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Time harmonic interactions in non local thermoelastic solid with two temperatures

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Abstract. The present investigation is concerned with two dimensional deformation in a non local thermoelastic solid with two temperatures due to time harmonic sources. The nonlocal thermoelastic solid is homogeneous with the effect of two temperature parameters. Fourier transforms are used to solve the problem. The bounding surface is subjected to concentrated and distributed sources. The analytical expressions of displacement, stress components and conductive temperature are obtained in the transformed domain. Numerical inversion technique has been applied to obtain the results in the physical domain. Numerical simulated results are depicted graphically to show the effect of nonlocal parameter and frequency on the components of displacements, stresses and conductive temperature. Some special cases are also deduced from the present investigation.

Keywords: thermoelasticity; nonlocality; nonlocal theory of thermoelasticity; Eringen model of nonlocal theories; two temperature; time harmonic sources

1. Introduction

The nonlocal theory of thermoelasticity considers that the various physical quantities defined at a point are not just a function of the values of independent constitutive variables at that point only but a function of their values over the whole body. So the nonlocal theory can be termed as a generalization of the classical field theory. Nonlocal effects are dominant in nature. If the effects of strains at points other than the reference point are neglected, classical theory is recovered. Nonlocality is an essential characteristic in solid state physics as nonlocal attractions of atoms are common. Nonlocal continuum theories can describe the material properties from microscopic scales to the size of the lattice parmeter. So the nonlocal theory can satisfactorily explain some phenomena related to atomic scales. It is due to combined contribution of many researchers that the concept of nonlocality has developed to such an extent.

Edelen and Law (1971) discussed a theory of nonlocal interactions. Edelen *et al*. (1971) discussed the consequences of global postulate of energy balance and obtained the constitutive equations for the nonlinear theory. They called this nonlinear theory of nonlocal elasticity as protoelasticity. Eringen and Edelen (1972) developed the nonlocal elasticity theory via using the global balance laws and the second law of thermodynamics. This theory contains information about long range forces of atoms as according to this theory the stress field at a particular point is affected due to the strain at all the other point of the body also. These are characterized by the presence of nonlocality residuals of fields (like body forces, internal energy, entropy etc). They proved that the stress field at a particular point is affected due to the strain at all the other point of the body also. Wang and Dhaliwal (1993) established a reciprocity relation and addressed certain issues addressing nonlocal thermoelasticity. Artan (1996) proved the superiority of the nonlocal theory by comparing the results of local and nonlocal elasticity theories. Marin (1996) derived the generalized solutions in elasticity and Marin (1998) discussed their contributions on uniqueness in thermoelastodynamics.

Polizzotto (2001) assumed an attenuation function and used it to further refine the Eringen model of nonlocal elasticity theory. Eringen (2002) developed nonlocal continuum field theories for prevalent nonlocal intermolecular attractions in material bodies. He presented a unified approach to the development of the basic field equations for nonlocal continuum field theories. Othman and Abbas (2012) developed a solution of thermal-shock problem of generalized thermoelasticity of a non-homogeneous isotropic hollow cylinder with energy dissipation. Othman *et al*. (2015) studied the effects of rotation on the general model of the equations of generalized thermo-micro stretch for a homogeneous isotropic elastic half-space solid. Belkorissat *et al*. (2015) presented a new nonlocal hyperbolic refined plate model for free vibration properties of functionally graded plates. Ebrahimi and Shafiei (2016) investigated the size dependent vibration behavior of a rotating functionally graded (FG) Timoshenko nanobeam based on Eringen's nonlocal theory. Marin *et al*. (2017a, 2017b) discussed various results and problems for elastic dipolar bodies. Othman and Marin (2017) studied the effect of thermal loading due to laser

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pulse on thermoelastic porous medium under G-N theory.

Bellifa *et al*. (2017) developed a nonlocal zeroth-order shear deformation theory. Khetir *et al*. (2017) proposed a new nonlocal trigonometric shear deformation theory. Lata (2018a and 2018b) studied the plane waves in a layered medium of two semi-infinite nonlocal solids with anisotropic thermoelasic medium depicting the nonlocal parameter effects graphically. Karami *et al*. (2018) developed a three dimensional elasticity theory in conjunction with nonlocal strain gradient theory. Mokhtar *et al*. (2018) formulated a novel simple shear deformation theory for buckling analysis of single layer graphene sheet using the nonlocal differential constitutive relations of Eringen. Benahmed *et al*. (2019) presented an efficient higher order nonlocal beam theory for the critical buckling of functionally graded nanobeams with porosities. Soleimani *et al*. (2019) investigated the effects of inevitable out of plane defects on the postbuckling behavior of single layered graphene sheets under in-plane loadings based on nonlocal first order shear deformation theory. Belmahi *et al*. (2019) studied the forced vibration of nanobeam of a single-walled carbon nanotube (SWCNTs) surrounded by a polymer matrix according to the Euler-Bernoulli beam model and with the application of the non-local continuum or elasticity theory.

Thermoelasticity with two temperatures is one of non-classical theories of thermodynamics of elastic solids with only difference being the consideration of thermal effects. Chen and Gurtin (1968) suggested that in case of bodies being deformable there is a dependence on two distinct temperatures, namely the thermodynamic temperature and the conductive temperature. Youssef (2005) obtained the uniqueness theorem for equations of two temperature generalized thermoelasticity. Youssef and Al-Lehaibi (2007), after investigating various problems, gave an indication that the two temperature generalized thermoelasticity is more realistic in describing the state of an elastic body as compared to one temperature. Abbas and Zenkour (2014) constructed a mathematical model of two-temperature generalized thermoelasticity with relaxation times in the context of Youssef model.

Abbas (2014) obtained a general solution to the field equations of two-temperature generalized thermoelastic theory. Atwa and Jahangir (2014) investigated the effect of two temperatures on plane waves propagating through a generalized thermo-microstretch elasic solid. Sharma *et al*. (2015) studied the deformation in transversely isotropic thermoelastic solids with two temperatures due to time harmonic sources. Said and Othman (2016) applied a general model of equations of the two-temperature theory of generalized thermoelasticity to study the wave propagation in a fibre reinforced magneto-thermoelastic medium. Kumar *et al*. (2016a, 2016b) studied the disturbances in a homogeneous transversely iosotropic thermoelastic rotating medium with two temperatures, in the presence of Hall currents and magnetic field due to thermomechanical sources. Lata and Singh (2019) focused on the study of deformation due to inclined load in nonlocal thermoelastic solid with two temperatures.

The deformation at any point of the medium is useful for analysis of deformation field around mining tremors and for theoretical considerations of volcanic and seismic sources. The purpose of present paper is to determine the expression for components of displacements, normal stresses, tangential stresses and conductive temperature, when time harmonic mechanical or thermal source is applied under the effects of local and nonlocal parameters. As most of the structural elements of heavy industries are mostly under the effects of mechanical and thermal stresses, so the present model is useful for understanding the nature of interaction between mechanical and thermal fields using the concept of nonlocality.

2. Basic equations

Following Youseff (2005) and Eringen (2002), the equations of motion and the constitutive relations in a homogeneous non local thermoelastic solid with two temperatures are given by

$$(\lambda + 2\mu)\nabla(\nabla.\boldsymbol{u}) - \mu\,(\nabla \times \nabla \times \boldsymbol{u}) - \beta\nabla\theta = (1 - \epsilon^2\nabla^2)\rho\frac{\partial^2\boldsymbol{u}}{\partial t^2}, \tag{1}$$

$$K^*\nabla^2\varphi = \rho\, C^*\frac{\partial\theta}{\partial t} + \beta\theta_0\frac{\partial}{\partial t}\,(\nabla.u), \tag{2}$$

where,

$$\theta = (1 - a\nabla^2)\,\varphi, \tag{3}$$

$$t_{ij} = \lambda u_{k,k}\delta_{ij} + \mu\big(u_{i,j} + u_{j,i}\big) - \beta\theta\delta_{ij}. \tag{4}$$

where λ, μ are material constants, ϵ is the nonlocal parameter, ρ is the mass density, $u = (u_1, u_2, u_3)$ is the displacement vector, $\boldsymbol{\varphi}$ is the conductive temperature, a is two temperature parameter, θ is absolute temperature and θ_0 is reference temperature, K^* is the coefficient of the thermal conductivity, C^* the specific heat at constant strain, $\beta = (3\lambda + 2\mu)\alpha$, where α is coefficient of liner thermal expansion, e_{ij} are components of strain tensor, e_{kk} is the dilatation, δ_{ij} is the Kronecker delta, t_{ij} are the components of stress tensor.

3. Formulation of the problem

We consider a homogeneous non local isotropic thermoelastic body in an initially undeformed state at temperature θ_0. We take a rectangular Cartesian co-ordinate system (x_1, x_2, x_3) with x_3 axis pointing normally into the half space. We restrict our analysis to two dimensional problem with

$$\boldsymbol{u} = (u_1, 0, u_3). \tag{5}$$

Using Eq.(5) in Eqs.(1)–(2), yields

$$(\lambda + \mu)\frac{\partial e}{\partial x_1} + \mu\,\nabla^2 u_1 - \beta\frac{\partial\theta}{\partial x_1} = (1 - \epsilon^2\nabla^2)\,\rho\frac{\partial^2 u_1}{\partial t^2}, \tag{6}$$

$$(\lambda + \mu)\frac{\partial e}{\partial x_3} + \mu\,\nabla^2 u_3 - \beta\frac{\partial\theta}{\partial x_3} = (1 - \epsilon^2\nabla^2)\,\rho\frac{\partial^2 u_3}{\partial t^2}, \tag{7}$$

$$K^{*}\nabla^{2}\varphi = \rho C^{*}\frac{\partial\theta}{\partial t} + \beta\theta_0\frac{\partial e}{\partial t}. \quad (8)$$

where, $e = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}$, $\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}$.

we define the following dimensionless quantities

$$(x_1', x_3') = \frac{\omega_1}{c_2}(x_1, x_3),\ (u_1', u_3') = \frac{\omega_1}{c_2}(u_1, u_3),\ t_{ij}' = \frac{t_{ij}}{\beta T_0},\ t' = \omega_1 t,\ a' = \frac{\omega_1^2}{c_2^2}a,\ K_n' = \frac{c_2}{\lambda\omega_1}K_n,\ F_1' = \frac{F_1}{\beta T_0} \quad (9)$$

and $F_2' = \frac{F_2}{\beta T_0}$.

where,

$$c_2^2 = \frac{\mu}{\rho} \text{ and } \omega_1 = \frac{\rho C^* c_2^2}{K^*}.$$

Upon introducing the quantities defined by Eq.(9) in equations Eqs.(6)-(8), and suppressing the primes, yields

$$\left(\frac{\lambda+2\mu}{\mu}\right)\frac{\partial^2 u_1}{\partial x_1^2} + \left(\frac{\lambda+\mu}{\mu}\right)\frac{\partial^2 u_3}{\partial x_1\partial x_3} + \frac{\partial^2 u_1}{\partial x_3^2} - \beta\frac{\theta_0}{\mu}\frac{\partial\theta}{\partial x_1} = (1-\epsilon^2\nabla^2)\frac{\partial^2 u_1}{\partial t^2}, \quad (10)$$

$$\left(\frac{\lambda+2\mu}{\mu}\right)\frac{\partial^2 u_3}{\partial x_3^2} + \left(\frac{\lambda+\mu}{\mu}\right)\frac{\partial^2 u_1}{\partial x_1\partial x_3} + \frac{\partial^2 u_3}{\partial x_1^2} - \beta\frac{\theta_0}{\mu}\frac{\partial\theta}{\partial x_3} = (1-\epsilon^2\nabla^2)\frac{\partial^2 u_3}{\partial t^2}. \quad (11)$$

Also, $K^{*}\nabla^{2}\varphi = \rho C^{*}\frac{\partial\theta}{\partial t} + \beta\theta_0\frac{\partial e}{\partial t}$.

$$\Rightarrow \nabla^2\varphi - \frac{\partial}{\partial t}(1-a\nabla^2)\varphi = \frac{\beta {c_2}^2}{K^*{\omega_1}^2}\frac{\partial}{\partial t}\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}\right). \quad (12)$$

The initial and regularity conditions are given by

$$u_1(x_1, x_3, 0) = 0 = \dot{u}_1(x_1, x_3, 0),$$

$$u_3(x_1, x_3, 0) = 0 = \dot{u}_3(x_1, x_3, 0),$$

$$\varphi(x_1, x_3, 0) = 0 = \dot{\varphi}(x_1, x_3, 0) \text{ for } x_3 \geq 0, -\infty < x_1 < \infty,$$

$$u_1(x_1, x_3, t) = u_3(x_1, x_3, t) = \varphi(x_1, x_3, t) = 0 \text{ for } t > 0 \text{ when } x_3 \to \infty.$$

Assuming the harmonic behavior as

$$(u_1, u_3, \varphi)(x_1, x_3, t) = (u_1, u_3, \varphi)(x_1, x_3)e^{i\omega t}. \quad (13)$$

where, ω is the angular frequency.

Using Eq.(13) in Eqs.(10)-(12),we obtain

$$\left(\frac{\lambda+2\mu}{\mu}\right)\frac{\partial^2 u_1}{\partial x_1^2} + \left(\frac{\lambda+\mu}{\mu}\right)\frac{\partial^2 u_3}{\partial x_1\partial x_3} + \frac{\partial^2 u_1}{\partial x_3^2} - \beta\frac{\theta_0}{\mu}\frac{\partial\theta}{\partial x_1} = (1-\epsilon^2\nabla^2)\,(-\omega^2 u_1), \quad (14)$$

$$\left(\frac{\lambda+2\mu}{\mu}\right)\frac{\partial^2 u_3}{\partial x_3^2} + \left(\frac{\lambda+\mu}{\mu}\right)\frac{\partial^2 u_1}{\partial x_1\partial x_3} + \frac{\partial^2 u_3}{\partial x_1^2} - \beta\frac{\theta_0}{\mu}\frac{\partial\theta}{\partial x_3} = (1-\epsilon^2\nabla^2)(-\omega^2 u_3), \quad (15)$$

$$\Rightarrow \nabla^2\varphi - \frac{\partial}{\partial t}(1-a\nabla^2)\varphi = \iota\omega\frac{\beta {c_2}^2}{K^*{\omega_1}^2}\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}\right). \quad (16)$$

Applying Fourier Transform defined by

$$\hat{f}(\boldsymbol{\xi}, x_3, \omega) = \int_{-\infty}^{\infty}\bar{f}(x_1, x_3, \omega)\, e^{i\xi x_1}dx_1. \quad (17)$$

on Eqs. (14)–(16), we obtain a system of equations,

$$\left[(1-\epsilon^2\omega^2)\frac{d^2}{dx_3^2} - (1+a_1)\boldsymbol{\xi}^2 + (1+\epsilon^2\boldsymbol{\xi}^2)\omega^2\right]\widehat{u_1} + \iota a_1\boldsymbol{\xi}\frac{\boldsymbol{d}}{\boldsymbol{dx_3}}\widehat{u_3} - \iota a_2\hat{\varphi} = 0, \quad (18)$$

$$\iota a_1\boldsymbol{\xi}\frac{\boldsymbol{d}}{\boldsymbol{dx_3}}\widehat{u_1} + \left[(1+a_1-\epsilon^2\omega^2)\frac{d^2}{dx_3^2} - \boldsymbol{\xi}^2 + (1+\epsilon^2\boldsymbol{\xi}^2)\omega^2\right]\widehat{u_3} - a_2\frac{\boldsymbol{d}}{\boldsymbol{dx_3}}\hat{\varphi} = 0, \quad (19)$$

$$\beta a_3\boldsymbol{\omega\xi}\widehat{u_1} - \iota\beta a_3\boldsymbol{\omega}\frac{\boldsymbol{d}}{\boldsymbol{dx_3}}\widehat{u_3} - \left[(1+\iota a\omega)\frac{d^2}{dx_3^2} - \boldsymbol{\xi}^2 - \iota\omega(1+a\boldsymbol{\xi}^2)\right]\hat{\varphi} = 0. \quad (20)$$

From Eq. (18), Eq. (19) and Eq. (20), we obtain a set of homogeneous equations which will have a nontrivial solution if determinant of coefficient $(\widehat{u_1}, \widehat{u_3}, \hat{\varphi})$ vanishes so as to give a characteristic equation as

$$\left[\frac{d^6}{dx_3^6} + Q\frac{d^4}{dx_3^4} + R\frac{d^2}{dx_3^2} + S\right](\widehat{u_1}, \widehat{u_3}, \hat{\varphi}) = 0. \quad (21)$$

where,

$$Q = \frac{1}{P}\{(1+a_1-\omega^2\boldsymbol{\epsilon}^2)(1+\iota a\omega)[(1+\epsilon^2\boldsymbol{\xi}^2)\omega^2 - (1+a_1)\boldsymbol{\xi}^2] - (1-\omega^2\epsilon^2)(1+a_1-\omega^2\boldsymbol{\epsilon}^2)[\boldsymbol{\xi}^2 + \iota\omega(1+a\boldsymbol{\xi}^2)] + (1-\omega^2\epsilon^2)(1+\iota a\omega)[(1+\epsilon^2\boldsymbol{\xi}^2)\omega^2 - \boldsymbol{\xi}^2] - \iota\beta a_2 a_3\omega[\xi(1+a_1-\omega^2\boldsymbol{\epsilon}^2) - \omega^2(+{a_1}^2\xi^2(1+\iota a\omega)\},$$

$$R = \frac{1}{P}\{(1-\omega^2\boldsymbol{\epsilon}^2)[\boldsymbol{\xi}^2 - (1+\epsilon^2\boldsymbol{\xi}^2)\omega^2][\boldsymbol{\xi}^2 + \iota\omega(1+a\boldsymbol{\xi}^2)] + (1+a_1-\omega^2\boldsymbol{\epsilon}^2)[\boldsymbol{\xi}^2(1+a_1) - \omega^2(1+\epsilon^2\boldsymbol{\xi}^2)][\boldsymbol{\xi}^2 + \iota\omega(1+a\boldsymbol{\xi}^2)] + \iota\beta a_2 a_3\omega[\xi(1+a_1-\omega^2\boldsymbol{\epsilon}^2) - (1+\epsilon^2\boldsymbol{\xi}^2)\omega^2 + (1-a_1)\boldsymbol{\xi}^2 + (1+\iota a\omega)(1+\epsilon^2\boldsymbol{\xi}^2)\omega^2[(1+\epsilon^2\boldsymbol{\xi}^2)\omega^2 - (2+a_1)\boldsymbol{\xi}^2] - a_1^2\xi^2[\boldsymbol{\xi}^2 + \iota\omega(1+a\boldsymbol{\xi}^2)]\},$$

$$S = \frac{1}{P}\{(2+a_1)(1+\epsilon^2\boldsymbol{\xi}^2)\omega^2\boldsymbol{\xi}^2[(1+\epsilon^2\boldsymbol{\xi}^2)\boldsymbol{\xi}^2 + \iota\omega(1+a\boldsymbol{\xi}^2)] - (1+a_1)\boldsymbol{\xi}^6 - \iota\omega(1+a\boldsymbol{\xi}^2)[(1+\epsilon^2\boldsymbol{\xi}^2)^2\omega^3 + (1+a_1)\boldsymbol{\xi}^3] - (1+\epsilon^2\boldsymbol{\xi}^2)^2\omega^4\boldsymbol{\xi}^2 + \iota\beta a_2 a_3\omega\xi^2[(1+\epsilon^2\boldsymbol{\xi}^2)\omega^2 - \boldsymbol{\xi}^2]\},$$

$$P = (1-\omega^2\boldsymbol{\epsilon}^2)(1+a_1-\omega^2\boldsymbol{\epsilon}^2)(1+\iota a\omega).$$

The roots of the Eq. (21) are $\pm\lambda_i (i = 1,2,3)$ satisfying the radiation condition that $\widehat{u_1}, \widehat{u_3}, \hat{\varphi} \to 0$ as $x_3 \to \infty$, the solutions of equation can be written as,

$$\widehat{u_1} = A_1 e^{-\lambda_1 x_3} + A_2 e^{-\lambda_2 x_3} + A_3 e^{-\lambda_3 x_3}, \quad (22)$$

$$\widehat{u_3} = d_1 A_1 e^{-\lambda_1 x_3} + d_2 A_2 e^{-\lambda_2 x_3} + d_3 A_3 e^{-\lambda_3 x_3}, \quad (23)$$

$$\hat{\varphi} = l_1 A_1 e^{-\lambda_1 x_3} + l_2 A_2 e^{-\lambda_2 x_3} + l_3 A_3 e^{-\lambda_3 x_3}. \quad (24)$$

where,

$$d_i = \frac{P^* \lambda_i^3 + Q^* \lambda_i}{S^* \lambda_i^4 + T^* \lambda_i^2 + U^*} \quad i = 1,2,3. \quad (25)$$

$$l_i = \frac{P^{**} \lambda_i^2 + Q^{**}}{S^* \lambda_i^4 + T^* \lambda_i^2 + U^*} \quad i = 1,2,3. \quad (26)$$

where,

$P^* = \iota a_1 \xi (1 + \iota a \omega)$,

$Q^* = -\iota a_1 \xi [\boldsymbol{\xi}^2 + \iota \omega (1 + a \boldsymbol{\xi}^2)] + \beta a_2 a_3 \omega$,

$S^* = (1 + a_1 - \omega^2 \boldsymbol{\epsilon}^2)(1 + \iota a \omega)$,

$T^* = (1 + \epsilon^2 \boldsymbol{\xi}^2)(1 + \iota a \omega)\omega^2 - (1 + a_1 - \omega^2 \boldsymbol{\epsilon}^2)[\boldsymbol{\xi}^2 + \iota \omega (1 + a \boldsymbol{\xi}^2)] - \iota \beta a_2 a_3 \omega$,

$U^* = [\boldsymbol{\xi}^2 + \iota \omega (1 + a \boldsymbol{\xi}^2)][\boldsymbol{\xi}^2 - (1 + \epsilon^2 \boldsymbol{\xi}^2)\omega^2]$,

$P^{**} = -\beta a_3 \omega \xi (1 - \omega^2 \boldsymbol{\epsilon}^2)$,

$Q^{**} = \beta a_3 \omega \xi [\boldsymbol{\xi}^2 - (1 + \epsilon^2 \boldsymbol{\xi}^2)\omega^2]$.

4. Applications

On the half-space ($x_3 = 0$) normal point force and thermal point source, which are assumed to be time harmonic, are applied. We consider two types of boundary conditions, as follows:

Case 1. The normal force on the surface of half-space

The boundary conditions are

$$(1)\ t_{33}(x_1, x_3, t) = -F_1 \psi_1(x) e^{i\omega t},$$

$$(2)\ t_{31}(x_1, x_3, t) = 0, \quad (27)$$

$$(3)\ \frac{\partial}{\partial x_3} \varphi(x_1, x_3, t) = 0 \text{ at } x_3 = 0.$$

where, F_1 is the magnitude of the force applied, $\psi_1(x)$ specify the source distribution function along x_1 axis.

Case 2. The thermal source on the surface of half-space

The boundary conditions in this case are

$$(1)\ t_{33}(x_1, x_3, t) = 0,$$

$$(2)\ t_{31}(x_1, x_3, t) = 0, \quad (28)$$

$$(3)\ \frac{\partial}{\partial x_3} \varphi(x_1, x_3, t) = F_2 \psi_1(x) e^{i\omega t} \text{ at } x_3 = 0.$$

where, F_2 is the constant force applied on the boundary, $\psi_1(x)$ specify the source distribution function along x_1 axis.

Subcase 1: Mechanical force

Using the dimensionless quantities defined by Eq. (9) and using Eqs. (3), (4), (13), (17) in Eq. (27) and substituting values of $\widehat{u_1}, \widehat{u_3}$ *and* $\hat{\varphi}$ from Eqs. (22)-(24), and solving, we obtain the components of displacement, normal stress, tangential stress and conductive temperature as

$$\widehat{u_1} = -\frac{F_1 \widehat{\psi_1}(\xi) e^{i\omega t}}{\Delta} \{ M_{11} e^{-\lambda_1 x_3} + M_{12} e^{-\lambda_2 x_3} + M_{13} e^{-\lambda_3 x_3} \}, \quad (29)$$

$$\widehat{u_3} = -\frac{F_1 \widehat{\psi_1}(\xi) e^{i\omega t}}{\Delta} \{ d_1 M_{11} e^{-\lambda_1 x_3} + d_2 M_{12} e^{-\lambda_2 x_3} + d_3 M_{13} e^{-\lambda_3 x_3} \}, \quad (30)$$

$$\hat{\varphi} = -\frac{F_1 \widehat{\psi_1}(\xi) e^{i\omega t}}{\Delta} \{ l_1 M_{11} e^{-\lambda_1 x_3} + l_2 M_{12} e^{-\lambda_2 x_3} + l_3 M_{13} e^{-\lambda_3 x_3} \}, \quad (31)$$

$$\Delta = \Delta_{11} M_{21} - \Delta_{12} M_{22} + \Delta_{13} M_{23}, \quad (32)$$

$$\widehat{t_{33}} = -\frac{F_1 \widehat{\psi_1}(\xi) e^{i\omega t}}{\beta \theta_0 \Delta} \{ N_{11} M_{11} e^{-\lambda_1 x_3} + N_{12} M_{12} e^{-\lambda_2 x_3} + N_{13} M_{13} e^{-\lambda_3 x_3} \}, \quad (33)$$

$$\widehat{t_{31}} = -\frac{\mu F_1 \widehat{\psi_1}(\xi) e^{i\omega t}}{\beta \theta_0 \Delta} \{ \Delta_{21} M_{11} e^{-\lambda_1 x_3} + \Delta_{22} M_{12} e^{-\lambda_2 x_3} + \Delta_{23} M_{13} e^{-\lambda_3 x_3} \}, \quad (34)$$

$$\widehat{t_{11}} = -\frac{\mu F_1 \widehat{\psi_1}(\xi) e^{i\omega t}}{\beta \theta_0 \Delta} \{ \mathrm{N}_{21} M_{11} e^{-\lambda_1 x_3} + \mathrm{N}_{22} M_{12} e^{-\lambda_2 x_3} + \mathrm{N}_{23} M_{13} e^{-\lambda_3 x_3} \}. \quad (35)$$

where,

$M_{11} = \Delta_{22}\Delta_{33} - \Delta_{32}\Delta_{23}$, $M_{12} = \Delta_{31}\Delta_{23} - \Delta_{33}\Delta_{21}$, $M_{13} = \Delta_{32}\Delta_{21} - \Delta_{31}\Delta_{22}$

$\Delta_{1j} = \lambda_j d_j (\lambda + 2\mu) + \beta l_j$, $\Delta_{2j} = \iota \xi d_j - \lambda_j$, $\Delta_{3j} = l_j \lambda_j$; $j = 1,2,3.$,

$M_{21} = \Delta_{22}\Delta_{33} + \Delta_{32}\Delta_{23}$, $M_{22} = \Delta_{31}\Delta_{23} + \Delta_{33}\Delta_{21}$, $M_{23} = \Delta_{32}\Delta_{21} + \Delta_{31}\Delta_{22}$.

$N_{1j} = -\lambda_j d_j (\lambda + 2\mu) - \beta \theta_0 (1 + a\xi^2) l_j + \beta \theta_0 a \lambda_j^2 l_j$,

$N_{2j} = \iota \xi (\lambda + 2\mu) - \beta \theta_0 (1 + a\xi^2) l_j + \beta \theta_0 a \lambda_j^2 l_j$; ; j = 1,2,3.

Subcase 2: Thermal source on the surface of half-space

Using the dimensionless quantities defined by Eq. (9) and using Eqs. (3), (4), (13), (17) in Eq. (28) and substituting values of $\widehat{u_1}, \widehat{u_3}$ *and* $\hat{\varphi}$ from Eqs. (22)-(24), and solving, we obtain the components of displacement, normal stress, tangential stress and conductive temperature as

$$\widehat{u_1} = -\frac{F_2 \widehat{\psi_1}(\xi) e^{i\omega t}}{\Delta} \{ [(\lambda + 2\mu) M_{31} + \beta N_{31}] e^{-\lambda_1 x_3} + [(\lambda + 2\mu) M_{32} + \beta N_{32}] e^{-\lambda_2 x_3} + [(\lambda + 2\mu) M_{33} + \beta N_{33}] e^{-\lambda_3 x_3} \}, \quad (36)$$

$$\widehat{u_3} = -\frac{F_2\widehat{\psi_1}(\xi)e^{i\omega t}}{\Delta}\{d_1\,[(\lambda+2\mu)M_{31} + \beta N_{31}]e^{-\lambda_1 x_3} + d_2[(\lambda+2\mu)M_{32} + \beta N_{32}]e^{-\lambda_2 x_3} + d_3[(\lambda+2\mu)M_{33} + \beta N_{33}]e^{-\lambda_3 x_3}\}, \quad (37)$$

$$\hat{\varphi} = -\frac{F_2\widehat{\psi_1}(\xi)e^{i\omega t}}{\Delta}\{l_1\,[(\lambda+2\mu)M_{31} + \beta N_{31}]e^{-\lambda_1 x_3} + l_2[(\lambda+2\mu)M_{32} + \beta N_{32}]e^{-\lambda_2 x_3} + l_3[(\lambda+2\mu)M_{33} + \beta N_{33}]e^{-\lambda_3 x_3}\}, \quad (38)$$

$$\Delta = \Delta_{11}M_{21} - \Delta_{12}M_{22} + \Delta_{13}M_{23}, \quad (39)$$

$$\widehat{t_{33}} = -\frac{F_2\widehat{\psi_1}(\xi)e^{i\omega t}}{\beta\theta_0\Delta}\{N_{11}\,[(\lambda+2\mu)M_{31} + \beta N_{31}]e^{-\lambda_1 x_3} + \mathrm{N}_{12}\,[(\lambda+2\mu)M_{32} + \beta N_{32}]e^{-\lambda_2 x_3} + \mathrm{N}_{13}\,[(\lambda+2\mu)M_{33} + \beta N_{33}]e^{-\lambda_3 x_3}\}, \quad (40)$$

$$\widehat{t_{31}} = -\frac{\mu F_1\widehat{\psi_1}(\xi)e^{i\omega t}}{\beta\theta_0\Delta}\{\nabla_{21}\,[(\lambda+2\mu)M_{31} + \beta N_{31}]e^{-\lambda_1 x_3} + \nabla_{22}\,[(\lambda+2\mu)M_{32} + \beta N_{32}]e^{-\lambda_2 x_3} + \nabla_{23}\,[(\lambda+2\mu)M_{33} + \beta N_{33}]e^{-\lambda_3 x_3}\}, \quad (41)$$

$$\widehat{t_{11}} = -\frac{F_2\widehat{\psi_1}(\xi)e^{i\omega t}}{\beta\theta_0\Delta}\{\mathrm{N}_{21}\,[(\lambda+2\mu)M_{31} + \beta N_{31}]e^{-\lambda_1 x_3} + \mathrm{N}_{22}\,[(\lambda+2\mu)M_{32} + \beta N_{32}]e^{-\lambda_2 x_3} + \mathrm{N}_{23}\,[(\lambda+2\mu)M_{33} + \beta N_{33}]e^{-\lambda_3 x_3}\}. \quad (42)$$

where,

$N_{31} = \nabla_2\nabla_{23} - \nabla_3\nabla_{22}$, $N_{32} = \nabla_1\nabla_{23} - \nabla_3\nabla_{21}$, $N_{33} = \nabla_1\nabla_{22} - \nabla_2\nabla_{21}$.

$\nabla_{2j} = \iota\xi d_j - \lambda_j$, $\nabla_j = l_j$, $\nabla_{3j} = \lambda_j d_j$; $j = 1,2,3$.

$M_{31} = \nabla_{32}\nabla_{23} - \nabla_{33}\nabla_{22}$, $M_{32} = \nabla_{33}\nabla_{21} - \nabla_{31}\nabla_{22}$, $M_{33} = \nabla_{31}\nabla_{22} - \nabla_{22}\nabla_{21}$.

$N_{1j} = -\lambda_j d_j(\lambda+2\mu) - \beta\theta_0(1+a\xi^2)l_j + \beta\theta_0 a{\lambda_j}^2 l_j$; $j = 1,2,3$.

$N_{2j} = \iota\xi(\lambda+2\mu) - \beta\theta_0(1+a\xi^2)l_j + \beta\theta_0 a{\lambda_j}^2 l_j$; $j = 1,2,3$.

$\Delta_{1j} = \lambda_j d_j(\lambda+2\mu) + \beta l_j$, $\Delta_{3j} = l_j\lambda_j$; $j = 1,2,3$.

$M_{21} = \Delta_{22}\Delta_{33} + \Delta_{32}\Delta_{23}$, $M_{22} = \Delta_{31}\Delta_{23} + \Delta_{33}\Delta_{21}$, $M_{23} = \Delta_{32}\Delta_{21} + \Delta_{31}\Delta_{22}$.

a) Green's function

To synthesize the Green's function, i.e. the solution due to concentrated normal force and thermal source on the half-space is obtained by setting

$$\psi_1(x) = \delta(x).$$

Applying the Fourier transform defined by Eq.(17) gives

$$\widehat{\psi_1}(\xi) = 1. \quad (43)$$

b) Influence function

The method to obtain the half-space influence function, i.e. the solution due to distributed load applied on the half space is obtained by setting

$$\psi_1(x) = \begin{cases} 1\ if\ \ |x| \le m \\ 0\ if\ \ |x| > m \end{cases}.$$

The Fourier transform of $\psi_1(x)$ with respect to the pair (x, ξ) for the case of a uniform strip load of non-dimensional width 2m applied at origin of co-ordinate system $x_1 = x_3 = 0$ in the dimensionless form after suppressing the primes is given by

$$\widehat{\psi_1}(\xi) = \left[{2\sin(\xi m)}/{\xi}\right];\ \xi \neq 0. \quad (44)$$

The expressions for displacement, stresses and conductive temperature can be obtained for concentrated load and uniformly distributed normal force for mechanical or thermal forces by replacing $\widehat{\psi_1}(\xi)$ from Eq. (43) and Eq. (44) in Eqs. (29)-(42) respectively.

5. Particular cases

- If $a = 0$, then from Eqs. (29)–(35) and Eqs. (36)-(42), we obtain the corresponding expressions for displacements, stresses and conductive temperature for nonlocal isotropic solid without two temperature.
- If $\epsilon = 0$, then from Eqs. (29)–(35) and Eqs. (36)-(42), we obtain the corresponding expressions for displacements, stresses and conductive temperature for isotropic solid without nonlocal effects and with two temperature.
- If $\epsilon = a = 0$, then from Eqs. (29)–(35) and Eqs. (36)-(42), we obtain the corresponding expressions for displacements, stresses and conductive temperature for isotropic solid without nonlocal effects and two temperature.

6. Inversion of the transformation

To obtain the solution of the problem in physical domain, we must invert the transforms in Eqs. (29)–(35) and Eqs. (36)-(42). Here the displacement components, normal and tangential stresses and conductive temperature are functions of x_3 and the parameters of Fourier transform ξ and hence are of the form $f(\xi, x_3)$. To obtain the function $f(x_1, x_3)$ in the physical domain, we first invert the Fourier transform as used by Sharma et al. (2008), using

$$f(x_1,x_3) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\xi x_1}\hat{f}(\xi,x_3)d\xi = \frac{1}{2\pi}\int_{-\infty}^{\infty}|\cos(\xi x_1)f_e - i\sin(\xi x_1)f_0|\,d\xi. \quad (45)$$

where, f_e and f_0 are respectively the even and odd parts of $\hat{f}(\xi, x_3)$. The method for evaluating this integral is described in Press et al. (1986). It involves the use of Romberg's integration with adequate step size. The results

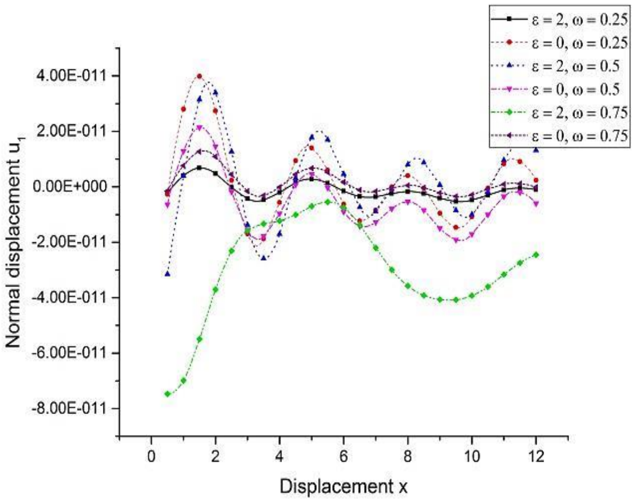

Fig. 1 Variation of normal displacement u_1 with displacement x (concentrated force)

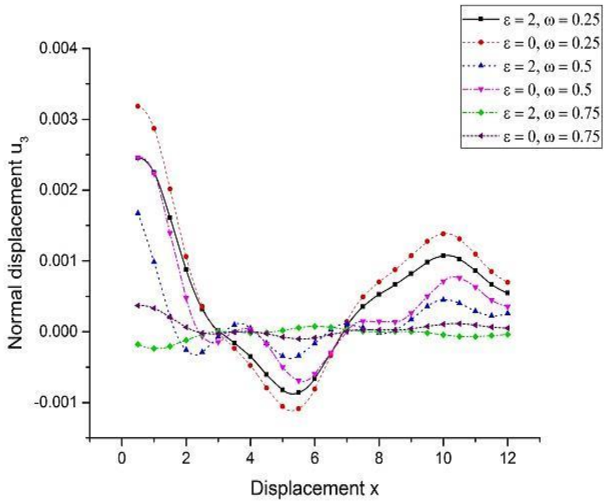

Fig. 2 Variation of normal displacement u_3 with displacement x (concentrated force)

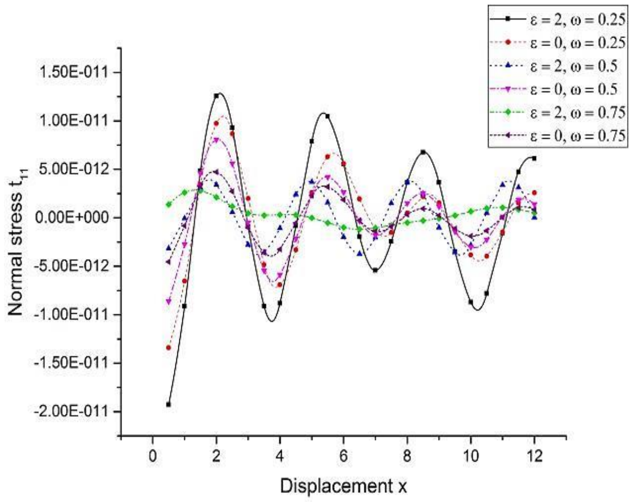

Fig. 3 Variation of normal stress t_{11} with displacement x (concentrated force)

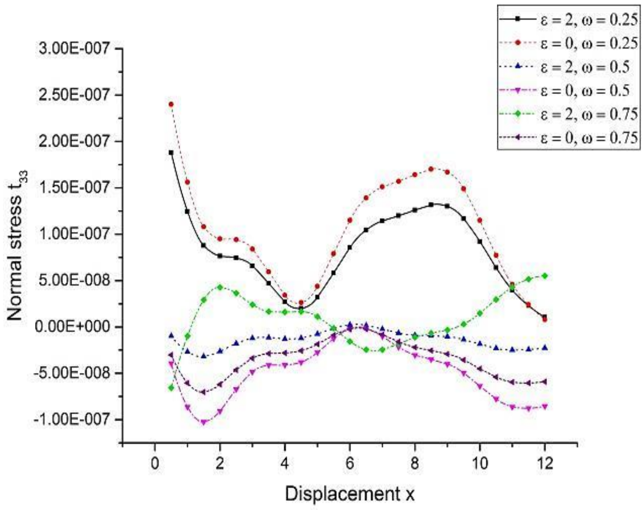

Fig. 4 Variation of normal stress t_{33} with displacement x (concentrated force)

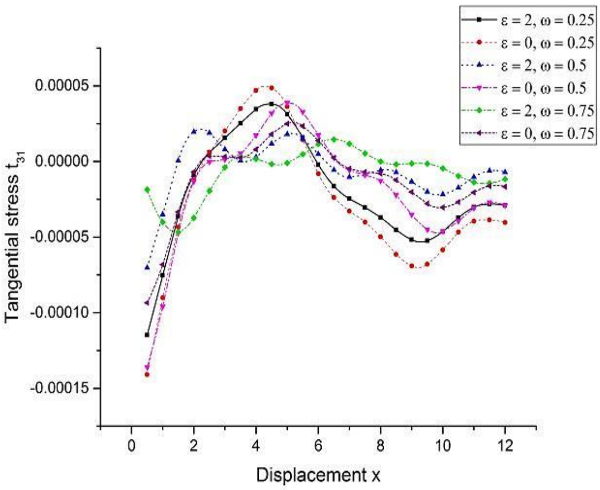

Fig. 5 Variation of tangential stress t_{31} with displacement x (concentrated force)

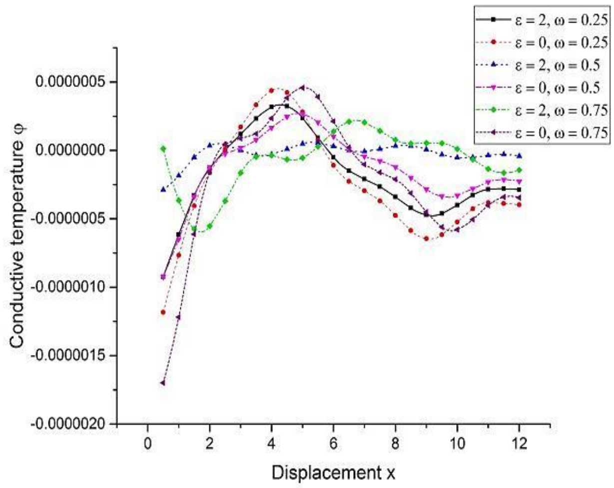

Fig. 6 Variation of conductive temperature φ with displacement x (concentrated force)

from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero are also used.

7. Numerical results and discussion

Magnesium material is chosen for the purpose of numerical calculation which is transversely isotropic and and according to Dhaliwal and Singh (1980), physical data for which is given as

$\lambda = 9.4 \times 10^{10} Nm^{-2},\ \mu = 3.278 \times 10^{10} Nm^{-2}, K^* = 1.7 \times 10^2 Wm^{-1}K^{-1},\ \rho = 1.74 \times\ 10^3 Kgm^{-3}, T_0 = 298\ K, C^* = 10.4 \times 10^2 JKg^{-1}deg^{-1}, \omega_1 = 3.58,\ a = 0.05.$

A comparison of values of normal displacements u_1 and u_3, normal stresses $t_{11}\ and\ t_{33}$, tangential stress t_{31} and conductive temperature φ for a transversely isotropic thermoelasic solid with distance x has been made for the local parameter $(\epsilon = 0)$ and nonlocal parameter $(\epsilon = 2)$ and is presented graphically for the non-dimensional frequencies $\omega = .25,\ \omega = .5\ and\ \omega = .75$

(1) The solid black colored line with centre symbol square, blue colored dotted line with centre symbol as triangle and green colored dash dot dot line with centre symbol diamond corresponds to nonlocal parameter $(\epsilon = 2)$ with frequencies $\omega = .25,\ \omega = .5\ and\ \omega = .75$ respectively.

(2) The dashed red colored line with centre symbol circle, magenta colored dash-dot line with centre symbol inverted triangle and purple colored short dashed line with centre symbol left pointing triangle, respectively represents local parameter $(\epsilon = 0)$ with frequencies $\omega = .25,\ \omega = .5\ and\ \omega = .75$.

a) Normal force on the surface of half-space Concentrated force

1) Fig. 1, shows the variations in values of normal displacement u_1. It is clear that the values of u_1 follow oscillatory pattern. For $\epsilon = 0$, the variations are small as compared to $\epsilon = 2$ for $\omega = .5$ while the opposite for other two frequencies. Fig. 2 depicts the variation of values of normal displacement u_3. The pattern is oscillatory with small variations for $\omega = .75$ owing to scale of graph. There is a clear difference between values for local and non-local parameters. Fig. 3 and Fig. 4 describe the variations of normal stress $t_{11}\ and\ t_{33}$ with respect to displacement. For both local and non-local parameters, the behavior is oscillatory for all the frequencies but the nonlocality effects can be clearly noticed. Fig. 5 shows the variation of tangential stress t_{31}. Here too the behavior followed is oscillatory with more variations for $\omega = .25$ as compared to $\omega = .5\ and\ .75$. Nonlocality is having a say for all the frequencies. Fig. 6 illustrates the variation of conductive temperature φ. The variation in values of $\omega = .5\ and\ \epsilon = 2$ is small owing to scale of graph but behavior is oscillatory with non-local parameter causing the difference in values for all frequencies.

b) Thermal source on the surface of half-space Concentrated thermal source

Fig. 7 shows the variations in values of normal displacements u_1 . The behavior followed is purely oscillatory. For $\omega = .5\ and\ \omega = .75, \epsilon = 0$ behavior is oscillatory but less visible due to scale of graph. For $\omega = .25$ the behavior is oscillatory but decreases sharply towards the end. Fig. 8 depicts the variations in values of normal displacements u_3. The behavior is oscillatory with more deflections for $\omega = .5\ and\ \epsilon = 2$. Fig. 9 explains the variations of values of normal stress t_{11}. The behavior followed is oscillatory. For $\omega = .5, \epsilon = 0$ and $\omega = .75, \epsilon = 0$ the behavior is oscillatory but with a decreasing curve graph. Similarly, for $\omega = .75\ and\ \epsilon = 2$, the behavior is oscillatory but with increasing effects. The effects of non-local parameter are clearly noticeable. Fig. 10 describes the variations of normal stress t_{33}.Nonlocality is visibly causing the differences as compared to local parameter but the behavior is mostly oscillatory with less oscillations at some frequencies due to scale of graph. Fig. 11 illustrates the variation of tangential stress $t_{31}$with the behavior being oscillatory. Difference in local and nonlocal parameters is clearly there at all the values. For $\omega = .75\ and\ \epsilon = 0$ the oscillations are maximum. Fig. 12 depicts the variation of conductive temperature φ. The behavior is oscillatory in nature. Non-local parameter is affecting the values for all frequencies.

8. Conclusion

In the present discussion the numerical results have been depicted graphically showing the effects of nonlocal parameter on the components of displacements, stresses and conductive temperature. From above investigation it is observed that there is a significant impact on normal displacements, normal stress, tangential stress and conductive temperature due to nonlocality. The variation of the components is dependent upon the nonlocal parameters as well as the variations in the frequencies. It is observed from the figures (1-12) that the trends in the variations of the characteristics mentioned are similar with difference in their magnitude when the mechanical forces or thermal sources are applied. Although the theory of thermoelasticity has a long history but concept of nonlocality has not been used to much extent till now. As thermoelasticity is very important in the field of aeronautics as the high velocities of modern aircraft give rise to aerodynamic heating which produces intense thermal stresses and thus reduces the strength of the aircraft structure. Likewise, in the technology of modern propulsive systems, the high temperatures associated with combustion processes are due to thermal stresses. But such effects can be reduced to a higher level using concept of nonlocality. Also, the results of this paper can contribute to a greater extent for the researchers working in the field of material science, geophysics, acoustics etc. and for the theoretical considerations of the seismic and volcanic sources as it is clear from above observations that nonlocality plays a major role.

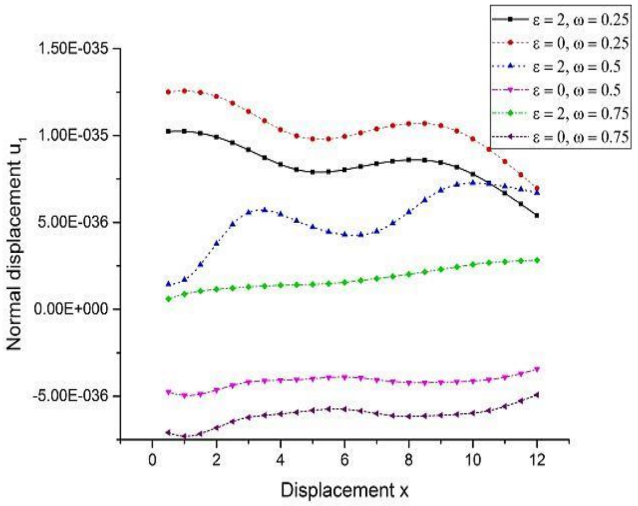

Fig. 7. Variation of normal displacement u_1 with displacement x (concentrated thermal source)

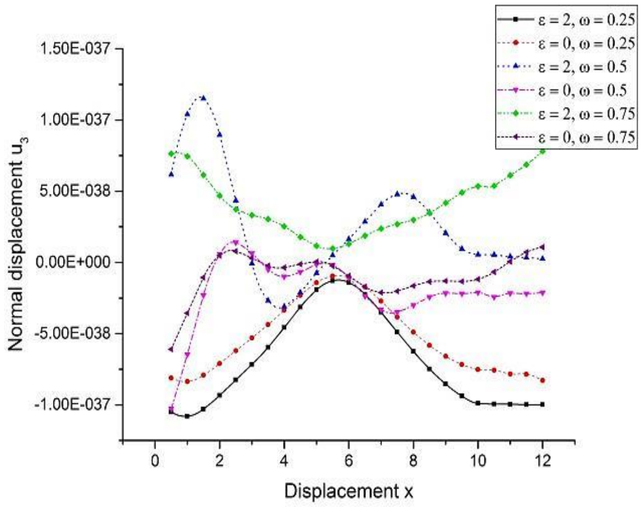

Fig. 8. Variation of normal displacement u_3 with displacement x (concentrated thermal source)

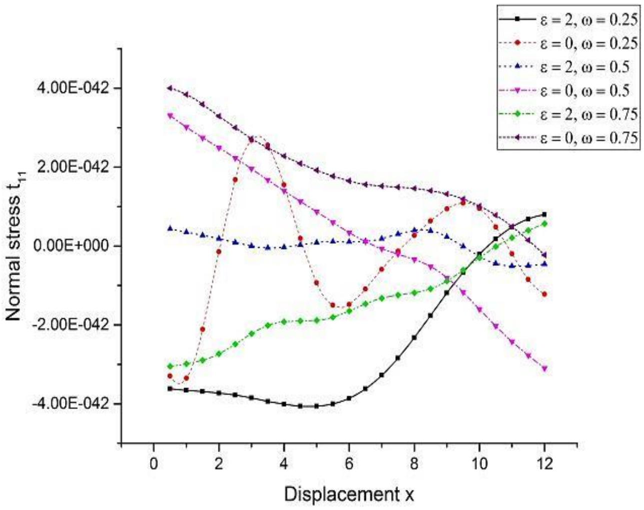

Fig. 9. Variation of normal stress t_{11} with displacement x (concentrated thermal source)

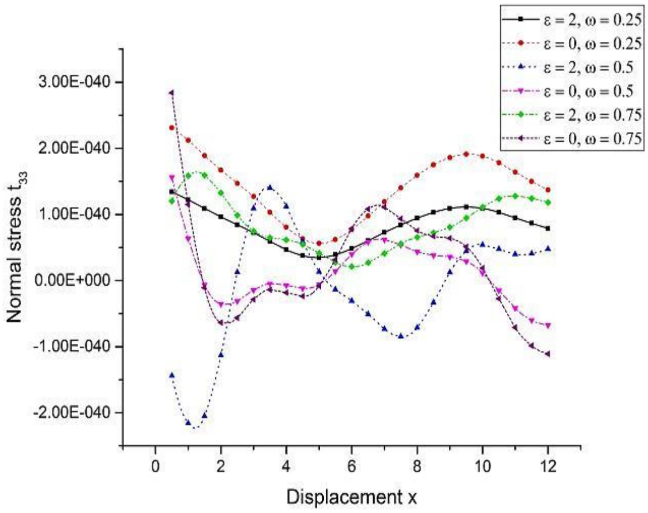

Fig. 10. Variation of normal stress t_{33} with displacement x (concentrated thermal source)

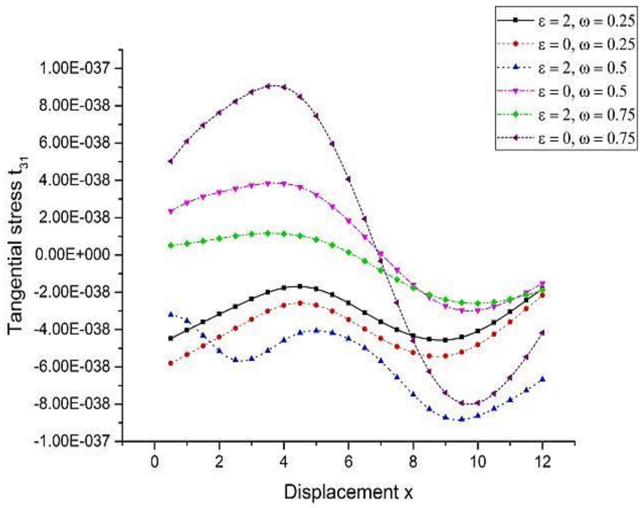

Fig. 11. Variation of tangential stress t_{31} with displacement x (concentrated thermal source)

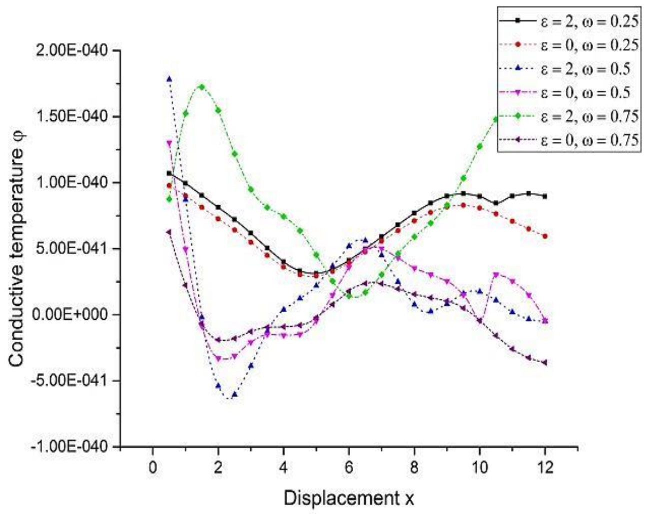

Fig. 12. Variation of conductive temperature φ with displacement x (concentrated thermal source)

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