.0131
	Mindlin (1951)	FSDT	0.0032	0.1985	0.2356	0.0091	0.0131
	Kirchhoff (1850)	CPT	0.0032	0.1976	0.2357	0.0091	0.0130

[(a)] Results taken from reference Ghugal and Sayyad (2013)

thickness ratios. It must be noted again that the present method underestimates the values of transverse maximum displacement $\bar{w}$, in-plane normal stress $\bar{\sigma}_y$ and in-plane shear stress $\bar{\tau}_{xy}$ compared to other shear deformation theories. Comparison of transverse shear stresses for the (0°/90°/0°) square laminated plate subjected to line load is shown in Table 8. The examination of results reveals that the present theory overestimates the value of maximum transverse shear stress $\bar{\tau}_{xz}^{CR}$ as the obtained constitutive relations are used in comparison with previously published results.

5. Conclusions

In this paper, the analysis of the bending stress analysis of simply supported cross-ply laminated plates subjected to transverse parabolic load and line load is presented using a novel higher-order shear deformation plate theory considering the stretching effect, in which the axial displacements involve an undetermined integral component. The present theory satisfies the zero shear stress boundary conditions on the top and bottom surfaces of the plate without using any shear correction factors. A Navier-type analytical method was used to solve the governing differential equations. Several examples of simply supported antisymmetric and symmetric cross-ply laminated composite plates for different values of thickness ratio have been presented to validate the proposed theory, and it is found that the proposed theory with five unknowns is more accurate for thick and moderately thick antisymmetric cross-ply laminated composite plates and is characterized by less computational cost as compared to other shear deformation theories with six or more unknowns. Finally, it is interesting to consider other types of materials in future to improve this study (Daouadji 2017, Ayat *et al*. 2018, Chemi *et al*. 2018, Belmahi *et al*. 2018, Hussain *et al*. 2019ab, Karami *et al*. 2019a,b,c,d,e, Khiloun *et al*. 2019, Mahmoudi *et al*. 2019, Medani *et al*. 2019, Meksi *et al*. 2019, Salah *et al*. 2019, Sahouane *et al*. 2019, Semmah *et al*. 2019, Zarga *et al*. 2019, Zaoui *et al*. 2019, Balubaid *et al*. 2019, Tounsi *et al*. 2020).

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Table 6 Comparison of transverse shear stresses $\bar{\tau}_{xz}, \bar{\tau}_{yz}$ for simply supported three-layered (0°/90°/0°) square laminated plate subjected to parabolic load

a/h	Theory	Model	$\bar{\tau}_{xz}^{CR}$	$\bar{\tau}_{xz}^{EE}$	$\bar{\tau}_{yz}^{CR}$	$\bar{\tau}_{yz}^{EE}$
			(0)	(0)	(0)	(0)
4	Present	TSDT	0.0780	0.1391	0.1897	0.0254
	Ref [(a)]	TSDT	0.1125	0.1007	0.1234	0.0900
	Reddy (1984)	HSDT	0.1124	0.1122	0.1245	0.1153
	Mindlin (1951)	FSDT	0.0763	0.2132	0.0954	0.1546
	Kirchhoff (1850)	CPT	—	0.2379	—	0.1093
10	Present	TSDT	0.0938	0.1992	0.2011	0.0875
	Ref [(a)]	TSDT	0.1489	0.1912	0.0969	0.1010
	Reddy (1984)	HSDT	0.1431	0.1965	0.0961	0.1088
	Mindlin (1951)	FSDT	0.0835	0.2328	0.0735	0.1183
	Kirchhoff (1850)	CPT	—	0.2379	—	0.1093
100	Present	TSDT	0.1065	0.2369	0.2082	0.1089
	Ref [(a)]	TSDT	0.1649	0.2370	0.0910	0.1090
	Reddy (1984)	HSDT	0.1558	0.2371	0.0891	0.1092
	Mindlin (1951)	FSDT	0.0854	0.2379	0.0682	0.1093
	Kirchhoff (1850)	CPT	—	0.2379	—	0.1093

[(a)] Results taken from reference Ghugal and Sayyad (2013)

Table 7 Comparison of inplane displacement $\bar{u}$, transverse displacement $\bar{w}$, in-plane normal stresses $\bar{\sigma}_x, \bar{\sigma}_y$ and in-plane shear stress $\bar{\tau}_{xy}$ for simply supported three-layered (0°/90°/0°) square laminated plate subjected to line load

a/h	Theory	Model	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$
			$(-h/2)$	(0)	$(-h/2)$	$(-h/2)$	$(-h/2)$
4	Present	TSDT	0.0148	2.7021	2.6878	0.0919	0.0763
	Ref [(a)]	TSDT	0.0151	3.5799	2.7128	0.1711	0.0930
	Reddy (1984)	HSDT	0.0149	3.6635	2.6460	0.1427	0.0981
	Mindlin (1951)	FSDT	0.0096	3.1246	0.9299	0.0798	0.0780
	Kirchhoff (1850)	CPT	0.0116	0.7624	1.0815	0.0175	0.0563
10	Present	TSDT	0.0121	1.1151	1.7180	0.0636	0.0617
	Ref [(a)]	TSDT	0.0123	1.3349	1.7217	0.0838	0.0656
	Reddy (1984)	HSDT	0.0123	1.3332	1.6534	0.0606	0.0665
	Mindlin (1951)	FSDT	0.0113	1.1720	1.0530	0.0301	0.0622
	Kirchhoff (1850)	CPT	0.0116	0.7624	1.0815	0.0175	0.0563
100	Present	TSDT	0.0116	0.7660	1.0948	0.0192	0.0565
	Ref [(a)]	TSDT	0.0116	0.7685	1.0930	0.0199	0.0568
	Reddy (1984)	HSDT	0.0116	0.7685	1.0912	0.0187	0.0569
	Mindlin (1951)	FSDT	0.0116	0.7665	1.0791	0.0180	0.0567
	Kirchhoff (1850)	CPT	0.0116	0.7624	1.0815	0.0175	0.0563

[(a)] Results taken from reference Ghugal and Sayyad (2013)

Table 8 Comparison of transverse shear stresses $\bar{\tau}_{xz}, \bar{\tau}_{yz}$ for simply supported three-layered (0°/90°/0°) square laminated plate subjected to line load

a/h	Theory	Model	$\bar{\tau}_{xz}^{CR}$	$\bar{\tau}_{xz}^{EE}$	$\bar{\tau}_{yz}^{CR}$	$\bar{\tau}_{yz}^{EE}$
			(0)	(0)	(0)	(0)
4	Present	TSDT	0.4389	0.5456	2.5800	20.9528
	Ref [(a)]	TSDT	0.3118	0.4254	2.1435	13.0485
	Reddy (1984)	HSDT	0.2981	0.4279	3.7532	7.6620
	Mindlin (1951)	FSDT	0.1723	0.4826	6.6573	10.5762
	Kirchhoff (1850)	CPT	—	0.5826	—	6.5485
10	Present	TSDT	0.4073	0.5524	6.6154	8.0586
	Ref [(a)]	TSDT	0.3783	0.5219	4.8317	5.8024
	Reddy (1984)	HSDT	0.3595	0.5258	6.1266	4.3926
	Mindlin (1951)	FSDT	0.2019	0.5625	6.2571	9.9911
	Kirchhoff (1850)	CPT	—	0.5826	—	6.5485
100	Present	TSDT	0.4013	0.5806	5.3960	6.3369
	Ref [(a)]	TSDT	0.4036	0.5791	6.0101	7.2112
	Reddy (1984)	HSDT	0.3814	0.5796	5.8537	7.2196
	Mindlin (1951)	FSDT	0.2092	0.5817	4.3858	7.1597
	Kirchhoff (1850)	CPT	—	0.5826	—	6.5485

[(a)] Results taken from reference Ghugal and Sayyad (2013)

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